FIITJEE

Solutions to JEE (Main)-2020

JEE–Main–2020 –Sept–3–Second–Shift

PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. **B**

Sol. $\vec{B}$ is $\perp$ to $\vec{E}$ and direction of propagation of wave. Also, $vB_0 = E_0$

$$B_0 = \frac{E_0}{v} = E_0\sqrt{\mu_0 \epsilon_0}$$

2. **A**

Sol. $\vec{P}_i = \vec{P}_f \Rightarrow 0.1 \times 20 = 2 \times v_x$

$\Rightarrow \quad v_x = 1$ m/s

$v_y = \sqrt{2gh} = \sqrt{2\times 10\times 1}$, KE = $\frac{1}{2}m\,(V_x^2 + V_y^2)$

$KE = \frac{1}{2}\times 2\times(1+20) = 21\,J$

3. **D**

Sol. $qV = \frac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}} \quad \Rightarrow \quad v \propto \sqrt{\frac{q}{m}}$

$\frac{v_H}{v_{He}} = \sqrt{\frac{1}{1}\times\frac{4}{1}} = \frac{2}{1}$

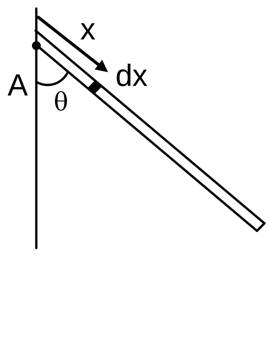
4. **C**

Sol. Torque of centrifugal force about A

$= \int d = \int [dm\,\omega^2(x\sin\theta)]\,(x\cos\theta)$

$= \int_0^{\ell} \frac{m}{\ell}dx\,\omega^2 x^2 \sin\theta\cos\theta$

$= \frac{m}{\ell}\omega^2\sin\theta\cos\theta\left[\frac{x^3}{3}\right]_0^{\ell} = \frac{m\omega^2\ell^2\sin\theta\cos\theta}{3}$



$\tau_{mg} = \tau_{centrifugal}$ (about A)

$mg\frac{\ell}{2}\sin\theta = \frac{m\omega^2\ell^2\sin\theta\cos\theta}{3}$

$\cos\theta = \frac{3g}{2\ell\omega^2}$

5. **C**

Sol. Solar constant = $\frac{\text{Energy}}{\text{Area}\times\text{Time}} = \frac{ML^2T^{-2}}{L^2\times T} = ML^oT^{-3}$

6. **C**

Sol. Heat lost by steam = Heat gained by water and calorimeter.

m × 540 + m × 1 × (100 – 31) = 200

540 m + 69 m = 1200

$$m = \frac{1200}{609} \approx 2$$

7. **C**

Sol. Multimeter will show deflection when current will flow to charge the capacitor.

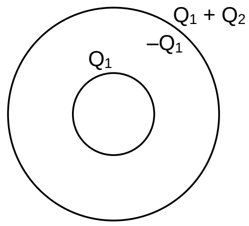
8. **A**

Sol. Power = Number of photons emitted per sec × Energy of 1 photon

$$P = n \times \frac{hc}{\lambda} \quad \Rightarrow \quad n \propto \lambda$$

9. **C**

Sol. $\sigma_1 = \sigma_2$



$$\frac{Q_1}{4\pi R^2} = \frac{Q_2 + Q_1}{4\pi(16R^2)}$$

$\Rightarrow$ $Q_1 + Q_2 = 16\, Q_1$

$\Rightarrow$ $15Q_1 = Q_2$

$$V(R) - V(4R) = \left(\frac{KQ_1}{R} + \frac{KQ_2}{4R}\right) - \left(\frac{KQ_1}{4R} + \frac{KQ_2}{4R}\right)$$

$$= \frac{3KQ_1}{4R} = \frac{3Q_1}{16\pi\in R}$$

10. **B**

Sol. $V_{max} = A'\omega' = A\omega$

$$A'\sqrt{\frac{k}{m/2}} = A\sqrt{\frac{k}{m}}$$

$$A' = \frac{A}{\sqrt{2}}$$

11. **C**

Sol. $\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{\lambda}\left[n_1L_1 - n_2L_2\right]$

12. **C**

Sol. $$\frac{50-40}{300} = -k\left[\frac{50+40}{2} - 20\right]$$

$$\frac{40-T}{300} - k\left[\frac{T+40}{2} - 20\right]$$

$\Rightarrow$ $\frac{10}{40-T} = \frac{25}{T} \times 2$

$\Rightarrow$ T = 200 – 5T

$\Rightarrow$ $T = \frac{100}{3} \approx 33°C$

13. **A**

Sol. Band gap energy = $\frac{hc}{\lambda} = \frac{1240\ \text{eV nm}}{400\ \text{nm}}$ = 3.1 eV

14. **D**

Sol. $\Delta Q = nC_p\Delta T \Rightarrow 160 = nC_p\ 50$

$\Delta Q = nC_v\Delta T \Rightarrow 240 = nC_v\ 100$

$\Rightarrow \quad \frac{C_p}{C_v} = \frac{16}{5} \times \frac{10}{24} = \frac{4}{3}$

$\Rightarrow \quad 1 + \frac{2}{f} = \frac{4}{3} \quad \Rightarrow f = 6$

15. **A**

Sol. Due to perfect diamagnetic sphere, effect of external field cannot be felt inside it.

