FIITJEE

Solutions to JEE (Main)-2020

JEE–Main–2020 –Sept–4–First–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. **D**

Sol. $g = \frac{Ax}{(x^2 + a^2)^{3/2}}$

$\Rightarrow \int_v^0 dV = -\int_x^\infty g\,dx$

$\Rightarrow O - V = -\left[\int \frac{Ax}{(a^2 + x^2)^{3/2}}\right]$

Let, $a^2 + x^2 = t^2$

$\Rightarrow 2xdx = 2t\,dt$

$\Rightarrow xdx = tdt$

$\Rightarrow V = \int \frac{At\,dt}{t^3} \Rightarrow -\frac{A}{t} \Rightarrow -\frac{A}{\sqrt{a^2 + x^2}}\Bigg|_x^\infty$

$\Rightarrow V = \frac{A}{\sqrt{a^2 + x^2}}$

2. **D**

Sol. $eVs = h\nu - \phi$

At $V_s = 0 \Rightarrow h\nu = \phi$

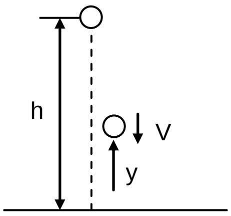
$\Rightarrow \phi = [6.62 \times 10^{-34}]\,[10^{14}]\,[5.5]$

$\Rightarrow \phi = \frac{[6.62 \times 10^{-34}]\,[10^{14}]\,[5.5]\,eV}{\left(1.6 \times 10^{-19}\right]}$

= 2.27

3. **C**

Sol.



$\Rightarrow V^2 = U^2 + 2gS$

$\Rightarrow V^2 = O + 2g\,(h - y)$

$\Rightarrow V^2 = 2gh - 2gy$

$\Rightarrow V = \sqrt{2gh - 2gy}$

4. **A**

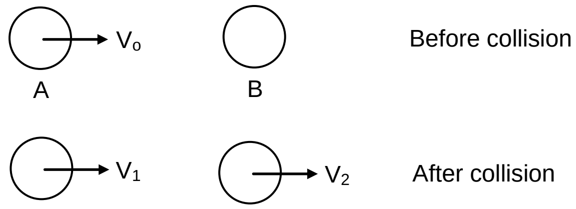
Sol. $K = \frac{\left(\frac{Q}{t}\right)\Delta x}{A\ \ \Delta T}$

$\Rightarrow \frac{(ML^2T^{-2})\,(L)}{(L^2)\,(\theta)\,(T)}$

$\Rightarrow M^1\,L^{1-}\,T^{-3}\,\theta^{-1}$

5. **D**

Sol.



$\Rightarrow \quad \vec{V}_1 = 2\overrightarrow{U_{cm}} - \overrightarrow{U_1}$

$\Rightarrow \quad 2\left[\dfrac{m/2\ \ V_o}{\dfrac{m}{2}+\dfrac{m}{3}}\right] - V_o$

$\Rightarrow \quad \dfrac{6}{5}V_o - V_o \Rightarrow \dfrac{V_o}{5}$

$\Rightarrow \quad \lambda_o = \dfrac{hc}{\left(\dfrac{m}{2}V_o\right)} \qquad \lambda_f = \dfrac{hc}{\left(\dfrac{M}{2}\dfrac{V_o}{5}\right)}$

$\Rightarrow \quad \Delta\lambda = \dfrac{8hc}{mV_o}$

6. **A**

Sol. $B_A = \dfrac{\mu_o I\theta}{4\pi R}$

$\Rightarrow \quad \dfrac{B_A}{B_B} = \dfrac{I_A\theta_A R_B}{I_B\theta_B R_A}$

$\Rightarrow \quad \dfrac{2\left(\dfrac{3\pi}{2}\right)(4)}{3\left[\dfrac{5\pi}{3}\right][2]}$

$\Rightarrow \quad \dfrac{6}{5}$.

7. **B**

Sol. $m(L) = m_1S_1\ (\Delta T)$

$\Rightarrow \quad m(3.4 \times 10^5) = (200)\ (4200)\ (25)$

$\Rightarrow \quad m = 61.7$

8. **A**

9. **D**

Sol. $E = (I)\ (t)\ (A) <\cos^2 \theta>$

$\Rightarrow \quad (3.3)\left[\dfrac{2\pi}{31.4}\right][3\times10^{-4}]\times\dfrac{1}{2}$

$\Rightarrow \quad 0.99 \times 10^{-4}$

10. **D**

Sol. $\vec{\tau} = \vec{\mu}\times\vec{B}$

$\Rightarrow \quad 0.018 = \mu(0.06)\ (\sin 30°)$

$\Rightarrow \quad \mu = 0.6$

$\Rightarrow \quad$ Work $= U_f - U_i$

$\Rightarrow \quad 2\mu B$

$\Rightarrow$ 7.2×10^{-2} J.

