# FIITJEE

# Solutions to JEE (Main)-2020

**JEE–Main–2020 –Jan–7–Second–Shift**
**PHYSICS, CHEMISTRY & MATHEMATICS**

## PART –A (PHYSICS)

1. An emf of 20 V is applied at time t = 0 to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time t = ∞ and at t = 40 s is close to: (take $e^2 = 7.389$)
(A) 1.06 (B) 0.84
(C) 1.46 (D) 1.15

Ans. **B**

Sol. $i = \frac{V}{r}\left(1 - e^{-Rt/L}\right)$

$= 4(1 - e^{-500\,t})$

At t = ∞

$i_1 = 4A$

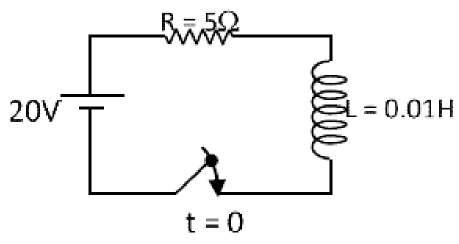
at t = 40s

$i_2 = 4(1 - e^{-20000})$

$= 4\left[1 - \frac{1}{(e^2)^{10000}}\right]$

$= 4\left[1 - \frac{1}{(7.389)^{10000}}\right]$

$\frac{i_1}{i_2}$ is slightly greater than 1.




2. A particle of mass m and charge q has an initial velocity $\vec{v} = v_0\hat{j}$. If an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{i}$ act on the particle, its speed will double after a time:
(A) $\frac{2mv_0}{qE_0}$ (B) $\frac{\sqrt{2}mv_0}{qE_0}$ (C) $\frac{\sqrt{3}mv_0}{qE_0}$ (D) $\frac{3mv_0}{qE_0}$

Ans. **C**

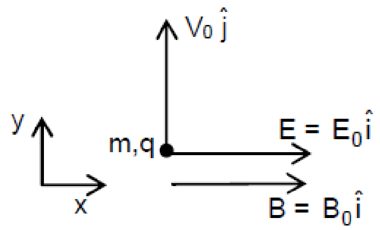
Sol. Since magnetic force cannot change the speed. So only electric field which is along x-direction will change the speed along x-direction only.

$v_x = \frac{E_o q}{m}t$ but $v_y = v_0$

$2v_0 = \sqrt{v_x^2 + v_y^2}$

$4v_0^2 = \frac{E_o^2 q^2 t^2}{m^2} + V_0^2$

$t = \frac{\sqrt{3}\,mv_o}{qE_o}$



3. In a building there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be

(A) 10 A (B) 20 A (C) 25 A (D) 15 A

Ans. **B**

Sol. Total power is (15 × 45) + (15 × 100) + (15 × 10) + (2 × 1000) = 4325 W.

So, current is $\frac{4325}{220}$ = 19.66 A

Answer is 20 Amp.

4. An electron (of mass m) and a photon have the same energy E in the range of a few eV. The ratio of the de-Broglie wavelength associated with the electron and the wavelength of the photon is (c = speed of light in vacuum)

(A) $c\,(2mE)^{1/2}$ (B) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$ (C) $\frac{1}{c}\left(\frac{2E}{m}\right)^{1/2}$ (D) $\left(\frac{E}{2m}\right)^{1/2}$

Ans. **B**

Sol. $\lambda_{electron} = \frac{h}{\sqrt{2mE}}$ ; $\frac{\lambda_{electron}}{\lambda_{photon}} = \frac{1}{c}\sqrt{\frac{E}{2m}}$

$\lambda_{photon} = \frac{hc}{E}$

5. A thin lens made of glass (refractive index = 1.5) of focal length f = 16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is $f_\ell$, then the ratio $f_\ell/f$ is closest to the integer:

(A) 17 (B) 1 (C) 9 (D) 5

Ans. **C**

Sol. $\frac{1}{f} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ …(i)

$\frac{1}{f_\ell} = \left(\frac{\mu_g}{\mu_\ell} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ …(ii)

$\frac{(i)}{(ii)}$ $\frac{f_\ell}{f} = \frac{\mu_g - 1}{\frac{\mu_g}{\mu_\ell} - 1} = \frac{1.5 - 1}{\frac{1.5}{1.41} - 1}$ = 8.875 ≈ 9.

6. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much les than the speed of sound). The observers hears 2 beats/sec. The oscillation frequency of each tuning fork is $\nu_0$ = 1400 Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to:

(A) $\frac{1}{4}$ m/s (B) 1 m/s (C) $\frac{1}{2}$ m/s (D) $\frac{1}{8}$ m/s

Ans. **A**

Sol. $f_1 = f_o\left(\frac{v}{v - v_s}\right)$ $f_1 - f_2 = 2$

$f_2 = f_o\left(\frac{v}{v + v_s}\right)$ $f_0 v\left[\frac{v + v_s - (v - v_s)}{v^2 - v_s^2}\right] = 2$ ; $\frac{2f_0 v\, v_s}{v^2 - v_s^2} = 2$

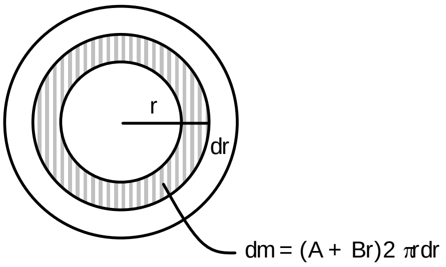
$\because$ $v_s << v$ $\therefore$ $v_s = \frac{1}{4}$ m/s

7. Mass per unit are of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is:

(A) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$
(B) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$
(C) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$
(D) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

Ans. **B**

Sol.




Moment of inertia of ring

$dI = dmr^2$

$$I=\int_0^a (A+Br)2\pi r dr \pi r^2$$

$$=2\pi A\int_0^a r^3 dr+2\pi B\int_0^a r^4 dr$$

$$=2\pi\left[A\frac{a^4}{4}+B\frac{a^5}{5}\right]$$

$$=2\pi a^4\left[\frac{A}{4}+\frac{Ba}{5}\right]$$

8. A planar loop of wire rotates in a uniform magnetic field. Initially, at t = 0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at:

(A) 2.5 s and 5.0 s
(B) 5.0 s and 10.0 s
(C) 2.5 s and 7.5 s
(D) 5.0 and 7.5 s

Ans. **A**

Sol. At any time t

$$\phi = BA\cos\frac{\pi t}{5} \qquad \left(\because \quad \omega=\frac{2\pi}{T}=\frac{2\pi}{10}=\frac{\pi}{5}\right)$$

$$\frac{d\phi}{dt}=-\frac{\pi}{5}BA\sin\left(\frac{\pi t}{5}\right)$$

$$e=\frac{\pi}{5}BA\sin\left(\frac{\pi t}{5}\right)$$

e will be maximum when $\frac{\pi t}{5}$ is $\frac{\pi}{2}$

t = 2.5 sec.

e will be minimum when $\frac{\pi t}{5}$ is $\pi, 0$

t = 0, 5 sec

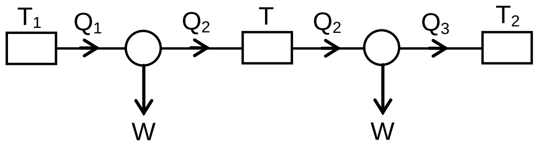
So, answer will be (1).

9. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, $T_1$ and $T_2$. The temperature of the hot reservoir of the first engine is $T_1$ and the temperature of the cold reservoir of the second engine is $T_2$. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to $T_1$ and $T_2$, if both the engines perform equal amount of work?

(A) $T=\sqrt{T_1T_2}$
(B) $T=\frac{T_1+T_2}{2}$
(C) $T=\frac{2T_1T_2}{T_1+T_2}$
(D) T = 0

Ans. **B**

Sol.