16. **B**

Sol. $\text{Density} = \frac{M}{V} = \frac{1.67 \times 10^{-27} \times A}{\frac{4}{3}\pi (1.3)^3 \times 10^{-45} \times A} \approx 10^{17}$

17. **C**

Sol. $E = \frac{G\, m_{enc}}{r^2} = \frac{G}{r^2}\int_0^r \rho_0 \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr$

$E = \frac{G}{r^2} 4\pi \rho_0 \left[\frac{r^3}{3} - \frac{r^5}{5R^2}\right]_0^r$

$= 4\pi G\rho_0 \left[\frac{r}{3} - \frac{r^3}{5R^2}\right]$

For E to be max, $\frac{dE}{dr} = 0$

$\Rightarrow \quad \frac{1}{3} - \frac{3r^2}{5R^2} = 0$

$r = \sqrt{\frac{5}{9}}\, R$

18. **A**

Sol. $i = \frac{E}{R} = \frac{1}{R}\frac{d\phi}{dt} = \frac{a^2}{R}\frac{dB}{dt}$

$i = \frac{(7.5)^2 \times 10^{-4} \times \pi \times 4 \times 10^{-6}}{1.23 \times 10^{-8} \times 0.3} \times 0.032$

= 0.61 A

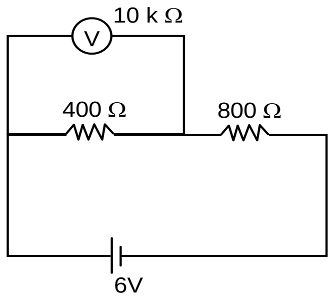
19. **B**

Sol. P = constant then $S = \sqrt{\frac{8P}{9m}}\, t^{3/2}$

$S \propto t^{3/2}$

20. **C**

Sol. Parallel of 10 kΩ and 400 Ω = 384.61 Ω

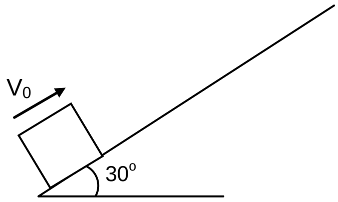


$$V_{400\,\Omega} = \frac{384.61}{384.61+800}\times 6$$

$= 1.95$ V

21. **346.00**

Sol. Upward journey



$$0 - v_0^2 = -2\left(\frac{g}{2}+\frac{mg\sqrt{3}}{2}\right)S$$

$$S = \frac{v_0^2}{g(1+\mu\sqrt{3})}$$

Downward journey

$$\frac{v_0^2}{4} - 0 = 2\left(\frac{g}{2}-\frac{4g\sqrt{3}}{2}\right)\frac{v_0^2}{g(1+\mu\sqrt{3})}$$

$$\frac{1}{4} = \frac{1-\sqrt{3}\,\mu}{1+\sqrt{3}\,\mu}$$

$$\frac{5}{3} = \frac{2}{2\sqrt{3}\,\mu} \quad \Rightarrow \quad \mu = \frac{\sqrt{3}}{5} = \frac{1732}{5}$$

$\Rightarrow \mu = 0.346$

$\Rightarrow$ I = 346.00

22. **20.00**

Sol. τ = B I N A sin θ

$\Rightarrow$ 1.5 = B × 0.5 × 500 × 3 × 10^{-4} × 1

$\Rightarrow$ B = 20.00

23. **8791**

Sol. Q_1 = mL = 100 gm × $\frac{80\text{ cal}}{\text{gm}} = 8000\text{ cal}$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \Rightarrow Q_2 = \frac{8000\times 300}{273} = 8791\text{ cal}$$

24. **25.00**

Sol. $I = 0 + ma^2 + m\left(\frac{a}{2}\right)^2$

$$I = ma^2 + \frac{ma^2}{4} = \frac{5ma^2}{4}$$

$\Rightarrow$ N = 25.00

25. **1.00**

Sol. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$-\frac{1}{v^2}\frac{dV}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

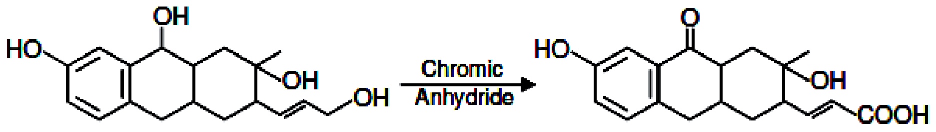
$$\frac{dv}{dt} = \frac{-v^2}{u^2}\frac{du}{dt}$$

$$V_I = -\left(\frac{10}{30}\right)^2 \times 9 = 1 \text{ cm/s}$$

PART – B (CHEMISTRY)

26. C

Sol.

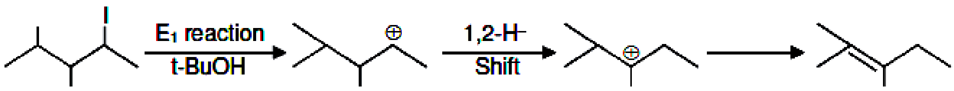


Product 'P'

3° Alcohol gives Red colour with ceric ammonium nitrate

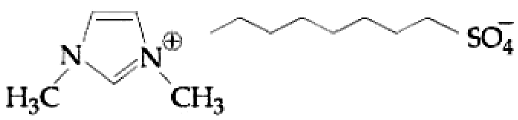
27. B

Sol.