11. **A**

Sol. $U_{initial} = \frac{k(4q)(q)}{(d/2)} + \frac{k(q)(-q)}{(d/2)}$

$\Rightarrow$ $\frac{6kq^2}{d}$

$\Rightarrow$ $U_{final} = \frac{4(4q)(q)}{\left(\frac{3d}{2}\right)} + \frac{k(q)(-q)}{(d/2)}$

$\Rightarrow$ $\frac{2}{3}\frac{kq^2}{d}$

$\Rightarrow$ $\Delta U = \left(\frac{2}{3} - 6\right)\frac{kq^2}{d}$ $\Rightarrow$ $\frac{-16}{3}\ \frac{kq^2}{d}$

12. **D**

13. **B**

Sol. Mono atomic $\longrightarrow$ $C_V = \frac{3R}{2}$ $C_p = \frac{5R}{2}$

Di- atomic $\longrightarrow$ $C_V = \frac{5R}{2}$ $C_p = \frac{7R}{2}$
(Rigid)

Di-atomic $\longrightarrow$ $C_V = \frac{7R}{2}$ $C_p = \frac{9R}{2}$
(Non-Rigid)

Tri–atomic $\longrightarrow$ $C_V = 3R$ $C_P = 4R$
(Rigid)

14. **A**

15. **B**

Sol. (n) $\lambda = 5$
(n, m): Integers
$\left(\frac{2m+1}{2}\right)\lambda = \frac{3}{2}$

$\Rightarrow$ $\frac{3/2}{5} = \frac{2m+1}{2n}$

$\Rightarrow$ $3n = 10m + 5$

N, m are integers.

So, m = 1, n = 5, $\lambda = 1$

m = 4 n = 15 $\lambda = \frac{1}{3}$

m = 7 n = 25 $\lambda = \frac{1}{5}$

16. **D**

Sol. $\rho vg - mg = ma$

$\Rightarrow$ $\frac{\rho vg}{m} = g + a$

$\Rightarrow \quad m = \dfrac{\rho v g}{g+a}$

$\Rightarrow \quad \dfrac{10^3\left(\dfrac{4}{3}\pi \times 10^{-6}\right)(9.8)}{9.898}$

$\Rightarrow$ 4.15 gm

17. **D**

Sol. $\vec{U} = 5\hat{j}$

$\vec{a} = 10\hat{i} + 4\hat{j}$

$\Rightarrow \quad \vec{S} = \vec{U}t + \dfrac{1}{2}(\vec{a})t^2$

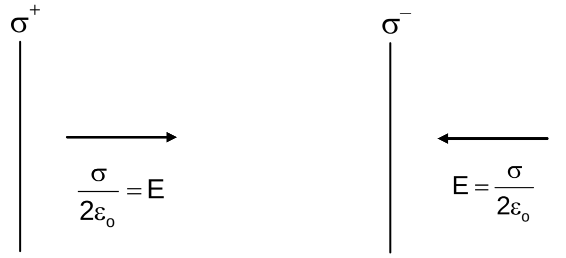
$\Rightarrow \quad 20\hat{i} + y_o\hat{j} = (5t^2)\hat{i} + (5t + 2t^2)\hat{j}$

$20 = 5t^2$; $y_o = 5t + 2t^2$

$t = 2$ $\Rightarrow$ 18 m.

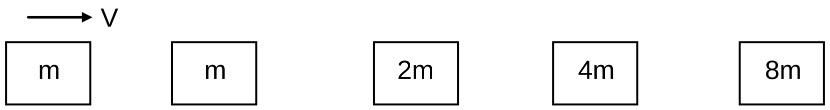
18. **D**

Sol. $\Rightarrow$ Electric field due to infinite sheet is uniform.



19. **D**

Sol.



$\Rightarrow$ mv = 16 mv$_1$

$\Rightarrow \quad V_1 = \dfrac{V}{16}$

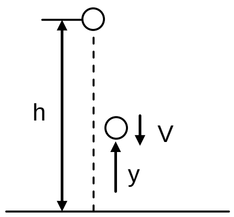
$\Rightarrow \quad \Delta k \text{ loss} = \dfrac{1}{2}mv^2 - \dfrac{1}{2}(16m)\left(\dfrac{V}{16}\right)^2$

$\Rightarrow \quad \dfrac{1}{2}mv^2\left(\dfrac{15}{16}\right)$

% loss = $\dfrac{15}{16} \times 100$ = 93.75%

20. **B**

Sol.



