KVPY – CLASS-XII
FULL TEST – 1
(OLTS-1819-T5-FT-1-KVPY-XII)

PART – I

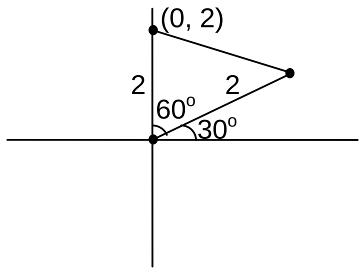
MATHEMATICS

1. If one vertex of an equilateral triangle of side 2 is the origin and another vertex lies on the line $x=\sqrt{3}y$, then the third vertex can be

(A) (0, 2)
(B) $\left(-\sqrt{3},\ -1\right)$
(C) (–2, –2)
(D) $\left(\sqrt{3},\ 1\right)$

Ans. A

Sol.



2. The minimum distance of the surface $xyz^2=2$ from the origin is equal to

(A) 0
(B) 1
(C) 2
(D) 5

Ans. C

Sol. Let $d=x^2+y^2+z^2$.

$$d=x^2+y^2+\frac{2}{xy}=\left(x-y\right)^2+2xy+\frac{2}{xy}\geq 2xy+\frac{2}{xy}\geq 4$$

3. If the equation $|\sec\theta|+|\operatorname{cosec}\theta|=c$ has solutions in θ, then the least integer value that c^2 can take is

(A) 4
(B) 6
(C) 8
(D) 10

Ans. C

Sol. $c^2=\dfrac{1}{\cos^2\theta}+\dfrac{1}{\sin^2\theta}+\dfrac{2}{|\sin\theta||\cos\theta|}$

$=4\operatorname{cosec}^2 2\theta+4|\operatorname{cosec}2\theta|$

$\geq 4+4$

$\Rightarrow c^2\geq 8$

4. If $x=\sum_{r=1}^{90} 2r\sin(2r)^\circ$, then the value of x is equal to

(A) $90\cot 1^\circ \operatorname{cosec} 1^\circ$ (B) $90\sec 1^\circ$

(C) $90\cot 1^\circ$ (D) none of these

Ans. C

Sol. $S = 1\sin 2^\circ + 2\sin 4^\circ + 3\sin 6^\circ + ... + 89\sin 178^\circ$

$S = 89\sin 178^\circ + 88\sin 176^\circ + 87\sin 174^\circ + ... + 1\sin 2^\circ$

Adding the two, we get

$$S = 90\left(\sin 2^\circ + \sin 4^\circ + ... + \sin 178^\circ\right) = 90\left(\frac{\sin 89^\circ}{\sin 1^\circ}\right)\sin 90^\circ = 90\cot 1^\circ$$

5. If $f(x) = \left(x^2+2x+3\right)^2 + 2\left(x^2+2x+3\right)+3$, then which of the following statements is correct?

(A) The equation $f(x)=0$ has four real roots.

(B) The equation $f(x)=0$ has two real roots and two imaginary roots.

(C) The minimum value of $f(x)$ equals 11.

(D) The minimum value of $f(x)$ equals 12.

Ans. C

Sol. Let $x^2+2x+3=t$.

$$f(x) = t^2+2t+3 = \left((x+1)^2+3\right)^2+2$$

6. If z_1, z_2 lie on $|z|=r$ and $\arg\left(\frac{z_1}{z_2}\right)=\frac{\pi}{3}$, then $\frac{(z_1+z_2)^2}{z_1z_2}$ is equal to

(A) 1 (B) 2

(C) 3 (D) none of these

Ans. C

Sol. $\frac{z_1}{z_2}+\frac{z_2}{z_1} = 2\cos\frac{\pi}{3} \Rightarrow \frac{(z_1+z_2)^2}{z_1z_2} = 4\cos^2\frac{\pi}{6} = 3$

7. The graph of a function $y=f(x)$ is symmetric about the lines $x=a$ and $x=b$ $(b<a)$. The period of the function is equal to

(A) $a+b$ (B) $a-b$

(C) $2(a+b)$ (D) $2(a-b)$

Ans. D

Sol. $f(a-x)=f(a+x)$ and $f(b-x)=f(b+x)$

$\Rightarrow f(2a-x)=f(2b-x)$

$\Rightarrow f(x)=f(2(a-b)+x)$

8. There are 3 bags. Bag 1 contains 2 red and a^2-4a+8 black balls, bag 2 contains 1 red and a^2-4a+9 black balls, and bag 3 contains 3 red and a^2-4a+7 black balls. A ball is drawn at random from a bag. The maximum value of the probability that it is a red ball is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$