28. A

Sol. excess water to liquid



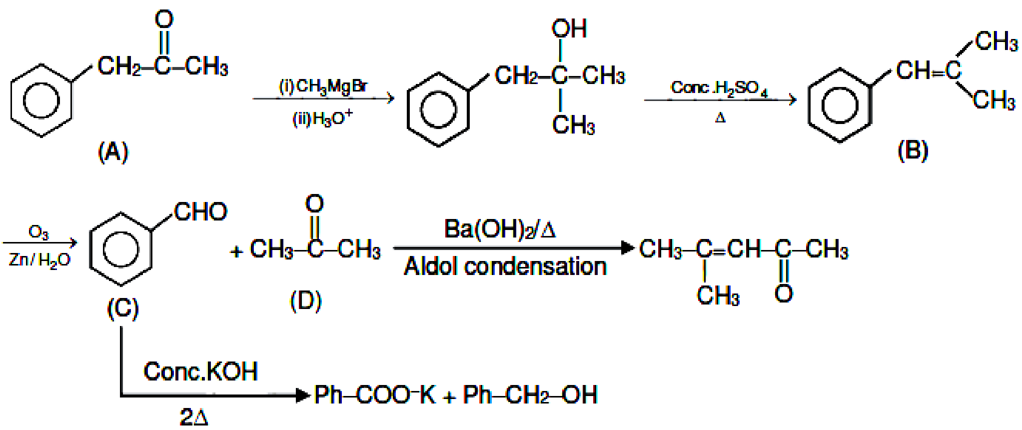
Due to presence of hydrophobic chain it forms micelle

29. D

Sol. As difference in 3rd and 4th ionisation energies is high so atom contains 3 valence electrons.

30. B

Sol.



31. B

Sol. For n = 1 value of ℓ = 0, 1, 2

For n = 2 value of ℓ = 0, 1, 2, 3

So, according to n + l rule the filling order of subshells will be:
1s 1p 2s 1d 2p 3s 2d 3p 4s ……..
(1) 1st noble gas will have configuration $1s^2\ 1p^6$ so atomic number will be 8.
(2) 1st alkali metal will have electronic configuration ⇒ $1s^1$ ⇒ (Z = 1)
(3) Electronic configuration of C (Z = 6) ⇒ $1s^2\ 1p^4$
(4) Z = 13, Electronic configuration = $1s^2\ 1p^6\ 2s^2\ 1d^3$

So it will not have half-filled electronic configuration

32. C

Sol. For H_2O_2

$$\text{Molarity} = \frac{\text{Volume strength}}{11.2} = \frac{5.6}{11.2} = 0.5\ \text{M}$$

$$\text{Molarity} = \frac{\%(w/w)\times 10\times d}{\text{GMM}}$$

$$0.5 = \frac{\%(w/w)\times 10\times d}{34}$$

$$\%(w/w) = \frac{0.5\times 34}{10} = 1.7$$

33. D

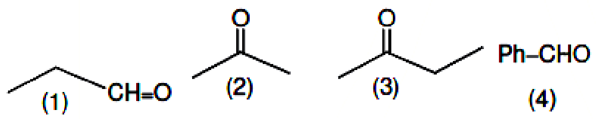
Sol. At equivalence point pH is 7 and pH increases with addition of NaOH so correct graph is (1).

34. B

Sol. (b) It is harmful for trees and plants
(c) It causes breathing problem in human being and animals

35. B

Sol. Rate of NAR $\alpha - \mathrm{I} - \mathrm{M}$ on substate



1 > 4 > 2 > 3

36. A

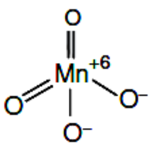
Sol. Conc. H_2SO_4 acts as dehydrating agent.
Molar mass of given complex = 266.5 g/mol.
On treating with conc. H_2SO_4 the mass
lost by the complex = $= \frac{13.5}{100}(266.5) \approx 36\text{g}$ = 2 moles of H_2O
Formula of the complex = $[Cr(H_2O)_4Cl_2]Cl.\ 2H_2O$

37. C

Sol.

Manganate

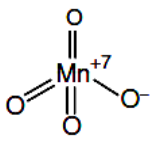
MnO_4^{2-}



Paramagnetic, green in colour,
Tetrahedral & contains $p\pi$-$d\pi$ bond

Permanganate

$MnO_4^{\ominus}$



Diamagnetic, purple in colour,
Tetrahedral & contains $p\pi$-$d\pi$ bond

38. B

Sol. S_N2 reaction depend upon –I, –M effect on substrate. On increase –I, –M, effect rate of S_N2 reaction increase.