(1) $\varepsilon = 3$

(2) $\varepsilon - Ir = 2.5$ V

$\Rightarrow$ Ir = 0.5

Now, IR = 2.5

$\Rightarrow \quad \dfrac{R}{r} = 5.$

$\Rightarrow \quad \frac{P_R}{R_r} = \frac{I^2R}{I^2r} = \frac{R}{r} = 5$

$\Rightarrow \quad P_r = \frac{0.5}{5} = 0.1$

21. **266.67**

Sol. $\Rightarrow \quad n_1T_1 + n_2T_2 = nT$

$\Rightarrow \quad (0.1)(200) + (0.05)(400) = (0.15)T$

$\Rightarrow \quad T = 266.67$

22. **6.25**

Sol. $M = \frac{1}{f_o}\left[1 + \frac{D}{Fe}\right]$

$\Rightarrow \quad 100 = \frac{20}{1}\left[1 + \frac{25}{Fe}\right]$

$\Rightarrow \quad 1 + \frac{25}{Fe} = 5$

$\Rightarrow \quad Fe = \frac{25}{4}$ = 6.25 cm

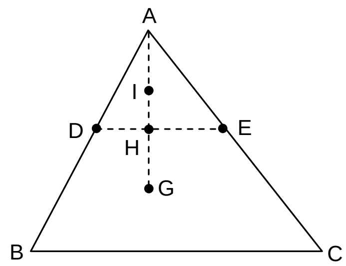
23. **11**

Sol.

$AH \rightarrow \frac{H}{2}$ $\qquad H = \frac{\sqrt{3}a}{2}$

$AG \rightarrow \frac{H}{\sqrt{3}}$ $\qquad AI = \frac{H}{2\sqrt{3}}$

$M_{ABC} = M$ $\qquad M_{ADE} = \frac{M}{4}$



$\Rightarrow \quad I_G = \frac{Ma^2}{12} - \left[\frac{M}{4}\frac{\left[\frac{a}{2}\right]^2}{12} + \frac{M}{4}\left[\frac{a}{2\sqrt{3}}\right]^2\right] = \left(\frac{11}{16}\right)\frac{Ma^2}{12}$

24. **10553.33**

Sol. $\frac{1}{\lambda_{min}} = R\left[\frac{1}{1} - \frac{1}{\infty}\right] = R$ $\qquad [n = \infty \rightarrow n = 1]$

$\frac{1}{\lambda_{max}} = R\left[\frac{1}{1} - \frac{1}{4}\right] = \frac{3R}{4}$ $\qquad [n = 2 \rightarrow n = 1]$

$\Rightarrow \quad \Delta\lambda \quad \Rightarrow \quad \frac{4}{3R} - \frac{1}{R} \quad \Rightarrow \quad \frac{1}{3R} = 340 \quad \ldots(1)$

For Paschan $\Rightarrow \frac{1}{\lambda_{min}} = R\left[\frac{1}{9}\right]$ $\qquad [n = \infty \rightarrow h = 3]$

$\Rightarrow \quad \frac{1}{\lambda_{max}} = R\left[\frac{1}{9} - \frac{1}{16}\right] = \frac{7R}{144}$

$\Rightarrow \quad \Delta\lambda = \frac{81}{7R} \quad \ldots(2)$

25. **20**

Sol. Angular momentum conservation:

$\Rightarrow$ $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

$\Rightarrow$ $\frac{MR^2}{2}\omega_o = \left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\omega_f$

$\Rightarrow$ $\omega_f = \frac{4}{5}\omega_o$

$\Rightarrow$ $KE_{final} = \frac{1}{2}(I_1 + I_2){\omega_f}^2 = \frac{MR^2\omega^2}{5}$

$\Rightarrow$ $KE_{initial} = \frac{1}{2}I_i\,\omega_0^2 = \frac{MR^2\omega^2}{4}$

$\Rightarrow$ % loss $\Rightarrow$ 20%.

PART –B (CHEMISTRY)

26. A

Sol. (a) In extraction of iron lime stone is added on a flux

$$CaCO_3 \longrightarrow CaO + CO_2$$

$$CO_2 + SiO_2 \longrightarrow CaSiO_3$$

(b) Extraction of silver

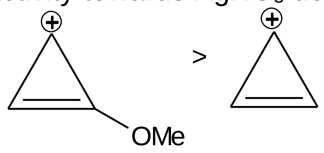
$$Ag + NaCN \longrightarrow \underset{\text{anionic complex}}{\left[Ag(CN)_2\right]^-}$$

(c) Nickel is purified by Mond's process

(d) Zr and Ti are purified by Van Arkel method by converting to volatile iodide.

