FIITJEE

Solutions to JEE (Main)-2020

JEE–Main–2020 –Jan–8–Second–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. A simple pendulum is being used to determine the vale of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1 s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is:
(A) 4.40% (B) 2.40%
(C) 3.40% (D) 5.40%

Ans. **A**

Sol. $\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta g}{g} + \frac{\Delta L}{L}\right)$

$\frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta L}{L} = 2\left(\frac{1}{50}\right) + \frac{0.1}{25.0}$

$= 4.4\%$

2. A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is:
(A) 99 J (B) 100 J
(C) 1 J (D) 90 J

Ans. **D**

Sol. For Carnot engine using as refrigerator

$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$

It is given $\eta = \frac{1}{10}$

$\Rightarrow \quad \eta = 1 - \frac{T_2}{T_1}$

$\Rightarrow \quad \frac{T_2}{T_1} = \frac{9}{10}$

So, $Q_2 = 90$ J (as W = 10 J)

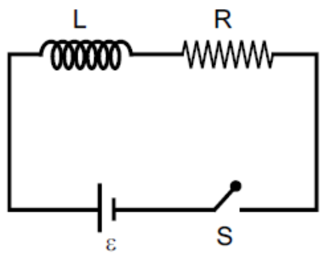
3. Consider two charged metallic spheres S_1 and S_2 of radii R_1 and R_2, respectively. The electric fields E_1 (on S_1) and E_2 (on S_2) on their surfaces are such that $E_1/E_2 = R_1/R_2$. Then the ratio of V_1 (on S_1) / V_2 (on S_2) of the electrostatic potentials on each sphere is:
(A) $(R_1/R_2)^2$ (B) $\left(\frac{R_1}{R_2}\right)^3$
(C) (R_2/R_1) (D) R_1/R_2

Ans. **A**

Sol. $\frac{E_1}{E_2} = \frac{r_1}{r_2}$

$$\frac{V_1}{V_2} = \frac{E_1 r_1}{E_2 r_2} = \frac{r_1}{r_2} \times \frac{r_1}{r_2} = \left(\frac{r_1}{r_2}\right)^2$$

4. As shown in the figure, a battery of emf ε is connected to an inductor L and resistance R in series. The switch is closed at t = 0. The total charge that flows from the battery, between t = 0 and t = t_c (t_c is the time constant of the circuit) is



(A) $\frac{\in L}{eR^2}$ (B) $\frac{\in L}{R^2}\left(1-\frac{1}{e}\right)$

(C) $\frac{\in R}{eL^2}$ (D) $\frac{\in L}{R^2}$

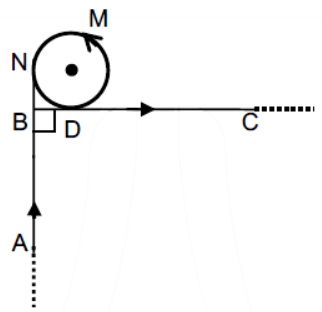
Ans. **A**

Sol. $q = \int_0^{T_c} i\,dt$

$$= \frac{\varepsilon}{R}\left[t - \frac{e^{-t/T_C}}{\frac{-1}{T_C}}\right]_0^{T_c} \quad ; \quad = \frac{\varepsilon}{R}\left[T_c + T_c e^{-1} - T_c\right]$$

$$= \frac{\varepsilon}{R} \times \frac{1}{e} \times \frac{L}{R} \quad ; \quad = \frac{\varepsilon L}{R^2 e}$$

5. A very long wire ABADMNDC is shown in figure carrying current I. AB and BC parts are straight, long and at right angle. At D wire forms a circular turn DMND of radius R. AB, BC parts are tangential to circular turn at N and D. Magnetic field at the centre of circle is:

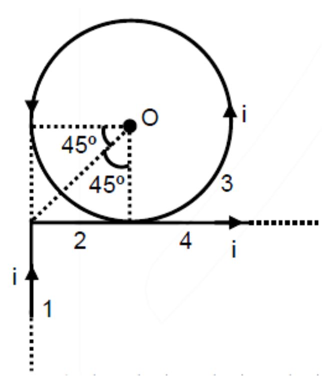


(A) $\frac{\mu_0 I}{2R}$ (B) $\frac{\mu_0 I}{2\pi R}\left(\pi - \frac{1}{\sqrt{2}}\right)$

(C) $\frac{\mu_0 I}{2\pi R}(\pi + 1)$ (D) $\frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$

Ans. **D**

Sol. $\vec{B}_0 = \left(\vec{B}_0\right)_1 + \left(\vec{B}_0\right)_2 + \left(\vec{B}_0\right)_3 + \left(\vec{B}_0\right)_4$



$$\frac{\mu_0 i}{4\pi R}\left[\sin 90^\circ - \sin 45^\circ\right] \otimes + \frac{\mu_0 i}{2R}\Theta + \frac{\mu_0 i}{4\pi R}(\sin 45^\circ + \sin 90^\circ)\Theta$$

$$= \frac{-\mu_0 i}{4\pi R}\left[1 - \frac{1}{\sqrt{2}}\right] + \frac{\mu_0 i}{2R} + \frac{\mu_0 i}{4\pi R}\left[\frac{1}{\sqrt{2}} + 1\right]\Theta$$

$$= \frac{\mu_0 i}{4\pi R}\left[-1 + \frac{1}{\sqrt{2}} + 2\pi + \frac{1}{\sqrt{2}} + 1\right]\Theta$$

$$= \frac{\mu_0 i}{4\pi R}\left[\sqrt{2} + 2\pi\right]\Theta = \frac{\mu_0 i}{2\pi R}\left[\frac{1}{\sqrt{2}} + \pi\right]\Theta$$

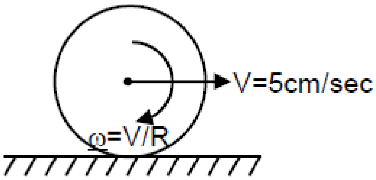
6. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is:

(A) 8.75×10^{-4} J (B) 8.75×10^{-3} J
(C) 6.25×10^{-4} J (D) 1.13×10^{-3} J

Ans. **A**

Sol. K.E. of the sphere = Translational K.E + Rotational K.E.

$= \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$ K = Radius of gyration



$\frac{1}{2}\times\frac{1}{2}\times\left(\frac{5}{100}\right)^2\left(1+\frac{2}{5}\right)$

$\frac{35}{4}\times10^{-4}$ J

7. In a double-slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}$th of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is:

(A) 0.672 (B) 0.568 (C) 0.760 (D) 0.853

Ans. **D**

Sol. $I = I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$

$$\frac{I}{I_0} = \cos^2\left[\frac{\frac{2\pi}{\lambda}\times\Delta x}{2}\right] = \cos^2\left(\frac{\pi}{8}\right)$$

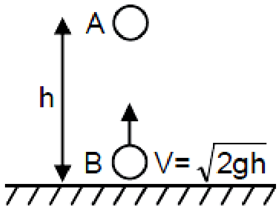
$\frac{I}{I_0} = 0.853$

8. A particle of mass m is dropped from a height h above the ground. At the same time another particle of same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is:

(A) $\frac{1}{2}$ (B) $\sqrt{\frac{1}{2}}$ (C) $\sqrt{\frac{3}{4}}$ (D) $\sqrt{\frac{3}{2}}$

Ans. **D**

Sol. Time for collision $t_1 = \frac{h}{\sqrt{2gh}}$



After t_1

$V_A = 0 - gt_1 = -\sqrt{\frac{gh}{2}}$

and $V_B = \sqrt{2gh} - gt_1 = \sqrt{gh}\left[\sqrt{2}-\frac{1}{\sqrt{2}}\right]$

at the time f collision

$\vec{P}_i = \vec{P}_f$

$\Rightarrow m\vec{V}_A + m\vec{V}_B = 2m\vec{V}_f$

$\Rightarrow -\sqrt{\frac{gh}{2}} + \sqrt{gh}\left[\sqrt{2}-\frac{1}{\sqrt{2}}\right] = 2\vec{V}_f$