39. B

Sol. $$P_{gas} = \frac{n_{gas}RT}{V}$$

as n, T & V constant So

$$P_{H_2} = P_{O_2} = P_{He} = 2\,atm$$

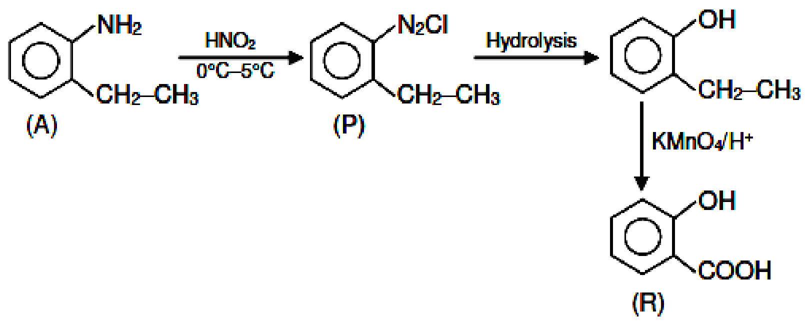
So, $P_{Total} = P_{H_2} + P_{O_2} + P_{He} = 6$ atm

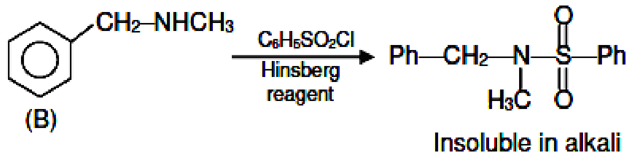
40. D

Sol. Charge / radius ratio of Be and Al is same because of diagonal relationship. Remaining statements are correct.

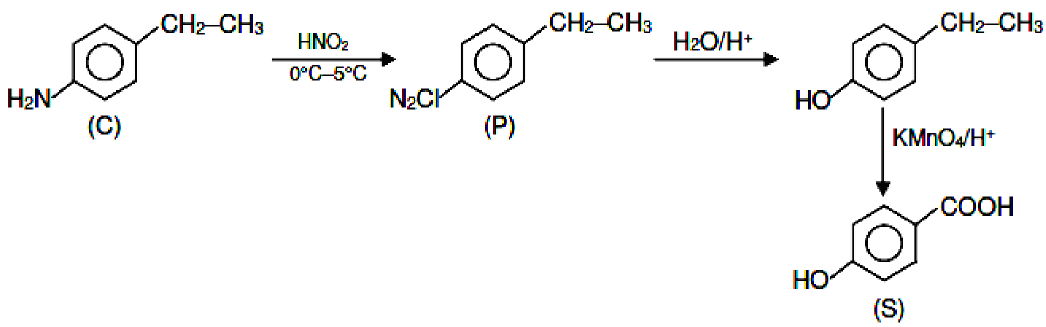
41. B

Sol.





Insoluble in alkali



42. C

Sol. Due to inter molecular H-Bonding in B, than A, B is more soluble and having more B.P point than A.

43. D

Sol. For a given reaction, $\text{rate} = -\frac{1}{2}\frac{dn_A}{dt} = -\frac{1}{3}\frac{dn_B}{dt} = -\frac{2}{3}\frac{dn_C}{dt}$

$$\text{rate} = \frac{dn_A}{dt} = \frac{2}{3}\frac{d\,n_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$$

44. A

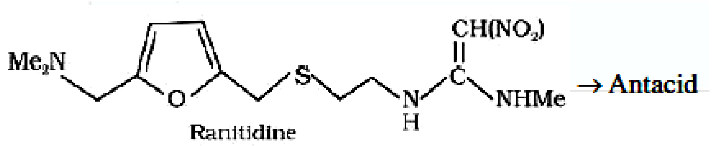
Sol. $[Ru(en)_3]Cl_2 \Rightarrow Ru^{2+} = 4d^6 = t_{2g}^6, e_g^0$

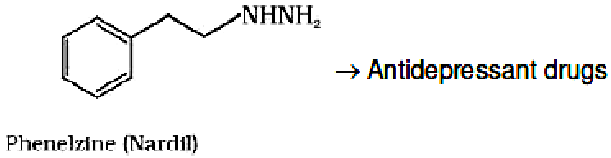
$[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+} = 3d^6 = t_{2g}^4, e_g^2$

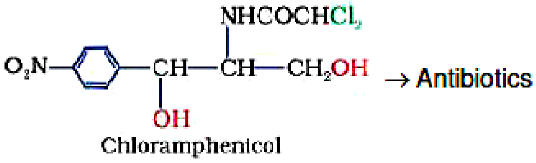
So, correct answer is (1).

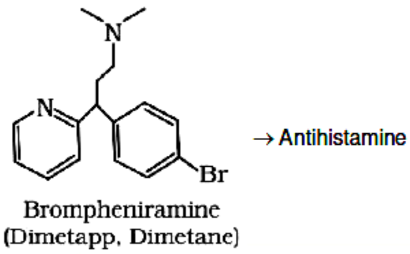
45. D

Sol.









