FIITJEE

Solutions to JEE (Main)-2020

JEE–Main–2020 –Jan–8–First–Shift

PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be

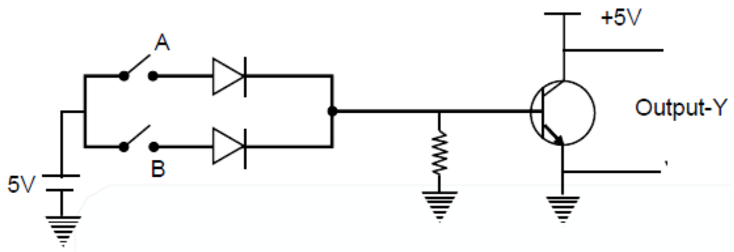
(A) 60° (B) 15°
(C) 45° (D) 30°

Ans. **D**

Sol. $V = \frac{1}{\sqrt{\mu\varepsilon}}$; $n = \sqrt{\mu_r \varepsilon_r} = 2$

$\sin c = \frac{1}{2}$; c = 30°

2. Boolean relation at the output state-Y for the following circuits is:

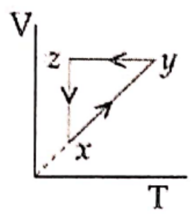


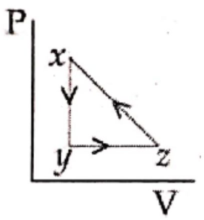
(A) $\overline{A} \cdot \overline{B}$ (B) A + B
(C) $\overline{A} + \overline{B}$ (D) $A \cdot B$

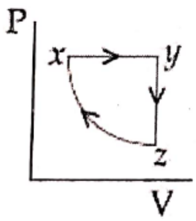
Ans. **A**

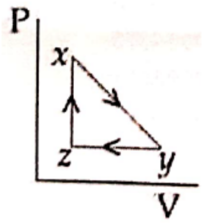
Sol. First part of figure shown is OR gate and
Second part of figure shown is NOT gate
So, Y_P = OR + NOT = NOR gate
$Y = \overline{A + B} = \overline{A} \cdot \overline{B}$

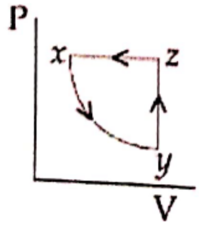
3. A thermodynamic cycle xyzx is shown on a V-T diagram. The P-V diagram that best describes this cycle is: (diagrams are schematic and not to scale)



(A) 

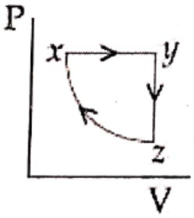
(B) 

(C) 

(D) 

Ans. **B**

Sol.



4. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length ℓ. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is:

(A) $\frac{m\ell\omega^2}{k+m\omega}$

(B) $\frac{m\ell\omega^2}{k+m\omega^2}$

(C) $\frac{m\ell\omega^2}{k-\omega m}$

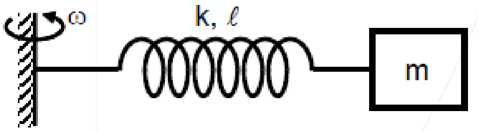
(D) $\frac{m\ell\omega^2}{k-m\omega^2}$

Ans. **D**

Sol. $m\omega^2(\ell + x) = kx$

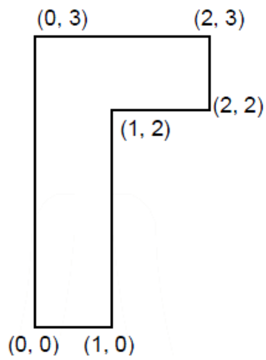
$\left(\frac{\ell}{x}+1\right)=\frac{k}{m\omega^2}$

$x=\frac{\ell m\omega^2}{k-m\omega^2}$



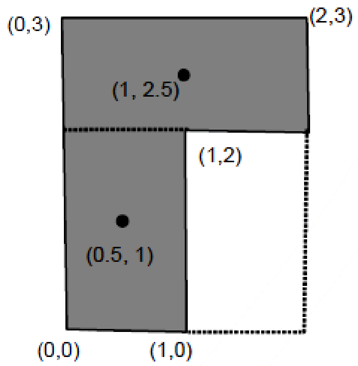
5. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. (the coordinates of the same are shown in figure) are:

(A) (0.75 m, 0.75 m)
(B) (0.75 m, 1.75 m)
(C) (1.25 m, 1.50 m)
(D) (1 m, 1.75 m)



Ans. **B**

Sol.



$$\vec{r}_{cm} = \frac{1\times\left(\frac{\hat{i}}{2}+\hat{j}\right)+1\times\left(\hat{i}+\frac{5\hat{j}}{2}\right)}{2}$$

$$\vec{r}_{cm} = \frac{3}{4}\hat{i}+\frac{7}{4}\hat{j}$$

6. When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated form another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is

(A) 4 eV (B) 1.5 eV
(C) 2 eV (D) 3 eV

Ans. **A**

Sol. Relation between De-Broglie wavelength and K.E. is

$$\lambda = \frac{h}{\sqrt{2(KE)m_e}} \Rightarrow \lambda \propto \frac{1}{\sqrt{KE}}$$

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{KE_B}}{\sqrt{KE_A}} \Rightarrow \frac{1}{2} = \sqrt{\frac{T_A - 1.5}{T_A}}$$

$\Rightarrow$ T_A 2 eV

$\therefore$ $KE_B = 2 - 1.5 = 0.5$ eV

$\phi_B = 4.5 - 0.5 = 4$ eV

7. In finding the electric field using Gauss law the formula $|\vec{E}| = \frac{q_{enc}}{\varepsilon_0 |A|}$ is applicable. In the formula ε_0 is permittivity of free space, A is the area of Gaussain surface and q_{enc} is charge enclosed by the Gaussian surface. This equation can be used in which of the following situation? Only when the Gaussian surface is an

(A) equipotential surface and $|\vec{E}|$ is constant on the surface.

(B) For any choice of Gaussian surface.

(C) Only when $|\vec{E}|$ = constant on the surface.

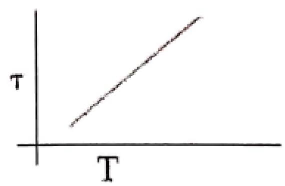
(D) Only when the Gaussian surface is an equipotential surface.

Ans. **A**

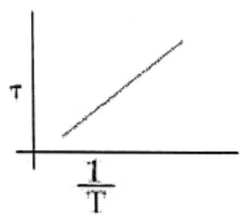
Sol. Magnitude of electric field is constant & the surface is equipotential.

