FIITJEE

Solutions to JEE (Main)-2020

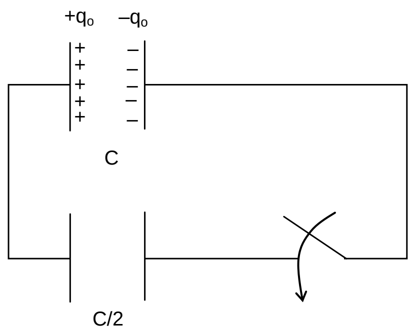
JEE–Main–2020 –Sept–4–Second–Shift

PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. **A**

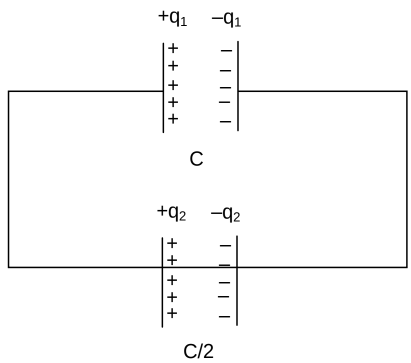
Sol. $Q_o = CV_o$



$U_i = \frac{1}{2}CV_0^2$

$q_1 = \frac{q_o C}{C + \frac{C}{2}} = \frac{2q_o}{3}$

$q_2 = \frac{q_o \frac{C}{2}}{C + \frac{C}{2}} = \frac{q_o}{3}$; $U_f = \frac{q_1^2}{2c} + \frac{q_2^2}{\frac{2c}{2}}$



$= \frac{4q_0^2}{9 \times 2c} + \frac{q_0^2}{9c}$

$= \frac{6q_0^2}{18c} = \frac{q_0^2}{3c} = \frac{cv_0^2}{3}$

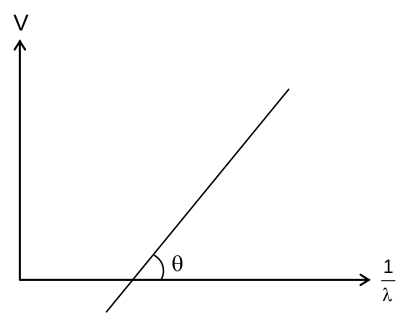
Energy loss in the process = $U_i - U_f$

$= \frac{1}{2}CV_0^2 - \frac{CV_0^2}{3}$

$= \frac{CV_0^2}{6}$

2. **D**

Sol. $eV_{slop} = \frac{hc}{\lambda} - \phi$



Slope of curve

$\tan\theta = \frac{hc}{e} = \text{constant}$

as intensity of incident radiation is increased, there will be no effect on graph

3. **A**

Sol. $I_1 \propto R_1^2$ & $I_2 \propto R_2^2$

$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2 = \frac{1}{\alpha^2} = \frac{1}{16}$$

$\alpha = 4$

4. **None of the options**

Sol. I = 0.8 kg M^2, $\left|\vec{\mu}\right|$ = 20 Am^2

$U_i = -\vec{\mu} \cdot \vec{B} = 0$

$U_f = -\mu B \cos(30°) = -20 \times 4 \times \frac{\sqrt{3}}{2}$

$U_i - U_f = 40\sqrt{3} = \frac{1}{2} I\omega^2 = 0.4\, \omega^2$

$\omega^2 = 100\sqrt{3}$; $\omega = 10\,(3^{1/4})$

5. **A**

Sol. Binding energy = $(50\ M_p + 70\ M_n - M_{sn})C^2$

= $(50.3915 + 70.6069 - 119.902199)UC^2$

= $(1.0962\ U)C^2$

= 931 × 1.0962 MeV

Binding energy per neucleon

$= \frac{931 \times 1.0962}{120}$ MeV

= 8.5 MeV

6. **C**

Sol. $x = \frac{IFV^2}{WL^4}$

$I = [ML^2]$

$F = [MLT^{-2}]$

$V^2 = [L^2T^{-2}]$

$W = [M\ L^2T^{-2}]$

$Q^4 = [L^4]$

$X = [M\ L^{-1}\ T^{-2}]$

7. **C**

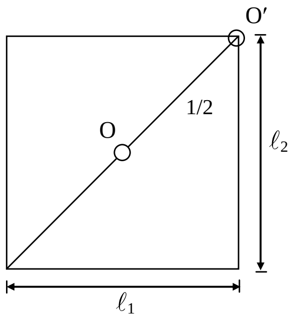
Sol. $\ell_1 = 0.6$ M

$\ell_2 = 0.8$ M

$\sqrt{\ell_1^2 + \ell_2^2} = 1$

MI about 0 = $\frac{M}{12}\left(\ell_1^2 + \ell_2^2\right)$

$I_1 = \frac{M}{12}$



MI about O' = $\frac{M}{12}\left(\ell_1^2 + \ell_2^2\right) + \frac{M\left(\ell_1^2 + \ell_2^2\right)}{4}$

$I_2 = \frac{M}{12} + \frac{M}{4} = \frac{4m}{12}$; $\frac{I_1}{I_2} = \frac{1}{4}$

8. **A**

9. **A**

Sol. $E = E_o(1 - ax^2)$

$F = qE_o$

$\text{acceleration} = \frac{F}{m} = \frac{qE_o}{m}(1 - ax^2) = v\frac{dv}{dx}$

$\frac{qE_o}{m}\int_0^x (1 - ax^2)dx = \int_0^0 vdv$; $\frac{qE_0}{M}\left(x - \frac{ax^3}{3}\right) = 0$