$V_f = 0$

and height from ground = $h-\frac{1}{2}gt_1^2 = h-\frac{h}{4}=\frac{3h}{4}$

So time = $\sqrt{2\times\frac{\left(\frac{3h}{4}\right)}{g}}=\sqrt{\frac{3h}{2g}}$

9. A particle moves such that its position vector $\vec{r}(t)=\cos\omega\hat{i}+\sin\omega t\hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle:
(A) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin.
(B) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$
(C) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin.
(D) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

Ans. **A**

Sol. $\vec{r}=\cos\omega t\hat{i}+\sin\omega t\hat{j}$

$\vec{v}=\frac{d\vec{r}}{dt}=\omega(-\sin\omega t\hat{i}+\cos\omega t\hat{j})$

$\vec{a}=\frac{d\vec{v}}{dt}=-\omega^2(\cos\omega t\hat{i}+\sin\omega t\hat{j})$

$\vec{a}=-\omega^2\vec{r}$ $\quad\therefore\quad$ $\vec{a}$ is antiparallel to $\vec{r}$

$\vec{v}\cdot\vec{r}=\omega(-\sin\omega t\cos\omega t+\cos\omega t\sin\omega t)=0$

So, $\vec{V}\perp\vec{r}$

10. Consider a mixture of n moles of helium gas and 2n moles of oxygen gas (molecules taken to be rigid) as an ideal gas. It C_P/C_V value will be
(A) 19/13 (B) 40/27 (C) 67/45 (D) 23/15

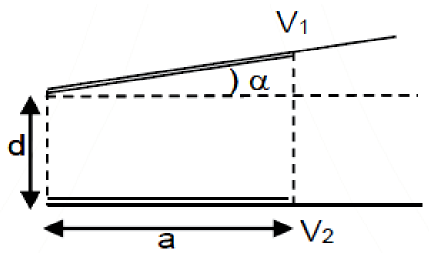
Ans. **A**

Sol. $\gamma_{mix}=\frac{n_1c_{p_1}+n_2c_{p_2}}{n_1c_{v_1}+n_2c_{v_2}}$

$=\frac{n\left(\frac{5}{2}R\right)+2n\left(\frac{7}{2}R\right)}{n\left(\frac{3}{2}R\right)+2n\left(\frac{5}{2}R\right)}$

$=\frac{5+14}{3+10}=\frac{19}{13}$

11. A capacitor is made of two square plats each of side ‘a’ making a very small angle a between them, as shown in figure. The capacitance will be close to:



(A) $\frac{\epsilon_0 a^2}{d}\left(1+\frac{\alpha a}{d}\right)$ (B) $\frac{\epsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$

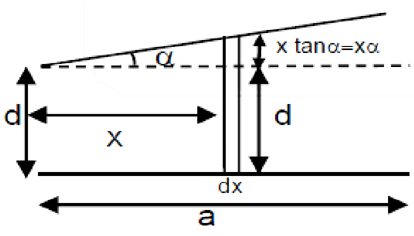
(C) $\frac{\epsilon_0 a^2}{d}\left(1-\frac{3\alpha a}{2d}\right)$ (D) $\frac{\epsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$

Ans. **D**

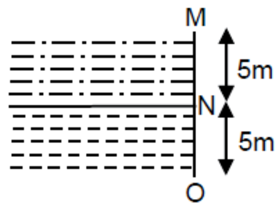
Sol. $dc = \frac{\varepsilon_0 a dx}{d + \alpha x}$

$\Rightarrow \quad c = \frac{\varepsilon_0 a}{\alpha}\left[\ln(d + \alpha x)\right]_0^a$

$= \frac{\varepsilon_0 a}{\alpha}\ln\left(1 + \frac{\alpha a}{d}\right) \approx \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$



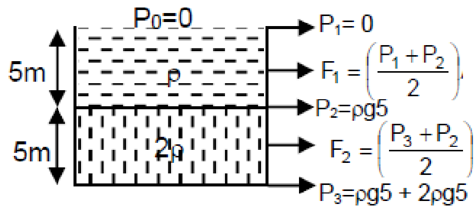
12. Two liquids of densities ρ_1 and ρ_2($\rho_2 = 2\rho_1$) are filled up behind a square wall of inside 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing):



(A) 1/2 (B) 1/4
(C) 2/3 (D) 1/3

Ans. **B**

Sol. $\frac{F_1}{F_2} = \frac{1}{4}$



13. A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z-direction. At a particular point in space and time, the magnetic field is given by $\vec{B} = 5 \times 10^{-8}\,\hat{j}$ T. The corresponding electric field $\vec{E}$ is (speed of light c = 3 × 10^8 ms^{-1})

(A) $-1.66 \times 10^{-16}\,\hat{i}$ V / m (B) $-15\,\hat{i}$ V / m
(C) $15\,\hat{i}$ V / m (D) $1.66 \times 10^{-16}\,\hat{i}$ V / m

Ans. **C**

Sol. $\frac{E}{B} = c$

E = B × c

= 15 N/c

14. An electron (mass m) with initial velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$ is in an electric field $\vec{E} = -E_0\hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, its de-Broglie wavelength at time t is given by

(A) $\frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}}$ (B) $\frac{\lambda_0}{\sqrt{2 + \frac{e^2E^2t^2}{m^2v_0^2}}}$ (C) $\frac{\lambda_0\sqrt{2}}{\sqrt{1 + \frac{e^2E^2t^2}{m^2v_0^2}}}$ (D) $\frac{\lambda_0}{\sqrt{1 + \frac{e^2E^2t^2}{2m^2v_0^2}}}$

Ans. **A**

Sol. Initially $m\left(\sqrt{2}\,v_0\right) = \frac{h}{\lambda_0}$

Velocity as a function of time = $v_0\hat{i} + v_0\hat{j} + \frac{eE_0}{m}t\hat{k}$

So wavelength $\lambda = \frac{h}{m\sqrt{2v_0^2 + \frac{e^2E_0^2}{m^2}t^2}}$

$$\lambda = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2}{2m^2v_0^2}t^2}}$$

15. A transverse wave travels on a taut steel wire wit a velocity of v when tension in it is 2.06×10^4 N. When the tension is changed to T, the velocity changed to v/2. The value of T is close to:
(A) 10.2×10^2 N
(B) 30.5×10^4 N
(C) 5.15×10^3 N
(D) 2.50×10^4 N

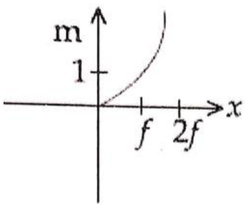
Ans. **C**

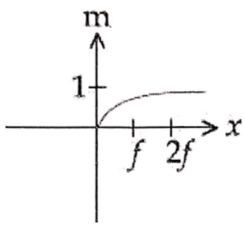
Sol. $v \propto \sqrt{T}$

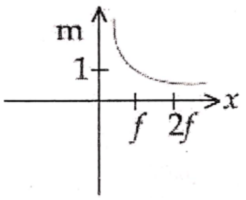
$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \quad \Rightarrow \quad \frac{v}{(v/2)} = \sqrt{\frac{2.06 \times 10^4}{T}}$$

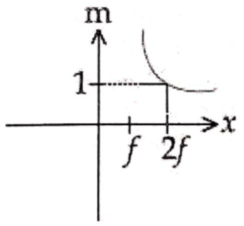
$$\Rightarrow \quad T = \frac{2.06 \times 10^4}{4} N = 0.515 \times 10^4 \text{ N}$$

16. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by (Graphs are drawn schematically and are not to scale)

(A) 

(B) 

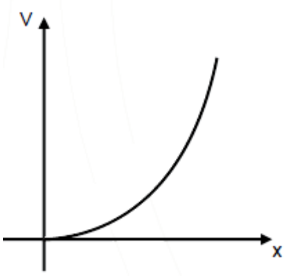
(C) 

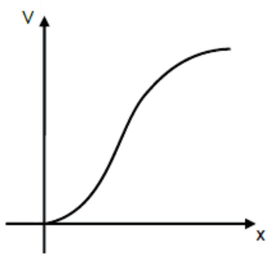
(D) 

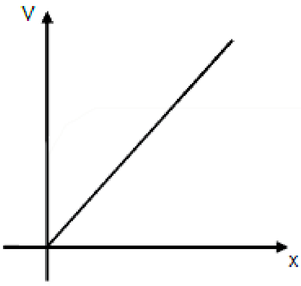
Ans. **D**

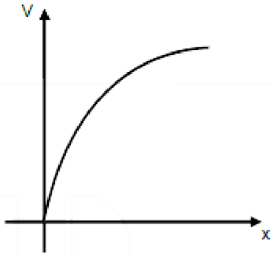
Sol. At focus, magnification is ∞.