27. D

Sol. Reactivity towards $AgNO_3$ depend on stability of carbocation formed



$$CH_3\overset{\oplus}{C}HCH_3 > CH_3\overset{\oplus}{C}HCH_2NO_2$$

28. B

Sol. Balmer series lies in the visible region.

29. C

Sol. Cr^{2+} and Fe^{2+} both have 4 unpaired electrons.

30. B

Sol. $$2Pb(NO_3)_2 \xrightarrow{\Delta} 2PbO + 2NO_2 + O_2$$

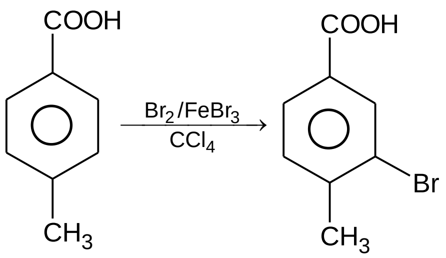
$$2NO_2 \longrightarrow N_2O_4 \text{colourless}$$

$$N_2O_4 + 2NO \longrightarrow \underset{\text{Blue solid}}{2N_2O_3}$$

Oxidation state in N_2O_3 is +3

31. C

Sol.



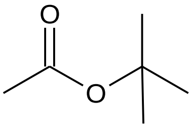
32. D

Sol. $O^{2-} > F^- > Na^+ > Mg^{2+}$. Greater the Zeff, smaller is size.

33. D

Sol. A – B has lowest potential energy and it means it has stronger bond.

34. A

Sol. 

$$\xrightarrow{H_2SO_4} CH_3COOH + (CH_3)_3C - OH$$

$$\downarrow ZnCl_2 / HCl$$

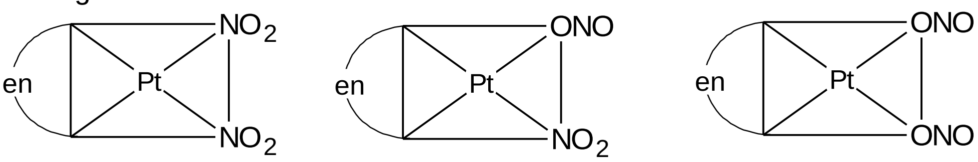
Turbidity

35. D

Sol. At equilibrium rate of forward and backward reaction becomes equal.

36. B

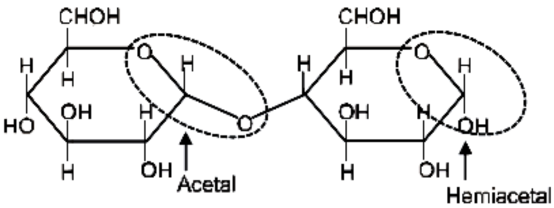
Sol. Two linkage isomers



37. B

Sol. Correct IUPAC name is 4–Bromo–2–methylcyclopentane carboxylic acid

38. A

Sol. 

Maltose

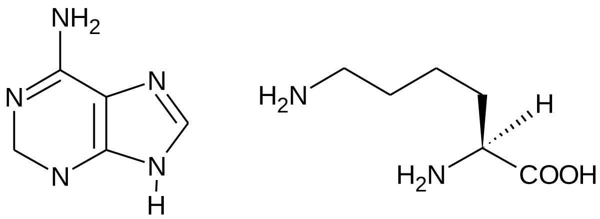
39. C

Sol. E_{ext} > 1.1 V, cell reaction is reversed

i.e. $Cu \longrightarrow Cu^{2+}$ anode

$Zn^{2+} + 2e^- \longrightarrow Zn$ cathode

40. B

Sol. 

Adenine Lysine

Both contains $-NH_2$ group

41. D

Sol. (i) Foam – Froth (e)

(ii) Gel – Jellies (c)

(iii) Aerosol – smoke(a)

(iv) Emulsion – milk(f)

42. A

Sol. U and H are temperature dependent

$C_{P,m} - C_{V,m} = R$ (for 1 mole of ideal gas)

$dU = C_V dt$

43. A

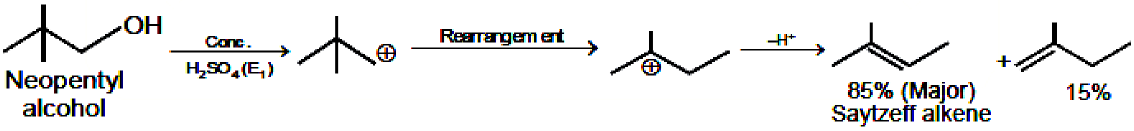
Sol. $4Li + O_2 \longrightarrow 2Li_2O$ oxide

$2Na + O_2 \longrightarrow Na_2O_2$ peroxide

$K + O_2 \longrightarrow KO_2$ superoxide

44. B

Sol.