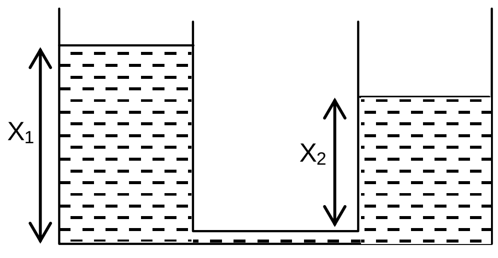
$x\left(1 - \frac{ax^3}{3}\right) = 0$; $x = 0$ & $x = \sqrt{\frac{3}{a}}$

10. **D**

Sol. $i_1 = \frac{8}{8} = 1$ A

11. **C**

Sol. $U_i = sx_1dg\frac{x_1}{2} + sx_2dg\frac{x_2}{2}$



$U_f = \frac{S(x_1 + x_2)gd}{2}\left(\frac{x_1 + x_2}{4}\right)x2$

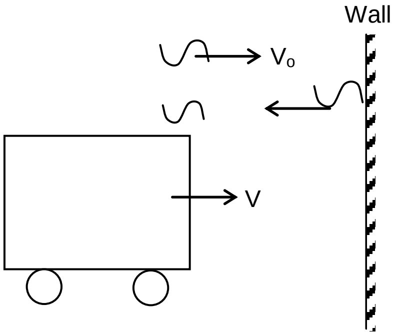
$= \frac{s(x_1 + x_2)^2 gd}{4}$

$$U_i - U_f = \frac{5gd}{4}\left\{2x_1^2 + 2x_2^2 - (x_1 + x_2)^2\right\}$$

$$= \frac{5gd}{4}(x_1 - x_2)^2$$

12. **A**

Sol. $f_1 = 420$ Hz



$$f_2 = \left(\frac{V_o + V}{V_o - V}\right) f_1 = 490$$

$$\left(\frac{330 + V}{300 - V}\right) 420 = 490$$

(330 + V) 6 = 7 (330 – V)

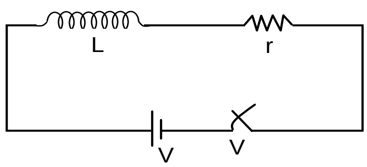
13 V = 330

$$V = \frac{33}{13}(m/s)$$

= 91 (km/hr)

13. **4**

Sol.



$$i = \frac{V}{r}\left\{1 - e^{\frac{-t}{L}}\right\}$$

$$i_{max} = \frac{V}{r}$$

$$U = \frac{1}{2}Li^2$$

$$= \frac{1}{2}L\frac{V^2}{r^2}\left\{1 - e^{-rt/L}\right\} = \frac{1}{n} \times \frac{L}{2}\frac{V^2}{r^2}$$

$$1 - e^{-rt/L} = \frac{1}{\sqrt{n}} \quad ; \quad e^{-rt/L} = 1 - \frac{1}{\sqrt{n}} = \frac{\sqrt{n} - 1}{\sqrt{n}}$$

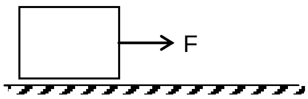
$$e^{rt/L} = \ln\frac{\sqrt{n}}{\sqrt{n} - 1} \quad ; \quad \frac{rt}{L} = \ln\frac{\sqrt{n}}{\sqrt{n-1}}$$

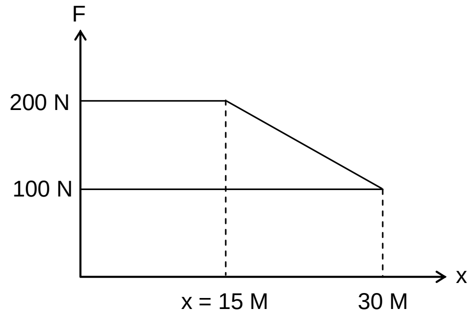
$$T = \frac{L}{r}\ \ln\left(\frac{\sqrt{n}}{\sqrt{n-1}}\right)$$

14. **C**

15. **A**

Sol.





Network done is equal to area under F-x curve

$= 200 \times 15 + \frac{1}{2} \times 100 \times 15 + 100x15$

$= 4500 + 750$

$= 5250$

16. **B**

Sol. From aerie's law $x = \frac{c}{T}$

$I = x H$

$I_1 = x_1 H_1$

$I_2 = x_2 H_2$

$$\frac{I_2}{I_1} = \frac{\chi_2 H_2}{\chi_1 H_1}$$

$$\frac{I_2}{I_1} = \frac{\chi_2 H_2}{\chi_1 H_1}$$

$$\frac{I_2}{I_1} = \frac{T_1 H_2}{T_2 H_1} = \frac{4 \times 0.3}{24 \times 0.4} = \frac{1}{8}$$