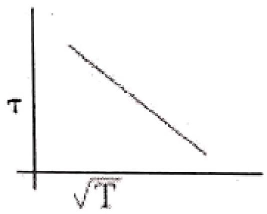
8. The plot that depicts the behavior of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale)

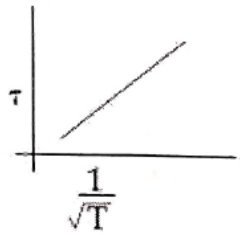
(A)



(B)



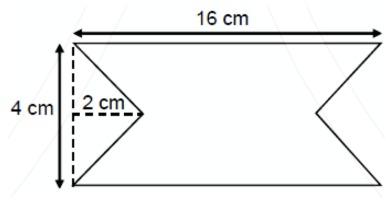
(C) 

(D) 

Ans. **D**

Sol. $\tau \propto \frac{1}{\sqrt{T}}$

9. At time t = 0 magnetic field of 1000 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5s, then induced EMF in the loops is:



(A) 28 μV
(B) 56 μV
(C) 48 μV
(D) 36 μV

Ans. **B**

Sol. $\varepsilon = \left|-\frac{d\varphi}{dt}\right| = \left|-\frac{AdB}{dt}\right|$

$= (16\times 4 - 4\times 2)\frac{(1000-500)}{5}\times 10^{-4}\times 10^{-4}$

$= 56\times\frac{500}{5}\times 10^{-8} = 56\times 10^{-6}$ V

10. The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant 'h', speed of light 'c' and Gravitational constant 'G' and ampere A is:

(A) $h^{1/3} G^{2/3} c^{1/3} A^{-1}$
(B) $h^{-2/3} c^{-1/3} G^{4/3} A^{-1}$
(C) $h^{2/3} c^{5/3} G^{1/3} A^{-1}$
(D) $h^{2} G^{3/2} c^{1/3} A^{-1}$

Ans. **Bonus**

Sol. $V = K (h)^a (I)^b (G)^c (C)^d$ (V is voltage)

We know, $[h] = ML^2T^{-1}$

$[I] = A$

$[G] = M^{-1} L^3 T^{-2}$

$[C] = L T^{-1}$

$[V] = M L^2 T^{-3} A^{-1}$

$M L^2 T^{-3} A^{-1} = (M L^2 T^{-1})^a (A)^b (M^{-1}L^3T^{-2})^c (LT^{-1})^d$

$M L^2 T^{-3} A^{-1} = M^{a-c} L^{2a+3c+d} T^{-a-2c-d} A^b$

$a - c = 1$ …(1)

$2a + 3c + d = 2$ …(2)

$-a - 2c - d = -3$ …(3)

$b = -1$ …(4)

On solving,

$c = -1$

$a = 0$

$d = 5, b = -1$

$V = K(h)^o (I)^{-1} (G)^{-1} (C)^5$

11. Consider a uniform rod of mass M = 4 m and length ℓ pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular sped of the rod-mass system just after the collision is

(A) $\frac{3}{7}\frac{v}{\ell}$ (B) $\frac{3\sqrt{2}}{7}\frac{v}{\ell}$

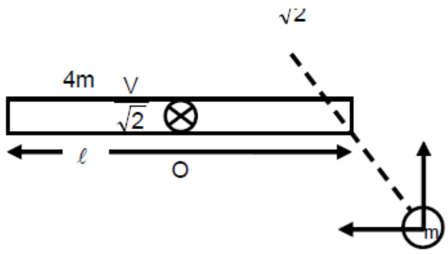
(C) $\frac{4}{7}\frac{v}{\ell}$ (D) $\frac{3}{7\sqrt{2}}\frac{v}{\ell}$

Ans. **B**

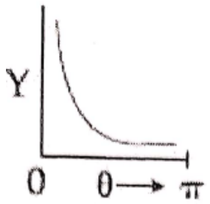
Sol. $L_{oi} = L_{of}$

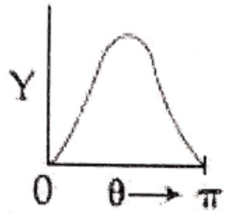
$$\frac{mv}{\sqrt{2}} \times \frac{\ell}{2} = \left[\frac{4m\ell^2}{12} + \frac{m\ell^2}{4}\right] \times \omega$$

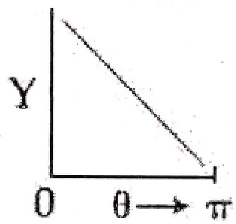
$$\omega = \frac{6v}{7\sqrt{2}\,\ell} = \frac{3\sqrt{2}\,v}{7\ell}$$

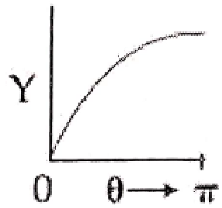


12. The graph which depicts the results of Rutherford gold foil experiment with α-particles is
θ : Scattering angle
Y : Number of scattered α-particles detected
(Plots are schematic and not to scale)

(A)


(B)


(C)


(D)


Ans. **A**

Sol. $N \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$

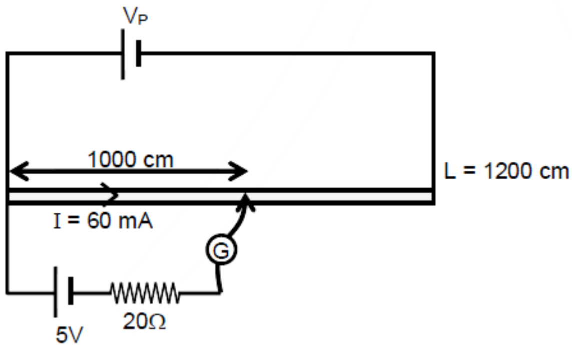
13. The length of a potentiometer wire is 1200 cm and it carries a current of 60 mA. For a cell of emf 5 V and internal resistance of 20 Ω, the null point on it is found to be at 1000 cm. The resistance of whole wire is:

(A) 60 Ω (B) 100 Ω

(C) 80 Ω (D) 120 Ω

Ans. **B**

Sol.



Potential gradient = $\frac{5}{1000} = \frac{V_P}{1200}$

$V_P = 6$ V

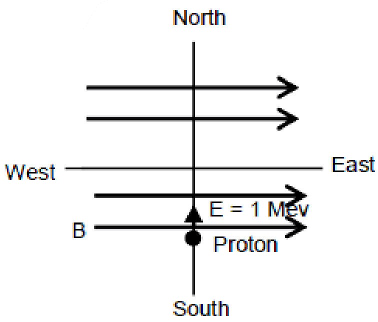
and $R_P = \frac{V_P}{I} = \frac{6}{60 \times 10^{-3}}$ = 100 Ω

14. Proton with kinetic energy of 1 MeV moves from south the north. It gets an acceleration of 10^{12} m/s^2 by an applied magnetic field (west to east). The value of magnetic field: (Rest mass of proton is 1.6×10^{-27} kg)

(A) 0.71 mT (B) 7.1 mT
(C) 0.071 mT (D) 71 mT

Ans. **A**

Sol.



$\because$ K.E. = $1.6 \times 10^{-13} = \frac{1}{2} \times 1.6 \times 10^{-27} v^2$

v = $\sqrt{2} \times 10^7$ m/s

$\therefore$ Bqv = ma

$B = \frac{1.6 \times 10^{-27} \times 10^{12}}{1.6 \times 10^{-19} \times \sqrt{2} \times 10^7}$

= 0.71×10^{-3} T

So, 0.71 mT

15. The magnifying power of a telescope with tube length 60 cm is 5. What is the focal length of its eye piece?

(A) 40 cm (B) 10 cm
(C) 20 cm (D) 30 cm

Ans. **B**

Sol. $m = \frac{f_o}{f_e}$; $5 = \frac{f_o}{f_e}$

$f_o = 5 f_e$; $f_o + f_e = 60$

$6f_e = 60$; $f_e = 10$

16. Consider a solid sphere of radius R and mass density $p(r) = p_0\left(1 - \frac{r^2}{R^2}\right)$, $0 < r \le R$. The minimum density of a liquid in which it will float is

(A) $\frac{p_0}{5}$ (B) $\frac{2p_0}{5}$
(C) $\frac{2p_0}{3}$ (D) $\frac{p_0}{3}$

Ans. **B**

Sol. $\rho = \rho_0\left(1 - \frac{r^2}{R^2}\right)$ $0 < r \le R$

mg = B

$\int \rho(4\pi r^2 dr) = \rho_L \frac{4}{3}\pi R^3$; $\int_0^R \rho_0 \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr = \rho_L \frac{4}{3}\pi R^3$