(C) $\frac{2}{9}$ (D) $\frac{4}{9}$

Ans. A

Sol. The required probability is $P(A)=\frac{1}{3}\frac{6}{a^2-4a+10}$

9. Let a, b, c be positive real numbers forming an A.P. If the equation $ax^2+bx+c=0$ has real roots, then

(A) $\left|\sqrt{\frac{a}{c}}-\sqrt{\frac{c}{a}}\right|\ge 2\sqrt{3}$ (B) $\left|\sqrt{\frac{a}{c}}-\sqrt{\frac{c}{a}}\right|\le 2\sqrt{3}$

(C) $\left|\sqrt{\frac{a}{c}}+\sqrt{\frac{c}{a}}\right|\ge 2\sqrt{3}$ (D) $\left|\sqrt{\frac{a}{c}}+\sqrt{\frac{c}{a}}\right|\le 2\sqrt{3}$

Ans. A

Sol. $D\ge 0 \Rightarrow b^2-4ac\ge 0$

$(a+c)^2-16ac\ge 0$

$\frac{a}{c}+\frac{c}{a}-2\ge 12 \Rightarrow \left|\sqrt{\frac{a}{c}}-\sqrt{\frac{c}{a}}\right|\ge 2\sqrt{3}$

10. The smallest natural number n that satisfies $12^{200}<n^{300}$ is

(A) 5 (B) 6

(C) 7 (D) 8

Ans. B

Sol. $12^{200}<n^{300}\Rightarrow 144^{100}<\left(n^3\right)^{100}\Rightarrow 144<n^3$

$\Rightarrow$ The smallest number is 6.

11. The equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$ has

(A) no solution (B) only one solution

(C) only two solutions (D) more than two solutions

Ans. D

Sol. Put $\sqrt{x-1}=t$ or $x=t^2+1$ so that the given equation becomes

$\sqrt{t^2+4-4t}+\sqrt{t^2+9-6t}=1$ or $\sqrt{(t-2)^2}+\sqrt{(t-3)^2}=1$

or $|t-2|+|t-3|=1$

This equation is satisfied for all values of t lying between 2 and 3 i.e. $2\le t\le 3$.

Thus, the given equation is satisfied for all values of x lying between 5 and 10.

12. If $a = \log_{12} 18$, $b = \log_{24} 54$, then the value of $ab + 5(a-b)$ is equal to

(A) 0 (B) 4

(C) 1 (D) none of these

Ans. C

Sol. We have $a = \log_{12} 18 = \dfrac{\log_2 18}{\log_2 12} = \dfrac{1+2\log_2 3}{2+\log_2 3}$ and $b = \log_{24} 54 = \dfrac{\log_2 54}{\log_2 24} = \dfrac{1+3\log_2 3}{3+\log_2 3}$

Putting $x = \log_2 3$, we have

$$ab + 5(a-b) = \frac{1+2x}{2+x}.\frac{1+3x}{3+x} + 5\left(\frac{1+2x}{2+x} - \frac{1+3x}{3+x}\right)$$

$$= \frac{6x^2 + 5x + 1 + 5(-x^2+1)}{(x+2)(x+3)} = \frac{x^2+5x+6}{(x+2)(x+3)} = 1$$

13. If the (n + 1) numbers a, b, c, d, … are all different and each of them is a prime number, then the number of different factors (other than 1) of a^m b c d... $(m \in N)$ is equal to

(A) $m - 2^n$ (B) $(m+1)2^n$

(C) $(m+1)2^n - 1$ (D) none of these

Ans. C

Sol. Number of different factors of a^m b c d...

$$= (m+1)\{(1+1)(1+1)(1+1)\text{... n factors}\} - 1$$

$$= (m+1)2^n - 1$$

14. If $(1+x)^n = \sum_{r=0}^{n} a_r x^r$, $b_r = 1 + \dfrac{a_r}{a_{r-1}}$ and $\prod_{r=1}^{n} b_r = \dfrac{(101)^{100}}{100!}$, then n is equal to