46. 10

Sol. Phosphinic acid is hypo phosphorous acid (H_3PO_2).

$$NaOH + H_3PO_2 \longrightarrow NaH_2PO_2 + H_2O$$

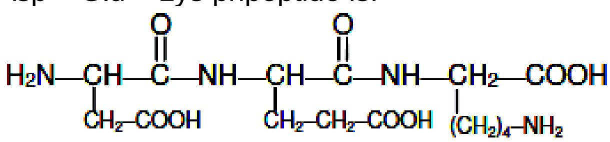
For neutrization

$(N_1V_1)_{acid} = (N_2V_2)_{base}$

$0.1 \times 10 = 0.1 \times (V_{mL})_{NaOH}$

47. 5

Sol. Asp – Glu – Lys pripeptide is:



No. of CO group = 5

48. 177

Sol. For isotonic solution

$i_1C_1 = i_2C_2$ {for protein i = 1}

$C_1 = C_2$

$$\frac{0.73 \times 1000}{M_A \times 250} = \frac{1.65}{M_B \times 1}$$

$$\frac{M_A}{M_B} = \frac{0.73 \times 4}{1.65} = 1.77 = 177 \times 10^{-2}$$

49. 60

Sol. According to Faraday law

$W = ZIt \times \eta$ Where η = efficiency

or $W = \frac{E}{96500} \times I \times t \times \eta$

Putting values

$$.104 = \frac{\left(52/3\right) \times 2 \times 8 \times 60 \times \eta}{96500} \Rightarrow \eta = 0.6$$

So percentage efficiency = 60%

50. 25

Sol.

Number of mole of X = $\frac{6.022 \times 10^{22}}{6.022 \times 10^{23}} = \frac{10}{\text{Molar mass of X}}$

So molar mass of X = 100g

Molarity = $\frac{5}{100 \times 2} = 0.025M$

Ans. = 0.025 M

$M = 25 \times 10^{-3}$

So P = 25

PART–C (MATHEMATICS)

51. C

Sol. $488 = \frac{n}{2}\left[2\left(\frac{100}{5}\right) + (n-1)\left(\frac{2}{5}\right)\right]$

$488 = \frac{n}{2}(101 - n)$

$\Rightarrow \quad n^2 - 101n + 2440 = 0$

$\Rightarrow \quad n = 61 \quad \text{or} \quad 40$

For $n = 40 \quad \Rightarrow \quad T_n > 0$

For $n = 61 \quad \Rightarrow \quad T_n < 0$

$T_n = \frac{100}{5} + (61-1)\left(-\frac{2}{5}\right) = -4$

52. A

Sol. $|z_1 - 1| = \text{Re}(z_1)$ Let $z_1 x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$(x_1 - 1)^2 + y_1^2 = x_1^2$

$y_1^2 - 2x_1 + 1 = 0 \qquad \ldots\ldots\ldots\ldots(1)$

$|z_2 - 1| = \text{Re}(z_2)$

$(x_2 - 1)^2 + y_2^2 = x_2^2$

$y_2^2 - 2x_2 + 1 = 0 \qquad \ldots\ldots\ldots\ldots..(2)$

$y_1^2 - y_2^2 - 2(x_1 - x_2) = 0$

$(y_1 - y_2)(y_1 + y_2) = 2(x_1 - x_2)$

$y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right) \qquad \ldots\ldots\ldots\ldots.(3)$

$\arg(z_1 - z_2) = \frac{\pi}{6}$

$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{6}$

$\frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}} \qquad \ldots\ldots\ldots.(4)$

$\therefore \quad y_1 + y_2 = 2\sqrt{3}$

$\Rightarrow \quad \text{Im}(z_1 + z_2) = 2\sqrt{3}$

53. D

Sol. $a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right) = k$

$a = \frac{k}{\cos\theta}, \ b = \frac{k}{\cos\left(\theta + \frac{2\pi}{3}\right)}, \ c = \frac{k}{\cos\left(\theta + \frac{4\pi}{3}\right)}$

$$ab+bc+ca=k^2\frac{\left[\cos\left(\theta+\frac{4\pi}{3}\right)+\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)\right]}{\cos\left(\theta+\frac{4\pi}{3}\right)\cos\theta\cos\left(\theta+\frac{2\pi}{3}\right)}$$

$$=k^2\left[\frac{\cos\theta+2\cos(\theta+\pi).\cos\left(\frac{\pi}{3}\right)}{\cos\theta.\cos\left(\theta+\frac{2\pi}{3}\right).\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

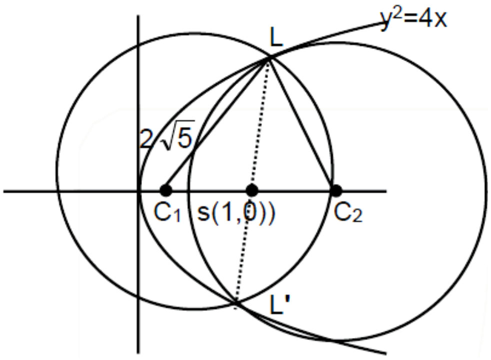
$$=k^2\left[\frac{\cos\theta-2\cos\theta.\frac{1}{2}}{\cos\theta.\cos\left(\theta+\frac{2\pi}{3}\right).\cos\left(\theta+\frac{4\pi}{3}\right)}\right]=0$$

$$\cos\phi=\frac{\left(a\hat{i}+b\hat{j}+c\hat{k}\right).\left(b\hat{i}+c\hat{j}+a\hat{k}\right)}{\sqrt{a^2+b^2+c^2}.\sqrt{b^2+c^2+a^2}}=ab+bc+ca=0$$

$$\phi=\frac{\pi}{2}$$

54. C

Sol. $C_1C_2=2C_1S=2\sqrt{20-4}=8$



55. A

Sol. For R_1 let $a=1+\sqrt{2}, b=1-\sqrt{2}, c=8^{1/4}$

$aR_1b \Rightarrow a^2+b^2=\left(1+\sqrt{2}\right)^2+\left(1-\sqrt{2}\right)^2=6\in Q$