$W = Q_1 - Q_2 = Q_2 - Q_3$

$Q_2 = \frac{Q_1 + Q_3}{2}$

$2 = \frac{Q_1}{Q_2} + \frac{Q_3}{Q_2}$

$2 = \frac{T_1}{T} + \frac{T_2}{T}$

$T = \frac{T_1 + T_2}{2}$

10. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10\ ms^{-2}$ at the north pole and the radius of the earth = 6400 km)

(A) 194.66 N (B) 194.32 N
(C) 195.32 N (D) 195.66 N

Ans. **C**

Sol. $W_{equator} = W_{pole} - m\omega^2 R$

$= 196 - 19.6 \times \left(\frac{2\pi}{24 \times 60 \times 60}\right)^2 \times 6400 \times 10^3$

$= 195.33$ N

11. The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and $\mu_0$ is the magnetic permeability of vacuum, is

(A) $ML^2T^{-1}$ (B) $ML^2T^{-2}$
(C) $ML^{-1}T^{-2}$ (D) $MLT^{-2}$

Ans. **C**

Sol. Magnetic energy per unit volume = $\frac{B^2}{2\mu_0}$

$\frac{\text{Energy}}{\text{Volume}} = \frac{ML^2T^{-2}}{L^3} = (ML^{-1}T^{-2})$

12. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is:

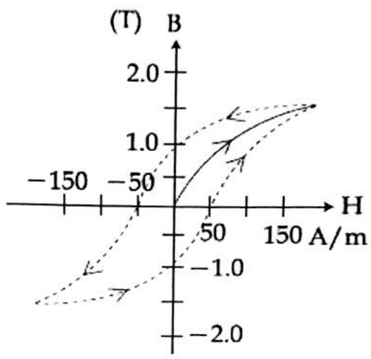
(A) 4.9 mm (B) 6.9 mm
(C) 3.9 mm (D) 5.9 mm

Ans. **D**

Sol. The distance between two successive bright fringes is fringe width ($\beta$).

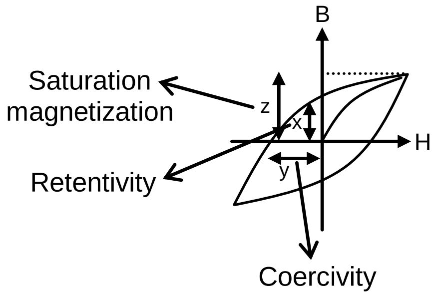
$\beta = \frac{\lambda D}{d} = \frac{589 \times 10^{-9} \times 1.5}{0.15 \times 10^{-3}}$ = 5.9 mm

13. The figure gives experimentally measured B vs H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are
(A) 1.0 T, 50 A / m and 1.5 T
(B) 1.5 T, 50 A / m and 1.0 T
(C) 150 A / m, 1.0 T and 1.5 T
(D) 1.5 T, 50 A / m and 1.0 T




Ans. **A**

Sol.




x = retentivity
y = coercivity
z = saturation magnetization

14. The activity of a radioactive sample falls from 700 $s^{-1}$ to 500 $s^{-1}$ in 30 minutes. Its half life is close to

(A) 66 min (B) 52 min (C) 62 min (D) 72 min

Ans. **C**

Sol. $A = A_o e^{-\lambda t}$

$500 = 700 e^{-\lambda t}$

$$\lambda t = \ell n \frac{7}{5}$$

$$\frac{\ell n 2}{t_{1/2}} \times 30 = \ell n \frac{7}{5}$$

$$\because \quad t_{1/2} = \frac{\ell n 2 \times 30}{\ell n \frac{7}{5}}$$

$t_{1/2}$ = 61.8 min ≈ 62 min.

15. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is

(A) $\frac{3}{4}$ (B) $\frac{81}{256}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{9}{16}$

Ans. **D**

Sol. Using equation of continuity

$A_1V_1 = A_2V_2$

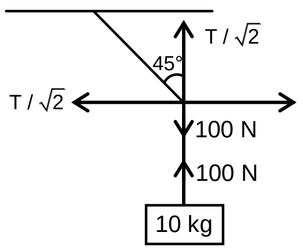
$$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{4.8}{6.4}\right)^2 = \frac{9}{16}$$

16. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals: (Take g = 10 $ms^{-2}$ and the rope to be massless)

(A) 100 N (B) 75 N (C) 70 N (D) 90 N

Ans. **A**

Sol.




$\frac{T}{\sqrt{2}} = 100$ ; $\frac{T}{\sqrt{2}} = F$

F = 100 N

17. The electric field of a plane electromagnetic wave is given by

$$\vec{E} = E_0 \frac{\hat{i}+\hat{j}}{\sqrt{2}} \cos(kz + \omega t)$$

At t = 0, a positively charged particle is at the point (x, y, z) = $\left(0, 0, \frac{\pi}{k}\right)$. If its instantaneous velocity at (t = 0) is $v_0\hat{k}$, the force acting on it due to the wave is:

(A) zero (B) parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(C) parallel to $\hat{k}$ (D) antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

Ans. **D**

Sol. Electric field at t = 0 & (x, y, z) = $\left(0, 0, \frac{\pi}{k}\right)$ is

$\vec{E} = -\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)E_0$ and $\vec{F} = \vec{E}q$

$\therefore \quad \vec{F} = -\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)q$

Which is antiparallel to $\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

18. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from $\tau_1$ to $\tau_2$. If $\frac{C_p}{C_v} = \gamma$ for the gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by

(A) 2 (B) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$ (C) $\left(\frac{1}{2}\right)^{\gamma}$ (D) $\frac{1}{2}$

Ans. **Bonus**

Sol. Relaxation time $(\tau) \propto \frac{V}{\sqrt{T}}$

and $\quad T \propto \dfrac{1}{V^{\gamma-1}}$

$\tau \propto V^{1+\frac{\gamma-1}{2}} \quad ; \quad \tau \propto V^{\frac{1+\gamma}{2}}$

$\dfrac{\tau_f}{\tau_i} = \left(\dfrac{2V}{V}\right)^{\frac{1+\gamma}{2}} \quad ; \quad \dfrac{\tau_f}{\tau_i} = (2)^{\frac{1+\gamma}{2}}$

19. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. the frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10\ m/s^2$) must be at least:

(A) 56300 W (B) 66000 W (C) 62360 W (D) 48000 W

Ans. **B**

Sol. Net force on motor will be

$F_{min} = [920 + 68(10)]g + 6000$

$= 22000$ N

So, required power for motor

$P_{min} = \vec{F}_{min} \cdot \vec{v}$

$= 22000 \times 3$

$= 66000$ watt

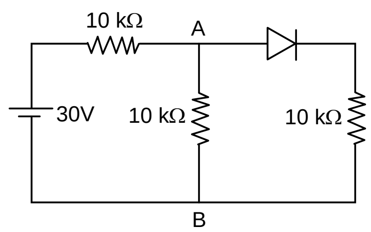
20. In the figure, potential difference between A and B is:

(A) 5 V

(B) 10 V

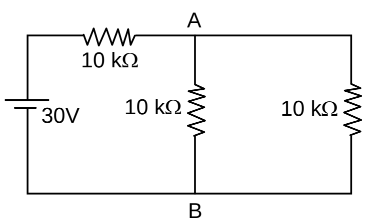
(C) zero

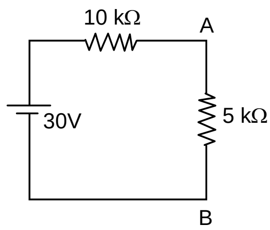
(D) 15 V




Ans. **B**

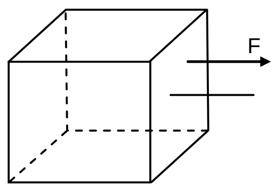
Sol. In forward bias diode act as a short circuit wire. Hence, the equivalent circuit is now.