45. B

Sol. Z = 101 belong to actinoids
104 belong to group 4

46. 600

Sol. $P_T = X_A\left(P_A^0 - P_B^0\right) + P_B^0$

ATQ

$550 = \frac{1}{4}\left(P_A^0 - P_B^0\right) + P_B^0$

$2200 = P_A^0 - P_B^0 + 4P_B^0$ …… (1)

$560 = \frac{1}{5}\left(P_A^0 - P_B^0\right) + P_B^0$

$2200 = P_A^0 - P_B^0 + 5P_B^0$ …… (2)

$P_A^0 + 3P_B^0 = 2200$

$P_A^0 \pm 4P_B^0 = 2800$

$P_B^0 = 600$ $P_A^0 = 400$ mm Hg

47. 59.51

Sol. first order reaction

$K = \frac{2.303}{t}\log\frac{a_0}{a_0 - x}$

$K = \frac{2.303}{90}\log\frac{a_0}{0.25a_0}$ ....(1)

= 0.0154

$t = 60\% = \frac{2.303}{K}\log\frac{a_0}{a_0}$....(2)

$= \frac{2.303}{0.0154}\times(1-0.602) = 59.51$ mins ≈ 60

48. 3400

Sol. $N_2 + 3H_2 \longrightarrow 2NH_3$

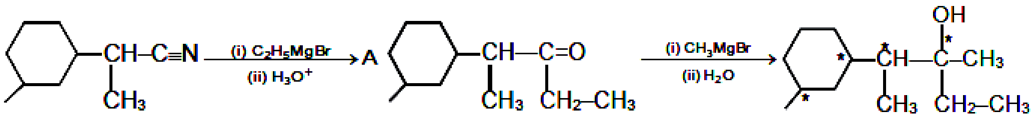
$\frac{2.8\times10^3}{28}$ $\frac{1\times10^3}{2}$

$\frac{0.1\times10^3\,mol}{LR}$ $0.5\times10^3\,mol$

Mass of NH_3 produced = $0.2 \times 10^3 \times 17$ = 3.4 kg = 3400 g

49. 4

Sol.



50. 85

Sol. eq of H_2O_2 = eq of $KMnO_4$

$$= \frac{0.316}{158} \times 5 = 0.01$$

$$= \frac{0.01 \times 17}{0.2} \times 100 = 85\%$$

PART–C (MATHEMATICS)

51. D

Sol. $x^2 - 3x + p = 0$

$\alpha, \beta, \gamma, \delta$ in G.P.

$\alpha + \alpha r = 3$ ……………..(1)

$x^2 - 6x + q = 0$

$\alpha r^2 + \alpha r^3 = 6$ …………..(2)

(2) $\div$(1) $\Rightarrow$ $r^2 = 2$

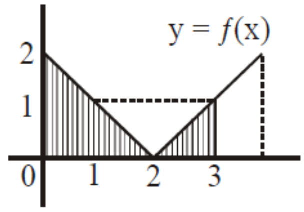
So, $\frac{2q+p}{2q-p} = \frac{2r^5+r}{2r^5-r} = \frac{2r^4+1}{2r^4-1} = \frac{9}{7}$

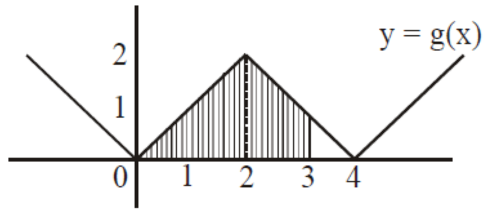
52. C

Sol. $\int_0^3 g(x) - f(x) = \int_0^3 \left||x-2|-2\right| dx - \int_0^3 |x-2| dx$

$= \left(\frac{1}{2}\times 2\times 2 + 1 + \frac{1}{2}\times 1\times 1\right) - \left(\frac{1}{2}\times 2\times 2 + \frac{1}{2}\times 1\times 1\right)$

$= \left(2 + 1 + \frac{1}{2}\right) - \left(2 + \frac{1}{2}\right) = 1$





53. C

Sol. $f(2) = 8, f'(2) = 5, f'(x) \geq 1, f''(x) \geq 4, \forall x \in (1, 6)$

Using LMVT

$f''(x) = \frac{f'(5) - f'(2)}{5-2} \geq 4 \Rightarrow f'(5) \geq 17$ ………..(1)