17. A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale)

(A) 

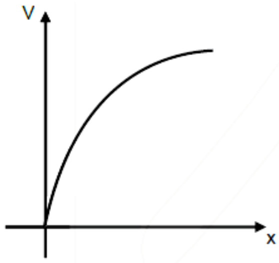
(B) 

(C) 

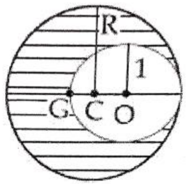
(D) 

Ans. **D**

Sol. $V^2 = \frac{2qE}{m}x$



18. As shown in figure when a spherical cavity (centered at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e. on the surface of the cavity. R can be determined by the equation:



(A) $(R^2 + R + 1)(2 - R) = 1$
(B) $(R^2 + R - 1)(2 - R) = 1$
(C) $(R^2 - R + 1)(2 - R) = 1$
(D) $(R^2 - R - 1)(2 - R) = 1$

Ans. **B**

Sol. $M_1 = \frac{4}{3}\pi R^3 \rho$; $M_2 = \frac{4}{3}\pi (1)^3 (-\rho)$

$$X_{com} = \frac{M_1 X_1 + M_2 X_2}{M_1 + M_2}$$

$$\Rightarrow \frac{\left[\frac{4}{3}\pi R^3 \rho\right] 0 + \left[\frac{4}{3}\pi (1)^3 (-\rho)\right][R-1]}{\frac{4}{3}\pi R^3 \rho + \frac{4}{3}\pi (1)^3 (-\rho)} = -(2-R)$$

$$\Rightarrow \frac{(R-1)}{(R^3-1)} = (2-R) \qquad (R \neq 1)$$

$$\frac{(R-1)}{(R-1)(R^2+R+1)} = 2-R$$

$(R^2 + R + 1)(2 - R) = 1$

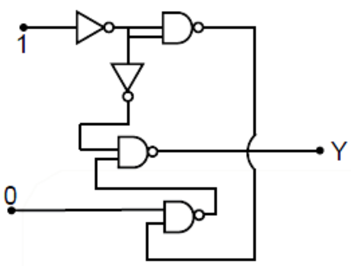
Alternative:

$M_{remaining}(2 - R) = M_{cavity}(1 - R)$

$\Rightarrow (R^3 - 1^3)(2 - R) = 1^3[R - 1]$

$\Rightarrow (R^2 + R + 1)(2 - R) = 1$

19. In the given circuit, value of Y is
(A) 0
(B) 1
(C) toggles between 0 and 1
(D) will not execute



Ans. **A**

Sol. $Y = \overline{\overline{AB} \cdot A}$

$= \overline{\overline{AB}} + \overline{A}$

$= AB + \overline{A}$

= 0 + 0 = 0

20. A galvanometer having a coil resistance 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V.
(A) 9.9 kΩ (B) 7.9 kΩ
(C) 10 kΩ (D) 8.9 kΩ

Ans. **A**

Sol. $V_g = i_g R_g = 0.1$ V

V = 10 V

$R = R_g\left(\frac{V}{V_g} - 1\right)$

= 100 × 99 = 9.9 KΩ

21. A ball is dropped form the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is _________.

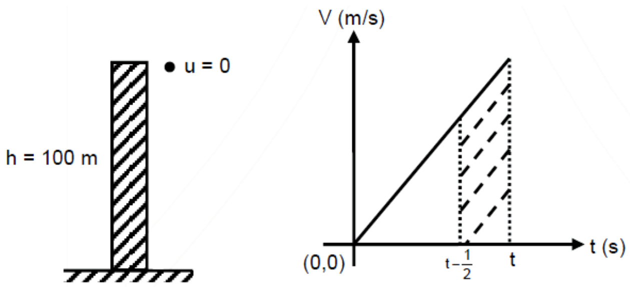
Ans. **8.00 or 2888.00**

Sol. Area of shaded trapezium

$= \frac{g\left[t - \frac{1}{2} + t\right]}{2} \times \frac{1}{2} = 19$ ...(1)

$\frac{1}{2}gt^2 = 100$ ...(2)

$\Rightarrow \quad t = \sqrt{\frac{200}{g}}$



$g\left[2t - \frac{1}{2}\right] = 76 \quad \Rightarrow \quad \frac{76}{g} = \frac{\left[4\sqrt{\frac{200}{g}} - 1\right]}{2}$

g = 8 m/s^2

22. The first member of the Balmer series of hydrogen atom has a wavelength of 6561 Å. The wavelength of the second member of the Balmer series (in nm) is _________.

Ans. **486**

Sol. $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda_1} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

$$\frac{1}{\lambda_2} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

$$\lambda_2 = \frac{20}{27} \times 6561 \text{ Å} = 4860 \text{ Å}$$

$= 486$ nm

23. The series combination of two batteries, both of the same emf 10 V, but different internal resistance of 20 Ω and 5 Ω, is connected to the parallel combination of two resistors 30 Ω and R Ω. The voltage difference across the battery of internal resistance 20 Ω is zero, the value of R (in Ω) is _________.

Ans. **30**

Sol. $V_1 = \varepsilon_1 - I, r_1$

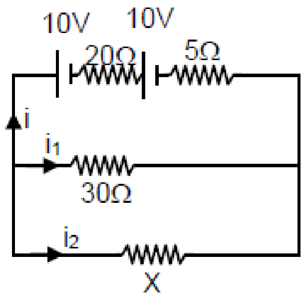
$0 = 10 - I \times 20$

$i = 0.5$ A

$V_2 = \varepsilon_2 - ir_2$

$= 10 - 0.5 \times 5$

$V_2 = 7.5$ V



$$0.5 = \frac{7.5}{30} + \frac{7.5}{x}$$

$$0.5 = 0.25 + \frac{7.5}{x}$$

$$\frac{7.5}{x} = 0.25 \;;\quad x = \frac{7.5}{0.25} = 30$$

24. An asteroid is moving directly towards the centre of the earth. When at a distance of 10 R (R is the radius of the earth) from he earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity form the earth is 11.2 km/s)? Given your answer to the nearest integer in kilometer/s _________.

Ans. **16**

Sol. $KE_i + PE_i = KE_f + PE_f$

$$\frac{1}{2}mu_0^2 + \left(-\frac{GMm}{10R}\right) = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$$

$$v^2 = u_0^2 + \frac{2GM}{R}\left[1 - \frac{1}{10}\right]$$

$$v = \sqrt{u_0^2 + \frac{9}{5}\frac{GM}{R}}$$

$$= \sqrt{12^2 + \left(\frac{9}{5}\right)\frac{(11.2)^2}{2}}$$

$$= \sqrt{144 + 0.9(11.2)^2} = \sqrt{256.896}$$

$= 16.028$ km/s $\simeq 16$

25. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in liters) are taken from each container and mixed (assume no loss of heat during the process)

C_1	C_2	C_3	T
1ℓ	2ℓ	—	60°C
—	1ℓ	2ℓ	30°C
2ℓ	—	1ℓ	60°C
1ℓ	1ℓ	1ℓ	θ

The value of θ (in °C to the nearest integer) is _________.