$\int_0^R \rho_0 4\pi \left(r^2 - \frac{r^4}{R^2}\right) dr = \rho_0 4\pi \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)_0^R = \rho_L \frac{4}{3}\pi R^3$; $\frac{2}{5}\rho_0 = \rho_L$

17. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitor are individually connected to a voltage source of 1 V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be

(A) 3.2 μF (B) 8.4 μF
(C) 1.6 μF (D) 4.2 μF

Ans. **C**

Sol. Given: $c_1 + C_2 = 10\ \mu F$ …(i)

$$4\left(\frac{1}{2}C_1V^2\right) = \frac{1}{2}C_2V^2$$

$\Rightarrow$ $4C_1 = C_2$ …(ii)

From equation (i) and (ii)

$C_1 = 2\mu F$

$C_2 = 8\mu F$

If they are in series

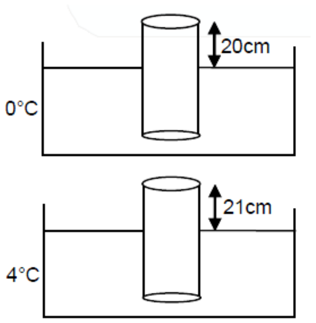
$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = 1.6\ \mu F$$

18. A leak proof cylinder of length 1 m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4°C, relative to the density at T = 0°C is close to:

(A) 1.01 (B) 1.26
(C) 1.04 (D) 1.03

Ans. **A**

Sol.

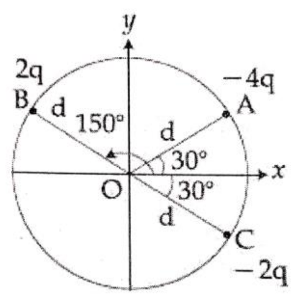


$mg = A(80)\ \rho_{0^\circ C}\ g$

$mg = A(79)\ \rho_{4^\circ C}\ g$

$$\frac{\rho_{4^\circ C}}{\rho_{0^\circ C}} = \frac{80}{79} = 1.01$$

19. Three charged particles A, B and C with charges –4q, 2q and –2q are present on the circumference of a circle of radius d. The charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is:

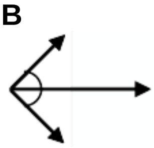


(A) $\dfrac{2\sqrt{3}q}{\pi\varepsilon_0 d^2}$ (B) $\dfrac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$

(C) $\dfrac{3\sqrt{3}q}{4\pi\varepsilon_0 d^2}$ (D) $\dfrac{\sqrt{3}q}{4\pi\varepsilon_0 d^2}$

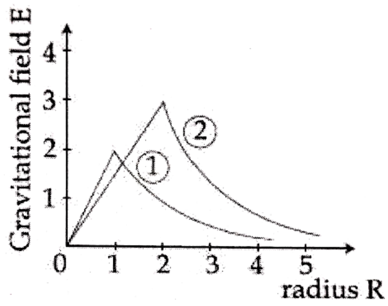
Ans. **B**

Sol.



$$E_{net} = \frac{4kq}{d^2} \times 2\cos 30^o = \frac{q\sqrt{3}}{\pi\varepsilon_0 d^2}$$

20. Consider two solid spheres of radii $R_1 = 1m$, $R_2 = 2m$ and masses M_1 and M_2, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{M_1}{M_2}$ is:



(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{2}{3}$

Ans. **C**

Sol. $3 = \frac{Gm_2}{2^2}$; $2 = \frac{Gm_1}{1^2}$

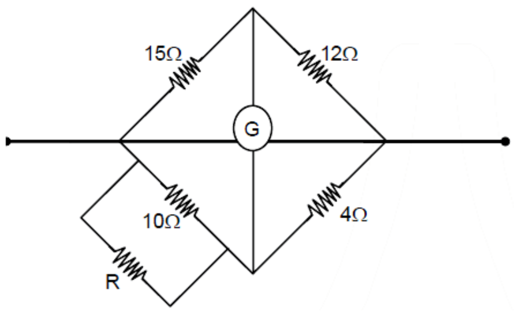
$\therefore \quad \frac{3}{2} = \frac{1}{4}\frac{m_2}{m_1}$

$\frac{m_1}{m_2} = \frac{1}{6}$

21. Four resistances of 15 Ω, 12 Ω, 4 Ω and 10 Ω respectively in cyclic order to form Whetstone's network. The resistance that is to be connected in parallel with the resistance of 10 Ω to balance the network is ___________.

Ans. **10**

Sol.



$\frac{10\,R}{10+R} \times 12 = 15 \times 4$

on solving

R = 10 Ω

22. A particle is moving along the x-axis with its coordinate with time 't' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate a function of time given by $y(t) = 5 - 8t^3$. At t = 1 s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is ___________.

Ans. **580**

Sol. $X_1 = -3t^2 + 8t + 10$

$\vec{v}_1 = (-6t + 8)\hat{i} = 2\hat{i}$

$Y_2 = 5 - 8t^3$

$\vec{v}_2 = -24t^2\,\hat{j}$

$\sqrt{v} = |\,\vec{v}_2 - \vec{v}_1| = \left|-24\,\hat{j} - 2\,\hat{i}\,\right|$

$\sqrt{v} = \sqrt{24^2 + 2^2}$

v = 580

23. A body A, of mass m = 0.1 kg has an initial velocity of $3\hat{i}\,ms^{-1}$. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}\,ms^{-1}$. After collision, A moves with a velocity $\vec{v} = 4(\hat{i} + \hat{j})$. The energy of B after collision is written as $\frac{x}{10}$J. The value of x is ___________.

Ans. **1**

Sol. For elastic collision $KE_i = KE_r$

$\frac{1}{2}m\times 25+\frac{1}{2}\times m\times 9=\frac{1}{2}m\times 32+\frac{1}{2}mv^2$

$34 = 32 + v^2$

$KE=\frac{1}{2}\times 0.1\times 2=0.1J=\frac{1}{10}$

x = 1.

24. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is ___________ Hz.

Ans. **106**

Sol. $V=\sqrt{\frac{B}{\rho}}$

$\frac{V_{pipe}}{V_{air}}=\frac{\sqrt{\frac{B}{2\rho}}}{\sqrt{\frac{B}{\rho}}}=\frac{1}{\sqrt{2}}$

$V_{pipe}=\frac{V_{air}}{\sqrt{2}}$

$f_n=\frac{(n+1)V_{pipe}}{2\ell}$; $f_1-f_0=\frac{V_{pipe}}{2\ell}=\frac{300}{2\sqrt{2}}$

= 105.75 Hz (If $\sqrt{2}=1.41$)

= 106.05 Hz (If $\sqrt{2}=1.414$)

25. A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is ___________.

Ans. **60**

Sol. $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$R_1 = \propto$

$R_2 = -30$ cm

$\frac{1}{f}=(1.5-1)\left(\frac{1}{\infty}-\frac{1}{-30}\right)$

$\frac{1}{f}=\frac{0.5}{30}$; f = 60 cm

PART –B (CHEMISTRY)

26. The complex that can show fac- and mer-isomers is :
(A) $[CoCl_2(en)_2]$
(B) $[Co(NH_3)_3(NO_2)_3]$
(C) $[Pt(NH_3)_2Cl_2]$
(D) $[Co(NH_3)_4Cl_2]^+$

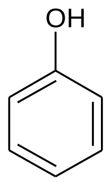
Ans. B

Sol. $[M\ a_3b_3]$ – type complex compound show fac-and mer-isomer.
$\therefore$ $[Co(NH_3)_3(NO_2)_3]$

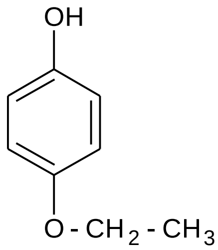
27. Arrange the following compounds in increasing order of C – OH bond length:
methanol, phenol, p-ethoxyphenol
(A) phenol < methanol < p-ethoxyphenol
(B) methanol < p-ethoxyphenol < phenol
(C) methanol < phenol < p-ethoxyphenol
(D) phenol < p-ethoxyphenol < methanol

Ans. D

Sol.