$$I_2 = \frac{T_1}{8} = \frac{6}{8} = 0.75$$

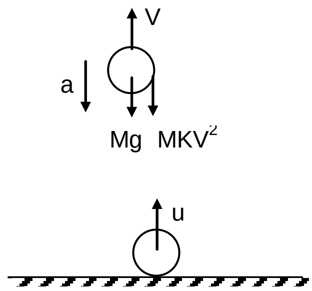
17. **D**

Sol. Escape velocity = $\sqrt{\frac{2GM}{R}} = V_{esf}$

Orbital speed = $\sqrt{\frac{GM}{R}} = V_o$

$$\frac{V_o}{V_{esp}} = \frac{1}{\sqrt{2}}$$

18. **D**



Sol. $Mg + MkV^2 = ma = -mv\dfrac{dV}{dx}$

$Vdv = (-)\,(g + kV^2)dx$

$$\int_a^0 \frac{Vdv}{g+kv^2} = \int_0^x -dx$$

$$\frac{\ln (g+kV^2)\Big|_u^0}{2k} = -x$$

$$\ln\left(\frac{2}{g+ku^2}\right) = -2kx$$

$$X = \frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$$

19. **A**

Sol. ΔQ = heat supplied

ΔW = work done

ΔU = change in internal energy

(i) adiabatic	(B) $\Delta\theta = 0$
(ii) isothermal	(D) $\Delta U = 0$
(iii) isochoric	(A) $\Delta W = 0$
(iv) isobaric	(C) $\Delta U \neq 0$, $\Delta W \neq = 0$, $\Delta Q \neq 0$

20. **C**

Sol. $B\dfrac{\Delta V}{V} = \Delta P$

$$\frac{\Delta V}{V} = \frac{\Delta P}{B} = \frac{4\times 10^9}{8\times 10^{10}} = \frac{1}{20}$$

$V = \ell^3$

$dV = 3\ell^2\, d\ell$

$$\frac{dV}{V} = \frac{3\ell^2}{\ell^3}d\ell = \frac{3d\ell}{\ell}$$

$$\frac{\Delta V}{V} = \frac{3\Delta\ell}{\ell} \quad ; \quad \frac{\Delta\ell}{\ell^2} = \frac{\Delta V}{3V} = \frac{1}{60}$$

$$\%\ \frac{\Delta\ell}{\ell} = \frac{100}{60} = 1.67\%$$

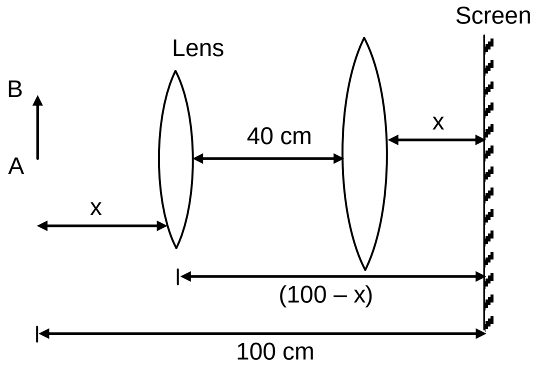
21. **20.00**

Sol. Distance moved = Area under curve

$$= \frac{1}{2}\times 8\times 5 = 20$$

22. **476**

Sol. $P = \frac{1}{f} = \left(\frac{N}{100}\right)D$



2x + 40 = 100

x = 30 cm

100 – x = 70 cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{70} - \frac{1}{-30} = \frac{1}{f}$

$\frac{1}{f} = \frac{1}{70} + \frac{1}{30} = \frac{3+7}{210} = \frac{1}{21}$

f = 21 cm = 0.21 M

Power = $\frac{1}{f} = \frac{1}{0.21}$; $D = \left(\frac{100}{21}\right)D$

$\frac{N}{100}D = \frac{100}{21}D$

N = $\frac{10000}{21}$ = 476.19

$N \simeq 476$

23. **150**

Sol. T = constant

PV = nRT

PdV + vdP = 0

$dV = (-)\frac{vd\rho}{P}$

$|\Delta V| = V\frac{\Delta P}{P}$

$V\frac{\Delta P}{P} = \frac{nR\Delta T}{P}$

$\Delta T = \frac{V}{nR}\Delta P$

$C = \frac{V}{nR} = \frac{T}{P} = \frac{300}{p}$ = 150

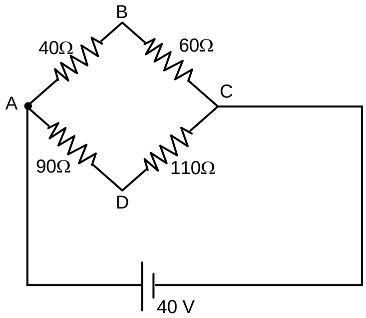
P = constant

PdV = nRdT

$\Delta v = \frac{nR\Delta T}{P}$

24. **2.00**

Sol. $i_1 = \frac{40}{100} = \frac{2}{5}$



$i_2 = \frac{40}{200} = \frac{1}{5}$

$V_A - V_B = 40\ i_1 = 40 \times \frac{2}{5}$

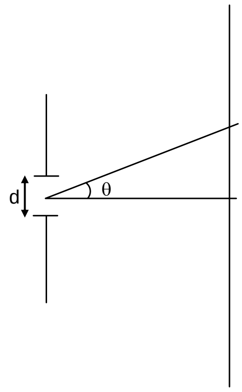
$V_A - B_B = 16$

$V_A - V_D = 90I_2 = \frac{90}{5} = 18$

$V_B - V_D = 18 - 16 = 2$ volt

25. **200.00**

Sol. $\lambda = 6 \times 10^{-7}$ M



$d = 6 \times 10^{-5}$ M

$I = I_o \left\{\frac{\sin(B)}{\beta}\right\}^2$; $\beta = \frac{\pi d \sin\theta}{\lambda}$