$f'(x) = \frac{f(5) - f(2)}{5-2} \geq 1 \Rightarrow f(5) \geq 11$ ……….(2)

Therefore $f'(5) + f(5) \geq 28$

54. A

Sol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b); \frac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$ ……….(i)

Now, $\phi(t) = \frac{5}{12} + t - t^2 = \frac{8}{12} - \left(t - \frac{1}{2}\right)^2$

$\phi(t)_{max} = \frac{8}{12} = \frac{2}{3} = e \Rightarrow e^2 = 1 - \frac{b^2}{a^2} = \frac{4}{9}$ ……….(ii)

$\Rightarrow a^2 = 81$ (From (i) and (ii)

So, $a^2 + b^2 = 81 + 45 = 126$

55. C

Sol. $\bar{x} = 10$

$\Rightarrow \bar{x} = \dfrac{63 + a + b}{8} = 10$

$\Rightarrow a + b = 17$ ....(1)

Since, variance is independent of origin.
So, we subtract 10 from each observation.

So, $\sigma^2 = 13.5 = \dfrac{79 + (a-10)^2 + (b-10)^2}{8}$

$\Rightarrow a^2 + b^2 - 20(a+b) = -171$

$\Rightarrow a^2 + b^2 = 169$ ....(2)

From (1) and (2) ; a = 12 and b = 5

56. B

Sol. $f(x) = \vec{a}.(\vec{b} \times \vec{c}) = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$

$= x^3 - 27x + 26$

$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$ and $f''(-3) < 0$

$\Rightarrow$ local maxima at $x = x_0 = -3$

Thus, $\vec{a} = -3\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} - \hat{k}$, and $\vec{c} = 7\hat{i} - 2\hat{j} - 3\hat{k}$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 9 - 5 - 26 = -22$

57. C

Sol. $\left(a + \sqrt{2}b\cos x\right)\left(a - \sqrt{2}b\cos y\right) = a^2 - b^2$

$\Rightarrow a^2 - \sqrt{2}\,ab\cos y + \sqrt{2}\,ab\cos x - 2b^2\cos x\cos y = a^2 - b^2$

Differentiating both sides:

$0 - \sqrt{2}ab\left(-\sin y\dfrac{dy}{dx}\right) + \sqrt{2}ab(-\sin x) - 2b^2\left[\cos x\left(-\sin y\dfrac{dy}{dx}\right) + \cos y(-\sin x)\right] = 0$

At $\left(\dfrac{\pi}{4}, \dfrac{\pi}{4}\right)$:

$ab\dfrac{dy}{dx} - ab - 2b^2\left(-\dfrac{1}{2}\dfrac{dy}{dx} - \dfrac{1}{2}\right) = 0$

$\Rightarrow \dfrac{dx}{dy} = \dfrac{ab + b^2}{ab - b^2} = \dfrac{a+b}{a-b};\ a,b > 0$

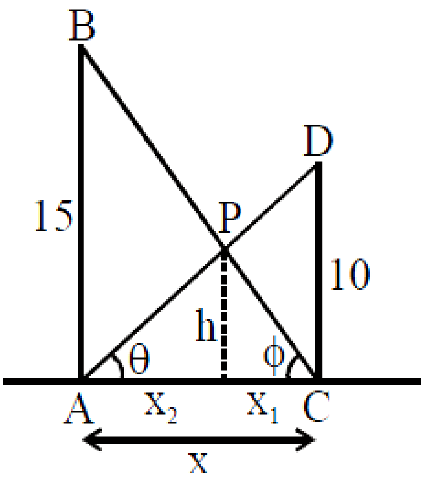
58. B

Sol. $\tan\theta = \frac{10}{x} = \frac{h}{x_2} \Rightarrow x_2 = \frac{hx}{10}$

$\tan\phi = \frac{15}{x} = \frac{h}{x} \Rightarrow x_1 = \frac{hx}{15}$

Now, $x_1 + x_2 = x = \frac{hx}{15} + \frac{hx}{10}$

$\Rightarrow 1 = \frac{h}{10} + \frac{h}{15} \Rightarrow h = 6$



59. D

Sol. $1 + (1 - 2^2.1) + (1 - 4^2.3) + ... + (1 - 20^2.19)$

$= \alpha - 220\beta$

$= 11 - (2^2.1 + 4^2.3 + ... + 20^2.19)$

$= 11 - 2^2.\sum_{r=1}^{10} r^2(2r - 1) = 11 - 4\left(\frac{110^2}{2} - 35 \times 11\right)$

$= 11 - 220(103)$

$\Rightarrow \alpha = 11, \beta = 103$

60. C

Sol. Let TV (r) denotes truth value of a statement r.