Ans. **50**

Sol. $1\theta_1 + 2\theta_2 = (1 + 2)\,60$

$\theta_1 + 2\theta_2 = 180$ …(1)

$0 \times \theta_1 + 1 \times \theta_2 + 2 \times \theta_3 = (1 + 2)\,30$

$\theta_2 + 2\theta_3 = 90$ …(2)

$2 \times \theta_1 + 0 \times \theta_2 + 1 \times \theta_3 = (2 + 1)\,60$

$2\theta_1 + \theta_3 = 180$ …(3)

and $\theta_1 + \theta_2 + \theta_3 = (1 + 1 + 1)\,\theta$ …(4)

from (1) + (2) + (3)

$3\theta_1 + 3\theta_2 + 3\theta_3 = 450 \Rightarrow \theta_1 + \theta_2 + \theta_3 = 150$

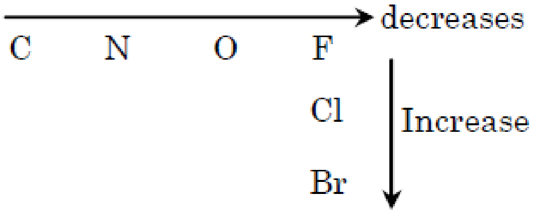
From (4) equation $150 = 3\theta \Rightarrow \theta = 50°C$

PART –B (CHEMISTRY)

26. The increasing order of the atomic radii of the following elements is:
(a) C (b) O (c) F (d) Cl (e) Br
(A) (a) < (b) < (c) < (d) < (e) (B) (d) < (c) < (b) < (a) < (e)
(C) (c) < (b) < (a) < (d) < (e) (D) (b) < (c) < (d) < (a) < (e)

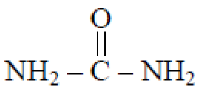
Ans. C

Sol. Generally in a period L $\rightarrow$ R Atomic radius decreases and in a Group T $\rightarrow$ B Atomic radius increases.



$Br > Cl > C > O > F$
$e > d > a > b > c$

27. Kjeldahl's method cannot be used to estimate nitrogen for which of the following compounds?

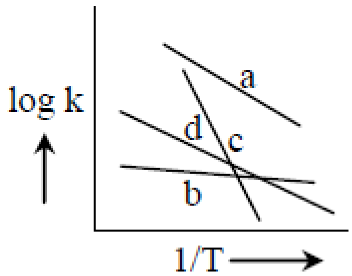
(A) 

(B) $C_6H_5NH_2$
(C) $C_6H_5NO_2$
(D) $CH_3CH_2 - C \equiv N$

Ans. C

Sol. Kjedahl's method cannot be used to Test nitrogen in nitro and diazo present in ring because nitrogen in nitro cannot convert into Ammonium sulphate.

28. Consider the following plots of rate constant versus $\frac{1}{T}$ for four different reactions. Which of the following order is correct for the activation energies of these reactions?



(A) $E_c > E_a > E_d > E_b$
(B) $E_b > E_d > E_c > E_a$
(C) $E_a > E_c > E_d > E_b$
(D) $E_b > E_a > E_d > E_c$

Ans. A

Sol. $k = Ae^{-\frac{Ea}{RT}}$

$$\left[\text{Log} \equiv \text{LogA} - \frac{Ea}{2.303RT}\right]$$

$$\text{Slope} = -\frac{Ea}{2.303RT}$$

Slope c > a > d > b

$E_c > E_a > E_d > E_b$

29. For the following Assertion and Reason, the correct option is :

Assertion : The pH of water increase with increase in temperature.

Reason : The dissociation of water into H^+ and OH^- is an exothermic reaction

(A) assertion is not true, but reason is true
(B) both assertion and reason are false
(C) but the reason is not the correct explanation for the assertion
(D) both assertion and reason are true, and the reason is the correct explanation for the assertion

Ans. B

Sol. $kw = [H^+][OH^-]$

$T\uparrow kw\uparrow$

For pure water $[H^+] = [OH^-]$

$kw = [H^-]^2 \Rightarrow [H^+] = \sqrt{kw}$

on increasing T kw $\uparrow$ $[H^+]$ $\uparrow$ pH $\downarrow$

Dissociation of H_2O is endothermic

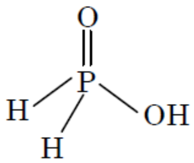
$H_2O \rightleftharpoons H^+ + OH^- \quad \Delta H = +ve$

30. White phosphorus on reaction with concentrated NaOH solution in an inert atmosphere of CO_2 gives phosphine and compound (X). (X) on acidification with HCl gives compound (Y). The basicity of compound (Y) is :

(A) 2
(B) 4
(C) 3
(D) 1

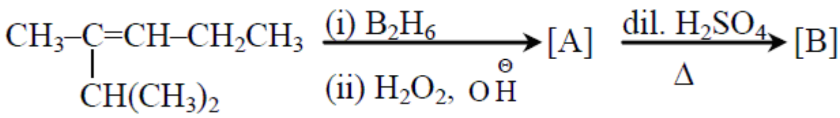
Ans. D

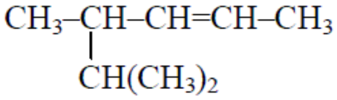
Sol. $P_4 + NaOH + H_2O \longrightarrow PH_3 + NaH_2PO_2 \xrightarrow{HCl} H_3PO_2$

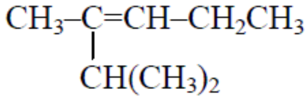


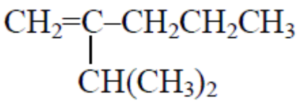
Basicity = 1

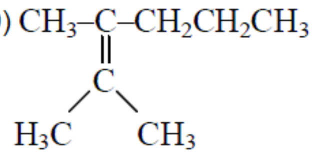
31. The major product [B] in the following sequence is :



(A) 

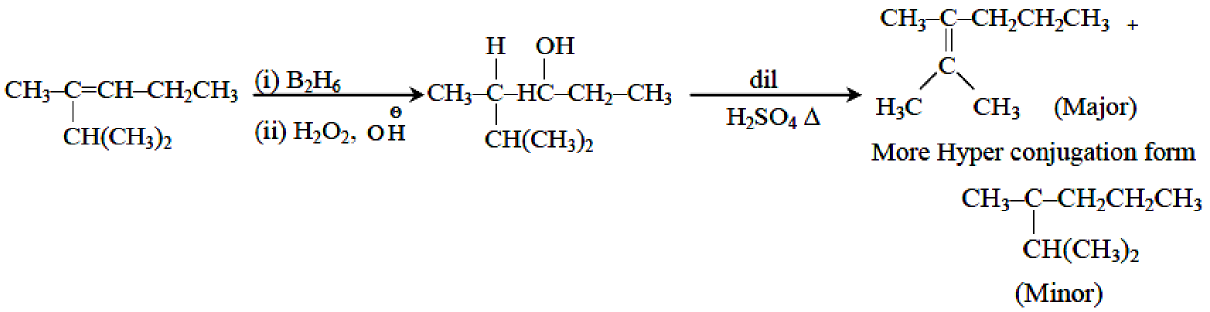
(B) 

(C) 

(D) 

Ans. D

Sol.



32. Among (a) – (d), the complexes that can display geometrical isomerism are:

(a) $[Pt(NH_3)_3]Cl^+$ (b) $[Pt(NH_3)Cl_5]^-$

(c) $[Pt(NH_3)_2Cl(NO_2)]$ (d) $[Pt(NH_3)_4ClBr]^{2+}$

(A) (d) and (a) (B) (c) and (d)

(C) (b) and (c) (D) (a) and (b)

Ans. B

Sol. $[Pt(NH_3)_2Cl(NO_2)]$ & $[Pt(NH_3)_4ClBr]^{2+}$

$[M\ A_2\ BC]$ type $[M\ A_4\ BC]$ type

33. For the following Assertion and Reason, the correct option is :

Assertion: For hydrogenation reactions, the catalytic activity increases from Group 5 to Group 11 metals with maximum activity shown by Group 7-9 elements.

Reason : The reactants are most strongly adsorbed on group 7-9 elements. Both assertion and reason are true and the reason is the correct

(A) Assertion is not true, but reason is true

(B) Both assertion and reason are false

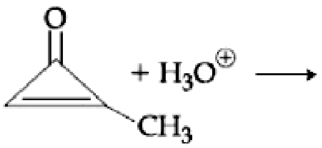
(C) but the reason is not the correct explanation for the assertion

(D) Both assertion and reason are true, and the reason is the correct explanation for the assertion

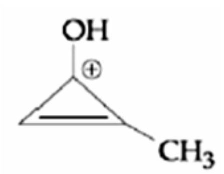
Ans. A

Sol. For hydrogenation reaction catalytic activity increase because reactants are more strongly adsorbed on group 7-9 element, So Assertion & Reason both are correct.