So, $V_{ab} = \dfrac{30}{5+10} \times 5 = 10$ V.

21. Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \dfrac{b}{a}$ for box not to topple before moving is _________.




Ans. **50.00**

Sol. For no toppling

$F\left(\dfrac{a}{2} + b\right) \le mg\dfrac{a}{2}$

$\mu\frac{a}{2} + \mu b \le \frac{a}{2}$

$0.2\ a + 0.4\ b \le 0.5a$

$0.4b \le 0.3a$

$b \le \frac{3a}{4}$

$b \le 0.75a$ (in limiting case)

But is not possible as maximum value of b can be equal to 0.5a only.

$\therefore \left(100\frac{b}{a}\right)_{max} = 50.00$

22. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/g and heat of fusion of ice is 80 cal/g], the value of M is _________.

Ans. **40**

Sol. $M_{ice} L_f + m_{ice} (40 - 0) C_w = m_{steam} L_v + m_{steam} (100 - 40) C_w$

$\Rightarrow$ 200[80 + 40(1)] = M[540 + 60(1)]

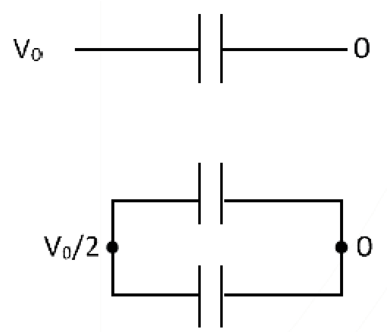
$\Rightarrow$ 200(120) = M(600)

M = 40 gm.

23. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) _________.

Ans. **6**

Sol. Common potential after connection.




$$V_{common} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{60 \times 20 + 0}{120} = 10 \text{ Volt}$$

Loss of energy = $\frac{1}{2}CV^2 - \frac{1}{2}(2C) \times V_{Common}^2$

$= \frac{1}{2} \times 60 \times 10^{-12} \times (20)^2 - 60 \times 10^{-12} \times (10)^2$

$= 60 \times 10^{-12} (200 - 100)$

$= 6000 \times 10^{-12}$

= 6 nJ

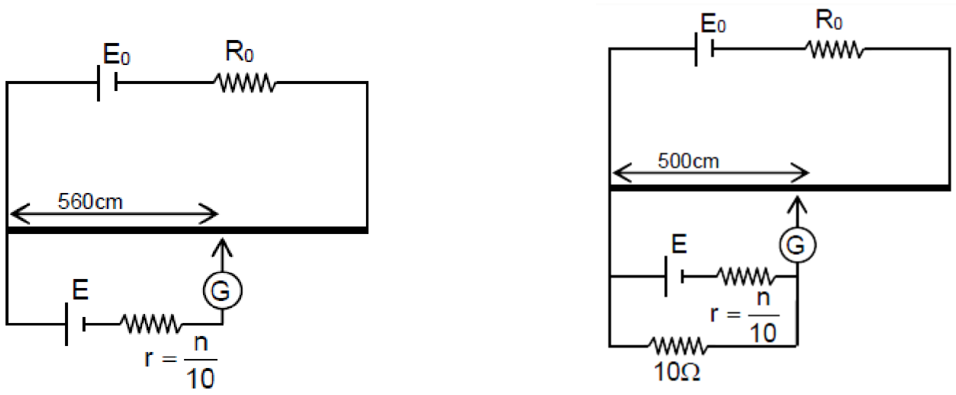
24. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of 10 Ω is connected in parallel to the cell, the balancing length changes by 60 cm. If the internal resistance of the cell is $\frac{N}{10}\Omega$, where N is an integer then value of N is _________.

Ans. **12**

Sol. $r_{in} = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right)R_{ext} = \frac{60}{500} \times 10$

$r = \frac{6}{5} = 1.2\ \Omega$

n = 12




25. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle $\theta$ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is, _________.

Ans. **90°**

Sol. $|\vec{P} + \vec{Q}| = |\vec{P}|$

$P^2 + Q^2 + 2PQ\cos\theta = P^2$

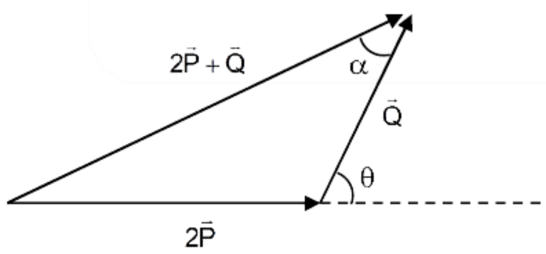
$\Rightarrow \quad Q + 2P\cos\theta = 0$

$\Rightarrow \quad \cos\theta = -\frac{Q}{2P}$

$\tan\alpha = \frac{2P\sin\theta}{2P\cos\theta + Q} = \infty$

$\because \quad [2P\cos\theta + Q = 0]$

$\alpha = 90°$



# PART –B (CHEMISTRY)

26. In the following reactions, products(A) and (B) respectively are

$$\underset{\text{(Hot and Conc)}}{NaOH + Cl_2} \longrightarrow (A) + \text{side products}$$

$$\underset{\text{(Dry)}}{Ca(OH)_2}\, Cl_2 \longrightarrow (B) + \text{side products}$$

(A) $NaClO_3$ and $Ca(OCl)_2$
(B) $NaOCl$ and $Ca(OCl)_2$
(C) $NaOCl$ and $Ca(ClO_3)_2$
(D) $NaClO_3$ and $Ca(ClO_3)_2$

Ans. A

Sol. $6NaOH + 3Cl_2 \longrightarrow 5NaCl + NaClO_3 + 3H_2O$

$2Ca(OH)_2 + Cl_2 \longrightarrow Ca(OCl)_2 + CaCl_2 + H_2O$

27. The refining method used when the metal and the impurities have low and high melting temperatures, respectively is
(A) zone refining
(B) vapour phase refining
(C) liquation
(D) distillation

Ans. C

Sol. Concept based.

28. The redox reaction among the following is
(A) reaction of $[Co(H_2O)_6]Cl_3$ with $AgNO_3$
(B) combination of dinitrogen with dioxygen at 2000 K
(C) reaction of $H_2SO_4$ with NaOH
(D) formation of ozone from atmospheric oxygen in the presence of sunlight

Ans. B

Sol. $N_2 + O_2 \longrightarrow 2NO$

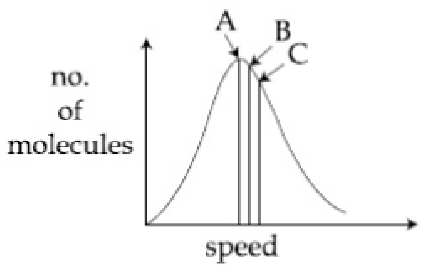
Oxidation state changes.

29. Within each pair of elements F & Cl, S & Se and Li & Na, respectively, the elements that release more energy upon an electron gain are
(A) F, Se and Na
(B) Cl, Se and Na
(C) F, S and Li
(D) Cl, S and Li

Ans. D

Sol. Cl > F (exception)
S > Se (size effect)
Li > Na (size effect)

30. Identify the correct labels of A, B and C in the following graph from the options given below




Root mean square speed($V_{rms}$); most probable speed($V_{mp}$); Average speed($V_{av}$)
(A) A-$V_{av}$, B-$V_{rms}$, C-$V_{mp}$
(B) A-$V_{rms}$, B-$V_{mp}$, C-$V_{av}$
(C) A-$V_{mp}$, B-$V_{rms}$, C-$V_{av}$
(D) A-$V_{mp}$, B-$V_{av}$, C-$V_{rms}$

Ans. D

Sol. $V_{rms} > V_{av} > V_{mp}$

31. For the reaction

$2H_2(g) + 2NO(g) \longrightarrow N_2(g) + 2H_2O$, the observed rate expression is, rate = $k_f[NO]^2[H_2]$.