Phenol



CH_3 - OH
Methanol

Resonance possible   P-ethoxy phenol   Resonance absence

Resonance possible

C – OH bond length: CH_3 – OH > P – ethoxy phenol > Phenol

28. Among the gases (a) – (e), the gases that cause greenhouse effect are :
(a) CO_2
(b) H_2O
(c) CFCs
(d) O_2

(A) (a), (b), (c) and (d)
(B) (a), (b), (c) and (e)
(C) (a) and (d)
(D) (a), (c), (d) and (e)

Ans. B

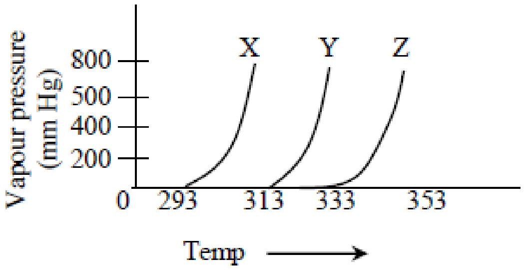
Sol. Except O_2, remarginal CO_2, O_3, H_2O, CFCs are green house gases.

29. The third ionization enthalpy is minimum for :
(A) Ni
(B) Co
(C) Mn
(D) Fe

Ans. D

Sol. $_{26}Fe = [Ar]3d^6 4S^2$
Among Ni, Co, Mn, Fe
Fe having minimum third ionization energy

30. A graph of vapour pressure and temperature for three different liquids X, Y, and Z is shown below:



The following inferences are made :
(a) X has higher intermolecular interactions compared to Y.
(b) X has lower intermolecular interactions compared to Y.
(c) Z has lower intermolecular interactions compared to Y.
The correct inference (s) is / are :

(A) (c) (B) (a) and (c)
(C) (b) (D) (a)

Ans. C

Sol. At a fixed temperature, X having more vapour pressure as compared to Y. So, intermolecular interaction is lower as compared to Y.

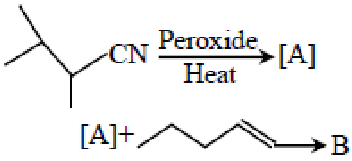
31. When gypsum is heated to 393 K, it forms :

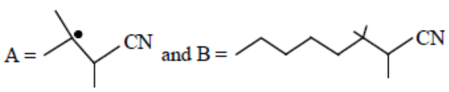
(A) $CaSO_4 . 0.5H_2O$ (B) Dead burnt plaster
(C) Anhydrous $CaSO_4$ (D) $CaSO_4 . 5\ H_2O$

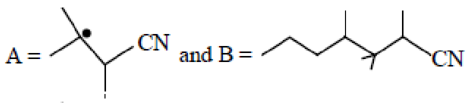
Ans. A

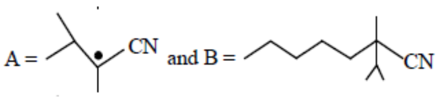
Sol. $\text{Gypsum} \xrightarrow{393\ K} CaSO_4 \frac{1}{2} H_2O$

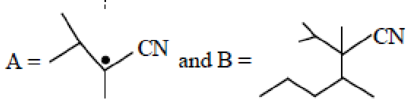
32. The major products A and B in the following reactions are :



(A) 

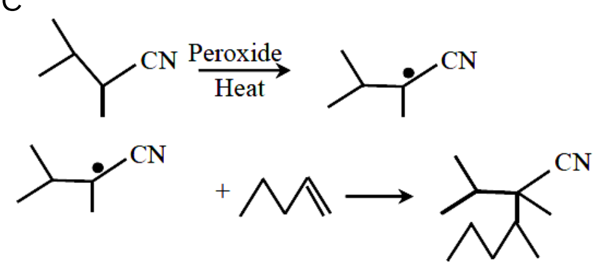
(B) 

(C) 

(D) 

Ans. C

Sol.

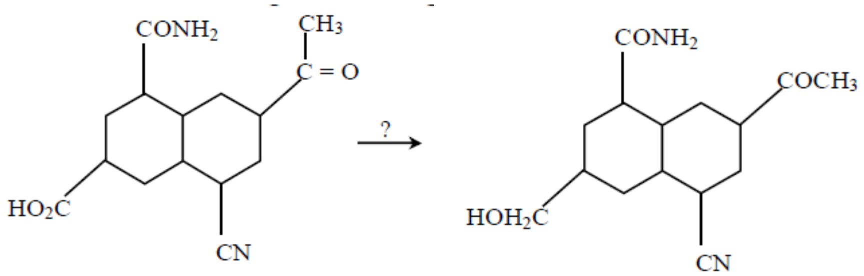


33. As per Hardy-Schulze formulation, the flocculation values of the following for ferric hydroxide sol are in the order :
(A) $AlCl_3 > K_3[Fe(CN)_6] > K_2CrO_4 > KBr = KNO_3$
(B) $K_3[Fe(CN)_6] < K_2CrO_4 < KBr = KNO_3 = AlCl_3$
(C) $K_3[Fe(CN)_6] < K_2CrO_4 < AlCl_3 < KBr < KNO_3$
(D) $K_3[Fe(CN)_6 > AlCl_3 > K_2CrO_4 > KBr > KNO_3$

Ans. B

Sol. According to Hardy-Schulze Rule coagulation value $\alpha \dfrac{1}{\text{Coagulation Power}}$.

34. The most suitable reagent for the given conversion is :



(A) $LiAlH_4$
(B) B_2H_6
(C) H_2/Pd
(D) $NaBH_4$

Ans. B

Sol. Diborane(B_2H_6) is used to reduced carboxylic acid to alcohol.

35. The predominant intermolecular forces present in ethyl acetate, a liquid, are:
(A) hydrogen bonding and London dispersion
(B) Dipole-dipole and hydrogen bonding
(C) London dispersion, dipole-dipole and hydrogen bonding
(D) London dispersion and dipole-dipole

Ans. D

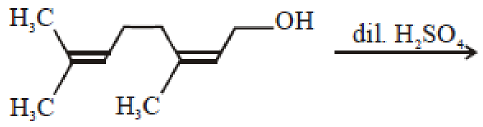
Sol. Ethyl acetate $\left(\begin{array}{c} CH_3 - \underset{\underset{O}{\|}}{C} - OC_2H_5 \end{array}\right)$ is polar molecule, so dipole-dipole and London Forces will be present between them.

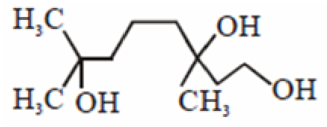
36. A flask contains a mixture of isohexane and 3-methylpentane. One of the liquids boils at 63 °C while the other boils at 60 °C. What is the best way to separate the two liquids and which one will be distilled out first?
(A) fractional distillation, 3-methylpentane
(B) simple distillation, isohexane
(C) simple distillation, 3-methylpentane
(D) fractional distillation, isohexane

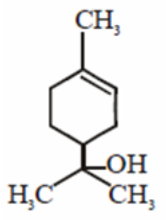
Ans. D

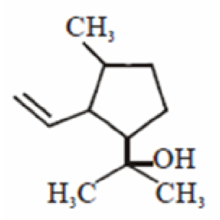
Sol. Isohexane and 3-Methylpentane having same molecular formula. Isohexane boil at 60°C and 3-Methyul pentane boils at 63°C. Both having low boiling point difference so fractional distillation is useful for separation and isohexane having low boiling point so comes out first.