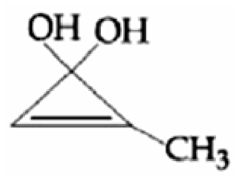
34. The major product in the following reaction is

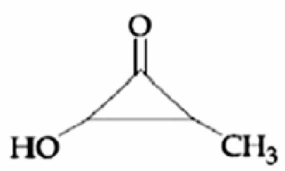


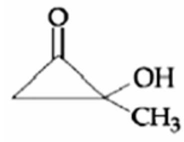
(A)



(B)

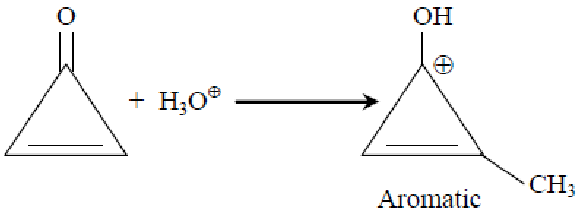


(C) 

(D) 

Ans. A

Sol.



35. Among the reactions (a)-(d), the reaction(s) that does/do not occur in the blast furnace during the extraction of iron is/are:

(a) $CaO + SiO_2 \longrightarrow CaSiO_3$

(b) $3Fe_2O_3 + CO \longrightarrow 2Fe_3O_4 + CO_2$

(c) $FeO + SiO_2 \longrightarrow FeSiO_3$

(d) $FeO \longrightarrow Fe + \frac{1}{2}O_2$

(A) (c) and (d)
(B) (a)
(C) (a) and (d)
(D) (d)

Ans. A

Sol. $CaO + SiO_2 \rightarrow CaSiO_3$

Used as flux.

$3Fe_2O_3 + CO \rightarrow 2Fe_3O_4 + CO_2$

Reduction done by CO.

Hence these two r × n take place but

$FeO \rightarrow Fe + \frac{1}{2}O_2$ & $FeO + SiO_2 \rightarrow FeSiO_3$

does not take place.

36. A metal (A) on heating in nitrogen gas gives compound B. B on treatment with H_2O gives a colourless gas which when passed through $CuSO_4$ solution gives a dark blue-violet coloured solution. A and B respectively, are:

(A) Mg and $Mg(NO_3)_2$
(B) Na and $NaNO_3$
(C) Mg and Mg_3N_2
(D) Na and Na_3N

Ans. C

Sol. $A + N_2 \longrightarrow \text{nitride (B)} \xrightarrow{H_2O} NH_3 + CuSO_4 \longrightarrow$ Blue-violet coloured sol.

$\underset{(A)}{3Mg} + N_2 \longrightarrow \underset{(B)}{Mg_3N_2} \xrightarrow{H_2O} Mg(OH)_2 + NH_3$

37. Two monomers in maltose are:

(A) α-D-glucose and α-D-galactose
(B) α-D-glucose and β-D-glucose
(C) α-D-glucose and α-D-glucose
(D) α-D-glucose and α-D-Fructose

Ans. C

Sol. Maltose on hydrolysis give 2 mole of α-D-glucose, because in maltose glucosidic linkage is present in between C_1 & C_4 of α-D-glucose.

38. The correct order of the calculated spin-only magnetic moments of complexes (A) to (D) is :

(a) $Ni(CO)_4$
(b) $[Ni(H_2O)_6]Cl_2$
(c) $Na_2[Ni(CN)_4]$
(d) $PdCl_2(PPh_3)_2$

(A) (c) ≈ (d) < (b) < (a)
(B) (a) ≈ (c) < (b) ≈ (d)
(C) (a) ≈ (c) ≈ (d) < (b)
(D) (c) < (d) < (b) < (a)

Ans. C

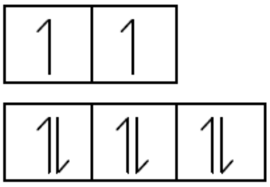
Sol. (a) $Ni(CO)_4$

$Ni = 3d^8\ 4s^2$ [S F L] $\quad M = 0$

(b) $[Ni(H_2O)_6]\ Cl_2$

$Ni^{2+} = 3d^8\ 45°$

Weak field ligand



No. of unpaired electron = 2

$M = \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8}$ B.M

(c) $Na_2\ [Ni(CN)_4]$

$Ni^{2+} - 3d^8$ (S F L)

paining take place (n = 0)

M = 0

(d) $Pd\ Cl_2\ (PPh_3)_2$

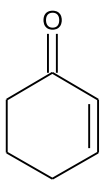
$Pd^{2+} = 4d^8$

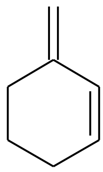
square plane

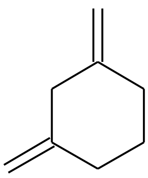
M = 0

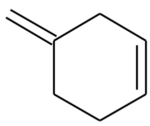
39. An unsaturated hydrocarbon X absorbs two hydrogen molecules on catalytic hydrogenation, and also gives following reaction:

$X \xrightarrow[Zn/H_2O]{O_3} A \xrightarrow{[Ag(NH_3)_2]^+}$ B(3-oxo-hexanedicarboxylic acid) X will be :

(A) 

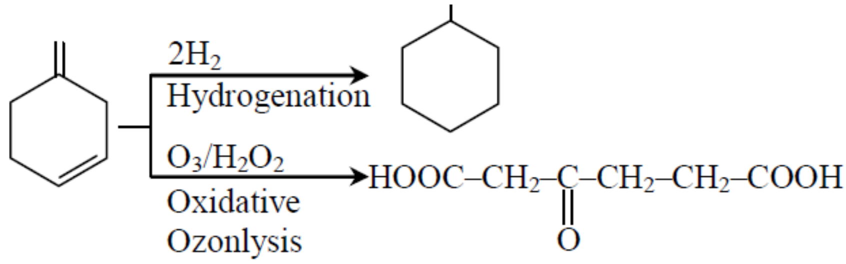
(B) 

(C) 

(D) 

Ans. D

Sol.



40. Hydrogen has three isotopes (a), (b) and (c). If the number of neutron(s) in (a), (b) and (c) respectively, are (x), (y) and (z), the sum of (x), (y) and (z) is:

(A) 4 (B) 3
(C) 2 (D) 1

Ans. B

Sol.

	^1_1H	^2_1H	^3_1H
no. of neutron	0 (x)	1 (y)	1 (z)

$= x + y + z$
$= 0 = 1 + 2 = 3$

41. Which of the following compound is likely to show both Frenkel and Schottky defects in its crystalline form?

(A) CsCl (B) AgBr
(C) ZnS (D) KBr

Ans. B

Sol. AgBr show both Frenkel and Schottky defect.

42. Arrange the following bonds according to their average bond energies in descending order:

C–Cl, C–Br, C–F, C–I

(A) C–F > C–Cl > C–Br > C–I (B) C–I > C–Br > C–Cl > C–F
(C) C–Cl > C–Br > C–I > C–F (D) C–Br > C–I > C–Cl > C–F

Ans. A

Sol.
$$\text{Bond energy} \propto \frac{1}{\text{Bond length}}.$$
Down the group size increases
Bond energy C–F > C–Cl > C–Br > C–I

43. Preparation of Bakelite proceeds via reactions:

(A) Electrophilic substitution and dehydration
(B) Condensation and elimination
(C) Electrophilic addition and dehydration
(D) Nucleophilic addition and dehydration

Ans. A

Sol. Formation of Bakelite follows electrophillic substitution of phenol and formaldehyde followed by dehydration.

