FIITJEE

Solutions to JEE (Main)-2020

JEE–Main–2020 –Jan–7–First–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. A polarizer-analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer-analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero is

(A) 18.4° (B) 45°
(C) 71.6° (D) 90°

Ans. **A**

Sol. $I = I_0 \cos^2\theta$

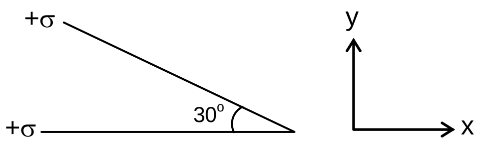
$\frac{I_0}{10} = I_0 \cos^2\theta$

$\cos\theta = \frac{1}{\sqrt{10}} = 0.31 < \frac{1}{\sqrt{2}}$ which is 0.707

So, $\theta > 45°$ and $90 - \theta < 45°$ so only one option is correct i.e. 18.4°

Angle rotated should be = 90° – 71.6° = 18.4°.

2. Two infinite planes each with uniform surface charge density $+\sigma$ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by:



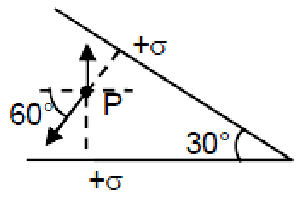
(A) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$ (B) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

(C) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}+\frac{\hat{x}}{2}\right]$ (D) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

Ans. **A**

Sol. $\vec{E} = \frac{\sigma}{2\varepsilon_0}\cos 60^o(-\hat{x}) + \left[\frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\sin 60^o\right](\hat{y})$

$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y} - \frac{1}{2}\hat{x}\right]$



3. A long solenoid of radius R carries a time (t) dependent current $I(t) = I_0 t(1 - t)$. A ring of radius 2R is placed cordially wear its middle. During the time internal $0 \le t \le 1$, the induced current (I_R) and the induced EMF (V_R) in the ring changes as:

(A) At t = 0.25 direction of I_R reverses and V_R is maximum.
(B) Direction of I_R remains unchanged and V_R is zero at t = 0.25
(C) Direction of I_R remains unchanged and V_R is maximum at t = 0.5
(D) At t = 0.5 direction of I_R reverses and V_R is zero.

Ans. **D**

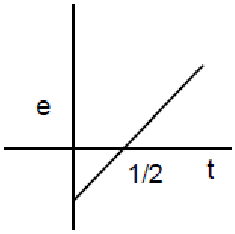
Sol. $I = I_0t - I_0t^2$

$\phi = BA$

$\phi = \mu_0 nIA$

$V_R = \frac{d\phi}{dt} = -\mu_0 nAI_0\ (1-2t)$

$V_R = 0$ at $t = \frac{1}{2}$ and $I_R = \frac{V_R}{\text{Resistance of loop}}$



4. Visible light of wavelength 6000×10^{-8} cm falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ_1 then θ_1 is close to:

(A) 20° (B) 30°

(C) 25° (D) 45°

Ans. **C**

Sol. For 2^{nd} minima

$d \sin\theta = 2\lambda$

$\sin\theta = \frac{\sqrt{3}}{2}$ (given)

$\Rightarrow \quad \frac{\lambda}{d} = \frac{\sqrt{3}}{4} \qquad \ldots(i)$

So for 1^{st} minima is

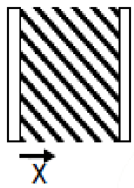
$d \sin\theta = \lambda$

$\sin\theta = \frac{\lambda}{d} = \frac{\sqrt{3}}{4}$ (from equation (i))

$\theta = 25.65°$ (from sin table)

$\theta \approx 25°$.

5. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(n) = k(1 + \alpha n)$ where 'x' is he distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression:



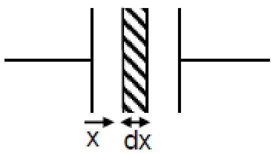
(A) $\frac{AK\varepsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$ (B) $\frac{A\varepsilon_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$

(C) $\frac{AK\varepsilon_0}{d}(1+\alpha d)$ (D) $\frac{A\varepsilon_0 K}{d}\left[1+\left(\frac{\alpha d}{2}\right)^2\right]$

Ans. **A**

Sol. Capacitance of element

Capacitance of element, $C' = \frac{K(1+\alpha x)\varepsilon_0 A}{dx}$



$\sum \frac{1}{C'} = \int_0^d \frac{dx}{K\varepsilon_0 A(1+\alpha x)}$

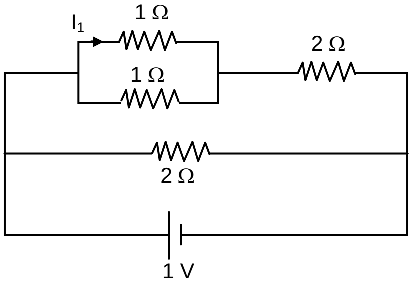
$\frac{1}{C} = \frac{1}{K\varepsilon_0 A\alpha} \ell n(1+\alpha d)$

Given: $\alpha d << 1$

$\frac{1}{C} = \frac{1}{K\varepsilon_0 A\alpha}\left(\alpha d - \frac{\alpha^2 d^2}{2}\right) \quad ; \quad \frac{1}{C} = \frac{d}{K\varepsilon_0 A}\left(1-\frac{\alpha d}{2}\right)$

$C = \frac{K\varepsilon_0 A}{d}\left(1+\frac{\alpha d}{2}\right)$

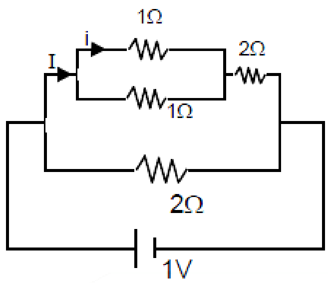
6. The current I_1 (in A) flowing through 1 Ω resistor in the following circuit is
(A) 0.25
(B) 0.4
(C) 0.2
(D) 0.5



Ans. **C**

Sol. $I = \frac{1}{2.5} = 0.4$ A

$I = \frac{I}{2} = 0.2$ A



7. The radius of gyration of a uniform rod of length ℓ, about an axis passing through a point $\frac{\ell}{4}$ away from the centre of the rod, an perpendicular to it is:

(A) $\frac{1}{8}\ell$ (B) $\sqrt{\frac{7}{48}}\ell$

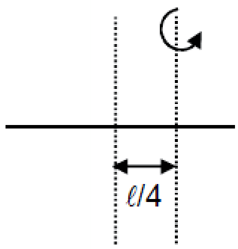
(C) $\frac{1}{4}\ell$ (D) $\sqrt{\frac{3}{8}}\ell$

Ans. **B**

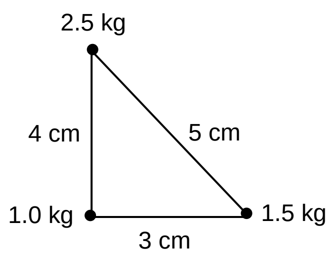
Sol. $\frac{M\ell^2}{12} + M\left(\frac{\ell}{4}\right) = MK^2$

$\frac{\ell^2}{12} + \frac{\ell^2}{16} = K^2$

$K = \sqrt{\frac{7}{48}}\ell$



8. Three point particles of masses 10 kg, 1.5 kg and 2.5 kg are placed at there corners of a right angle triangle of sides 4.0 cm, 3.0 ;cm and 5.0 cm as shown in the figure. The centre of mass of the system is at a point:
(A) 0.6 cm right and 2.0 cm above 1 kg mass.
(B) 2.0 cm right and 0.9 cm above 1 kg mass.
(C) 1.5 cm right and 1.2 cm above 1 kg mass.
(D) 0.9 cm right and 2.0 cm above 1 kg mass.