$\theta = \frac{\pi}{2}$; $\beta = \frac{\pi d}{\lambda}$

$= \frac{\pi\ 6 \times 10^{-5}}{6 \times 10^{-7}} = 100\ \pi$

So at ∞ also minima will form total number of minima = 2 × 100 = 200

PART –B (CHEMISTRY)

26. D

Sol. $[Co(H_2O)_3F_3]$ $Co^{3+} = 3d^6 4s^0 \Rightarrow t_{2g}^{2,1,1}, e_g^{1,1}$

CFSE = $[-0.4n_{t_{2g}} + 0.6n_{eg}]\Delta_0 + n(P)$

$= [-0.4 \times 4 + 0.6 \times 2]\Delta_0 + 0 = -0.4\ \Delta_0$

27. C

Sol. In Calcination and roasting CO_2 and SO_2 are released which are responsible for Global warning and acid rain.

28. C

Sol. Charge(q) = $\frac{it}{96500}F = \frac{1 \times 15 \times 60}{96500} = \frac{900}{96500} = \frac{9}{965}F = 0.0093F$

No. of moles of Au^+ = 0.025 & No. of moles of Ag^+ = 0.025

Species with higher value of SRP will get deposited first at cathode.

(i) $Au^+(aqs) + e^- \longrightarrow Au(s)$

0.025 0.0093 mole

So only Au will get deposited

29. B

Sol. Due to resonance C-Cl bond in option B is shortest.

30. A

Sol. H can easily gain electron to form its anion.

31. A

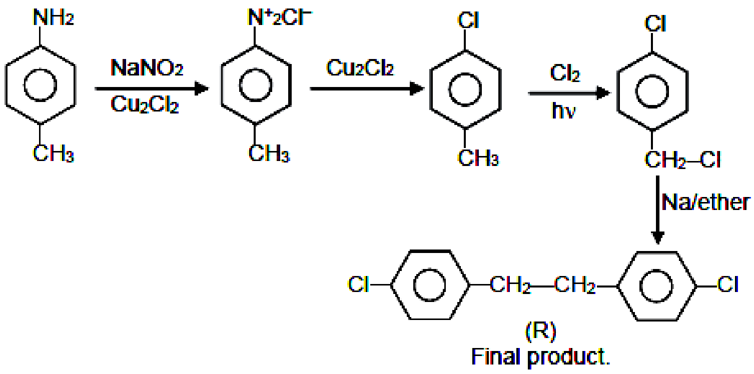
Sol. On adding reaction equilibrium constant will get multiplied.

32. C

Sol. S_N1 reaction depends on carbocation stability and cation form in 3 will be most stable.

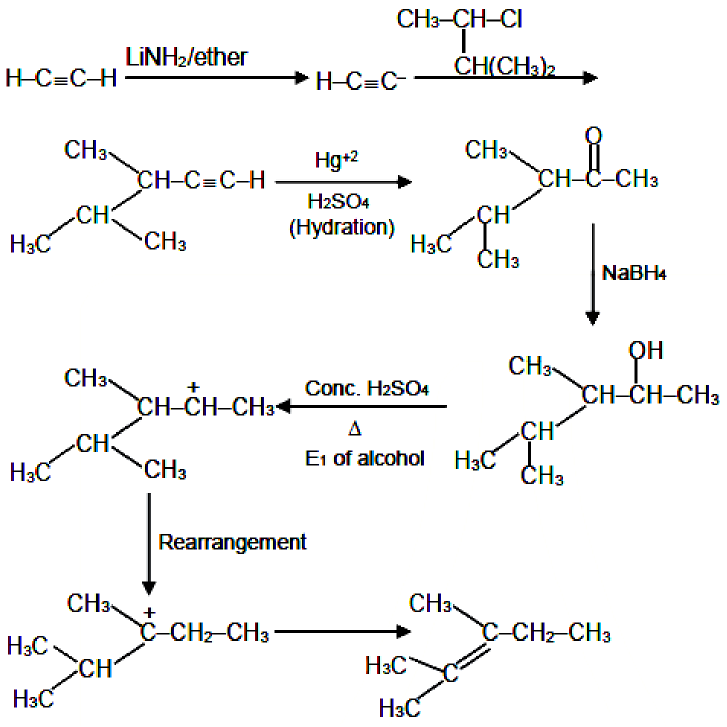
33. C

Sol.



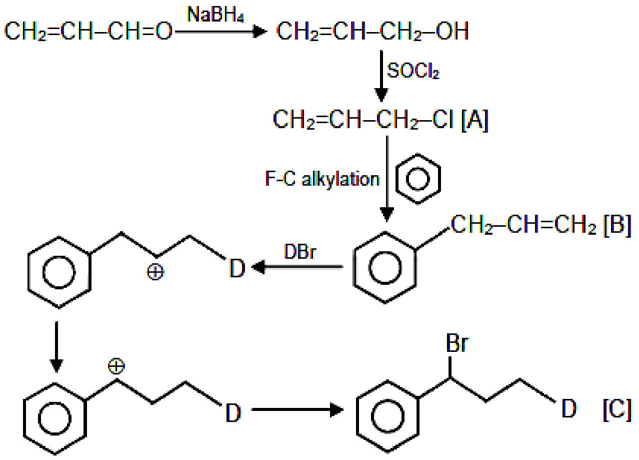
34. D

Sol.