The rate expression for the reverse reaction is:

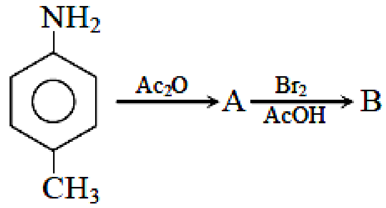
(A) $k_b[N_2][H_2O]$
(B) $k_b[N_2][H_2O]^2/[NO]$
(C) $k_b[N_2][H_2O]^2/[H_2]$
(D) $k_b[N_2][H_2O]^2$

Ans. C

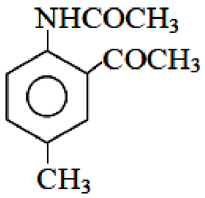
Sol. Al equilibrium

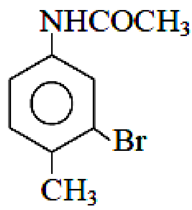
$$K_f[H_2][NO]^2 = \frac{K_b[N_2][H_2O]^+}{[H_2]}$$

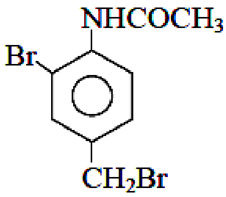
32. In the following reaction sequence

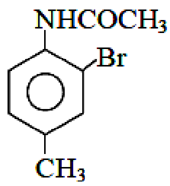



The major product B is

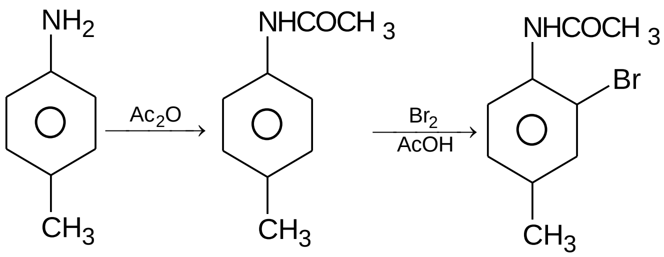
(A)



(B)



(C)



(D)


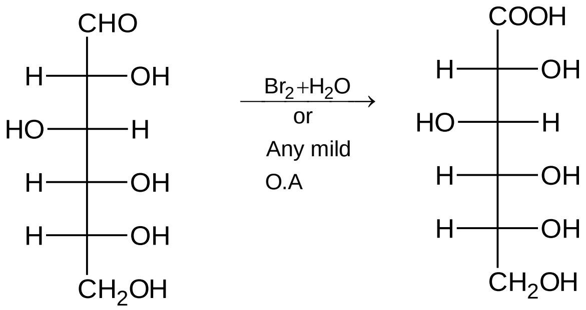

Ans. D

Sol.



33. Which of the following statements is correct?

(A) Gluconic acid is a dicarboxylic acid
(B) Gluconic acid can form cyclic(acetal/hemiacetal) structure
(C) Gluconic acid is a partial oxidation product of glucose
(D) Gluconic acid is obtained by oxidation of glucose with $HNO_3$

Ans. C

Sol.


34. The equation that is incorrect is

(A) $\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaI} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{NaBr}$
(B) $\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaCl} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{KCl}$
(C) $\left(\Lambda_m^0\right)_{H_2O} = \left(\Lambda_m^0\right)_{HCl} + \left(\Lambda_m^0\right)_{NaOH} - \left(\Lambda_m^0\right)_{NaCl}$
(D) $\left(\Lambda_m^0\right)_{KCl} - \left(\Lambda_m^0\right)_{NaCl} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{NaBr}$

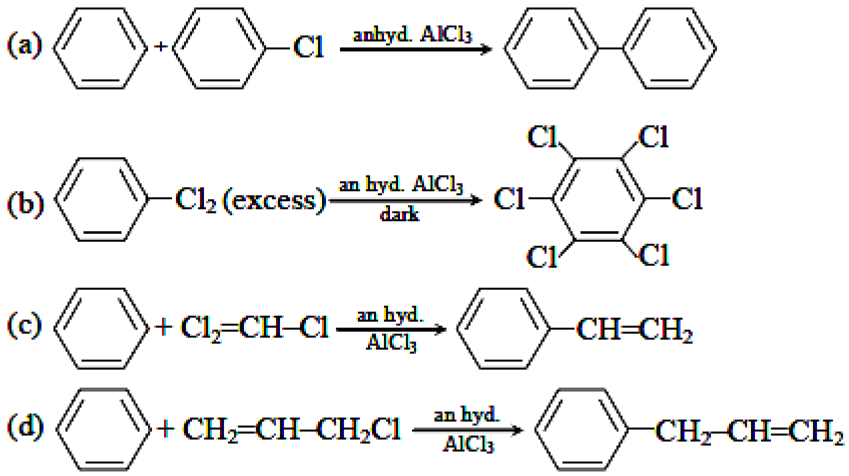
Ans. A

Sol. Each ion makes a definite contribution irrespective of the other ion

$$\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaI} = \Lambda_{m Br^-}^0 - \Lambda_{m I^-}^0$$

$$\left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{NaBr} = \Lambda_{m K^+}^0 - \Lambda_{m Na^+}^0$$

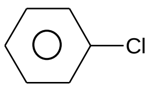
35. Consider the following reactions:




Which of these reactions are possible?

(A) (b) and (d)
(B) (a) and (d)
(C) (a) and (b)
(D) (b), (c) and (d)

Ans. A

Sol. Carbon-chlorine bond cleavage is not possible in  and $CH_3 = CH - Cl$

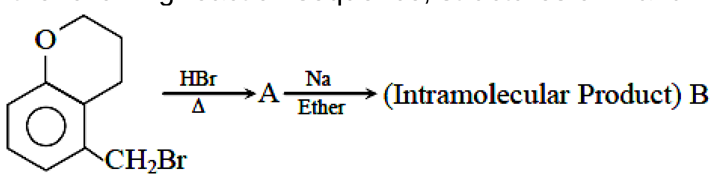

36. The number of possible optical isomers for the complexes $MA_2B_2$ with $sp^3$ and $dsp^2$ hybridized metal atom, respectively is:
[Note: A and B are unidentate neutral and unidentate monoanionnic ligands, respectively]

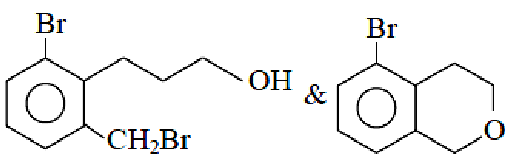
(A) 2 and 2
(B) 0 and 2
(C) 0 and 1
(D) 0 and 0

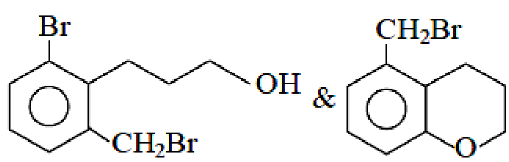
Ans. D

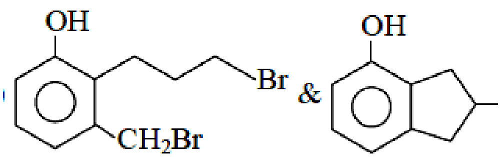
Sol. $MA_2B_2$ shows geometrical and not optical isomerism.