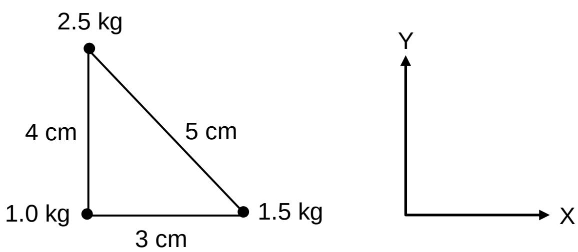


Ans. **D**

Sol. Take 1 kg mass at origin

$X_{cm} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5} = 0.9$ cm

$Y_{cm} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5} = 2$ cm



9. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ. The magnetic flux through the area is given by ϕ_0. Which of the following is correct?

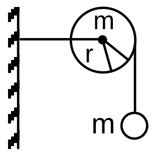
(A) $\phi_i = -\phi_0$
(B) $\phi_i > \phi_0$
(C) $\phi_i = \phi_0$
(D) $\phi_i < \phi_0$

Ans. **A**

Sol. As magnetic field lines always form a closed loop, hence every magnetic field line creating magnetic flux in the inner region must be passing through the outer region. Since flux in two regions are in opposite direction,

$\therefore \quad \phi_i = -\phi_0$

10. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be



(A) $r\sqrt{\frac{3}{4gh}}$
(B) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$
(C) $r\sqrt{\frac{3}{2gh}}$
(D) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$

Ans. **D**

Sol. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$v = \omega R$ (no slipping)

$$mgh = \frac{1}{2}m\omega^2R^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$$

$$mgh = \frac{3}{4}m\omega^2R^2$$

$$\omega = \sqrt{\frac{4gh}{3R^2}} = \frac{1}{R}\sqrt{\frac{4gh}{3}}$$

11. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye piece should be close to

(A) 22 mm
(B) 33 m
(C) 12 m
(D) 2 mm

Ans. **A**

Sol. **Case-I:**

If final image is at least distance of clear vision

$$M.P. = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) \;;\; 375 = \frac{150}{5}\left[1 + \frac{25}{f_e}\right]$$

$$\frac{375}{30} = 1 + \frac{25}{f_e} \quad ; \quad \frac{345}{30} = \frac{25}{f_e}$$

$$f_e = \frac{750}{345} = 2.17\text{ cm} ;\; f_e \approx 22\text{ mm}$$

Case-II:

In final image is at infinity

$$M.P. = \frac{L}{f_0}\left(\frac{D}{f_e}\right) = 375$$

$f_e = 22$ mm

12. Two moles of an ideal gas with $\frac{C_P}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_P}{C_V} = \frac{4}{3}$. The value of $\frac{C_P}{C_V}$ for the mixture is:

(A) 1.42 (B) 1.47
(C) 1.45 (D) 1.50

Ans. **A**

Sol. $$\gamma_{\text{mixture}} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \frac{n_1 \frac{\gamma_1 R}{\gamma_1 - 1} + n_2 \frac{\gamma_2 R}{\gamma_2 - 1}}{\frac{n_1 R}{\gamma_1 - 1} + \frac{n_2 R}{\gamma_2 - 1}}$$

on rearranging we get

$$\frac{n_1 + n_2}{\gamma_{mix} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \;;\quad \frac{5}{\gamma_{mix} - 1} = \frac{3}{1/3} + \frac{2}{2/3}$$

$$\frac{5}{\gamma_{mix} - 1} = 9 + 3 = 12 \quad \Rightarrow \gamma_{\text{mixture}} = \frac{17}{12} + 1 + \frac{5}{12} \;;\quad \gamma_{mix} = 1.42$$

13. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the young's modulus of wire is 16×10^{11} Nm^{-2}, the extension of wire over its natural length is:

(A) 0.03 mm (B) 0.01 mm
(C) 0.02 mm (D) 0.04 mm

Ans. **A**

Sol. $$v = \sqrt{\frac{T}{\mu}}$$

$T = \mu v^2$; $\frac{\mu v^2}{A} = Y \frac{\Delta \ell}{\ell}$

$$\Delta \ell = \frac{\mu v^2 \ell}{AY}$$

after substituting value of μ, v, ℓ, A and Y we get

$\Delta \ell$ = 0.03 mm

14. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If γ = 1.40, the work done by air is: ($3^{1.4}$ = 4.6555) [Take air to be an ideal gas]

(A) 48 J (B) 90.5 J
(C) 100.8 J (D) 60.7 J

Ans. **B**

Sol. P_1 = 1 atm, T_1 = 273 K

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_2 = P_1 \left[\frac{V_1}{V_2}\right]^{\gamma}$$

$$= 1\text{atm}\left(\frac{1}{3}\right)^{1.4}$$

Now work done = $\frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$ = 88.7 J

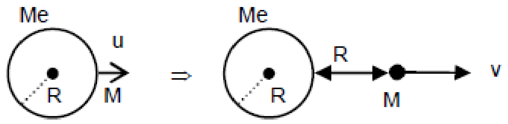
Closes answer is 90.5 J.

15. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant ; M is the mass of the earth)

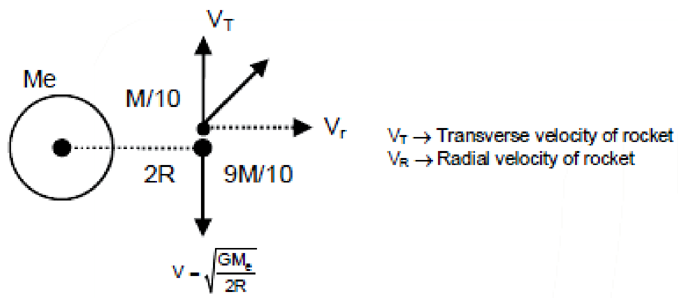
(A) $\frac{m}{20}\left(u-\sqrt{\frac{2GM}{3R}}\right)^2$
(B) $\frac{3m}{8}\left(u+\sqrt{\frac{5GM}{6R}}\right)^2$
(C) $\frac{m}{20}\left(u^2-\frac{113}{200}\frac{GM}{R}\right)$
(D) $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$

Ans. **D**

Sol. $\frac{-GM_eM}{R}+\frac{1}{2}Mu^2=\frac{-GM_eM}{2R}+\frac{1}{2}Mv^2$



$v=\sqrt{u^2-\frac{GM_e}{R}}$



$\frac{M}{10}V_T=\frac{9M}{10}\sqrt{\frac{GM_e}{2R}}$; $\frac{M}{10}V_r=M\sqrt{u^2-\frac{GM_e}{R}}$

Kinetic energy = $\frac{1}{2}\frac{M}{10}\left(V_T^2+V_r^2\right)=\frac{M}{20}\left(81\frac{GM_e}{2R}+100u^2-100\frac{GM_e}{R}\right)$

= $\frac{M}{20}\left(100u^2-\frac{119GM_e}{2R}\right)$

= $5M\left(u^2-\frac{119GM_e}{200R}\right)$

16. The time period of revolution of electron in its ground state orbit in a hydrogen atom is 1.6×10^{-16} s. The frequency of revolution of the electron is its first excited state (in s^{-1}) is