35. A

Sol.



36. A

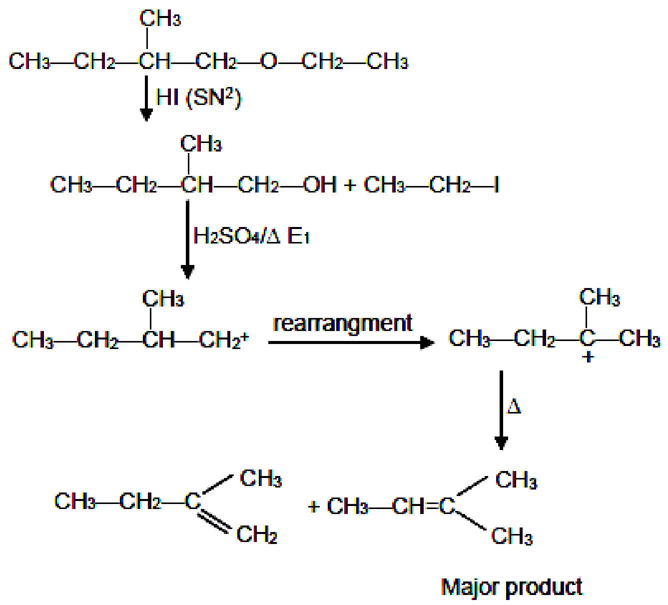
Sol. Egg white will stabilize blue ink easily.

37. B

Sol. Cobalt has 2 unpaired electron

38. B

Sol.



39. A
Sol. Terfenadine act as antihistamine.

40. B
Sol. BeO is hexagonal wurtzite type structure.

41. C
Sol. $KMnO_4$ oxidise HCl to Cl_2.

42. C
Sol. In $[Ni(CN)_4]^{2-}$ hybridization is dsp^2 remaining are SP^3d^2

43. A
Sol. $\underset{sp^3d^2}{XeF_4} + SbF_5 \longrightarrow \underset{sp^3d}{[XeF_3]^+}[SbF_6]^-$

44. C
Sol. For hydrogen atom :

For Lyman series $n_1 = 1$ & $n_2 = \infty$

$$\frac{1}{\lambda_H} = R_H\left[\frac{1}{1} - \frac{1}{\infty}\right] \qquad \text{So,} \quad \lambda = \frac{1}{R_H}$$

For He^+ ion

Balmer series $n_1 = 2$ & $n_2 = 3$

$$\frac{1}{\lambda_{He^+}} = R_H \times Z^2\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\frac{1}{\lambda_{He^+}} = R_H \times 4 \times \frac{5}{36}$$

$$\frac{1}{\lambda_{He^+}} = \frac{5}{9}R_H = \left(\frac{5}{9}\right)\frac{1}{\lambda}$$

$$(\lambda_{He^+}) = \frac{9}{5}\lambda$$

45. C
Sol. P_{ext} is zero so W = zero

46. 84297 J
Sol.
$$k = Ae^{-\frac{Ea}{RT}}$$

$$\ln\left(\frac{K_2}{K_1}\right) = \frac{Ea}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln(5) = \frac{Ea}{8.314}\left[\frac{1}{300} - \frac{1}{315}\right]$$

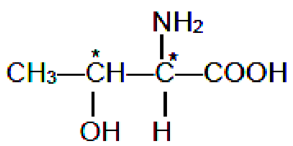
$$1.6094 = \frac{Ea}{8.314}\left[\frac{15}{300 \times 315}\right]$$

$$Ea = 84297J$$

47. 19
Sol.
(i) $2Fe^{2+} + H_2O_2 \longrightarrow 2Fe^{3+} + 2OH^-$

(ii) $2MnO_4^- + 5H_2O_2 + 6H^+ \longrightarrow 2Mn^{2+} + 5O_2 + 8H_2O$

So sum of $(x + y + x^1 + y^1 + z^1) = 2 + 2 + 2 + 5 + 8 = 19$

48. 2
Sol.


Threonine have two chiral carbon atom.

49. 10
Sol. Equivalent of solute = 0.1 × 0.1

Mole of solute ($Na_2CO_3.xH_2O$) = $[0.1 \times 0.1]\frac{1}{2}$

Mass of $Na_2CO_3.xH_2O$ = $[0.1 \times 0.1]\frac{1}{2} \times [106 + 18x] = 1.43$

$\Rightarrow$ $[106 + 18x = 286$

$18x = 180$

$x = 10$

50. 167
Sol.
$$\Pi = i\,CRT = i\left[\frac{n}{V}\right]RT$$

$$\Pi_{final} = \frac{(\pi_1 V_1) + (\pi_2 V_2)}{V_1 + V_2}$$

$$\Pi_{final} = \frac{(0.1 \times 1) + (0.2 \times 2)}{3}$$

$$= \frac{(0.1 + 0.4)}{3} = \frac{0.5}{3} = \frac{500}{3} \times 10^{-3}\,atm$$

so X = 167

PART–C (MATHEMATICS)

51. A

Sol. $(2+\alpha)^4 = \left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^4$

$= \left[\sqrt{3}e^{i\left(\frac{\pi}{6}\right)}\right]^4$

$= 9\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

$= \frac{-9}{2}+\frac{9\sqrt{3}i}{2}$

$\Rightarrow 0+9\left(\frac{-1+i\sqrt{3}}{2}\right)$

$\therefore a=0, b=9$

Answer A

52. C

Sol. Contrapositive of $p \to q$ is $\sim q \to \sim p$.