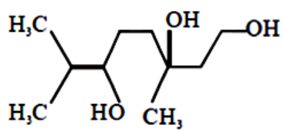
37. The major product of the following reaction is :



(A) 

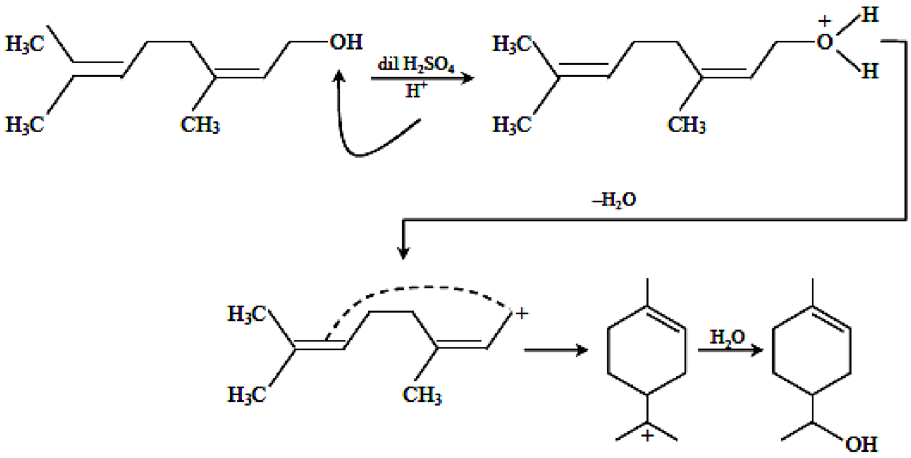
(B) 

(C) 

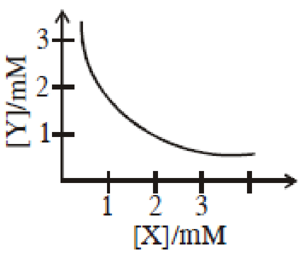
(D) 

Ans. D

Sol.



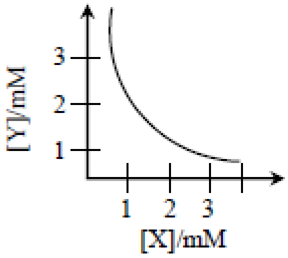
38. The stoichiometry and solubility product of a salt with the solubility curve given below is, respectively:



(A) XY, 2×10^{-5} M^3
(B) XY_3, 1×10^{-9} M^3
(C) XY_2, 4×10^{-9} M^3
(D) X_2Y, 2×10^{-9} M^3

Ans. C

Sol.



$XY_2 \rightarrow X^{+2} + 2y^-$

$Ksp = [X^{+2}]\,[X^-]^2$

$= (10^{-3})\,(2 \times 10^{-3})^2$

$= 4 \times 10^{-9}$

39. The rate of a certain biochemical reaction at physiological temperature (T) occurs 10^6 times faster with enzyme than without. The change in the activation energy upon adding enzyme is :

(A) – 6 (2.303)RT (B) + 6RT

(C) – 6RT (D) + 6(2.303)RT

Ans. A

Sol. Without catalyst: $K = Ae^{-Ea/RT}$ . . . . . (A)

Presence catalyst: $10^6\ K = Ae^{-Ec/RT}$ . . . . . (B)

Eq (B) – Eq(A) $10^6 = e^{-(E - Ec)/RT}$

$$\frac{E - Ec}{RT} = 2.303 \times 6$$

$\Delta E = Ec - E = (-2.303) \times 6RT$

40. The strength of an aqueous NaOH solution is most accurately determined by titrating :
(Note : consider that an appropriate indicator is used)

(A) Aq. NaOH in a burette and aqueous oxalic acid in a conical flask

(B) Aq. NaOH in a pipette and aqueous oxalic acid in a burette

(C) Aq. NaOH in a volumetric flask and concentrated H_2SO_4 in a conical flask

(D) Aq. NaOH in a burette and concentrated H_2SO_4 in a conical flask

Ans. A

Sol. Aq. NaOH in a burette and aqueous oxalic acid in a conical flask.

41. For the Balmer series in the spectrum of H atom, $\bar{\nu} = R_H \left\{ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right\}$ the correct statements among (I) to (IV) are :

(I) As wavelength decreases, the lines in the series converge

(II) The integer n1 is equal to 2

(III) The lines of longest wavelength corresponds to $n_2 = 3$

(IV) The ionization energy of hydrogen can be calculated from wave number of these lines

(A) (II), (III), (IV) (B) (I), (III), (IV)

(C) (I), (II), (III) (D) (I), (II), (IV)

Ans. C

Sol. $\bar{\nu} = \frac{1}{\lambda} = Rz^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

For H atom z = 1

$\bar{\nu} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For Balmer series: $n_1 = 2$

If $n_2 = 3$ $\therefore \frac{1}{\lambda} = Rz^2\left(\frac{1}{4} - \frac{1}{9}\right)$

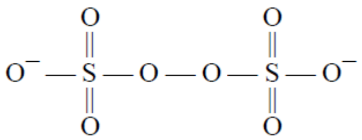
$\frac{1}{\lambda} = R\frac{5}{36}$

$\lambda_{max} = \frac{36}{5R}$

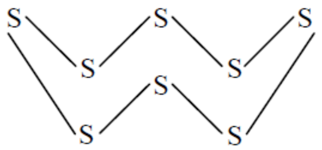
42. The number of bonds between sulphur and oxygen atoms in $S_2O_8^-$ and the number of bonds between sulphur and sulphur atoms in rhombic sulphur, respectively, are :
(A) 4 and 8 (B) 4 and 6
(C) 8 and 6 (D) 8 and 8

Ans. D

Sol.



Number of bond between sulphur and oxygen = 8



Number of bond between sulphur and sulphur = 8

43. Which of the following statement is not true for glucose?
(A) Glucose exists in two crystalline forms α and β
(B) Glucose gives Schiff's test for aldehyde
(C) Glucosse reacts with hydroxylamine to form oxime
(D) The pentaacetate of glucose does not react with hydroxylamine to give oxime

Ans. B

Sol. Open chain form of glucose not give schiff's test.

44. The first ionization energy (in kJ/mol) of Na, Mg, Al and Si respectively, are :
(A) 496, 577, 786, 737 (B) 786, 737, 577, 496
(C) 496, 577, 737, 786 (D) 496, 737, 577, 786

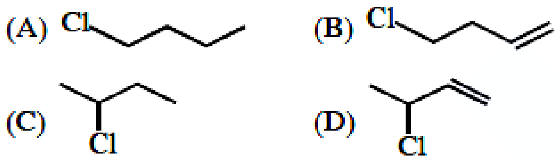
Ans. D

Sol. Order of I.E: Na < Al < Mg < Si

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

496 577 737 786

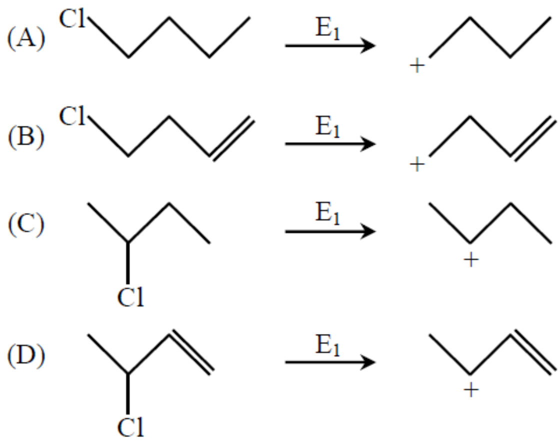
45. The decreasing order of reactivity towards dehydrohalogenation (E_1) reaction of the following compounds is :



(A) D > B > C > A (B) B > A > D > C
(C) B > D > C > A (D) B > D > A > C

Ans. A

Sol.