Now, if TV (p) = TV (q) = T

$\Rightarrow TV(S_1) = F$

Also, if TV (p) = T and TV (q) = F

$\Rightarrow TV(S_2) = T$

61. A

Sol. $u = \frac{2z + i}{z - ki}$

$= \frac{2x^2 + (2y+1)(y-k)}{x^2 + (y-k)^2} + i\frac{(x(2y+1) - 2x(y-k))}{x^2 + (y-k)^2}$

Since Re (u) + Im (u) = 1

$\Rightarrow 2x^2 + (2y+1)(y-k) + x(2y+1) - 2x(y-k) = x^2 + (y-k)^2$

$\left.\begin{matrix} P(0, y_1) \\ Q(0, y_2) \end{matrix}\right\rangle \Rightarrow y^2 + y - k - k^2 = 0 \left\langle \begin{matrix} y_1 + y_2 = -1 \\ y_1 y_2 = -k - k^2 \end{matrix}\right.$

$\because PQ = 5$

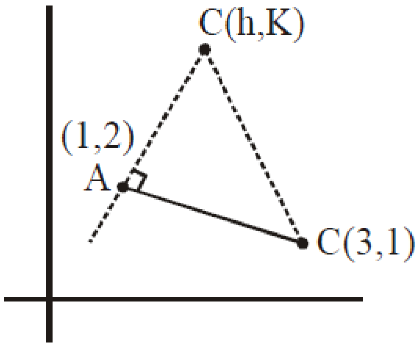
$\Rightarrow |y_1 - y_2| = 5 \Rightarrow k^2 + k - 6 = 0$

$\Rightarrow k = -3, 2$

So, $k = 2 (k > 0)$

62. A

Sol. $\left(\frac{K-2}{h-1}\right)\left(\frac{1-2}{3-1}\right)=-1 \Rightarrow K=2h$ ....(1)



$\because [\Delta ABC]=5\sqrt{5}$

$\Rightarrow \frac{1}{2}\left(\sqrt{5}\right)\sqrt{(h-1)^2+(K-2)^2}=5\sqrt{5}$ ....(2)

$\Rightarrow h=2\sqrt{5}+1(h>0)$

63. D

Sol. Since (3, 3) lies on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{9}{a^2}-\frac{9}{b^2}=1$ ....(1)

Now, normal at (3, 3) is $y-3=-\frac{a^2}{b^2}(x-3)$,

which passes through $(9, 0) \Rightarrow b^2=2a^2$ ....(2)

So, $e^2=1+\frac{b^2}{a^2}=3$

Also, $a^2=\frac{9}{2}$ (From (i) and (ii))

Thus, $\left(a^2, e^2\right)=\left(\frac{9}{2}, 3\right)$

64. B

Sol. $\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx=\int\left(\frac{x}{\cos x}\right).\frac{x\cos x\,dx}{(x\sin x+\cos x)^2}$

$=\frac{x}{\cos x}\left(-\frac{1}{x\sin x+\cos x}\right)+\int\left(\frac{\cos x+x\sin x}{\cos^2 x}\right)\left(\frac{1}{x\sin x+\cos x}\right)dx$

$=-\frac{x\sec x}{x\sin x+\cos x}+\int \sec^2 x\,dx$

$=-\frac{x\sec x}{x\sin x+\cos x}+\tan x+C$

65. B

Sol. $A^2=\begin{pmatrix}\cos 2\theta & i\sin 2\theta\\ i\sin 2\theta & \cos 2\theta\end{pmatrix}$

Similarly, $A^5=\begin{pmatrix}\cos 5\theta & i\sin 5\theta\\ i\sin 5\theta & \cos 5\theta\end{pmatrix}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

(1) $a^2+b^2=\cos^2 5\theta-\sin^2 5\theta=\cos 10\theta=\cos 75^o$

(2) $a^2-d^2=\cos^2 5\theta-\cos^2 5\theta=0$

(3) $a^2-b^2=\cos^2 5\theta+\sin^2 5\theta=1$

(4) $a^2-c^2=\cos^2 5\theta+\sin^2 5\theta=1$

66. C

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6 = {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + {}^{30}C_6$

$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + \left({}^{30}C_6 + {}^{30}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + \left({}^{31}C_6 + {}^{31}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{50}C_7 - {}^{30}C_7$