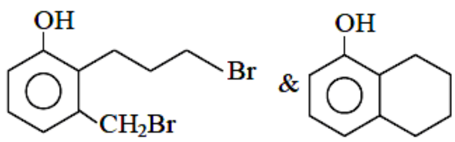
37. In the following reaction sequence, structures of A and B respectively will be



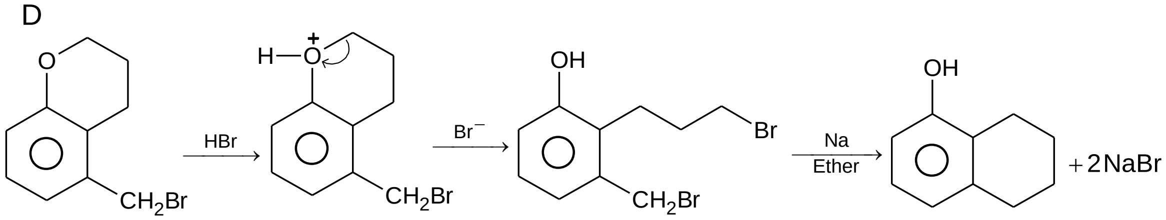

(A) 


(B) 

(C) 


(D) 


Ans. D

Sol. 

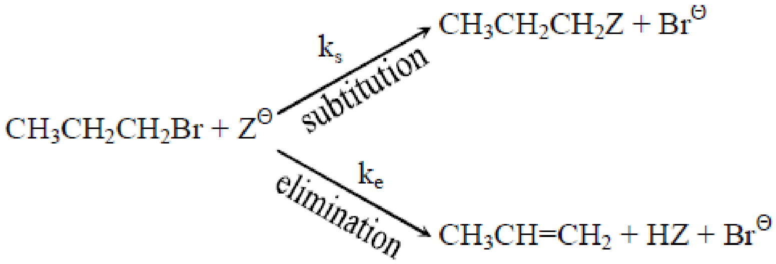

38. The bond order and the magnetic characteristics of $CN^-$ are

(A) 3, paramagnetic (B) 2½, diamagnetic
(C) 3, diamagnetic (D) 2½, paramagnetic

Ans. C

Sol. $CN^- \Rightarrow$ 14 electrons, so B.O = 3 and diamagnetic.

39. For the following reactions:




where,

$Z^{\ominus} = CH_3CH_2O^{\ominus}$ (A) or $H_3C-\underset{CH_3}{\overset{CH_3}{C}}-O^{\ominus}$ (B).

$k_s$ and $k_e$ are respectively, the rate constant for substitution and elimination, and $\mu = \frac{k_s}{k_e}$, the correct option is______________

(A) $\mu_B > \mu_A$ and $k_e(A) > k_e(B)$ (B) $\mu_A > \mu_B$ and $k_e(A) > k_e(B)$
(C) $\mu_A > \mu_B$ and $k_e(B) > k_e(A)$ (D) $\mu_B > \mu_A$ and $k_e(B) > k_e(A)$

Ans. C

Sol. $H_3C-\underset{CH_3}{\overset{CH_3}{C}}-O^-$

Is bulky base, so elimination is dominating.

40. Among the statements (a - d), the incorrect ones are:

(a) Octahedral Co(III) complexes with strong field ligands have very high magnetic moments
(b) When $\Delta_0 < P$, the d-electron configuration of Co(III) in an octahedral complex is $t_{eg}^4 e_g^2$
(c) Wavelength of light absorbed by $[Co(en)_3]^{3+}$ is lower than that of $[CoF_6]^{3-}$
(d) If the $\Delta_0$ for an octahedral complex of Co(III) is 18,000 $cm^{-1}$, the $\Delta_t$ for its tetrahedral complex with the same ligand will be 16,000 $cm^{-1}$

(A) (b) and (c) only (B) (a) and (b) only
(C) (c) and (d) only (D) (a) and (d) only

Ans. D

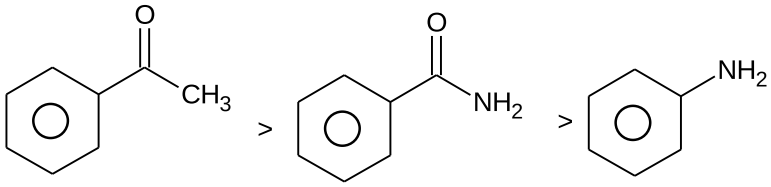
Sol. In strong ligand field $Co^{3+}$ will have $t_{2g}^{6}e_{g}^{0}$ of configuration and $\Delta t = \frac{4}{9}\Delta_0$

41. A Chromatography column, packed with silica gel as stationary phase, was used to separate a mixture of compounds consisting of (a) benzanilide, (b) aniline and (c) acetophenone. When the column is eluted with a mixture of solvents, hexane; ethyl acetate(20:80), the sequence of obtained compounds is

(A) (b), (c) and (a)
(B) (a), (b) and (c)
(C) (c), (a) and (b)
(D) (b), (a) and (c)

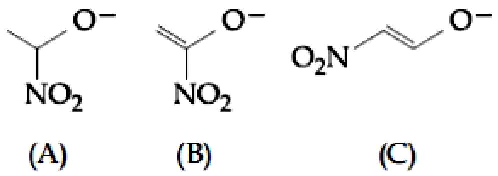
Ans. C

Sol.



Aniline has higher viscosity due to intermolecular H-bonding.

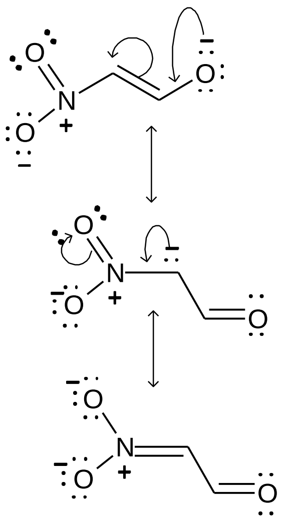
42. The correct order of stability for the following alkoxides is



(A) (B) > (C) > A
(B) (C) > (A) > (B)
(C) (C) > (B) > (A)
(D) (B) > (A) > (C)

Ans. C

Sol.



C has maximum resonating structure than in B

43. Among statements (a) – (d), the correct ones are

(a) decomposition of hydrogen peroxide gives dioxygen.

(b) like hydrogen peroxide, compounds such as $KClO_3$, $Pb(NO_3)_2$ and $NaNO_3$ when heated liberated dioxygen.

(c) 2-Ethylanthraquinone is useful for the industrial preparation of hydrogen peroxide

(d) Hydrogen peroxide is used for the manufacture of sodium perborate

(A) (a), (b), (c) and (d)
(B) (a) and (c) only
(C) (a), (b) and (c) only
(D) (a), (c) and (d) only

Ans. A

Sol. Conceptual.

44. The ammonia($NH_3$) released on quantitative reaction of 0.6 g urea ($NH_2CONH_2$) with sodium hydroxide(NaOH) can be neutralized by
(A) 100 mL of 0.1 N HCl (B) 100 mL of 0.2 N HCl
(C) 200 mL of 0.2 N HCl (D) 200 mL of 0.4 N HCl

Ans. B

Sol. Moles of HCl = moles of $NH_3$

$= 2 \times$ moles of urea $= 2 \times \frac{0.6}{60} = 0.02$

$\Rightarrow$ N.V = 0.02

45. Two open beakers one containing a solvent and the other containing a mixture of that solvent with a non-volatile solute are together sealed in a container. Over time
(A) the volume of the solution increases and the volume of the solvent decreases
(B) the volume of the solution and the solvent does not change
(C) the volume of the solution does not change and the volume of the solvent decreases
(D) the volume of the solution decreases and the volume of the solvent increases

Ans. A

Sol. Vapour pressure over solvent is greater than that over solution.