Order of stability : D > B > C > A

46. The volume (in mL) of 0.125 M $AgNO_3$ required to quantitatively precipitate chloride ions in 0.3 g of $[Co(NH_3)_6]Cl_3$ is ———

$^{M}[Co(NH_3)_6Cl_3] = 267.46$ g/mol

$^{M}AgNO_3 = 169.87$ g/mol

Ans. 26.92

Sol. $\left[Co\left(NH_3\right)_6\right]Cl_3 + 3\,AgNO_3 \longrightarrow 3\,AgCl$

$$\frac{\text{Mole of }\left[Co\left(NH_3\right)_6\right]Cl_3}{1} = \frac{\text{Mole of }AgNO_3}{3}$$

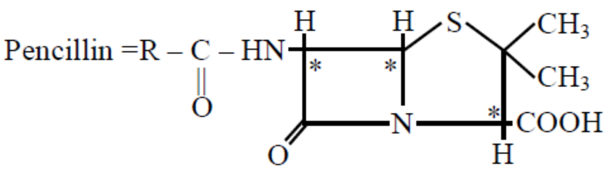
$$\frac{0.3}{267.46} = \frac{0.125 \times v \times 10^{-3}}{3}$$

V = 26.92 mL

47. The number of chiral centres in penicillin is ———-

Ans. 3.00

Sol.



48. What would be the electrode potential for the given half cell reaction at pH = 5 ?

Ans. 1.52

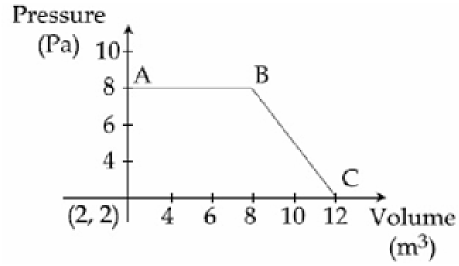
Sol. $E = E_0 - \frac{0.0591}{4}\log\left[H^+\right]^4$

$E = 1.23 + 0.0591 \times pH$

$E = 1.23 + 0.0591 \times (5)$

$E = 1.52$

49. The magnitude of work done by a gas that undergoes a reversible expansion along the path ABC shown in the figure is ——————



Ans. 48.00

Sol. $|w| = \frac{1}{2}(6+10) = 48\,J$

50. Ferrous sulphate heptahydrate is used to fortify foods with iron. The amount (in grams) of the salt required to achieve 10 ppm of iron in 100 kg of wheat is ——————

Atomic weight : Fe = 55.85; S = 32.00; O = 16.00

Ans. 496.00

Sol. $PPM = \frac{\text{Mass of Iron}}{\text{Mass of wheat}} \times 10^6$

$10 = \frac{\text{Mass of Iron}}{100 \times 10^3} \times 10^6$

Mass of Fe = 1 gm

Mole of Fe = $\frac{1}{56}$

$FeSO_4\ 7H_2O$ contain = 1 mole of Fe atom

$\therefore$ 56 g in 1 mole

1 g in $\frac{1}{56}$ mole

$\therefore$Mass = $\frac{1}{56} \times 277.85 = 496$

PART–C (MATHEMATICS)

51. Let the volume of a parallelepiped whose coterminous edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}, \vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ be 1 cu. Unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be:

(A) $\frac{7}{6\sqrt{3}}$ (B) $\frac{5}{7}$

(C) $\frac{5}{3\sqrt{3}}$ (D) $\frac{7}{6\sqrt{6}}$

Ans. D

Sol. $\pm 1 = \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} \Rightarrow = -\lambda + 3 = \pm 1$

$\Rightarrow \lambda = 2$ or $\lambda = 4$

For $\lambda = 4$

$\cos\theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$

52. Let $f(x) = x\cos^{-1}(-\sin|x|), x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then which of the following is true?

(A) $f'(0) = -\frac{\pi}{2}$

(B) f is not differentiable at x = 0

(C) f' is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$

(D) f' is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$

Ans. B

Sol. $f'(x) = x\left(\pi - \cos^{-1}(\sin|x|)\right) = x\left(\pi - \left(\frac{\pi}{2} - \sin^{-1}(\sin|x|)\right)\right) = x\left(\frac{\pi}{2} + |x|\right)$

$$f(x) = \begin{cases} x\left(\frac{\pi}{2} + x\right) & x \ge 0 \\ x\left(\frac{\pi}{2} - x\right) & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{\pi}{2} + 2x & x \ge 0 \\ \frac{\pi}{2} - 2x & x < 0 \end{cases}$$

$f'(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$ and decreasing in $\left(\frac{-\pi}{2}, 0\right)$

53. Let $y = y(x)$ be a solution of the differential equation, $\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1$. If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(\frac{1}{\sqrt{2}}\right)$ is equal to:

(A) $-\frac{\sqrt{3}}{2}$
(B) $-\frac{1}{\sqrt{2}}$
(C) $\frac{1}{\sqrt{2}}$
(D) $\frac{\sqrt{3}}{2}$

Ans. C

Sol. $\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0 \Rightarrow \sin^{-1}y + \sin^{-1}x = c$

At $x = \frac{1}{2}, y = \frac{\sqrt{3}}{2} \Rightarrow c = \frac{\pi}{2} \Rightarrow \sin^{-1}y = \cos^{-1}x$

Hence $y\left(\frac{-1}{\sqrt{2}}\right) = \sin\left(\cos^{-1}\frac{-1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

54. $\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ is equal to:

(A) $\frac{1}{e}$
(B) e
(C) e^2
(D) $\frac{1}{e^2}$

Ans. D

Sol. Let $\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}} = e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left\{\frac{3x^2+2}{7x^2+2}-1\right\}} = e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left\{\frac{-4x^2}{7x^2+2}\right\}} = e^{\frac{-4}{2}} = e^{-2}$

55. Let two points be A (1, –1) and B (0, 2). If a point $P(x', y')$ be such that the area of $\Delta PAB = 5$ sq. units and it lies on the line, $3x + y - 4\lambda = 0$, then a value of λ is:

(A) 3
(B) 4
(C) 1
(D) –3

Ans. A

Sol. $D = \frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix}$

$-2(1-x') + (y'+x') = \pm 10$

$-2 + 2x' + y' + x' = \pm 10$

$3x' + y' = 12$ or $3x' + y' = -8$

$\lambda = 3, -2$

56. If the equation, $x^2 + bx + 45 = 0 (b \in R)$ has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then:

(A) $b^2 + b = 12$
(B) $b^2 - b = 42$

(C) $b^2 - b = 30$ (D) $b^2 + b = 72$

Ans. C

Sol. Let $z = \alpha \pm i\beta$ be roots of the equation

So $2\alpha = -b$ and $\alpha^2 + \beta^2 = 45, (\alpha+1)^2 + \beta^2 = 40$

So $(\alpha+1)^2 - \alpha^2 = -5$

$\Rightarrow 2\alpha + 1 = -5$

$\Rightarrow 2\alpha = -6$

So b = 6

Hence $b^2 - b = 30$

57. For $a > 0$, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b(0 < b < c)$ intersect the chord OP and the x- axis at points Q and R, respectively. If the line $x = b$ bisects the area bounded by the curves, C_1 and C_2, and the area of $\Delta OQR = \frac{1}{2}$, then 'a' satisfies the equation:

(A) $x^6 - 12x^3 - 4 = 0$ (B) $x^6 - 12x^3 + 4 = 0$

(C) $x^6 + 6x^3 - 4 = 0$ (D) $x^6 - 6x^3 + 4 = 0$

Ans. B

Sol. $\int_0^b \left(\sqrt{ax} - \frac{x^2}{a}\right)dx = \frac{a^2}{6}$

$\Rightarrow \frac{2}{3}\sqrt{a}.b^{\frac{3}{2}} - \frac{b^3}{3a} = \frac{a^2}{6}$ ....(i)

also area of $\Delta OQR = \frac{1}{2}$

$\frac{1}{2}b^2 = \frac{1}{2} \quad \Rightarrow \quad b = 1$

Put in (i)

$\Rightarrow 4a\sqrt{a} - 2 = a^3$

$\Rightarrow a^6 + 4a^3 + 4 = 16a^3$

$\Rightarrow a^6 - 12a^3 + 4 = 0$

58. The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is:

(A) $\frac{7}{2}\sqrt{30}$ (B) $3\sqrt{30}$

(C) 3 (D) $2\sqrt{30}$

Ans. B

Sol. $\vec{w} = \hat{i} + \hat{j} - 3\hat{k}$

$\vec{p} = 3\hat{i} - \hat{j} + \hat{k}$

$\vec{q} = -3\hat{i} + 2\hat{j} + 4\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} i & j & k \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

S.D. $= \dfrac{\left|\overrightarrow{AD}\left(\vec{p}\times\vec{q}\right)\right|}{\left|\vec{p}\times\vec{q}\right|} = \dfrac{|36+225+9|}{\sqrt{36+225+9}} = 3\sqrt{30}$

59. Which one of the following is a tautology?

(A) $P \wedge (P \vee Q)$ (B) $P \vee (P \wedge Q)$

(C) $(P \wedge (P \to Q)) \to Q$ (D) $Q \to (P \wedge (P \to Q))$

Ans. C

Sol.

P	Q	$P \to Q$	$P \wedge (P \to Q)$	$(P \wedge (P \to Q)) \to Q$	$Q \to (P \wedge (P \to Q))$	$P \wedge Q$	$P \vee (P \wedge Q)$	$P \vee Q$	$P \wedge (P \wedge Q)$
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T	T	T
F	T	T	F	T	F	F	F	T	F
F	F	T	F	T	T	F	F	F	F

60. For which of the following ordered pairs (μ, δ), the system of linear equations

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$ is inconsistent?

$4x + 4y + 4z = \delta$

(A) (3, 4) (B) (1, 0)

(C) (4, 3) (D) (4, 6)

Ans. C

Sol. Note $D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix} \left(R_3 \to R_3 - 2R_1 + 3R_2\right)$

$= \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$

Now let $P_3 = 4x + 4y + 4z - \delta = 0$. If the system has solutions it will have infinite solution, so

$P_3 = \alpha P_1 + \beta P_2$

Hence $3\alpha + \beta = 4$ and $4\alpha + 2\beta = 4 \Rightarrow \alpha = 2$ and $\beta = -2$

So for infinite solution $2\mu - 2 = \delta \Rightarrow$ for $2\mu \neq \delta + 2$

System inconsistent

61. If $\int \dfrac{\cos x\, dx}{\sin^3 x \left(1+\sin^6 x\right)^2 3} = f(x)\left(1+\sin^6 x\right)^{1/\lambda} + c$ where c is a constant of integration, then $\lambda f\left(\dfrac{\pi}{3}\right)$ is equal to:

(A) –2 (B) $-\dfrac{9}{8}$

(C) $\dfrac{9}{8}$ (D) 2

Ans. A

Sol. $\sin x = t$

$\cos x\, dx = dx$

$$I = \int \frac{dt}{t^3\left(1+t^6\right)^{\frac{2}{3}}} = \int \frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

Put $1+\frac{1}{t^6} = t^3 \Rightarrow \frac{dt}{t^7} = \frac{-1}{2}r^2\, dr$

$$-\frac{1}{2}\int \frac{r^2\, dr}{r^2} = -\frac{1}{2}r + c$$

$$= -\frac{1}{2}\left(\frac{\sin^6 x + 1}{\sin^6 x}\right)^{\frac{1}{3}} + c$$

$$= -\frac{1}{2\sin^2 x}\left(1+\sin^6 x\right)^{\frac{1}{3}} + c$$

$f(x) = -\frac{1}{2}\operatorname{cosec}^2 x$ and $\lambda = 3$

$\Rightarrow \lambda\, f\left(\frac{\pi}{3}\right) = -2$

62. The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}, x \in (-1,1)$, is

(A) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$
(B) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$
(C) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$
(D) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

Ans.

Sol. $y = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$

$\frac{1+y}{1-y} = \frac{8^{2x}}{8^{-2x}}$

$8^{4x} = \frac{1+y}{1-y}$

$4x = \log_8\left(\frac{1+x}{1-y}\right)$

$x = \frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)$

$f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$

63. Let A and B be two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then, which of the following is TRUE?

(A) $P\left(\frac{A}{B}\right) = \frac{2}{3}$
(B) $P\left(\frac{A'}{B'}\right) = \frac{1}{3}$

(C) $P\left(\frac{A}{B'}\right)=\frac{1}{3}$ (D) $P\left(\frac{A}{(A\cup B)}\right)=\frac{1}{4}$

Ans. C

Sol. A and B are independent events so $P\left(\frac{A}{B'}\right)=\frac{1}{3}$

64. The locus of a point which divides the line segment joining the point (0, –1) and a point on the parabola, $x^2=4y$, internally in the ratio 1 : 2, is

(A) $9x^2-12y=8$ (B) $4x^2-3y=2$

(C) $x^2-3y=2$ (D) $9x^2-3y=2$

Ans.

Sol. Let point P be $\left(2t,t^2\right)$ and Q be (h, k).

$h=\frac{2t}{3},k=\frac{-2+t^2}{3}$

Hence locus is $3k+2=\left(\frac{3h}{2}\right)^2\Rightarrow 9x^2=12y+8$

65. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where p = 0 and $q\neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to:

(A) –5 (B) 10

(C) –10 (D) –20

Ans. D

Sol. If each observation is multiplied with p and then q is subtracted

New mean $\overline{x}_1=p\overline{x}-q$

$\Rightarrow 10=p(20)-q$ …….(1)

and new standard deviations

$\sigma_2=|p|\sigma_1\Rightarrow 1=|p|(2)$

$\Rightarrow |p|=\frac{1}{2}\Rightarrow p=\pm\frac{1}{2}$

If $p=\frac{1}{2}$

then $q=0$ (from equation (1))

If $p=-\frac{1}{2}$

$q=-20$

66. If c is a point at which Rolle's theorem holds for the function, $f(x)=\log_e\left(\frac{x^2+\alpha}{7x}\right)$ in the interval $[3,4]$, where $\alpha\in R$, then $f''(c)$ is equal to:

(A) $-\frac{1}{12}$ (B) $-\frac{1}{24}$

(C) $\frac{\sqrt{3}}{7}$ (D) $\frac{1}{12}$

Ans. D

Sol. $f(3)=f(4)\Rightarrow \alpha=12$

$f'(x)=\dfrac{x^2-12}{x\left(x^2+12\right)}$

$\therefore \quad f'(c)=0$

$\therefore \quad c=\sqrt{12}$

$\therefore \quad f''(c)=\dfrac{1}{12}$

67. If a, b and c are the greatest values of ${}^{19}C_p, {}^{20}C_q$ and ${}^{21}C_r$ respectively, then:

(A) $\dfrac{a}{10}=\dfrac{b}{11}=\dfrac{c}{42}$
(B) $\dfrac{a}{11}=\dfrac{b}{22}=\dfrac{c}{21}$
(C) $\dfrac{a}{11}=\dfrac{b}{22}=\dfrac{c}{42}$
(D) $\dfrac{a}{10}=\dfrac{b}{11}=\dfrac{c}{21}$

Ans. C

Sol. We know, ${}^{n}C_r$ is max at middle term

$a={}^{19}C_p={}^{19}C_{10}={}^{19}C_{19}$

$b={}^{20}C_q={}^{20}C_{10}$

$c={}^{21}C_6={}^{21}C_{10}={}^{21}C_{11}$

$$\frac{a}{{}^{19}C_9}=\frac{b}{\frac{20}{10}.{}^{19}C_9}=\frac{c}{\frac{21}{11}.\frac{20}{10}{}^{19}C_9}$$

$$\frac{a}{1}=\frac{b}{2}=\frac{c}{42/11}$$

$$\frac{a}{11}=\frac{b}{22}=\frac{c}{42}$$

68. Let $f:R\rightarrow R$ be such that for all $x\in R\left(2^{1+x}+2^{1-x}\right), f(x)$ and $\left(3^x+3^{-x}\right)$ are in A.P., then the minimum value of $f(x)$ is:

(A) 3
(B) 4
(C) 2
(D) 0

Ans. A

Sol. $f(x)=\left(\dfrac{2^{1-x}+2^{1+x}+3^x+3^{-x}}{2}\right)$

Using $AM\geq GM$

$f(x)\geq 3$

69. Let the line $y=mx$ and the ellipse $2x^2+y^2=1$ intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co – ordinate axes at $\left(-\dfrac{1}{3\sqrt{2}},0\right)$ and $(0,\beta)$ then β is equal to:

(A) $\dfrac{2\sqrt{2}}{3}$
(B) $\dfrac{2}{3}$

(C) $\frac{2}{\sqrt{3}}$ (D) $\frac{\sqrt{2}}{3}$

Ans. D

Sol. Let P be $\left(x_1, y_1\right)$

Equation of normal at P is $\frac{x}{2x_1} - \frac{y}{y_1} = -\frac{1}{2}$

It passes through $\left(-\frac{1}{3\sqrt{2}}, 0\right) \Rightarrow \frac{-1}{6\sqrt{2}x_1} = -\frac{1}{2} \Rightarrow x_1 = \frac{1}{3\sqrt{2}}$

So $y_1 = \frac{2\sqrt{2}}{3}$ (as P lies in 1st Quadrant)

So $\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$

70. Let $f(x) = \left(\sin\left(\tan^{-1}x\right) + \sin\left(\cot^{-1}x\right)\right)^2 - 1, |x| > 1$. If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1}\left(f(x)\right)\right)$ and $y\left(\sqrt{3}\right) = \frac{\pi}{6}$, then $y\left(-\sqrt{3}\right)$ is equal to

(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$ (D) $-\frac{\pi}{6}$

Ans. A

Sol. Let $\tan^{-1}x = \theta$

$\Rightarrow \quad = \tan\theta \Rightarrow \sin\theta = \frac{x}{\sqrt{1+x^2}}$

$y = \left(\frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right)^2 - 1 \Rightarrow y = \frac{(x+1)^2}{1+x^2} - 1$

$\Rightarrow y = \frac{2x}{1+x^2} = f(x)$

Now $\frac{dy}{dx} = \frac{1}{2}\frac{1}{\sqrt{1-f^2}} \times f'(x) = \frac{1}{2}\frac{1}{\sqrt{1-\frac{4x^2}{\left(1+x^2\right)^2}}} f(x) = \frac{\left(1+x^2\right)}{2\left(x^2-1\right)} f'(x)$

$= \frac{1+x^2}{2\left(x^2-1\right)} \times 2\frac{1+x^2-2x^2}{\left(1+x^2\right)^2}$

$y = -\tan^{-1}x + c$

given $y\left(\sqrt{3}\right) = \frac{\pi}{6} \Rightarrow \frac{\pi}{6} = -\frac{\pi}{3} + c \Rightarrow c = \frac{\pi}{2}$

$y = -\tan^{-1}x + \frac{\pi}{2} = \cot^{-1}x$

Now $y\left(-\sqrt{3}\right) = \cot^{-1}x\left(-\sqrt{3}\right) = \frac{5\pi}{6}$

71. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y – axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then $|m|$ is equal to _________.

Ans. 4

Sol. $P \equiv (x_1, y_1)$

$$2yy' - 6x + y' = 0 \Rightarrow y' = \left(\frac{6x_1}{1+2y_1}\right)$$

$$\left(\frac{\frac{3}{2} - y_1}{-x_1}\right) = -\left(\frac{1+2y_1}{6x_1}\right)$$

$$9 - 6y_1 = 1 + 2y_1 \Rightarrow y_1 = 1$$

$$\therefore x_1 = \pm 2$$

$$\therefore \text{ Slope of tangent } = \left(\frac{\pm 12}{3}\right)$$

$$= \pm 4$$

$$\therefore |n| = 4$$

72. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that the most three of them are red is ________

Ans. 490

Sol. 0 Red, 1 Red, 2 Red, 3 Red

Number of ways $= {}^7C_4 + {}^5C_1 . {}^7C_3 + {}^5C_2 . {}^7C_2 + {}^5C_3 \, {}^7C_1 = 35 + 175 + 210 + 70 = 490$

73. The least positive value of 'a' for which the equation, $2x^2 + (a-10)x + \frac{33}{2} = 2a$ has real root is ________

Ans. 8

Sol. $D \geq 0$

$$(a-10)^2 - 4(2)\left(\frac{33}{2} - 2a\right) \geq 0$$

$$(a-10)^2 - 4(33 - 4a) \geq 0$$

$$a^2 - 4a - 32 \geq 0 \Rightarrow a \in (-\infty, -4] \cup [8, \infty)$$

74. The sum $\sum_{k=1}^{20}(1+2+3+....+k)$ is ________________

Ans. 1540

Sol. $= \sum_{k=1}^{50}\frac{k(k+1)}{2}$

$$= \frac{1}{2}\sum_{k=1}^{20}\left(k^2 + k\right)$$

$$= \frac{1}{2}\left[\frac{20(21)(41)}{6} + \frac{20(21)}{2}\right]$$

$$= \frac{1}{2}\left[\frac{420 \times 41}{6} + \frac{20 \times 21}{2}\right]$$
$$= \frac{1}{2}[2870 + 210]$$
= 1540

75. The number of all 3×3 matrices, A, with enteries from the set $\{-1, 0, 1\}$ such that the sum of the diagonal elements of AA^T is 3, is ______________.

Ans. 672

Sol. Let $A = \left[a_{ij}\right]_{3\times3}$

$$\text{tr}\left(AA^T\right) = 3$$
$$a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + \ldots\ldots.. + a_{33}^2 = 3$$

Possible cases

$$\left.\begin{array}{ll} 0,0,0,0,0,0,1,1,1 & \rightarrow 1 \\ 0,0,0,0,0,0,-1,-1,-1 & \rightarrow 1 \\ 0,0,0,0,0,0,1,1,-1 & \rightarrow 3 \\ 0,0,0,0,0,0,-1,1,-1 & \rightarrow 3 \end{array}\right\} {}^9C_6 \times 8 = 84 \times 8 = 672$$