44. The radius of the second Bohr orbit, in terms of the Bohr radius, a_0, in Li^{2+} is

(A) $\frac{4a_0}{3}$ (B) $\frac{2a_0}{9}$

(C) $\frac{2a_0}{3}$ (D) $\frac{4a_0}{9}$

Ans. A

Sol. We know

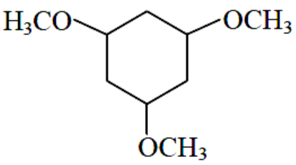
$$r = 0.529 \times \frac{n^2}{z} Å$$

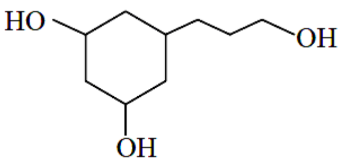
$$r = a_0 \times \frac{n^2}{z}$$

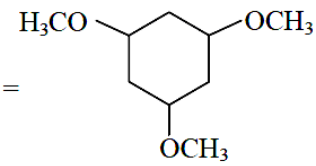
$n = 2$ $z = 3$

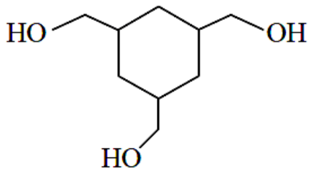
$$r = r = \frac{a_0 \times 4}{3} = \frac{4a_0}{3}$$

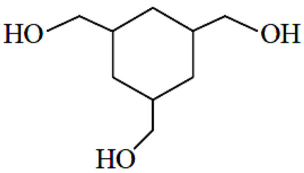
45. Among the compounds A and B with molecular formula $C_9H_{18}O_3$, A is having higher boiling point the B. The possible structures of A and B are:

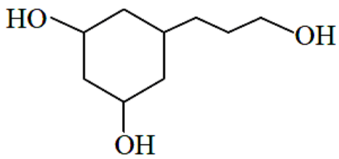
(A) A = 

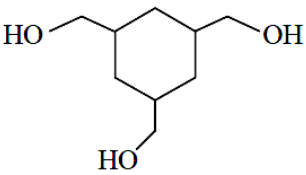
; B = 

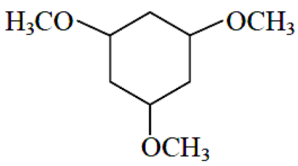
(B) A = 

; B = 

(C) A = 

; B = 

(D) A = 

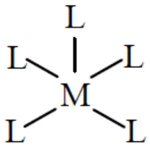
; B = 

Ans. D

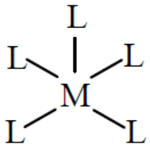
Sol. A is having higher boiling point than B. in case of A inter molecular H-bonding is possible while in case of B. intermolecular H-bonding is not possible hence have lower boiling point.

46. Complexes (ML_5) of metals Ni and Fe have ideal square pyramidal and trigonal bipyramidal geometries, respectively. The sum of the 90°, 120° and 180° L-M-L angles in the two complexes is _______

Ans. 20.00

Sol. 

$90° = 8$
$180° = 8$
total = 10



$12° = 3$
$90° = 6$
total = 1

47. At constant volume, 4 mol of an ideal gas when heated from 300 K to 500 K changes its internal energy by 5000 J. The molar heat capacity at constant volume is ____

Ans. 6.25

Sol. $\Delta U = ncv\ \Delta T$

$5000 = 4 \times C_v(500 - 300)$

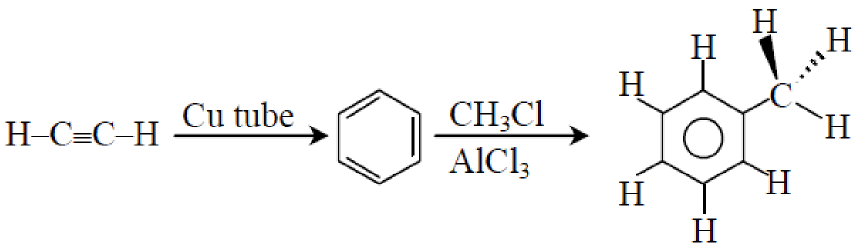
$C_v = 6.25\ J\ k^{-1}\ mol^{-1}$

48. In the following sequence of reactions the maximum number of atoms present in molecule 'C' in one plane is

$$A \xrightarrow[\text{Cu tube}]{\text{Red hot}} B \xrightarrow[\text{Anhydrous } AlCl_3]{CH_3Cl(1.eq.)} C$$ (A is a lowest molecular weight alkyne)

Ans. 13.00

Sol.



49. For an electrochemical cell $Sn(s)|Sn^{2+}$ (aq, 1M)||Pb^{2+} (aq, 1M)|Pb(s) the ratio $\frac{[Sn^{2+}]}{[Pb^{2+}]}$ when this cell attains equilibrium is _________.

$$\left(\text{Given}: E^0_{Sn^{2+}|Sn} = -0.14V, E^0_{Pb^{2+}|Pb} = -0.13V, \frac{2.303RT}{F} = 0.06\right)$$

Ans. 2.15

Sol. At $eqll^m$

$E_{Cell} = 0$

$E^0_{Cell} = 0.01\ V$

$Sn + Pb^{2+} \longrightarrow Sn^{2+} + Pb$

$$E_{Cell} = E^0_{Cell} - \frac{0.06}{n} \log^Q$$

$$0 = 0.01 - \frac{0.06}{2} \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$$

$$0.01 = \frac{0.06}{2} \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$$

$$\frac{1}{3} = \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$$

$$\frac{[Sn^{2+}]}{[Pb^{2+}]} = 10^{1/3} = 2.15$$

50. $NaClO_3$ is used, even in spacecraft, to produce O_2. The daily consumption of pure O_2 by a person is 492L at 1 atm, 300 K. How much amount of $NaClO_3$, in grams, is required to produce O_2 for the daily consumption of a person at 1 atm, 300 K? .

$NaClO_3(s) + Fe(s) \rightarrow O_2(g) + NaCl(s) + FeO(s)$

R = 0.082 L atm mol^{-1} K^{-1}

Ans. 2130.00

Sol. Moles of $NaCl_3$ = mole of O_2

Mole of O_2 = $\frac{PV}{RT} = \frac{1 \times 492}{0.082 \times 300} = 20$ mole

Mass of $NaClO_3$ = 20 × 106.5 = 2130 g

PART–C (MATHEMATICS)

51. If the 10^{th} term of an A.P. is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is:

(A) 100 (B) $50\frac{1}{4}$

(C) $100\frac{1}{2}$ (D) 50

Ans. C

Sol. $T_{10} = \frac{1}{20} = a + 9d$ ………..(i)

$T_{20} = \frac{1}{10} = a + 19d$ …………..(ii)

$\Rightarrow a = \frac{1}{200}, d = \frac{1}{200}$

$\Rightarrow S_{200} = \frac{200}{2}\left[\frac{2}{200} + \frac{199}{200}\right] = \frac{201}{2} = 100\frac{1}{2}$

52. Which of the following statement is a tautology?

(A) $\sim(p \wedge \sim q) \rightarrow p \vee q$ (B) $\sim(p \vee \sim q) \rightarrow p \vee q$

(C) $p \vee (\sim q) \rightarrow p \wedge q$ (D) $\sim(p \vee \sim q) \rightarrow p \wedge q$

Ans. B

Sol.

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim(p \vee \sim q)$	$p \vee q$	$\sim(p \vee \sim q) \rightarrow (p \vee q)$
T	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	F	F	T	T	T
F	F	T	T	T	F	F	T

53. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is:

(A) 4.02 (B) 3.98

(C) 4.01 (D) 3.99

Ans. D

Sol. $\frac{\sum x_i}{20} = 10$ ………….(i)

$\frac{\sum x_i^2}{20} - 100 = 4$ …………(ii)

$\sum x_i^2 = 104 \times 20 = 2080$

Actual mean $= \frac{200 - 9 + 11}{20} = \frac{202}{20}$

Variance $= \frac{2080 - 81 + 121}{20} = \left(\frac{202}{20}\right)^2$

$= \frac{2120}{20} - (10.1)^2 = 106 - 102.01 = 3.99$

54. The mirror image of the point (1, 2, 3) in a plane is $\left(-\frac{7}{3},-\frac{4}{3},-\frac{1}{3}\right)$. Which of the following points lies on this plane?