$= {}^{51}C_7 - {}^{30}C_7$

67. A

Sol. $[x]^2 + 2[x+2] - 7 = 0$

$\Rightarrow [x]^2 + 2[x] + 4 - 7 = 0$

$\Rightarrow [x] = 1, -3$

$\Rightarrow x \in [1, 2) \cup [-3, -2)$

68. A

Sol. $f(x) = \int_1^3 \frac{\sqrt{x}\,dx}{(1+x)^2} = \int_1^{\sqrt{3}} \frac{t.2t\,dt}{\left(1+t^2\right)^2}$ (put $\sqrt{x} = t$)

$= \left(-\frac{t}{1+t^2}\right)_1^{\sqrt{3}} + \left(\tan^{-1} t\right)_1^{\sqrt{3}}$ [Applying by parts]

$= -\left(\frac{\sqrt{3}}{4} - \frac{1}{2}\right) + \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{1}{2} - \frac{\sqrt{3}}{4} + \frac{\pi}{12}$

69. B

Sol. $\max\{n(A),\ n(B)\} \le n(A \cup B) \le n(U)$

$\Rightarrow 76 \le 76 + 63 - x \le 100$

$\Rightarrow -63 \le -x \le -39$

$\Rightarrow 63 \ge x \ge 39$

70. C

Sol. $x\frac{dy}{dx} - y = x^2(x\cos x + \sin x),\ x > 0$

$\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x) \Rightarrow \frac{dy}{dx} + Py = Q$

So, I.F. $= e^{\int -\frac{1}{x}dx}\ \frac{1}{|x|} = \frac{1}{x}(x > 0)$

Thus, $\frac{y}{x} = \int \frac{1}{x}\left(x(x\cos x + \sin x)\right)dx$

$\Rightarrow \frac{y}{x} = x\sin x + C$

$\because y(\pi) = \pi \Rightarrow C = 1$

So, $y = x^2 \sin x + x \Rightarrow (y)_{\pi/2} = \frac{\pi^2}{4} + \frac{\pi}{2}$

Also, $\frac{dy}{dx} = x^2 \cos x + 2x \sin x + 1$

$\Rightarrow \frac{d^2y}{dx^2} = -x^2 \sin x + 4x \cos x + 2 \sin x$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\frac{\pi}{2}} = -\frac{\pi^2}{4} + 2$

71. 8

Sol. Given $\left(2x^2 + 3x + 4\right)^{10} = \sum_{r=0}^{20} a_r x^r$ ....(1)

replace x by $\frac{2}{x}$ in above identity :-

$\frac{2^{10}\left(2x^2 + 3x + 4\right)^{10}}{x^{20}} = \sum_{r=0}^{20} \frac{a_r 2^r}{x^r}$

$\Rightarrow 2^{10} \sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^r x^{(20-r)}$ (from(i))

now, comparing coefficient of x^7 from both sides
(take r = 7 in L.H.S. and r = 13 in R.H.S.)

$2^{10} a_7 = a_{13} 2^{13} \Rightarrow \frac{a_7}{a_{13}} = 2^3 = 8$

72. 3

Sol. $D_1 = \begin{vmatrix} -7 & 4 & -1 \\ 8 & 1 & 5 \\ 15 & b & 6 \end{vmatrix} = 0 \Rightarrow b = -3$

$D = \begin{vmatrix} 1 & 4 & -1 \\ 3 & 1 & 5 \\ a & b & 6 \end{vmatrix} = 0 \Rightarrow 21a - 8b - 66 = 0$ ....(1)

$P : 2x - 3y + 6z = 15$

so required distance $= \frac{21}{7} = 3$

73. 5

Sol. $D = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0 \Rightarrow a = 8$

also, $D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix} = 0 \Rightarrow b = 3$

hence, $a - b = 8 - 3 = 5$

74. 10

Sol. Since, $\lim_{x\to 0}\frac{f(x)}{x}$ exist $\Rightarrow f(0)=0$

Now, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f(h)+xh^2+x^2h}{h}$ (take y = h)

$=\lim_{h\to 0}\frac{f(h)}{h}+\lim_{h\to .0}(xh)+x^2$

$\Rightarrow f'(x)=1+0+x^2$

$\Rightarrow f'(3)=10$

75. 3

Sol. We have, 1 – (probability of all shots result in failure) > $\frac{1}{4}$

$\Rightarrow 1-\left(\frac{9}{10}\right)^n>\frac{1}{4}$

$\Rightarrow \frac{3}{4}>\left(\frac{9}{10}\right)^n \Rightarrow n\ge 3$