$aR_1c \Rightarrow b^2+c^2=\left(1-\sqrt{2}\right)^2+\left(8^{1/4}\right)^2=3\in Q$

$aR_1c \Rightarrow a^2+c^2=\left(1+\sqrt{2}\right)^2+\left(8^{1/4}\right)^2=3+4\sqrt{2}\notin Q$

$\therefore$ R_1 is not transitive.

For R_2 let $a=1+\sqrt{2}, b=\sqrt{2}, c=1-\sqrt{2}$

$aR_2B \Rightarrow a^2+b^2=\left(1+\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2=5+2\sqrt{2}\notin Q$

$bR_2b \Rightarrow b^2+c^2=\left(\sqrt{2}\right)^2+\left(1-\sqrt{2}\right)^2=5-2\sqrt{2}\notin Q$

$aR_2c \Rightarrow a^2+c^2=\left(1+\sqrt{2}\right)^2+\left(1-\sqrt{2}\right)^2=6\in Q$

$\therefore$ R^2 is not transitive.

56. C

Sol. $\frac{k}{6}=\int_0^{\frac{1}{2}}\frac{x^2}{\left(1-x^2\right)^{3/2}}dx$ $\qquad$ $x=\sin\theta; dx=\cos\theta\, d\theta$

$$\Rightarrow \frac{k}{6}=\int_0^{\frac{\pi}{6}}\frac{\sin^2\theta}{\left(1-\sin^2\theta\right)^{3/2}}.\cos\theta\, d\theta$$

$$\Rightarrow \frac{k}{6}=\int_0^{\frac{\pi}{6}}\frac{\sin^2\theta}{\cos^3\theta}.\cos\theta\, d\theta$$

$$\Rightarrow \frac{k}{6}=\int_0^{\frac{\pi}{6}}\tan^2\theta\, d\theta=\int_0^{\frac{\pi}{6}}\left(\sec^2\theta-1\right)d\theta$$

$$\Rightarrow \frac{k}{6}=\left(\tan\theta-\theta\right)_0^{\pi/6}=\left(\frac{1}{\sqrt{3}}-\frac{\pi}{6}\right)=\frac{2\sqrt{3}-\pi}{6}$$

$$\Rightarrow k=2\sqrt{3}-\pi$$

57. C

Sol. $x^3dy+xydx=2ydx+x^2dy$

$$\Rightarrow \left(x^3-x^2\right)dy=\left(2-x\right)y\,dx$$

$$\Rightarrow \frac{dy}{y}=\frac{2-x}{x^2\left(x-1\right)}dx$$

$$\Rightarrow \int\frac{dy}{y}=\int\frac{2-x}{x^2\left(x-1\right)}dx \qquad \text{……………..(i)}$$

Let $\frac{2-x}{x^2\left(x-1\right)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$

$$\Rightarrow 2-x=A\left(x-1\right)+B\left(x-1\right)+Cx^2$$

$$\Rightarrow C=1, B=-2 \text{ and } A=-1$$

$$\Rightarrow \int\frac{dy}{y}=\int\left\{\frac{-1}{x}-\frac{2}{x^2}+\frac{1}{x-1}\right\}dx$$

$$\Rightarrow \ell ny=-\ell nx+\frac{2}{x}+\ell n|x-1|+C$$

$$\therefore\ y\left(2\right)=e$$

$$\Rightarrow 1=-\ell n2+1+0+C$$

$$\Rightarrow C=\ell n2$$

$$\Rightarrow \ell ny=-\ell nx+\frac{2}{x}+\ell n|x-1|+\ell n2$$

at $x=4$

$$\Rightarrow \ell ny\left(4\right)=-\ell n4+\frac{1}{2}+\ell n3+\ell n2$$

$$\Rightarrow \ell ny\left(4\right)=\ell n\left(\frac{3}{2}\right)+\frac{1}{2}=\ell n\left(\frac{3}{2}e^{1/2}\right)$$

$\Rightarrow y(4) = \frac{3}{2}e^{1/2}$

58. D

Sol. Total $= 9(10^4)$

Fav. Way $= {}^9C_2(2^5 - 2) + {}^9C_1(2^4 - 1) = 36(30) + 9(15) = 1080 + 135$

Probability $= \frac{36 \times 30 + 9 \times 15}{9 \times 10^4} = \frac{4 \times 30 + 15}{10^4} = \frac{135}{10^4}$

59. A

Sol. $S = 6a^2 \Rightarrow \frac{ds}{dt} = 12a.\frac{da}{dt} = 3.6$

$\Rightarrow 12(10)\frac{da}{dt} = 3.6$

$\Rightarrow \frac{da}{dt} = 0.03$

$V = a^3 \Rightarrow \frac{dv}{dt} = 3a^2.\frac{da}{dt}$

$= 3(10)^2.\left(\frac{3}{100}\right) = 9$

60. D

Sol. $f(0)f(1) \le 0$

$\Rightarrow \quad 2(\lambda^2 + 1 - 4\lambda + 2) \le 0 \qquad \Rightarrow \qquad 2(\lambda^2 - 4\lambda + 3) \le 0$