(A) 7.8×10^{14}
(B) 1.6×10^{14}
(C) 6.2×10^{15}
(D) 5.6×10^{12}

Ans. **A**

Sol. $T=\frac{2\pi}{\omega}=2\pi\frac{r}{v}\propto\frac{n^2}{\frac{1}{n}}\propto n^3$

$\therefore\quad \nu=\frac{1}{T}\propto\frac{1}{n^3}$

$\frac{\nu_2}{\nu_1}=\frac{1^3}{2^3}=\frac{1}{8}$

$\therefore \quad \nu_2 = \frac{1}{8} \times \frac{1}{T} = \frac{1}{8} \times \frac{10^{16}}{1.6}$ = 7.8 × 10^{14}

17. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the connect equivalence would be:

(A) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$
(B) $L \leftrightarrow k$, $C \leftrightarrow b$, $R \leftrightarrow m$
(C) $L \leftrightarrow m,\ C \leftrightarrow \frac{1}{k},\ R \leftrightarrow b$
(D) $L \leftrightarrow m$, $C \leftrightarrow k$, $R \leftrightarrow b$

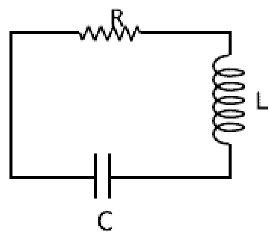
Ans. **C**

Sol. In damped oscillation

ma + bv + kx = 0

$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \qquad \ldots(i)$

In the circuit



$-iR - L\frac{di}{dt} - \frac{q}{c} = 0$

$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{c}q = 0 \qquad \ldots(ii)$

Comparing equation (i) and (ii)

$m = L,\ b = R,\ k = \frac{1}{c}$

18. If the magnetic field in a plane electromagnetic wav is given by $\vec{B}$ = 3 × 10^{-8} $\sin(1.6 \times 10^3 x + 48 \times 10^{10} t)j$ T, then what will be expression for electric field?

(A) $\vec{E} = \left(60 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\,\hat{k}\ V/m\right)$

(B) $\vec{E} = \left(3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\,\hat{i}\ V/m\right)$

(C) $\vec{E} = \left(9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\,\hat{k}\ V/m\right)$

(D) $\vec{E} = \left(3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\,\hat{j}\ V/m\right)$

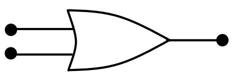
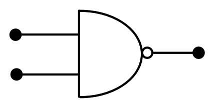
Ans. **C**

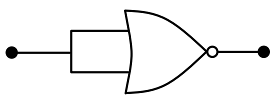
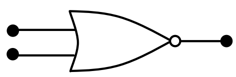
Sol. $\frac{E_0}{B_0} = C$ (speed of light in vacuum)

$E_0 = B_0C = 3 \times 10^{-8} \times 3 \times 10^8 = 9$ N/C

So, $\vec{E} = \left(9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\,\hat{k}\ V/m\right)$

19. Which of the following given a reversible operation?

(A) 
(B) 

(C) 
(D) 

Ans. **C**

Sol. A logic gate is reversible if we can recover input data fro the output eg. NOT gate.

20. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, g = 10 ms^{-2})
(A) 1.5 ms^{-1} (B) 2.0 ms^{-1}
(C) 1.7 m/s^{-1} (D) 1.9 m/s^{-1}

Ans. **D**

Sol. $4000 \times V + mg \times V = P$

$$\frac{60 \times 746}{2000 \times 4000} = V$$

$V = 1.86$ m/s ≈ 1.9 m/s.

21. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle is _________.

Ans. **600**

Sol. $n = \frac{W}{Q_h} = 1 - \frac{300}{900} = \frac{2}{3}$

$Q_h = \frac{3}{2} W = 1800$ J

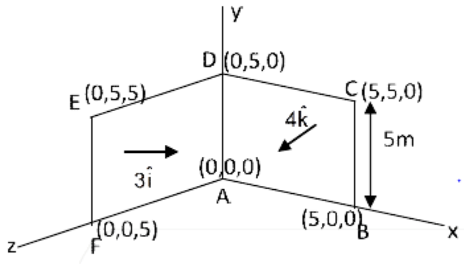
$Q_L = O_h - W = 600$ J

22. A loop ABCDEFA of straight edges has six corner points A (0, 0, 0), B (5, 0, 0), C(5, 5, 0), D(0, 5, 0), E(0, 5, 5) and F(0, 0, 5). The magnetic field in this region in $\vec{B} = (3\hat{i} + 4\hat{k})T$. The quantity of flux through the loop ABCDEFA (in Wb) is _________.

Ans. **175**

Sol. $\phi = \vec{B} \cdot \vec{A} = (3\hat{i} + 4\hat{k}) \cdot (25\hat{i} + 25\hat{k})$

$\phi = (3 \times 25) + (4 \times 25) = 175$ weber

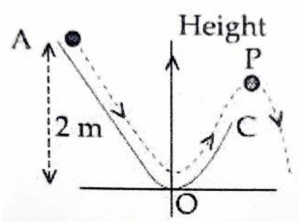


23. A non-isotropic solid metal cube has coefficients of linear expansion as: 5×10^{-5} / °C along the x = axis and 5×10^{-6}/°C along the y and the z-axis. If coefficient of volume expansion of the solid C × 10^{-6} /°C then the value of C is _________.

Ans. **60**

Sol. $V = 2\alpha_2 + \alpha_1$
$= 10 \times 10^{-6} + 5 \times 10^{-5}$
$= 60 \times 10^{-6}$ / °C

24. A particle (m = 1 kg) slides down a frictionless track (AOC) starting from rest of a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it leading its highest point P(height 1 m) the kinetic energy of the particle (in J) is: (Figure drawn is schematic and not to scale (take g = 10 ms^{-2})



Ans. **10**

Sol. $KE = PE_1 - PE_2 = mgh_1 - mgh_2$

25. A beam of electromagnetic radiation of intensity 6.4×10^{-5} W/cm^2 is comprised of wavelength, λ = 310 nm. It falls normally on a metal (work function φ = 2 eV) of surface area 1 cm^2. If one in 10^3 photons ejects an electron, total number of electrons ejected in is 10^x. (h_c = 1240 eVnm, 1 eV = 1.6×10^{-19} J), then x is _________.

Ans. **11**

Sol. Energy of photon, $E = \frac{1240}{310} = 4\text{eV} > 2\text{eV}$ (so photoelectric effect will take place)

$= 4 \times 1.6 \times 10^{-19} = 6.4 \times 10^{-19}$ Joule

Number of photons falling per second

$$= \frac{6.4 \times 10^{-5} \times 1}{6.4 \times 10^{-19}} = 10^{14}$$

Number of photoelectron emitted per second

$$= \frac{10^{14}}{10^3} = 10^{11}$$

PART –B (CHEMISTRY)

26. The relative strength of interionic/intermolecular forces in decreasing order is
(A) ion-dipole > dipole-dipole > ion-ion
(B) dipole-dipole > ion-dipole > ion- ion
(C) ion-ion > ion-dipole > dipole-dipole
(D) ion-dipole > ion-ion > dipole-dipole

Ans. C

Sol. Theory based.