46. Consider the following reactions:

$$NaCl + K_2Cr_2O_7 + \underset{(Conc.)}{H_2SO_4} \longrightarrow (A) + \text{side products}$$

$$(A) + NaOH \longrightarrow (B) + \text{Side products}$$

$$(B) + \underset{(dilute)}{H_2SO_4} + H_2O_2 \longrightarrow (C) + \text{Side products}$$

The sum of the total number of atoms in one molecule each of (A), (B) and (C) is _____

Ans. 18

Sol. $$NaCl + K_2Cr_2O_7 \xrightarrow{H^+} \underset{(A)}{CrO_2Cl_2} \xrightarrow{NaOH} \underset{(B)}{Na_2CrO_4} \xrightarrow[H^+]{H_2O_2} \underset{(C)}{CrO_5}$$

$CrO_2Cl_2 \rightarrow 5$
$Na_2CrO_4 \rightarrow 7$
$CrO_3 \rightarrow 6$

47. The standard heat of formation $\left(\Delta_f H^0_{298}\right)$ of ethane (in kJ/mol), if the heat of combustion of ethane, hydrogen and graphite are -1560, -393.5 and -286 kJ/mol respectively is _____

Ans. 192.5

Sol. $$2C(gr) + 3H_2(g) \longrightarrow C_2H_6(g)$$

$\Delta H_f = 2 \times \Delta H_{comb}(C) + 3 \times \Delta\sigma H_{comb}(H_2) - \Delta H_{Comb}(C_2H_6)$
$\Rightarrow \sigma H_f = 2(-286) + 3(-393.5) - (-1560) = 192.5$

48. The flocculation value of HCl for arsenic sulphide sol is 30 m mol $L^{-1}$. If $H_2SO_4$ is used for the flocculation of arsenic sulphide, the amount, in grams of $H_2SO_4$ in 250 mL required for the above purpose is __________
(Molecular mass of $H_2SO_4$ = 98 g/mol)

Ans. 0.3675

Sol. 30 m.mol $L^{-1}$ of HCl

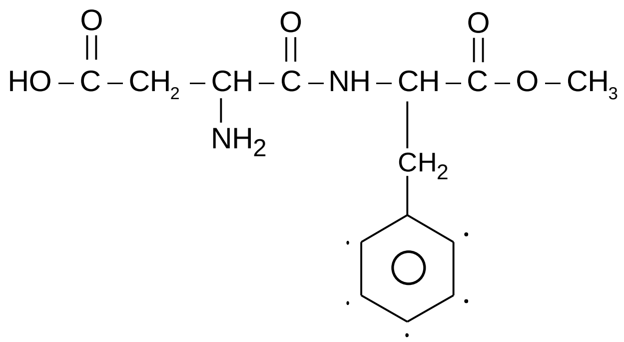
Then in 250 mL HCl will be $\frac{30}{4}$ m.mol

$H_2SO_4$ will be $\frac{1}{2} \times \frac{30}{4} \text{m.mol} = 98 \times \frac{1}{2} \times \frac{30}{4} \times 16^{-3} g = 0.3675$ g

49. The number of $sp^2$-hybridized carbons present in "Aspartame" is_________

Ans. 9

Sol.



The dotted ones are $sp^2$ carbons.

50. 3 g of acetic acid is added to 250 mL of 0.1 M HCl and the solution made up to 500 mL. To 20 mL of this solution ½ mL of 5 M NaOH is added. The pH of the solution is_________
[Given: $pK_a$ of acetic acid = 4.75, molar mass of acetic acid = 60 g/mol, log 3 = 0.4771]
Neglect any changes in volume.

Ans. 5.2271

Sol. 500 mL has HCl = $25 \times 10^{-3}$ mol = 25 m.mol
120 mL has = 1 m.mol

and 500 mL has $CH_3COOH$ = $\frac{1}{20} \times 10^3 \text{m.mol}$

so 20 mL has $\frac{10^3}{20} \times \frac{20}{500} = 2 \text{m.mol}$

NaOH added in 20 mL is $5 \times \frac{1}{2} = 2.5 \text{m.mol}$

So, $\underset{(\text{left})}{NaOH} + CH_3COOH \longrightarrow CH_3COOH + H_2O$

1.5 2 1.5

$\Rightarrow$ pH = $p^{K_a} + \log\frac{\text{salt}}{\text{acid}} = 4.75 + \log 0.4771 = 5.2271$

# PART–C (MATHEMATICS)

51. The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = \left(k^2 - 3\right) \cdot {}^{36}C_{r+1}$, where k is an integer, is:
(A) 3 (B) 6
(C) 4 (D) 2

Ans. C

Sol. $6 \cdot {}^{35}C_r = \left(k^2 - 3\right) \cdot \frac{36}{r+1} {}^{35}C_r$

$\Rightarrow k^2 - 3 = \frac{r+1}{6}, k^2 - 3 > 0$

(i) $k = \pm 2$ gives r = 5
(ii) $k = \pm 3$ gives r = 35
4 ordered pairs

52. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is:
(A) $2\sqrt{2}$ (B) $2\sqrt{7}$
(C) 4 (D) $2\sqrt{5}$

Ans. B

Sol. $y = -\frac{3x}{4} + 3\sqrt{2}$ line is tangent to ellipse

$\therefore c^2 = a^2m^2 + b^2$

$18 = \frac{9a^2}{16} + 9$

$a^2 = 16$

$\therefore e^2 = 1 - \frac{b^2}{a^2}$

$e = \frac{\sqrt{7}}{4}$

Distance between focii = 2ae
$= 2\sqrt{7}$

53. If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ....$ is (102)m, then m is equal to:
(A) 10 (B) 5
(C) 20 (D) 25

Ans. C

Sol. $3 + 4 + 8 + 9 + 13 + 14 + ......$ upto 40 terms
$\Rightarrow 7 + 17 + 27 + ........20$ terms

$S = \frac{20}{2}\left[2 \times 7 + 19 \times 10\right]$

$= 102 \times 20 = 102$ m

$\therefore m = 20$

54. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two $3 \times 3$ real matrices such that $b_{ij} = (3)^{(i+j-2)} a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is:

(A) $\frac{1}{3}$ (B) $\frac{1}{9}$

(C) $\frac{1}{81}$ (D) 3

Ans. B

Sol. $$|B| = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} 3^0 a_{11} & 3^1 a_{21} & 3^2 a_{31} \\ 3^1 a_{12} & 3^2 a_{22} & 3^3 a_{32} \\ 3^2 a_{13} & 3^3 a_{23} & 3^4 a_{33} \end{vmatrix}$$

$$81 = 3^3 . 3^3 . 3^2 |A|$$

$$\Rightarrow |A| = \frac{1}{9}$$

55. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0,1]$ is:

(A) $\frac{\sqrt{7}-2}{3}$ (B) $\frac{4-\sqrt{5}}{3}$

(C) $\frac{4-\sqrt{7}}{3}$ (D) $\frac{2}{3}$

Ans. C

Sol. Using LMVT

$$f'(c) = \frac{f(1) - f(0)}{1-0}$$

$$3c^2 - 8c + 8 = \frac{16-11}{1-0}$$

$$3c^2 - 8c + 3 = 0$$

$$c = \frac{4-\sqrt{7}}{3} \in (0, 1)$$

56. The value of $\alpha$ for which $4\alpha \int_{-1}^{2} e^{-\alpha|x|} dx = 5$, is

(A) $\log_e\left(\frac{3}{2}\right)$ (B) $\log_e\left(\frac{4}{3}\right)$

(C) $\log_e \sqrt{2}$ (D) $\log_e 2$

Ans. D

Sol. $$4\alpha \int_{-1}^{0} e^{\alpha x} dx + \int_{0}^{2} e^{-\alpha x} dx = 5$$

$$\Rightarrow 4\alpha\left(\frac{1-e^{-\alpha}}{\alpha} - \frac{e^{-2\alpha}-1}{-\alpha}\right) = 5$$

Let $e^{-\alpha} = t$

$\therefore 4t^2 + 4t - 3 = 0$

$\Rightarrow t = \frac{1}{2}$

$\alpha = \ell n 2$

57. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to:

(A) $-\frac{\sqrt{5}}{4}$
(B) $-\frac{\sqrt{5}}{2}$
(C) $\frac{2}{\sqrt{5}}$
(D) $\frac{\sqrt{5}}{2}$

Ans. B

Sol. Differentiating

$$y.\frac{-2x}{2\sqrt{1-x^2}} + \sqrt{1-x^2}.y' = \frac{x.2yy'}{2\sqrt{1-y^2}} - \sqrt{1-y^2}$$

Put $x = \frac{1}{2}, y = -\frac{1}{4}$ and $x, y = -\frac{1}{8}$

$$y' = \frac{-\sqrt{5}}{2}$$

58. Let $y = y(x)$ be the solution curve of the differential equation, $\left(y^2 - x\right)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x – axis at a point whose abscissa is:

(A) $2+e$
(B) –e
(C) 2
(D) 2 – e

Ans. D

Sol. $\frac{dx}{dy} + x = y^2$

I.F. $= e^{\int 1 dy} = e^y$

$\Rightarrow x.e^y = \int y^2.e^y\, dy$

$xe^y = y^2e^y - 2ye^y + 2e^y + C$

$\because y(0) = 1$

$\Rightarrow C = -e$

$\therefore xe^y = y^2e^y - 2ye^y + 2e^y - e$

Put $y = 0$

$\therefore x = 0 - 0 + 2 - e$

$\Rightarrow x = 2 - e$

59. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x\to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?

(A) x = 1 is a point of minima and x = –1 is a point of maxims of f.
(B) x = 1 is a point of maxima and x = –1 is a point of minimum of f
(C) f is an odd function

(D) $f(1)-4f(-1)=4$

Ans. B

Sol. $f'(x)=a(x+1)(x-1)x^2$

$f'(x)=ax^4-x^2f$

$f(x)=\frac{ax^5}{5}-\frac{ax^3}{5}+C$

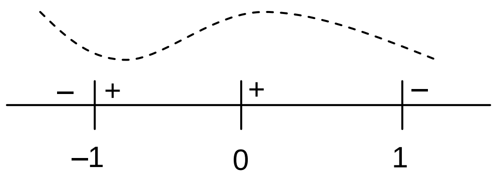
$\because f(0)=0 \Rightarrow c=0$

$\lim\limits_{x\to 0}\frac{f(x)}{x^3}=2$

$\Rightarrow a=-6$

$\therefore f'(x)=-6(x^2-1)(x^2)$

Minima at x = –1
Maxima at x = 1




60. The area (in sq. units of the region $\{(x,y)\in R^2 \,|\, 4x^2 \le y \le 8x+12\}$ is:

(A) $\frac{124}{3}$ (B) $\frac{125}{3}$

(C) $\frac{128}{3}$ (D) $\frac{127}{3}$

Ans. C

Sol. $4x^2=8x+12$

$x=-1,3$

Area $=\int_{-1}^{3}\left[(8x+12)-4x^2\right]dx$

$=\left[\frac{8x^2}{2}+12x-\frac{4x^3}{3}\right]_{-1}^{3}$

$=\frac{128}{3}$

61. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If $\lambda=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair, $(\lambda,\vec{d})$ is equal to:

(A) $\left(\frac{3}{2},3\vec{b}\times\vec{c}\right)$ (B) $\left(\frac{3}{2},3\vec{a}\times\vec{c}\right)$

(C) $\left(-\frac{3}{2},3(\vec{a}\times\vec{b})\right)$ (D) $\left(-\frac{3}{2},3\vec{a}\times\vec{b}\right)$

Ans. C

Sol. $3+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-\frac{3}{2}$

$\vec{d} = \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}$

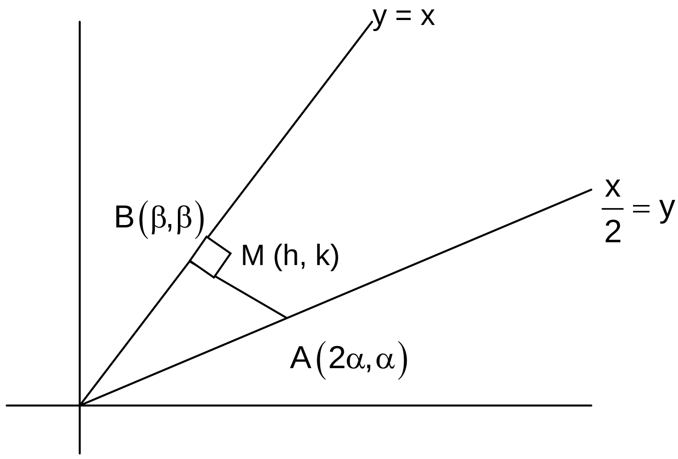
$\vec{d} = 3\left(\vec{a}\times\vec{b}\right)$

62. The locus of the mid – points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is:

(A) $7x - 5y = 0$
(B) $3x - 2y = 0$
(C) $2x - 3y = 0$
(D) $5x - 7y = 0$

Ans. D

Sol. Slope $AB = \frac{k-\alpha}{h-2\alpha} = -1$




$\Rightarrow \alpha = \frac{k+k}{3}$

also $\frac{\beta+2\alpha}{2} = h, \frac{\beta+\alpha}{2} = k$

$\alpha = 2h - 2k$

From (1) and (2)

$\frac{h+k}{3} = 2h - 2k$

$\Rightarrow 5h = 7k$

$\Rightarrow 5x = 7y$

63. Let $a_1, a_2, a_3, ........$ be a G.P. such that $a_1 < 0, a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then $\lambda$ is equal to:

(A) –171
(B) –513
(C) 171
(D) $\frac{511}{3}$

Ans. A

Sol. $a_1 + a_2 = 4 \Rightarrow a_1 + a_1 r = 4$ .........(i)

$a_3 + a_4 = 16 \Rightarrow a_1 r^2 + a_1 r^3 = 16$ .........(ii)

$\Rightarrow r = \pm 2$

$r = 2 \Rightarrow a_i = \frac{4}{3}$

$r = -2 \Rightarrow a_1 = -4$

$$\sum_{i=1}^{9} a_i = \frac{a\left(r^9 - 1\right)}{(r-1)} = (-4)\frac{\left((-2)^9 - 1\right)}{(-2-1)}$$

$= \frac{4}{3}(-513) = 4\lambda$

$\Rightarrow \lambda = -171$

64. Let $\alpha$ and $\beta$ be the roots of the equation $x^2 - x - 1 = 0$. If $P_k = (\alpha)^k + (\beta)^k, k \geq 1$, then which one of the following statements is not true?

(A) $p_5 = 11$
(B) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$
(C) $p_3 = p_5 - p_4$
(D) $p_5 = p_2 . p_3$

Ans. D

Sol. $p_5 = \alpha^5 + \beta^5$

$= (\alpha+1)^2 . \alpha + (\beta - 1)^2 . \beta$

$= 5\alpha + 5\beta + 6$

$= 5(1) + 6 = 11$

$p_2 = \alpha^2 + \beta^2 = \alpha + \beta + 2 = 3$

$p_3 = \alpha^3 + \beta^3 = (\alpha+1).\alpha + (\beta+1).\beta$

$= 1 + 3 = 4$

Hence $p_5 \neq p_2 . p_3$

65. If $\frac{3 + i\sin\theta}{4 - i\cos\theta}, \theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is:

(A) $-\tan^{-1}\left(\frac{3}{4}\right)$ (B) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

(C) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$ (D) $\tan^{-1}\left(\frac{4}{3}\right)$

Ans. B

Sol. Let $z = \frac{3 + i\sin\theta}{4 - i\cos\theta} \times \frac{(4 + i\cos\theta)}{(4 + i\cos\theta)}$