$(\lambda - 1)(\lambda - 3) \le 0$

$\Rightarrow \quad \lambda[1, 3]$

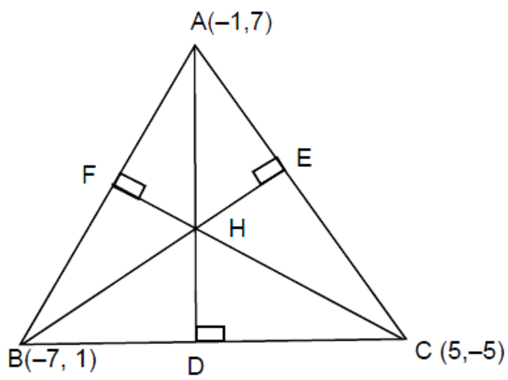
But at $\lambda = 1$, both roots are 1 so $\lambda \ne 1$

61. B

Sol. $m_{BC} = \frac{6}{-12} = -\frac{1}{2}$

$\therefore$ Equation of AD is $y - 7 = 2(x + 1)$

$y = 2x + 9$ ……………..(1)



$m_{AC} = \frac{12}{-6} = -2$

$\therefore$ Equation of BE is $y - 1 = \frac{1}{2}(x + 7)y$

$y = \frac{x}{2} + \frac{9}{2}$ ……………….(2)

by (1) and (2)

$2x + 9 = \frac{x + 9}{2}$

$\Rightarrow 4x + 18 = x + 9$

$\Rightarrow 3x = 9 \Rightarrow x = -3$

$\therefore y = 3$

62. B

Sol. $(p \wedge q)$ should be TRUE and $(\sim q \vee r)$ should be FALSE.

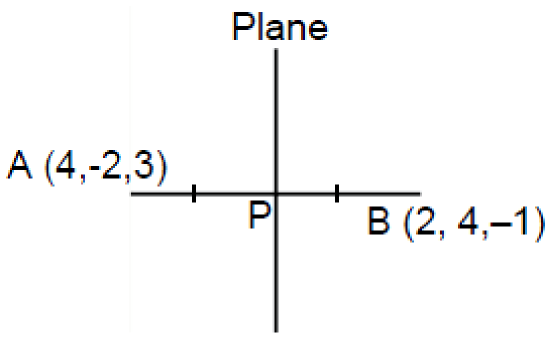
63. C

Sol. Mid point $P \equiv (3, 1, 1)$

Normal of plane is along the line AB.
D.R,'s of normal = 4 –2, –2 –4, 3 –1 (–1) = 2, –6, 4,
= 1, –3, 2

Plane $\rightarrow 1(x-3)-3(y-1)+2(z-1)=0$

$\Rightarrow \quad x-3y+2z-2=0$



64. A

Sol. $e_1 = \sqrt{1-\frac{b^2}{25}}; \; e_2 = \sqrt{1+\frac{b^2}{16}}$

$e_1 e_2 = 1$

$\Rightarrow (e_1 e_2)^2 = 1$

$\Rightarrow \left(1-\frac{b^2}{25}\right)\left(1+\frac{b^2}{16}\right) = 1$

$\Rightarrow 1+\frac{b^2}{16}-\frac{b^2}{25}-\frac{b^4}{25\times16} = 1$

$\Rightarrow \frac{9}{16.25}b^2 - \frac{b^4}{25.16} = 0$

$\Rightarrow \quad b^2 = 9$

$e_1 = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

$e_2 = \sqrt{1+\frac{9}{16}} = \frac{5}{4}$

$\alpha = 2(5)(e_1) = 8$

$\beta = 2(4)(e_2) = 10$

$(\alpha, \beta) = (8, 10)$

65. D

Sol. $|\text{adj}\,A| = |A|^2 = 9$

$\Rightarrow |A| = \pm 3 = \lambda \quad \Rightarrow \quad |\lambda| = 3$

$\Rightarrow |B| = |\text{adj}\,A|^2 = 81$

$\Rightarrow \left|\left(B^{-1}\right)^T\right| = \left|B^{-1}\right| = |B|^{-1} = \frac{1}{|B|} = \frac{1}{81} = \mu$

66. C

Sol. $I = \int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)dx$

$$\int \underset{I}{\tan^{-1}(\sqrt{x})}\underset{II}{dx} = x\tan^{-1}\sqrt{x} - \int \frac{1}{1+x}.\frac{1}{2\sqrt{x}}.x\,dx + C = x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{t.2t.dt}{1+t^2} + C\left(x = t^2\right)$$

$$= x\tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2}dt + C = x\tan^{-1}\sqrt{x} - t + \tan^{-1}t + C = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C \qquad \Rightarrow \qquad (Ax) = x+1 \Rightarrow B(x) = -\sqrt{x}$$