53. D

Sol. ${}^{N+5}C_{R-1} : {}^{N+5}C_R : {}^{N+5}C_{R+1}$

$= 5 \quad : \quad 10 \quad : \quad 4$

$2\left({}^{N+5}C_{R-1}\right) = {}^{N+5}C_R \Rightarrow 3R = N+6$

$7\left({}^{N+5}C_R\right) = 5\left({}^{N+5}C_{R+1}\right) \Rightarrow 12R = 18+5N$

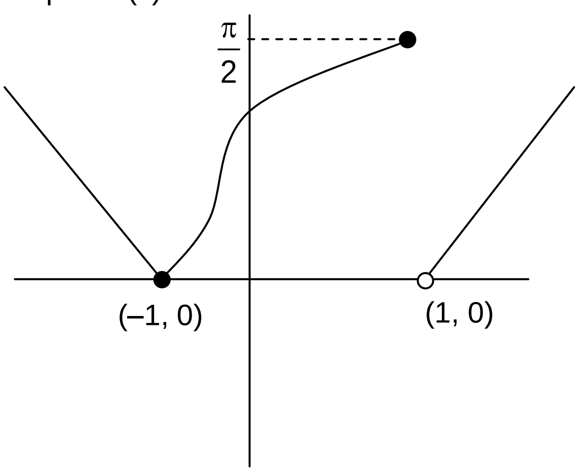
Solving: $N=6,\ R=4$

$\therefore$ Largest coefficient is ${}^{N+5}C_{R+1} = {}^{11}C_5 = 462$

Answer is Option D.

54. C

Sol. Graph of f(x) is



Option C is correct.

55. B

Sol. Here, $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = \frac{9}{2}$

Also, $\begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & \mu \end{vmatrix} = 0 \Rightarrow \mu = 5$

$\therefore$ Option B is correct.

56. A

Sol. Given: $3\alpha^2 - 10\alpha + 27\lambda = 0$ ……….(i)

$3\alpha^2 - 3\alpha + 6\lambda = 0$ …………(ii)

Subtract $-7\alpha + 21\lambda = 0$

$3\lambda = 0$

By (ii) $9\lambda^2 - 3\lambda + 2\lambda = 0$

$\Rightarrow \lambda = 0, \frac{1}{9}$

$\therefore \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \alpha = \frac{1}{3}, \gamma = 3$

$\therefore \frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}.3}{\frac{1}{9}} = 18$

57. B

Sol. By family of circle, passing through intersection of given circle will be member of $S + \lambda S, = 0$ family $(\lambda \neq 1)$

$(x^2 + y^2 - 6x) + \lambda(x^2 + y^2 - 4y) = 0$

$(\lambda + 1)x^2 + (\lambda + 1)y^2 - 6x - 4\lambda y = 0$

$x^2 + y^2 - \frac{6}{\lambda + 1} \times - \frac{4\lambda}{\lambda + 1} y = 0$

Centre $\left(\frac{3}{\lambda + 1}, \frac{2\lambda}{\lambda + 1}\right)$

Centre lies on i $2x - 3y + 12 = 0$

$2\left(\frac{3}{\lambda + 1}\right) - 3\left(\frac{2\lambda}{\lambda + 1}\right) + 12 = 0$

$6\lambda + 18 = 0$

$\lambda = -3$

Circle $x^2 + y^2 - 3x - 6y = 0$

58. A

Sol. $\int_{\pi/6}^{\pi/3} \left(\frac{\frac{d}{dx}\left(\tan^4 x\right)}{2} . \sin' 3x + \tan' x . \frac{\frac{d}{dx}\left(\sin^4 3x\right)}{2} \right)$

$$=\frac{1}{2}\int_{\pi/6}^{\pi/3}\frac{d}{dx}\left(\tan^4 x.\sin^4 3x\right)dx$$

$$=\frac{1}{2}\left[\tan^4 x.\sin^4 3x\right]_{\pi/6}^{\pi/3}$$

$$=\frac{1}{2}\left[\left(\sqrt{3}\right)^4.O-\frac{1}{\left(\sqrt{3}\right)^4}\right]$$

$$=-\frac{1}{18}$$

59. A

Sol. $A(\alpha,0),B(-\beta,0)$

$\Rightarrow D\left(\alpha,\alpha^2-1\right)$

Area (ABCD) = (AB) (AD)