$= \frac{12 - \sin\theta\cos\theta + i(4\sin\theta + 3\cos\theta)}{16 + \cos^2\theta}$

z is real

$\therefore 4\sin\theta + 3\cos\theta = 0$

$\Rightarrow \tan\theta = \frac{-3}{4} [\because \theta \text{lies is 2nd quadrant}]$

$\arg(\sin\theta + i\cos\theta) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$

$= \pi - \tan^{-1}\left(\frac{4}{3}\right)$

66. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to:

(A) 4 (B) $\frac{17}{8}$

(C) $\frac{17}{2}$ (D) $\frac{17}{4}$

Ans. B

Sol. Probability (at most two machines will be out of service) $= \left(\frac{3}{4}\right)^3 .k$

$\Rightarrow {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)^3 .k$

$\Rightarrow \frac{17}{8}.\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)^3.k$

$\Rightarrow k = \frac{17}{8}$

67. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. Then $(AB)^2$ is equal to:

(A) $\frac{56}{5}$ (B) $\frac{32}{5}$

(C) $\frac{64}{5}$ (D) $\frac{52}{5}$

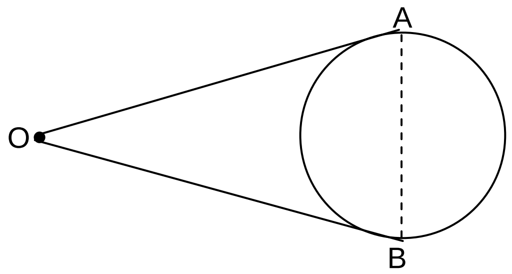
Ans. C

Sol. $OS = \sqrt{S_1}$

Radius = R = 2

Length of AB $= \frac{2RL}{\sqrt{L^2 + R^2}} = \frac{16}{\sqrt{20}}$

$AB^2 = \frac{64}{5}$




68. If $\theta_1$ and $\theta_2$ be respectively the smallest and the largest values of $\theta$ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to:

(A) $\frac{\pi}{3} + \frac{1}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{9}$ (D) $\frac{2\pi}{3}$

Ans. B

Sol. $2\frac{\cos^2\theta}{\sin^2\theta} - \frac{5}{\sin\theta} + 4 = 0$

$(2\sin\theta - 1)(\sin\theta - 2) = 0$

$\sin\theta = \frac{1}{2}$ only

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\downarrow \quad \downarrow$

$\theta_1 \quad \theta_2$

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \cos^2 3\theta \, d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(\frac{1 + \cos 6\theta}{2}\right) d\theta = \frac{\pi}{3}$$

69. The coefficient of $x^7$ in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1-x)^8 + ...... + x^{10}$ is:

(A) 120 (B) 420

(C) 330 (D) 210

Ans. C

Sol. $(1+x)^{10}+x(1+x)^{9}+x^{2}(1+x)^{8}+........+x^{10}$

$$=(1-x)^{10}\frac{\left[1-\left(\frac{x}{1+x}\right)^{11}\right]}{\left(1-\frac{x}{1+x}\right)}$$

$\Rightarrow (1+x)^{11}-x^{11}$

Coefficient of $x^7$ is ${}^{11}C_7=330$

70. Let A, B, C and D be four non – empty sets. The contrapositive statement of "If $A\subseteq B$ and $B\subseteq D$, then $A\subseteq C$" is:

(A) If $A\subseteq C$, then $B\subset A$ or $D\subset B$
(B) If $A\not\subseteq C$, then $A\not\subseteq B$ or $B\subseteq D$
(C) If $A\not\subseteq C$, then $A\not\subseteq B$ or $B\not\subseteq D$
(D) If $A\not\subseteq C$, then $A\subseteq B$ or $B\subseteq D$

Ans. C

Sol. If $A\subseteq B$ and $B\subseteq D$ then $A\subseteq C$
Contrapositive is
If $A\not\subseteq C$, then $A\not\subseteq B$ or $B\not\subseteq D$

71. If the function f defined on $\left(-\frac{1}{3},\frac{1}{3}\right)$ by $f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x\neq 0\\ k & \text{,when } x=0\end{cases}$ is continuous, then k is equal to ___________

Ans. 5

Sol. $\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}\left(\frac{1}{x}\ell n(1+3x)-\frac{1}{x}\ell n(1-2x)\right)$

$=\lim\limits_{x\to 0}\left(\frac{3\ell n(1+3x)}{3x}-\frac{2\ell n(1-2x)}{-2x}\right)$

$=3+2=5$

f is continuous

$\therefore \lim\limits_{x\to 0} f(x)=f(0)$

$\therefore f(0)=5=k$

72. If the system of linear equations,

$x+y+z=6$

$x+2y+3z=10$

$3x+2y+\lambda z=\mu$

has more than two solutions, then $\mu-\lambda^2$ is equal to ____________

Ans. 13

Sol. $\Delta=\begin{vmatrix}1&1&1\\1&2&3\\3&2&\lambda\end{vmatrix}=0$

$\Rightarrow 1(2\lambda-6)-1(\lambda-9)+1(-4)=0$

$\Rightarrow \lambda=1$

$$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 2\lambda + \mu = 16$

$\Rightarrow \mu = 14$

$\mu - \lambda^2 = 14 - 1 = 13$

73. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then xy is equal to ___________

Ans. 54

Sol. Mean $= 10 = \dfrac{3+7+9+12+13+20+x+y}{8}$

$16 = x + y$ ………(1)

Variance $\sigma^2 = 25 = \dfrac{\sum x_i^2}{8} - (\text{mean})^2$

$25 = \dfrac{3^2+7^2+9^2+12^2+13^2+20^2+x^2+y^2}{8} = 100$

$x^2 + y^2 = 148$ ……….(2)

$(x+y)^2 = x^2 + y^2 + 2xy$

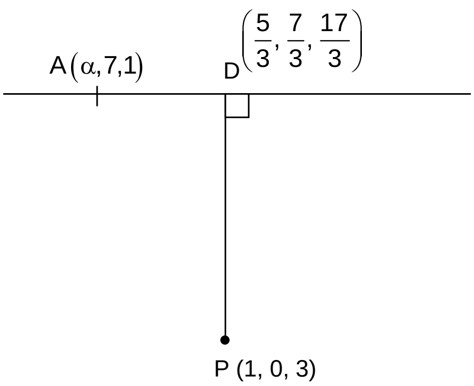
$256 = 148 + 2xy$

$x.y = 54$

74. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through $(\alpha, 7, 1)$ is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then $\alpha$ is equal to ___________

Ans. 4

Sol. $D\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$




$\overrightarrow{AD}.\overrightarrow{PD} = 0$

$\left(\left(\frac{5}{3} - \alpha\right)\hat{i} + \left(\frac{7}{3} - 7\right)\hat{j} + \left(\frac{17}{3} - 1\right)\hat{k}\right)\left(\frac{2}{3}\hat{i} + \frac{7}{3}\hat{j} + \frac{8}{3}\hat{k}\right) = 0$

$\left(\frac{5}{2} - \alpha\right)\frac{2}{3} + \frac{7}{3} \times \left(\frac{14}{3}\right) + \frac{14}{3} \times \frac{8}{3} = 0$

$\Rightarrow 3\alpha = 12$

$\alpha = 4$

75. Let $X = \{n \in N : 1 \le n \le 50\}$. If $A = \{n \in X : n \text{ is a multiple of } 2\}$ and $B = \{n \in X, n \text{ is a multiple of } 7\}$, then the number of elements in the smallest subset of X containing both A and B is ________

Ans. 29

Sol. $A = \{2, 4, 6, 8, .........50\} \Rightarrow 25$ element

$A = \{7, 14, 21, ..........49\} \Rightarrow 7$ elements

$A \cap B = \{14, 28, 42\} = 3$ elements

Required number of elements $= 25 + 7 - 3 = 29$