(A) (1, –1, 1) (B) (1, 1, 1)
(C) (–1, –1, –1) (D) (–1, –1, 1)

Ans. B

Sol. d.r of normal to the plane $\frac{10}{3},\frac{10}{3},\frac{10}{3}$

Midpoint of P and Q is $\left(\frac{-2}{3},\frac{1}{3},\frac{4}{3}\right)$ equation of plane $x+y+z=1$

55. If a hyperbola passes through the point P (10, 16) and it has vertices at $(\pm 6, 0)$ then the equation of the normal to it at P is:

(A) $2x+5y=100$ (B) $x+3y=58$
(C) $3x+4y=94$ (D) $x+2y=42$

Ans. A

Sol. Vertex is at $(\pm 6, 0)$

$\therefore a=6$

Let the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Putting point P (10, 16) on the hyperbola

$\frac{100}{36}-\frac{256}{b^2}=1 \quad \Rightarrow b^2=144$

$\therefore$ hyperbola is $\frac{x^2}{36}-\frac{y^2}{144}=1$

$\therefore$ equation of normal is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$

$\therefore$ putting we get $2x+5y=100$

56. If a line, $y=mx+c$ is a tangent to the circle, $(x-3)^2+y^2=1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2+y^2=1$ at the point $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$; then:

(A) $c^2+7c+6=0$ (B) $c^2-6c+7=0$
(C) $c^2+6c+7=0$ (D) $c^2-7c+6=0$

Ans. C

Sol. Slope of tangent to $x^2+y^2=1$ at $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

$x^2+y^2=1\ \left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

$x^2+y^2=1$

$2x+2yy'=0$ 2

$y'=-\frac{x}{y}=-1$

$y=mx+c$ is a tangent of $x^2+y^2=1$

So $y = x + c$

now distance of (3, 0) from $y = x + c$ is $\left|\frac{c+3}{\sqrt{2}}\right| = 1$

$c^2 + 6c + 9 = 2$
$c^2 + 6c + 7 = 0$

57. The area (in sq. units) of the region $\left\{(x, y) \in R^2 \le y \le 3 - 2x\right\}$, is

(A) $\frac{32}{3}$ (B) $\frac{29}{3}$
(C) $\frac{31}{3}$ (D) $\frac{34}{3}$

Ans. A

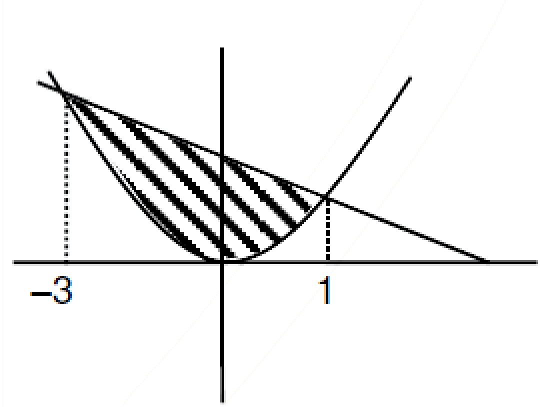
Sol. Point of intersection of $y = x^2$ and $y = -2x + 3$ is obtained by

$x^2 + 2x - 3 = 0$
$\Rightarrow x = -3, 1$

So, area $= \int_{-3}^{1} \left(3 - 2x - x^2\right) dx -$

$= 3(4) - 2\left(\frac{1^2 - 3^2}{2}\right) - \left(\frac{1^3 + 3^3}{3}\right)$

$= 12 + 8 - \frac{28}{3} = \frac{32}{3}$



58. Let S be the set of all functions $\int : [0,1] \to R$, which are continuous on $[0,1]$ and differentiable on (0, 1). Then for every $\int$ in S, there exists $a\,c \in (0,1)$, depending on f, such that:

(A) $|f(c) - f(1)| < (1 - c)|f'(c)|$ (B) $|f(c) + f(1)| < (1 + c)|f'(c)|$

(C) $|f(c) - f(1)| < |f'(c)|$ (D) $\frac{f(1) - f(c)}{1 - c} = f'(c)$

Ans. NA
Sol. NA

59. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c}.\vec{a} = 0$, then $\vec{c}.\vec{b}$ is equal to:

(A) $-\frac{1}{2}$ (B) $-\frac{3}{2}$

(C) -1 (D) $\frac{1}{2}$

Ans. A

Sol. $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{a} \times \left(\vec{b} \times \vec{a}\right)$

$-\left(\vec{a}.\vec{b}\right)\vec{c} = \left(\vec{a}.\vec{a}\right)\vec{b} - \left(\vec{a}.\vec{b}\right)\vec{a}$

$-4\vec{c} = 6\left(\hat{i} - \hat{j} + \hat{k}\right) - 4\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

$-4\vec{c} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{c} = -\frac{1}{2}\left(\hat{i} + \hat{j} + \hat{k}\right)$

$\vec{b}.\vec{c} = -\frac{1}{2}$

60. Let $f:(1,3) \to R$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where [x] denotes the greatest integer $\le x$. Then the range of f is:

(A) $\left(\frac{2}{5}, \frac{4}{5}\right]$
(B) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$
(C) $\left(\frac{3}{5}, \frac{4}{5}\right)$
(D) $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$

Ans. B

Sol. $f(x)\begin{cases} \frac{x}{x^2+1}; & x \in (1,2) \\ \frac{2x}{x^2+1}; & x \in [2, 3) \end{cases}$

$\therefore$ $f(x)$ is a decreasing function

$\therefore \quad y \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{6}{10}, \frac{4}{5}\right]$

61. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$ and $b = \sum_{k=0}^{100} \alpha^{3k}$, then a and b are the roots of the quadratic equation:

(A) $x^2 - 101x + 100 = 0$
(B) $x^2 - 102x + 101 = 0$
(C) $x^2 + 102x + 101 = 0$
(D) $x^2 + 101x + 100 = 0$

Ans. B

Sol. $\alpha = \omega, b = 1 + \omega^3 + \omega^6 + ... = 101$

$a = (1+\omega)\left(1 + \omega^2 + \omega^4 + ...\omega^{198} + \omega^{200}\right)$

$= (1+\omega)\frac{\left(1-\left(\omega^2\right)^{101}\right)}{1-\omega^2} = \frac{(1+\omega)(1-\omega)}{1-\omega^2} = 1$

Equation $x^2 - (101+1)x + (101) \times 1 = 0$

$\Rightarrow x^2 - 102x + 101 = 0$

62. $\lim_{x \to 0} \frac{\int_0^x t\sin(10t)dt}{x}$ is equal to:

(A) 0
(B) $\frac{1}{10}$
(C) $-\frac{1}{5}$
(D) $-\frac{1}{10}$

Ans. A

Sol. Using L' Hospital

$$\lim_{x\to 0}\frac{x\sin(10x)}{1}=0$$

63. The system of linear equations
$\lambda x+2y+2z=5$
$2\lambda x+3y+5z=8$
$4x+\lambda y+6z=10$ has:
(A) no solution when $\lambda=8$
(B) infinitely many solutions when $\lambda=2$
(C) no solution when $\lambda=2$
(D) a unique solution when $\lambda=-8$

Ans. C

Sol. $D=\begin{vmatrix}\lambda & 2 & 2\\ 2\lambda & 3 & 5\\ 4 & \lambda & 6\end{vmatrix}$

$D=(\lambda+8)(2-\lambda)$

for $\lambda=2$

$D_1=\begin{vmatrix}5 & 2 & 2\\ 8 & 3 & 5\\ 10 & 2 & 6\end{vmatrix}$

$=5[18-10]-2[48-50]+2(16-30)$

$=40+4-28\neq 0$

No solution for $\lambda=2$

64. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$, then:
(A) $\alpha+\beta=-30$
(B) $\alpha-\beta=60$
(C) $\alpha-\beta=-132$
(D) $\alpha+\beta=60$

Ans. C

Sol. $2\left[{}^6C_0x^6+{}^6C_2x^4\left(x^2-1\right)+{}^6C_4x^2\left(x^2-1\right)^2+{}^6C_6\left(x^2-1\right)^3\right]$