$\Rightarrow S=(2\alpha)\left(1-\alpha^2\right)=2\alpha-2\alpha^2$

$\frac{ds}{d\alpha}=2-6\alpha^2$

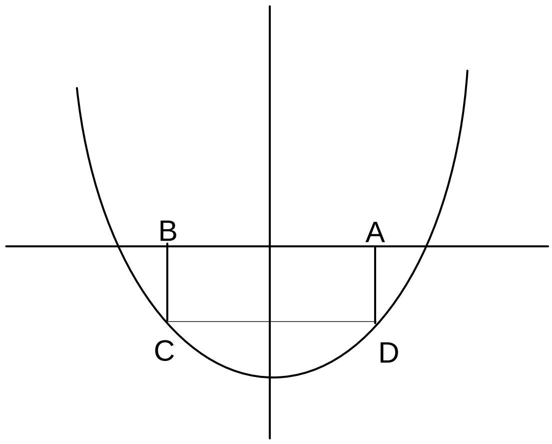
$=0\Rightarrow a^2$

$=\frac{1}{3}$

$\Rightarrow \alpha=\frac{1}{\sqrt{3}}$

Area = $2\alpha-2\alpha^2=\frac{2}{\sqrt{3}}-\frac{2}{3\sqrt{3}}$

$=\frac{4}{3\sqrt{3}}$



60. C

Sol. Let $A=\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}$

$Ax_1=B_1$

$a_1+a_2+a_3=1$

$b_1+b_2+b_3=0$

$c_1+c_2+c_3=0$

Similar $2a_3+a_3=0$ and $a_3=0$

$2b_2+b_3=2$ $\quad b_3=0$

$2c_2+c_3=0$ $\quad c_3=2$

$\therefore a_2=0,b_2=1,c_2=-1,$

$a_1=1,\ b_1=-1,\ c_1=-1$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix} \therefore |A| = 2$$

61. B

Sol. $\frac{dy}{dx} - \frac{y-3x}{\ln(y-3x)} + 3 = 0$

$\frac{dy}{dx} + 3 = \frac{y+3x}{\ln(y+3x)}$

$\frac{d}{dx}(y+3x) = \frac{y+3x}{\ln(y+3x)}$

$\int \frac{\ell n(y+3x)}{(y+3x)} d(y+3x) = \int dx$

Let $\ell n(y+3x) - t$

$\frac{1}{(y+3x)} d(y+3x) = dt$

$\int t\,dt = \int dx$

$\frac{t^2}{2} = x + c$

$\frac{(\ell n(y+3x))^2}{2} = x + c$

62. D

Sol. Since AM two positive quantities $\geq$ their G.M.

$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x}.2^{\cos x}}$

$= \sqrt{2^{\sin x + \cos x}} = \sqrt{2^{\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)}}$

$\geq \sqrt{2^{-\sqrt{2}}} \Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2.2^{1//\sqrt{2}} = 2^{1-1/\sqrt{2}}$

63. B

Sol Applying L – Hospital Rule

$\lim_{t \to x} \frac{2t f^2(x) - x^2(2f(t)f'(t))}{1}$

$\therefore 2 \times f^2(x) - x^2(2f(x)f'(x)) = 0$

$\Rightarrow f(x) - x f'(x) = 0$

$\Rightarrow \frac{f('x)}{f(x)} = \frac{1}{x} \Rightarrow \ell n f(x) = \ell n x + C$

At $x = 1, c = 1$

$\therefore \ell n f(x) = \ell n x + 1$

when $f(x) = 1$

then $\ell n x = -1$

$x = \frac{1}{e}$

64. D

Sol. Equation PQ

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$

Let $Q = (2\lambda+1, 3\lambda-2, -6\lambda+3)$

Q lies on $x - y + z = 5$

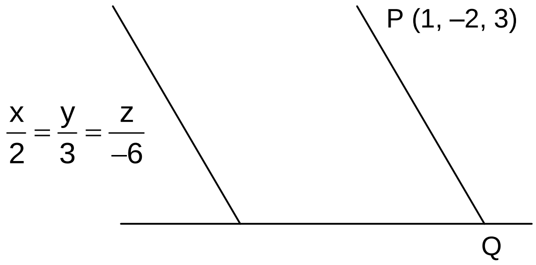
$= (2\lambda+1) - (3\lambda-2) + (-6\lambda+3) = 5$

$\Rightarrow \lambda = -\frac{1}{7}$

$Q = \left(\frac{5}{7}, \frac{-17}{7}, \frac{15}{7}\right)$

$\therefore PQ = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$

$PQ = 1$



65. C

Sol. sum 6 $\rightarrow$ (1, 5), (5, 1), (3, 3), (2, 4), (4, 2)

sum 7 $\rightarrow$ (1, 6), (6, 1), (5, 2), (2, 5), (3, 4), (4, 3)

$= P(A) + P(\bar{A}).P(\bar{B}).P(A) + P(\bar{A})P(\bar{B})P(\bar{A}).P(\bar{B}).P(A) + ...$

This is infinite G.P. with common ratio $P(\bar{A}) \times P(\bar{B})$

Probability of A wins $= \frac{P(A)}{1 - P(\bar{A})P(\bar{B})}$

$= \frac{\frac{5}{36}}{1 - \frac{31}{36}.\frac{30}{36}} = \frac{30}{61}$

66. C

Sol. $\tan 30^o = \frac{x}{y} = \frac{1}{\sqrt{3}} \Rightarrow y = \sqrt{3}x$ ....(i)

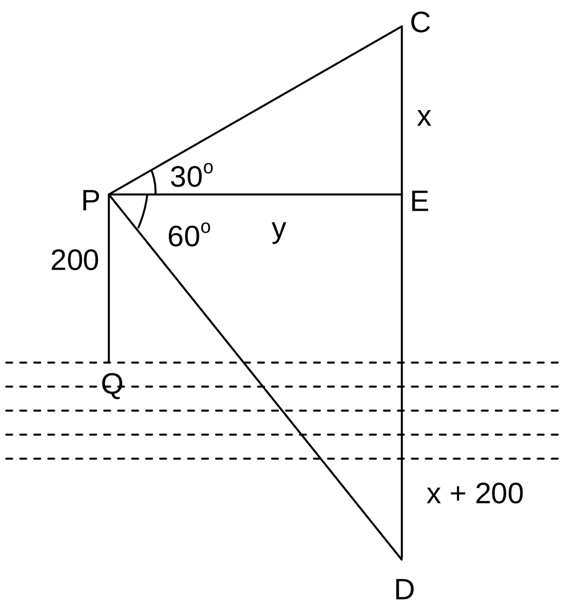
and $\tan 60^o = \frac{x+400}{y} \Rightarrow \sqrt{3}y$