67. D

Sol. $$\text{S.D.} = \sqrt{\frac{\sum_{i=1}^{10}(x_i - p)^2}{10} - \left(\frac{\sum_{i=1}^{10}(x_i - p)}{10}\right)^2}$$

$$\sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2} = \frac{9}{10}$$

68. C

Sol. $$T_{r+1} = {}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(-\frac{1}{3x}\right)^r$$

$$= {}^9C_r\left(\frac{3}{5}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$$ for the term independent of x put r = 6

$$\Rightarrow T_7 = {}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6$$

$$= {}^9C_3\left(\frac{1}{6}\right)^3 = \frac{9\times 8\times 7}{3\times 2\times 1}\left(\frac{1}{6}\right)^3 = \left(\frac{7}{18}\right)$$

69. D

Sol. $$f'(x) = k.x(x+1)(x-1) = k\left(x^3 - x\right)$$

$$\Rightarrow \quad f(x) = k\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C$$

$$\Rightarrow \quad f(0) = C$$

$$\Rightarrow \quad f(x) = f(0)$$

$$\Rightarrow \quad k\frac{\left(x^4 - 2x^2\right)}{4} + C = C$$

$$\Rightarrow \quad x^2\left(x^2 - 2\right) = 0$$

$$\Rightarrow \quad x = \left\{0, \sqrt{2}, -\sqrt{2}\right\}$$

70. D

Sol. $$\lim_{x\to a}\frac{\frac{1}{3}(a+2x)^{-2/3}.2 - \frac{1}{3}.(3x)^{-2/3}.3}{\frac{1}{3}(3a+x)^{-2/3}.-\frac{1}{3}(4x)^{-2/3}.4}$$

$$= \frac{\frac{1}{3}(3a)^{-2/3}.(2-3)}{\frac{1}{3}(4a)^{-2/3}.(1-4)} = \frac{3^{-2/3}}{4^{-2/3}}.\frac{1}{3}$$

$$= \frac{2^{4/3}}{9^{1/3}}.\frac{1}{3} = \frac{2}{3}.\left(\frac{2}{9}\right)^{1/3}$$

71. 04.00

Sol. For (1, 2) of $y^2 = 4x \Rightarrow t = 1, a = 1$

normal $\Rightarrow tx + y = 2at + at^3$

$\Rightarrow x + y = 3$ intersect x – axis at (3, 0)

$y = e^x \quad \Rightarrow \frac{dy}{dx} = e^x$

tangent $\Rightarrow y - e^c = e^c(x - c)$

at $(3,0) \Rightarrow 0 - e^c = e^c(3 - c) \Rightarrow c = 4$

72. 54.00

Sol. Let xyz be the three digit number

$x + y + z = 10,\ x \le 1,\ y \ge 0,\ z \ge 0$

$x - 1 = t \quad \Rightarrow \quad x = 1 + t \quad x - 1 \ge 0$

$t \ge 0$

$t + y + z = 10 - 1$

$t + y + z = 9, \quad 0 \le t, z, z \le 9$

total number of non negative integral solution $= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = \frac{11.10}{2} = 55$

But for t = 9, x = 10, so required number or integers = 55 – 1 = 54

73. 08.00

Sol. $x - 2y + 5z = 0$ ……………………(i)

$-2x + 4y + z = 0$ ……………………(ii)

$-7x + 14y + 9z = 0$ ……………………(iii)

$2\times(i) + (ii) \Rightarrow z = 0$

$\Rightarrow \quad x = 2y$

$\Rightarrow \quad 15 \le x^2 + y^2 + z^2 \le 150$

$\Rightarrow \quad 15 \le 4y^2 + y^2 \le 150$

$\Rightarrow \quad 3 \le y^2 \le 30$

$\Rightarrow \quad y = \pm 2, \pm 3, \pm 4, \pm 5$

$\Rightarrow$ 8 solutions.

74. 05.00

Sol. Normal of plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$\vec{n} = -\hat{i} + \hat{j} + \hat{k}$

D.R.'s = –1, 1, 1

Plane $\Rightarrow -1(x-1)+1(y-0)+1(z-0)=0$

$\Rightarrow x-y-z-1=0$

If (x, y, z) is foot of perpendicular of M (1, 0, 1) on the plane then

$$\Rightarrow \quad \frac{x-1}{1}=\frac{y-0}{-1}=\frac{z-1}{-1}=\frac{-(1-0-1-1)}{3}$$

$$x=\frac{4}{3},\ y=-\frac{1}{3},\ z=\frac{2}{3}$$

$$\alpha+\beta+\gamma=\frac{4}{3}-\frac{1}{3}+\frac{2}{3}=\frac{5}{3}$$

75. 39.00

Sol. $3, A_1, A_2, A_3, ...A_m, 243$

$$d=\frac{243-3}{m+1}=\frac{240}{m+1}$$

$3, G_1, G_2, G_3, 243$

$$r=\left(\frac{243}{3}\right)^{\frac{1}{3+1}}=(81)^{1/4}=3$$

$G_2 = A_4$

$$\Rightarrow \quad 3(3)^2=3+4\left(\frac{240}{m+1}\right)$$

$$\Rightarrow \quad 27=3+\frac{960}{m+1}$$

$\Rightarrow \quad m+1=40$

$\Rightarrow \quad m=39$