$=2\left[x^6+15\left(x^6-x^4\right)+15x^2\left(x^4-2x^2+1\right)+\left(-1+3x^2-3x^4+x^6\right)\right]$

$=2\left(32x^6-48x^4+18x^2-1\right)$

$\alpha=-96$ and $\beta=36$

$\therefore \alpha-\beta=-132$

65. The differential equation of the family of curves, $x^2=4b(y+b), b\in R$, is:
(A) $xy''=y'$
(B) $x(y')^2=x-2yy'$
(C) $x(y')^2=x+2yy'$
(D) $x(y')^2=2yy'-x$

Ans. C

Sol. $2x=4by'\Rightarrow b=\dfrac{x}{2y'}$

So, differential equation is $x^2=\dfrac{2x}{y}.y+\left(\dfrac{x}{y}\right)^2$

$$x^2 = \frac{2x}{y}.y + \left(\frac{x}{y}\right)^2 \Rightarrow x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx} + x$$

66. The length of the perpendicular from the origin, 0n the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2, 2) is:

(A) $2\sqrt{2}$
(B) $4\sqrt{2}$
(C) $\sqrt{2}$
(D) 2

Ans. A

Sol. $x^2 + 2xy - 3y^2 = 0$

$x^2 + 3xy - xy - 3y^2 = 0$

$(x-y)(x+3y) = 0$

$x - y = 0 \qquad x + 3y = 0$

(2, 2) satisfy $x - y = 0$

Normal $x + y = \lambda$

$\lambda = 4$

Hence $x + y = 4$

Perpendicular distance from origin $= \left|\frac{0+0-4}{\sqrt{2}}\right| = 2\sqrt{2}$

$= \left|\frac{0+0-4}{\sqrt{2}}\right| = 2\sqrt{2}$

67. If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is equal to:

(A) $4I - A$
(B) $A - 6I$
(C) $A - 4I$
(D) $6I - A$

Ans. B

Sol. Characteristics equation of matrix 'A' is $\begin{vmatrix} 2-x & 2 \\ 9 & 4-x \end{vmatrix} = 0 \Rightarrow x^2 - 6x - 10 = 0$

$\therefore \; A^2 - 6A - 10I = 0$

$\Rightarrow \; 10A^{-1} = A - 6I$

68. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is:

(A) 0.01
(B) 0.10
(C) 0.20
(D) 0.02

Ans. B

Sol. P (exactly one) $= \frac{2}{5}$

$\Rightarrow P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$

$P(A \cup B) = \frac{1}{2}$

$\Rightarrow P(A)+P(B)-P(A\cap B)=\frac{1}{2}$

$\therefore\ P(A\cap B)=\frac{1}{2}-\frac{2}{5}=\frac{5-4}{10}=\frac{1}{10}$

69. If $I=\int_1^2 \frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then:

(A) $\frac{1}{16}<I^2<\frac{1}{9}$
(B) $\frac{1}{9}<I^2<\frac{1}{8}$
(C) $\frac{1}{8}<I^2<\frac{1}{4}$
(D) $\frac{1}{6}<I^2<\frac{1}{2}$

Ans. B

Sol. $f(x)=\frac{1}{\sqrt{2x^3-9x^2+12x+4}}$

$f'(x)=\frac{-1}{2}=\frac{(6x^2-18x+12)}{(2x^3-9x^2+12x+4)^{\frac{3}{2}}}$

$=\frac{-6(x-1)(x-2)}{2(2x^3-9x^2+12x+4)^{\frac{3}{2}}}$

$f(1)=\frac{1}{3},\ f(2)=\frac{1}{\sqrt{8}}$

$\frac{1}{3}<I<\frac{1}{\sqrt{8}}$

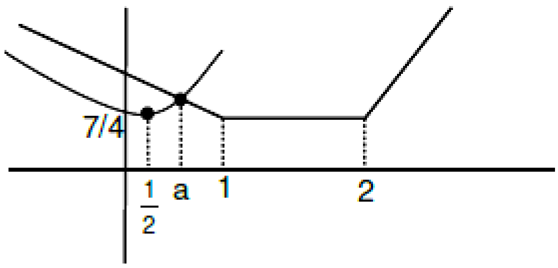
70. Let S be the set of all real roots of the equation, $3^x(3^x-1)+2=|3^x-1|+|3^x-2|$. Then S:

(A) contains at least four elements
(B) is a singleton
(C) is an empty set
(D) contains exactly two elements

Ans. B

Sol. Let $3^x=t$

$t(t-1)+2=|t-1|+|t-2|$



$t=a$

$3^x=a$

$x=\log_3 a$ so singleton set

71. Let $f(x)$ be a polynomial of degree 3 such that $f(-1)=10, f(1)=-6$, $f(x)$ has a critical point at $x=-1$ and $f'(x)$ has a critical point at x = 1. Then $f(x)$ has a local minima at x =________

Ans. 3

Sol. Let $f(x) = ax^3 + bx^2 + cx + d$

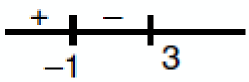
$a = \frac{1}{4} \quad d = \frac{35}{4}$

$b = \frac{-3}{4} \quad c = \frac{-9}{4}$

$\Rightarrow f(x) = a(x^3 - 3x^2 - 9x) + d$

$f'(x) = \frac{3}{4}(x^2 - 2x - 3)$

$\Rightarrow f'(x) = 0 \Rightarrow x = 3, -1$



local minima exist at x = 3

72. Let aline $y = mx(m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x – axis at the point Q. If area $(\Delta OPQ) = 4$ sq. units, then m is equal to ____________.

Ans. 0.5

Sol. $2ty = x + t^2$

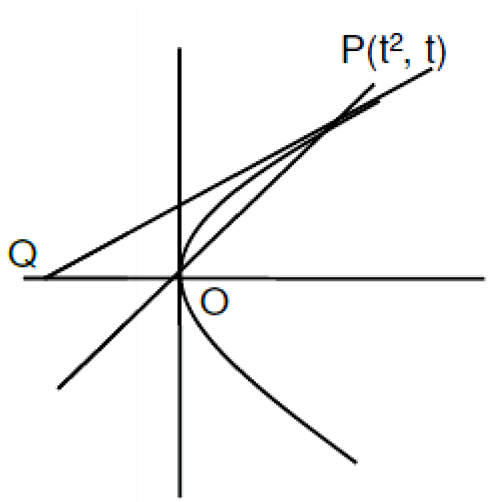
$Q(-t^2, 0)$

$\frac{1}{2}\left\|\begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix}\right\| = 4$

$|t|^3 = 8$

$t = \pm 2(t > 0)$

$m = \frac{1}{2}$



73. The sum $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to _____________.

Ans. 504

Sol. $\frac{1}{4}\left[\sum_{n=1}^{7}(2n^3 + 3n^2 + n)\right]$

$\frac{1}{4}\left[2\left(\frac{7.8}{2}\right)^2 + 3\left(\frac{7.8.15}{6}\right) + \frac{7.8}{2}\right]$

$\frac{1}{4}[2 \times 49 \times 16 + 28 \times 15 + 28]$

$\frac{1}{4}[1568 + 420 + 28] = 504$

74. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is ____________________.

Ans. 2454

Sol. EXAMINATION

2N, 2A, 2I, E, X, M, T, O

Case I All the different so $^{8}P_{4} = \frac{8!}{4!} = 8.7.6.5 = 1680$

Case II 2 same and 2 different so $^{3}C_{1}.{}^{7}C_{2}.\frac{4!}{2!} = 3.21.12 = 756$

Case III 2 same and 2 same so $^{3}C_{2}.\frac{4!}{2!.2!} = 3.6 = 18$

Total = 1680 + 756 + 18 = 2454

75. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to __________.

Ans. 1

Sol. $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$ and $\frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$

$$\tan\alpha = \frac{1}{7}$$

$$\sin\beta = \frac{1}{\sqrt{10}}$$

$$\tan\beta = \frac{1}{3}$$

$$\tan 2\beta = \frac{2.\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha \tan 2\beta} = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7}.\frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{26}} = 1$$