$= x + 400$ ....(ii)

Solving (i) and (ii), we get

$2x = 400, x = 200$

$\sin 30^o = \frac{x}{PC} = \frac{200}{PC} \Rightarrow PC = 400$



67. A

Sol. Given : $\frac{a}{e}=4$ and $\frac{1}{4}=1-\frac{b^2}{a^2}$

Solving : $a=2, b=\sqrt{3}$

Parametric co – ordinates are

$\left(2\cos\theta, \sqrt{3}\sin\theta\right)=\left(1,\beta\right)$

$\therefore \theta=60^o$

$\therefore$ Equation of normal is $a\,x\sec\theta$ – by $\operatorname{cosec}\theta=a^2-b^2$

$\Rightarrow \quad 4x-2y=1$

68. B

Sol. Let number of elements in T is R.

$\therefore$ $20R=500 \Rightarrow R=25$

and $6R=5N \Rightarrow N=30$

69. C

Sol. Any point (x, y) on perpendicular bisector equidistant from p and q

$\therefore (x-1)^2+(y-4)^2=(x-k)^2+(y-3)^2$

At $x=0$, $y=-4$

$\therefore$ $1+64=k^2+49$

$k^2=16$

70. A

Sol. Given,

$300=1+(N-1)d$

$\Rightarrow (N-1)d=299$

$\therefore (N,d)=(24,13)$ is the only possible pair

$\therefore$ $a_{20}=1+19(13)=248$ and, $S_{20}=\frac{1+248}{2}\times 20$

$=2490$

71. 18

Sol. Let $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$

$\hat{i}\times\left(\vec{a}\times\hat{i}\right)=\left(\hat{i}.\hat{i}\right)\vec{a}-\left(\vec{a}.\hat{i}\right)=y\hat{j}+z\hat{k}$

Similarly $\hat{j}\times\left(\vec{a}\times\hat{j}\right)=x\hat{i}+z\hat{k}$ and $\hat{k}\times\left(\vec{a}\times\hat{k}\right)=x\hat{i}+y\hat{k}$

$\left|\hat{i}\times\left(\vec{a}\times\hat{i}\right)\right|^2+\left|\hat{j}\times\left(\vec{a}\times\hat{j}\right)\right|^2+\left|\hat{k}\times\left(\vec{a}\times\hat{k}\right)\right|^2$

$=\left|y\hat{j}+z\hat{k}\right|^2+\left|x\hat{i}+z\hat{k}\right|^2+\left|x\hat{i}+y\hat{j}\right|^2=2|a|^2=2(9)=18$

72. 135

Sol. Ways of selecting correct questions $={}^6C_4=15$

Ways of doing them correct = 1

Ways of doing remaining 2 questions incorrect $=3^2=9$

$\therefore$ No. Of ways $=15\times 1\times 9=135$

73. 2

Sol. Let $P(2\cos\theta, 2\sin\theta)$

$\therefore Q(-2\cos\theta, -2\sin\theta)$

Given line $x + y - 2 = 0$

$\therefore \alpha = \frac{|2\cos\theta + 2\sin\theta - 2|}{\sqrt{2}}$

$\beta = \frac{|-2\cos\theta - 2\sin\theta - 2|}{\sqrt{2}}$

$\therefore \alpha\beta = \sqrt{2}(\cos\theta + \sin\theta - 1).\sqrt{2}(\cos\theta + \sin\theta + 1)$

$= 2\left|\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1\right| = 2|\sin 2\theta|$

Max $|\sin\theta| = 1$

$\therefore$ maximum $\alpha\beta = 2$.

74. 4

Sol.

xi	5	15	25
fi	2	x	2

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{10 + 15x + 50}{4 + x}$

$= \frac{60 + 15x}{4 + x} = 15$

$\sigma^2 = 50 = \frac{\sum f_i x_i^2}{\sum f_i} - (\bar{x})^2$

$50 = \frac{50 + 225x + 1250}{4 + x}(15)^2$

$50 = \frac{1300 + 225x}{4 + x} - 225$

$\Rightarrow 275(4 + x) - 1300 + 225x$

$\Rightarrow 50x = 200x \Rightarrow x = 4$

$50 = \frac{225x + 1250}{4 + x}(15)^2$

$50 = \frac{1300 + 225x}{4 + x} - 225$

$\Rightarrow 275(4 + x) = 1300 + 225x$

$\Rightarrow 50x = 200 \Rightarrow x = 4$

75. 21

Sol. Clearly, $\int_0^n \{x\}\, dx = \frac{n}{2}$

$\int_0^n [x]\, dx = 1 + 2 + 3.....n - 1$

$= \frac{n(n-1)}{2}$

$$\therefore \left(\frac{n(n-1)}{2}\right)^2 = \frac{n}{2}\{10n(n-1)\}$$

Solving, n = 21