27. Oxidation number of potassium in K_2O, K_2O_2 and KO_2 respectively is
(A) +1, +2 and +4
(B) +2, +1 and +½
(C) +1, +1 and +1
(D) +1, +4 and +2

Ans. C

Sol. $\underset{2x-2=0}{K_2O} \longrightarrow x=+1$

$\underset{2x-2=0}{K_2O_2} \longrightarrow x=+1$

$\underset{x-1=0}{KO_2} \longrightarrow x=+1$

28. At 35°C, the vapour pressure of CS_2 is 512 mm of Hg and that of acetone is 344 mm of Hg. A solution of CS_2 in acetone has a total vapour pressure of 600 mm of Hg. The false statement among the following is
(A) CS_2 and acetone are less attracted to each other than to themselves
(B) Heat must be absorbed in order to produce the solution at 35°C
(C) Raoult's law is not obeyed by this system
(D) a mixture of 100 mL CS_2 and 100 mL acetone has a volume of < 200 mL

Ans. D

Sol. The above mixture will show positive deviation from Raoult's law.

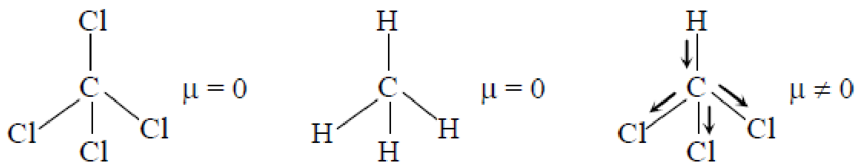
29. The atomic radius of Ag is closest to
(A) Ni
(B) Cu
(C) Au
(D) Hg

Ans. C

Sol. Theory based.

30. The dipole moments of CCl_4, $CHCl_3$ ad CH_4 are in the order
(A) $CHCl_3 < CH_4 = CCl_4$
(B) $CCl_4 < CH_4 < CHCl_3$
(C) $CH_4 = CCl_4 < CHCl_3$
(D) $CH_4 < CCl_4 < CHCl_3$

Ans. C

Sol.


31. In comparison to the zeolite process for the removal of permanent hardness, the synthetic resins method is
(A) less efficient as it exchanges only anions
(B) more efficient as it can exchange only cations
(C) less efficient as the resin cannot be regenerated
(D) more efficient as it can exchange both cation as well as anions

Ans. B

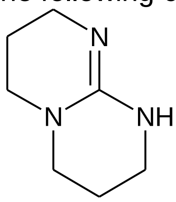
Sol. Theory based.

32. Amongst the following statements, that which was not proposed by Dalton was
(A) matter consists of invisible atoms
(B) when gases combine or reproduced in a chemical reaction they do so in a simple ratio by volume provided all gases are at the same temperature & pressure
(C) chemical reactions involve reorganisation of atoms. These are neither created nor destroyed in a chemical reaction
(D) All the atoms of a given element have identical properties including identical mass. Atoms of different elements differ in mass

Ans. B

Sol. Theory based.

33. The increasing order of pK_b for the following compounds will be

(a) $NH_2 - CH = NH_2$ (b)  (c) CH_3NHCH_3

(A) b < c < a
(B) c < a < b
(C) b < a < c
(D) a < b < c

Ans. B

Sol. $pK_b \propto \frac{1}{K_b} \propto \frac{1}{\text{Basic strength}}$

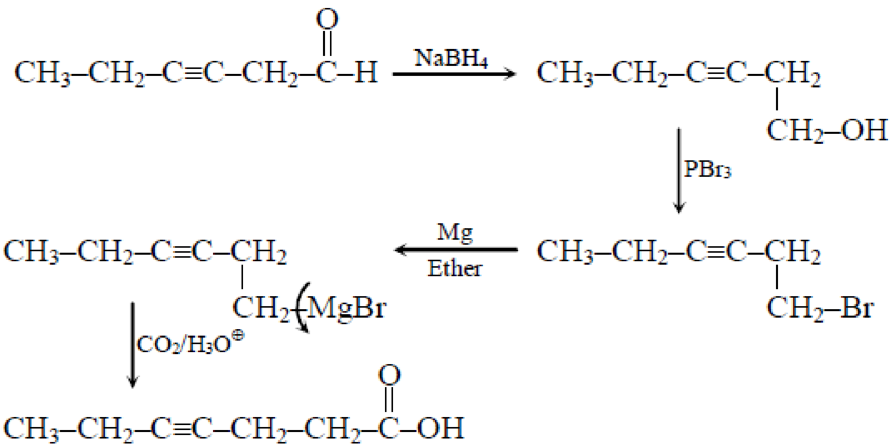
34. What is the product of following reaction?

$$\text{Hex} - 3 - \text{ynal} \xrightarrow[\substack{\text{(ii) } PBr_3 \\ \text{(iii) Mg/ether} \\ \text{(iv) } CO_2/H_3O^+}]{\text{(i) } NaBH_4} ?$$

(A) 
(B) 
(C) 
(D) 

Ans. C

Sol.



35. The number of orbitals associated with quantum number n = 5, $m_s = +\frac{1}{2}$ is
(A) 15
(B) 11
(C) 50
(D) 25

Ans. D

Sol. n=5 $\ell = n - 1 = 4$

0 1 2 3 4

$m \rightarrow -\ell$ to $+\ell$

$\rightarrow 0 \rightarrow 0$

$\rightarrow 1 \rightarrow -1, 0, 1$

$\rightarrow 2 \rightarrow -2, -1, 0, 1, 2$

$\rightarrow 3 \rightarrow -3, -2, -1, 0, 1, 2, 3$

$\rightarrow 4 \rightarrow -4, -3, -2, -1, 0, 1, 2, 3, 4$

Total 25 orbitals are possible

36. The purest form of commercial iron is
(A) scrap iron and pig iron
(B) wrought iron
(C) cast iron
(D) pig iron

Ans. B

Sol. Theory based.

37. The theory that can completely/properly explain the nature of bonding in $[Ni(CO)_4]$ is
(A) Werner's theory
(B) Crystal field theory
(C) Molecular orbital theory
(D) Valence bond theory

Ans. D

Sol. $[Ni(CO)_4]$ bonding diagram and energy can be explained by MOT clearly.

38. The IUPAC name of the complex is $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is
(A) Diamminechlorido(aminomethane)platinum(II)chloride
(B) Diamminechlorido(methanamine)platinum(II)chloride
(C) Diammine(methanamine)chlorido platinum(II)chloride
(D) Bisammine(methanamine)chloride platinum(II)chloride

Ans. B

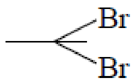
Sol. $[Pt(NH_3)_2Cl(NH_2CH_3)Cl$

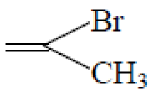
$x + 0 - 1 + 0 = +1$

$x = +2 \rightarrow Pt^{2+}$

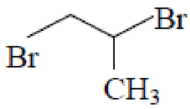
Diamminechlorido(methanamine)platinum(II)chloride

39. 1-methyl ethylene oxide when treated with an excess of HBr, produces

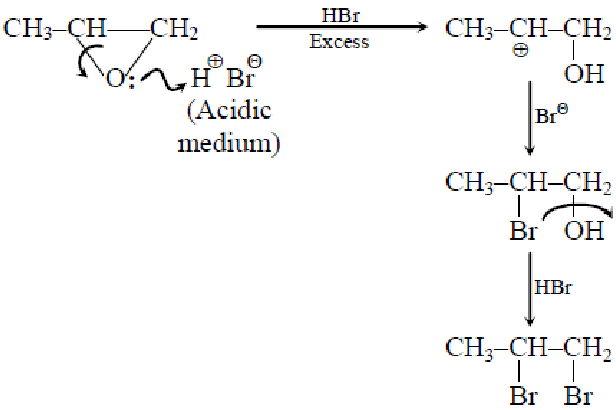
(A) 

(B) 

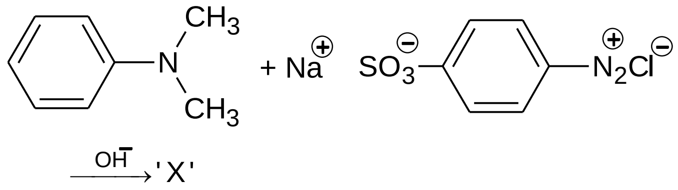
(C) 

(D) 

Ans. D

Sol.


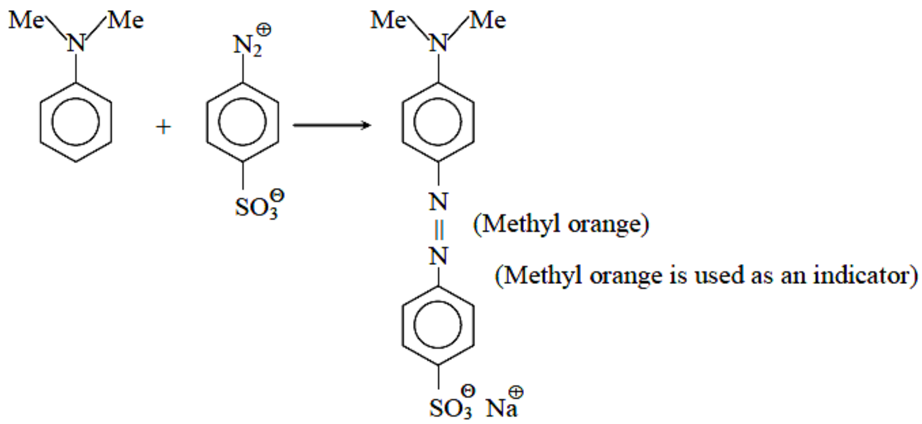
40. Consider the following reaction:



The product 'X' is used
(A) in protein estimation as an alternative to ninhydrin
(B) as food grade colourant
(C) in laboratory test for phenols
(D) in acid-base titration as an indicator

Ans. D

Sol.



41. Match the following

Column – I		Column – II	
(i)	Riboflamin	(a)	Beriberi
(ii)	Thiamine	(b)	Scurvy
(iii)	Pyridoxine	(c)	Cheilosis
(iv)	Ascorbic acid	(d)	Convulsions

(A) i → c, ii → a, iii → d, iv → b
(B) i → c, ii → d, iii → a, iv → b
(C) i → a, ii → d, iii → c, iv → b
(D) i → d, ii → b, iii → a, iv → c

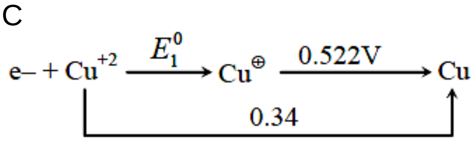
Ans. A

Sol. Theory based

42. Given that the standard potential (E^o) of Cu^{2+}/Cu and Cu^+/Cu are 0.34 V and 0.522 V respectively, the E^o of Cu^{2+}/Cu^+ is
(A) -0.158 V
(B) 0.182 V
(C) +0.158 V
(D) -0.182 V

Ans. C

Sol.



$$2 \times 0.34 = E_1^0 + 1 \times 0.522$$

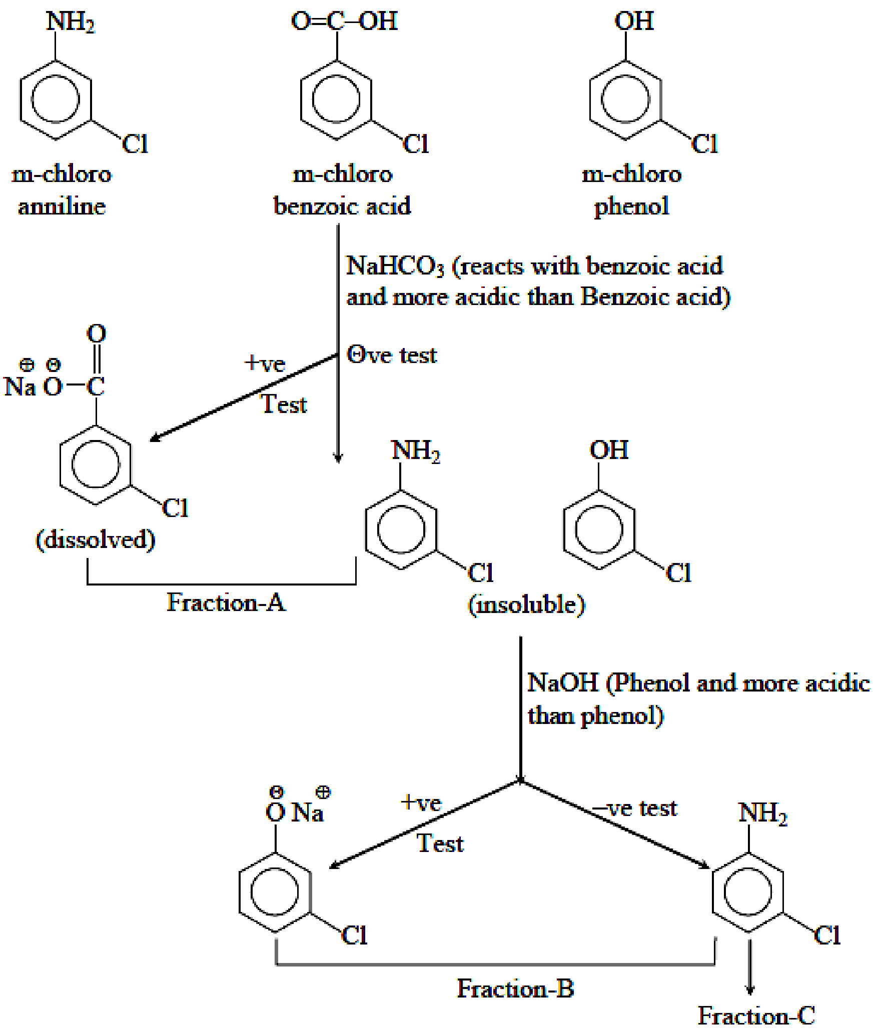
$$E_1^0 = 0.68 - 0.522$$

$$E_1^0 = 0.158$$

43. A solution of m-chloroaniline, m-chlorophenol and m-chlorobenzoic acid in ethyl acetate was extracted initially with a saturated solution of $NaHCO_3$ to give fraction A. The left over organic phase was extracted with dilute NaOH solution to give fraction B. The final organic layer was labelled as fraction C. Fraction A, B and C contain respectively
(A) m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline
(B) m-chlorophenol, m-chlorobenzoic acid and m-chloroaniline
(C) m-chlorobenzoic acid, m-chloroaniline and m-chlorophenol
(D) m-chloroaniline, m-chlorobenzoic acid and m-chlorophenol

Ans. A

Sol.



44. The order of electron gain enthalpy in kJ/mol of fluorine, chlorine, bromine and iodine, respectively are
(A) -333, -325, -349 and -296
(B) -333, -349, -325 and -296
(C) -349, -333, -325 and -296
(D) -296, -325, -333 and -349

Ans. B

Sol. Electron gain enthalpy increases with electro negativity chlorine has higher electron gain enthalpy than fluorine (exception)

45. Consider the following reactions, which of these reaction(s) will not produce Saytzeff product?

(a) $(CH_3)_3CCH(OH)CH_3 \xrightarrow{cons.H_2SO_4}$

(b) $(CH_3)_2CHCH\,(Br)CH_3 \xrightarrow{cons.KOH}$

(c) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{(CH_3)_3O^{\ominus}K^{\oplus}}$

(d) $(CH_3)_2\underset{\underset{OH}{|}}{C}-CH_2-CHO \xrightarrow{\Delta}$

Which of these reaction(s) will not produce Saytzeff product?

(A) (b) and (d) (B) (c) only
(C) (a), (c) and (d) (D) (d) only

Ans. B

Sol. Bulky base always prefer Hoffmann product

46. Two solutions A and B, each of 100 L was made by dissolution of 4g of NaOH and 9.8 g of H_2SO_4 in water, respectively. The pH of the resultant solution obtained from mixing 40 L of solution A and 10 L of solution B is

Ans. 10.60

Sol. $M_{(H_2SO_4)} = \frac{9.8}{98 \times 100} = 10^{-3}m$

$M_{NaOH} = \frac{4}{40 \times 100} = 10^{-3}m$

Equivalents of resulting solution[OH^-] $= \frac{40 \times 10^{-3} - 10 \times 10^{-3} \times 2}{50} = \frac{2}{5} \times 10^{-3}$

pOH = 3.39 | pH = 10.6

47. During the nuclear explosion, one of the product of ^{90}Sr with half life of 6.93 years. If 1 μg of ^{90}Sr absorbed in the bones of newly born baby in place of Ca, how much time, in years is required to reduce it by 90% if it is not lost metabolically

Ans. 23.03

Sol.

$$\frac{t_{90\%}}{t_{50\%}} = \frac{\ell n\frac{100}{10}}{\ell n2}$$

$$t_{90\%} = \frac{\ell n_{10}}{\ell n_2} \times t_{50\%} = \frac{6.93 \times \ell n10}{0.693} = 23.03 \text{ years}$$

48. Chlorine reacts with hot and concentrated NaOH and produces compound(X) and (Y). Compound(X) gives white precipitate with silver nitrate solution. The average bond order between Cl and O atoms in (Y) is

Ans. 1.67

Sol. $3Cl_2 + 6NaOH \longrightarrow 5NaCl + NaClO_3 + 3H_2O$

$NaCl + AgNO_3 \longrightarrow \underset{(White)}{AgCl} + NaNO_3$

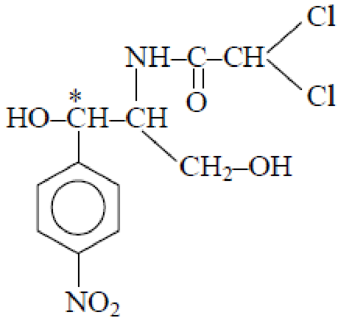
Y is $NaClO_3 \rightarrow ClO_3^-$

Bond order = $\frac{5}{3} = 1.67$

49. The number of chiral carbons in chloramphenicol is

Ans. 2

Sol.



50. For the reaction $A(l) \longrightarrow 2B(g)$

$\Delta U = 2.1$ Kcal, $\Delta S = 20$ cal K^{-1} at 300 K

Hence ΔG in Kcal is

Ans. -2.70

Sol. $\Delta H = \Delta U + \Delta ngRT$

$= 2.1 \times 10^3 + 2(B)\ (300) = 3300$ calories

$\Delta G = \Delta H - T\Delta S$

$= 3300 - 300(20) = -2700$ calories $= -2.7$ KCal

PART–C (MATHEMATICS)

51. If $f(a+b+1-x)=f(x)$, for all x, where a and b are fixed positive real numbers, then $\frac{1}{a+b}\int_a^b x(f(x)+f(x+1))dx$ is

(A) $\int_{a+1}^{b+1} f(x)dx$
(B) $\int_{a-1}^{b-1} f(x)dx$
(C) $\int_{a-1}^{b-1} f(x+1)dx$
(D) $\int_{a+1}^{b+1} f(x+1)dx$

Ans. C

Sol. Note : Answer given by NTS is (1) which is wrong

$$I=\frac{1}{(a+b)}\int_a^b \times\left[f(x)+f(x+1)\right]dx \quad ......(1)$$

$x \to a+b-x$

$$I=\frac{1}{(a+b)}\int_a^b (a+b-x)\left[f(a+b-x)+f(a+b+1-x)\right]dx$$

$$I=\frac{1}{(a+b)}\int_a^b (a+b-x)\left[f(x+1)+f(x)\right]dx \quad ......(2)$$

[$\because$ put $x \to x+1$ in given equation]

(1) + (2)

$$2I=\int_a^b \left[f(x+1)+f(x)\right]dx$$

$$2I=\int_a^b f(x+1)dx+\int_a^b f(x)dx$$

$$\int_a^b f(a+b+1-x)dx+\int_a^b f(x)dx$$

$$2I=2\int_a^b f(x)dx$$

$$I=\int_a^b f(x)dx$$

Put $x=t+1 \Rightarrow I=\int_{a-1}^{b-1} f(t+1)dt$

52. Let the function, $f:[-7,0]\to R$ be continuous on [–7, 0] and differentiable on (–7, 0). If $f(-7)=-3$ and $f'(x)\le 2$, for all $x\in(-7,0)$, then for all such functions f, $f(-1)+f(0)$ lies in the interval:

(A) $[-3, 11]$
(B) $(-\infty, 20]$
(C) $[-6, 20]$
(D) $(-\infty, 11]$

Ans. B

Sol. Using LMVT $x\in[-7,-1]$

$$\frac{f(-1)-f(-7)}{(-1+7)} \le 2$$

$$\frac{f(-1)+3}{6} \le 2 \Rightarrow f(-1) \le 9$$

Using LMVT for $x \in [-7,0]$

$$\frac{f(0)-f(-7)}{(0+7)} \le 2$$

$$\frac{f(0)+3}{7} \le 2 \Rightarrow f(0) \le 11$$

$$\therefore f(0)+f(-1) \le 20$$

53. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is:

(A) $\sqrt{3}$ (B) $3\sqrt{2}$

(C) $\frac{3}{\sqrt{2}}$ (D) $2\sqrt{3}$

Ans. B

Sol. $2ae = 6$ and $\frac{2a}{e} = 12$

$\Rightarrow ae = 3$ and $\frac{a}{e} = 6$

$\Rightarrow b^2 = a^2 - a^2e^2 = 18 - 9 = 9$

$\Rightarrow$ L.R. $= \frac{2b^2}{a} = \frac{2\times 9}{3\sqrt{2}} = 3\sqrt{2}$

54. An unbiased coin is tossed 5 time. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5, otherwise X takes the value –1. Then the expected value of X, is

(A) $-\frac{3}{16}$ (B) $-\frac{1}{8}$

(C) $\frac{1}{8}$ (D) $\frac{3}{16}$

Ans. D

Sol.

k	0	1	2	3	4	5
P (k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = number of times head occur consecutively

Now expectation

$$= \sum xP(k) = (-1)\times\frac{1}{32} + (-1)\times\frac{12}{32} + (-1)\times\frac{11}{32} + 3\times\frac{5}{32} + 4\times\frac{2}{32} + 5\times\frac{1}{32} = \frac{1}{8}$$

55. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is:

(A) 7 (B) $\frac{21}{2}$

(C) 16 (D) 27

Ans. C

Sol. Let terms be $a-2d, a-d, a, a+d, a+2d$

Sum $5a = 25 \Rightarrow a = 5$

Product = 2520

$(5-2d)(5-d)5(5+d)(5+2d) = 2520$

$\Rightarrow \left(25-4d^2\right)\left(25-d^2\right) = 504$

$\Rightarrow 625-100d^2-25d^2+4d^4 = 504$

$\Rightarrow 4d^4-125d^2+625-504 = 0$

$\Rightarrow 4d^4-125d^2+121 = 0$

$\Rightarrow 4d^2-121d^2-4d^2+121 = 0$

$\Rightarrow \left(d^2-1\right)\left(4d^2-121\right) = 0$

$\Rightarrow d = \pm 1,\ d = \pm\frac{11}{2}$

$d = \pm 1$, does not give $\frac{\{1\}2}{}$ as a term

$d = \pm 1$

$\therefore d = \frac{11}{2}$

$\therefore$ Largest term $= 5+2d = 5+11 = 16$

56. If the system of linear equations $2x+2ay+az=0,\ 2x+3by+bz=0, 2x+4cy+cz=0$ where $a,b,c \in R$ are non – zero and distinct; has a non – zero solution, then:

(A) $a+b+c=0$ (B) a, b, c are in A.P.

(C) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P. (D) a, b, c are in G.P.

Ans. C

Sol. For non – trivial solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$(3bc-4bc)-(2ac-4ac)+(2ab-3ab) = 0$

$-bc+2ac-ab = 0$

$ab+bc = 2ac$

a, b, c in H.P.

$\Rightarrow \frac{1}{a},\frac{1}{b},\frac{1}{c}$ in A.P.

57. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is:

(A) (6, 5, 2) (B) (4, 3, 2)

(C) (6, 5, –2) (D) (3, 4, –2)

Ans. C

Sol. Plane is $x+y-2z=3$

$$\Rightarrow \frac{x-2}{1}=\frac{y-1}{1}$$

$$=\frac{z-6}{-2}=\frac{-2(2+1-12-3)}{6}$$

$$\Rightarrow (x,y,z)=(6,5,-2)$$

58. If $\text{Re}\left(\frac{z-1}{2z+i}\right)=1$, where $z=x+iy$, then the point (x, y) lies on a:

(A) straight line whose slope is $\frac{3}{2}$

(B) circle whose diameter is $\frac{\sqrt{5}}{2}$

(C) straight line whose slope is $-\frac{2}{3}$

(D) circle whose centre is at $\left(-\frac{1}{2},-\frac{3}{2}\right)$

Ans. B

Sol. $$\left(\frac{z-1}{2z+i}\right)=\frac{(x-1)+iy}{2(x+iy)+i}=\frac{(x-1)+iy}{2x+(2y+1)i}\times\frac{2x-(2y+1)i}{2x-(2y+1)i}$$

$$\text{Re}\left(\frac{z+1}{2z+i}\right)=\frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2}=1$$

$$\Rightarrow 2x^2+2y^2-2x+y=4x^2+4y^2+4y+1$$

$$\Rightarrow 2x^2+2y^2+2x+3y+1=0$$

$$r=\sqrt{\frac{1}{4}+\frac{9}{16}-\frac{1}{2}}=\sqrt{\frac{4+9-8}{16}}=\frac{\sqrt{5}}{4}$$

59. If $y(\alpha)=\sqrt{2\left(\frac{\tan\alpha+\cot\alpha}{1+\tan^2\alpha}\right)+\frac{1}{\sin^2\alpha}}, \alpha\in\left(\frac{3\pi}{4},\pi\right)$ then $\frac{dy}{d\alpha}$ at $\alpha=\frac{5\pi}{6}$ is:

(A) 4

(B) $-\frac{1}{4}$

(C) –4

(D) $\frac{4}{3}$

Ans. A

Sol. $$y=\sqrt{\frac{2\cos^2\alpha}{\sin\alpha\cos\alpha}+\frac{1}{\sin^2\alpha}}$$

$$=\sqrt{2\cot\alpha+\text{cosec}^2\alpha}=|1+\cot\alpha|=-1-\cot\alpha$$

$$\frac{dy}{d\alpha}=\text{cosec}^2\alpha$$

$$\Rightarrow \text{at } \alpha=\frac{5\alpha}{6}, \frac{dy}{d\alpha}=4$$

60. Let α be a root of the equation $x^2+x+1=0$ and the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha^4\end{bmatrix}$, then the matrix A^{31} is equal to:

(A) A^3
(B) A
(C) I_3
(D) A^2

Ans. A

Sol. $$A^2=\frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

$\Rightarrow A^4=I$

$\Rightarrow A^{30}=A^{28}\times A^3=A^3$

61. If $g(x)=x^2+x-1$ and $(gof)(x)=4x^2-10x+5$, then $f\left(\frac{5}{4}\right)$ is equal to

(A) $\frac{3}{2}$
(B) $\frac{1}{2}$
(C) $-\frac{3}{2}$
(D) $-\frac{1}{2}$

Ans. D

Sol. $g(f(x))=f^2(x)+f(x)-1$

$$g\left(f\left(\frac{5}{4}\right)\right)=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1$$

$$-\frac{5}{4}=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1$$

$$f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)+\frac{1}{4}=0$$

$$\left(f\left(\frac{5}{4}\right)+\frac{1}{2}\right)^2=0$$

$$f\left(\frac{5}{4}\right)=-\frac{1}{2}$$

62. If $y=mx+4$ is a tangent to both the parabolas, $y^2=4x$ and $x^2=2by$, then b is equal to:

(A) –32
(B) –128
(C) –64
(D) 128

Ans. B

Sol. $y=mx+4$ ……..(i)

Tangent of $y^2=4x$

$y=mx+\frac{a}{m}\Rightarrow y=mx+\frac{1}{m}$ …..(ii)

$\therefore\ 4=\frac{1}{m}\Rightarrow m=\frac{1}{4}$

Line $y = \frac{1}{4}x + 4$ is also tangent to parabola $x^2 = 2by$,

$\therefore x^2 = 2b\left(\frac{x+16}{4}\right)$

$\Rightarrow 2x^2 - bx - 16b = 0$

$\Rightarrow D = 0 \Rightarrow b^2 - 4\times 2\times(-16b) = 0$

$\Rightarrow b^2 + 32\times 4b = 0$

$b = -128, b = 0$ (not possible)

63. The logical statement $(p \Rightarrow q)\wedge(q \Rightarrow \sim p)$ is equivalent to:

(A) $\sim q$ (B) p

(C) q (D) ~p

Ans. D

Sol.

p	q	$p \to q$	$-P$	$q \to -p$	$(p \to q)\wedge(q \to \sim p)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly $(p \to q)\wedge(q \to \sim p)$ is equivalent to $\sim p$

64. Let $x^k + y^k = a^k, (a, k > 0)$ and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is:

(A) $\frac{3}{2}$ (B) $\frac{4}{3}$

(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

Ans. D

Sol. $k.x^{k-1} + k.y^{k-1}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$

$\frac{dy}{dx} + \left(\frac{x}{y}\right)^{k-1} = 0$

$k - 1 = -\frac{1}{3}$

$k = 1 - \frac{1}{3} = \frac{2}{3}$

65. Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}.\lambda\tan x = (1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha+\beta) = 50$, then a value of λ is:

(A) 10 (B) 5

(C) $5\sqrt{2}$ (D) $10\sqrt{2}$

Ans. A

Sol. $(k+1)\tan^2 x - \sqrt{2}\lambda\tan x + (k-1) = 0$

$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1}$

$\tan\alpha\tan\beta = \frac{k-1}{k+1}$

$\tan(\alpha+\beta) = (k-1) = 0\frac{\sqrt{2}\lambda}{k+1}$

$\tan\alpha\tan\beta = \frac{k-1}{k+1}$

$\tan(\alpha+\beta) = (k-1) = 0\frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$

$\tan^2(\alpha+\beta) = \frac{\lambda^2}{2} = 50$

66. If $y = y(x)$ is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to:

(A) $1 + \log_e 2$ (B) $2e$

(C) $\log_e 2$ (D) $2 + \log_e 2$

Ans. A

Sol. $e^y = t$

$e^y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - t = e^x$

$IF = e^{\int -1.dx} = e^{-x}$

$t\left(e^{-x}\right) = \int e^x.e^{-x}dx$

$e^{y-x} = x + c$

Putting $x = 0, y = 0, c = 1$

$e^{y-x} = x + 1$

$y = x + \log_e(x+1)$

at $x = 1, y = 1 + \log_e 2$

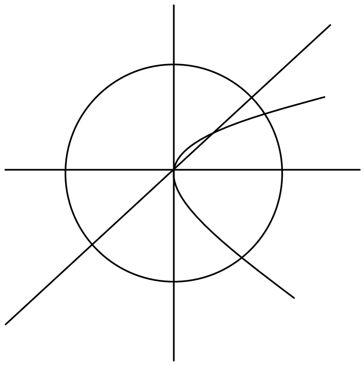
67. The area of the region, enclosed by the circle $x^2 + y^2 = 2$, which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is :

(A) $\frac{1}{3}(12\pi - 1)$ (B) $\frac{1}{3}(6\pi - 1)$

(C) $\frac{1}{6}(12\pi - 1)$ (D) $\frac{1}{6}(24\pi - 1)$

Ans. C

Sol. $A = 2\pi - \int_0^1 \sqrt{x} - x\,dx$



$2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right)_0^1$

$2\pi = \left(\frac{2}{3} - \frac{1}{2}\right) \Rightarrow 2\pi - \left(\frac{1}{6}\right) \Rightarrow \frac{12\pi - 1}{6}$

68. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}\,(\alpha, \beta \in R)$ lies in the plane of the vectors, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then:

(A) $\vec{a}.\hat{k} + 4 = 0$ (B) $\vec{a}.\hat{k} + 2 = 0$

(C) $\vec{a}.\hat{i} + 3 = 0$ (D) $\vec{a}.\hat{i} + 1 = 0$

Ans. B

Sol. For angle bisector $\vec{a} = \lambda\left(\hat{b} + \hat{c}\right)$ or $\vec{a} = \mu\left(\hat{b} - \hat{c}\right)$

$\vec{a} = \lambda\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} + \frac{\hat{i} + \hat{j} + 4\hat{k}}{3\sqrt{2}}\right)$

$= \frac{\lambda}{3\sqrt{2}}\left[3\hat{i} + 3\hat{j} + \hat{i} - \hat{j} + 4\hat{k}\right]$

$= \frac{\lambda}{3\sqrt{2}}\left[4\hat{i} + 2\hat{j} + 4\hat{k}\right]$

but $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$\frac{2\lambda}{3\sqrt{2}} = 2 \Rightarrow \lambda = 3\sqrt{2}$

$\vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

Now if $\vec{a} = \mu\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} - \frac{\hat{i} - \hat{j} + 4\hat{k}}{3\sqrt{2}}\right)$

$\vec{a} = \frac{\mu}{3\sqrt{2}}\left(3\hat{i} + 3\hat{j} - \hat{i} + \hat{j} - 4\hat{k}\right)$

$= \frac{\mu}{3\sqrt{2}}\left(2\hat{i} + 4\hat{j} - 4\hat{k}\right)$

Comparing $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$\frac{4\mu}{3\sqrt{2}} = 2 \Rightarrow \mu = \frac{3\sqrt{2}}{2}$

$\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a}.\hat{k} + 2 = 0$

$-2 + 2 = 0$

69. The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \ldots\ldots + 49^2 + 49 + 1$, is:

(A) 65 (B) 63

(C) 32 (D) 60

Ans. B

Sol. $\frac{(49)^{126}-1}{48}=\frac{\left((49)^{63}+1\right)\left(49^{63}-1\right)}{48}$

70. Total number of 6 – digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is

(A) 5^6 (B) 6!

(C) $\frac{5}{2}(6!)$ (D) $\frac{1}{2}(6!)$

Ans. C

Sol. 1, 3, 5, 7, 9

For digit to repeat we have ${}^5C_1$ choice

And six digits can be arrange in $\frac{6!}{2!}$ ways.

Hence total such number $=\frac{5}{2}(6!)$

71. Let S be the set of points where the function, $f(x)=\left|2-|x-3|\right|, x\in R$, is not differentiable. Then $\sum_{x\in S} f(f(x))$ is equal to _______

Ans. 3

Sol. $\because f(x)$ is non differentiable at $x=1,3,5$

$\sum f(f(x))=f(f(1))+f(f(3))+f(f(5))$

$=1+1+1$

= 3

72. If the sum of the coefficients of all even powers of x in the product $\left(1+x+x^2+......+x^{2n}\right)\left(1-x+x^2-x^3+.....+x^{2n}\right)$ is 61, then n is equal to

Ans. 30

Sol. Let $\left(1-x+x^2.....\right)\left(1-x+x^2.......\right)=a_0+a_1x+a_2x^2+....$

Putting x = 1

$1(2n+1)=a_0+a_1+a_2+......a_{2n}$ ………..(i)

Putting $x=-1$

$(2n+1)\times 1=a_0-a_1+a_2+......a_{2n}$ ………….(ii)

(i) + (ii)

$4n+2=2(a_0+a_2+......)$

$=2\times 61$

$\Rightarrow 2n+1=61 \Rightarrow n=30$

73. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then m + n is equal to ___________

Ans. 18

Sol. $\text{Var}(1,2,.......,n)=\frac{1^2+2^2+.......+n^2}{n}-\left(\frac{1+2+.......+n}{n}\right)^2=10$

$\Rightarrow \frac{(n+1)(2n+1)}{6}-\left(\frac{n+1}{2}\right)^2=10$

$\Rightarrow n^2 - 1 = 120$
$\Rightarrow n = 11$
$Var(2, 4, 6, ..., 2m) = 16 \Rightarrow var(1, 2, ..., m) = 4$
$\Rightarrow m^2 - 1 = 48 \Rightarrow m = 7 \Rightarrow m + n = 18$

74. $\lim_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ is equal to _________

Ans. 36

Sol. Let $3^{x/2} = t$

$$\lim_{t \to 3} \frac{t^2 + \frac{27}{t^2} - 12}{\frac{1}{t} - \frac{3}{t^2}} = \lim_{t \to 3} \frac{t^4 + 27 - 12t^2}{t - 3}$$

$$\lim_{t \to 3} \frac{(t^2 - 3)(t + 3)(t - 3)}{(t - 3)} = 6 \times 6 = 36$$

75. Let $A(1, 0), B(6, 2)$ and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$ is ______________

Ans. 5

Sol. P will be centroid of ΔABC

$$P\left(\frac{17}{6}, \frac{8}{3}\right) \Rightarrow PQ = \sqrt{\left(\frac{24}{6}\right)^2 + \left(\frac{9}{3}\right)} = 5$$