(A) 99 (B) 100

(C) 101 (D) 102

Ans. B

Sol. $(1+x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r = \sum_{r=0}^{n} a_r x^r$

$\therefore$ $a_r = {}^nC_r$

Also, $b_r = 1 + \dfrac{a_r}{a_{r-1}} = 1 + \dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n+1}{r}$

$$\therefore \quad \prod_{r=1}^{n} b_r = \prod_{r=1}^{n}\left(\frac{n+1}{r}\right) = \frac{(n+1)^n}{n!} = \frac{(101)^{100}}{100!}$$

$\therefore$ n = 100

15. If x, y, z are integers in A.P. lying between 1 and 9 and x51, y41 and z31 are three digit numbers, then the value of $\begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$ is equal to

(A) $x + y + z$ (B) $x - y + z$

(C) 0 (D) $x + 2y + z$

Ans. C

Sol. Let $\Delta = \begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$

$$\Delta = \begin{vmatrix} 5 & 4 & 3 \\ 100x+50+1 & 100y+40+1 & 100z+30+1 \\ x & y & z \end{vmatrix}$$

Apply $R_2 \to R_2 - (100R_3 + 10R_1)$.

$$\Delta = \begin{vmatrix} 5 & 4 & 3 \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$$

Applying $C_2 \to C_2 - \frac{1}{2}(C_1 + C_3)$, we get $\Delta = \begin{vmatrix} 5 & 0 & 3 \\ 1 & 0 & 1 \\ x & y - \frac{1}{2}(x+z) & z \end{vmatrix} = \begin{vmatrix} 5 & 0 & 3 \\ 1 & 0 & 1 \\ x & 0 & z \end{vmatrix} = 0$

16. If $(5,12)$ and $(24, 7)$ are the foci of an ellipse passing through the origin, then the eccentricity of the ellipse is

(A) $\sqrt{386}/13$ (B) $\sqrt{386}/38$

(C) $\sqrt{386}/25$ (D) $1/\sqrt{2}$

Ans. B

Sol. If the foci be $S(5, 12)$ and $S'(24, 7)$ and the ellipse passes through origin O, then SO $= 13$; $S'O =$ 25 and SS' = $\sqrt{386}$.

Now, $SO + S'O = 2a$ and SS' = 2ae

$$\therefore \quad e = \frac{SS'}{S'O + SO} = \frac{\sqrt{386}}{38}$$

17. The consecutive odd integers whose sum is $45^2 - 21^2$ are

(A) 43, 45, ..., 75 (B) 43, 45, ..., 79

(C) 43, 45, ..., 85 (D) 43, 45, ..., 89

Ans. D

Sol. Let n consecutive odd integers be $2m+1$, $2m+3$, $2m+5$, ..., $2m+2n-1$.

Given that $(2m+1)+(2m+3)+(2m+5)+...+(2m+2n-1) = 45^2 - 21^2$

$\Rightarrow \quad 2mn + (1+3+5+...+2n-1) = 45^2 - 21^2$

$\Rightarrow \quad 2mn + n^2 = 45^2 - 21^2$

$\Rightarrow \quad (n+m)^2 - m^2 = 45^2 - 21^2$

On comparing, $n + m = 45$, $m = 21$

18. The values of x and y satisfying the equation
$\sin^7 y = \left|x^3 - x^2 - 9x + 9\right| + \left|x^3 - x^2 - 4x + 4\right| + \sec^2 2y + \cos^4 y$ are

(A) $x = 1, y = n\pi, n \in I$

(B) $x = 1, y = 2n\pi + \frac{\pi}{2}, n \in I$

(C) $x = 1, y = 2n\pi, n \in I$

(D) $x = 1, y = 2n\pi + \frac{3\pi}{2}, n \in I$

Ans. B

Sol. $\sin^7 y = \left|x^3 - x^2 - 9x + 9\right| + \left|x^3 - x^2 - 4x + 4\right| + \sec^2 2y + \cos^4 y$

$= \left|(x-1)(x^2-9)\right| + \left|(x-1)(x^2-4)\right| + \sec^2 2y + \cos^4 y \ge 1$ **and** $\sin^7 y \le 1$

19. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^{\frac{1}{a}}\left(n^{a-\frac{1}{a}} + k^{a-\frac{1}{a}}\right)}{n^{a+1}}$ is equal to

(A) 1

(B) 2

(C) 3

(D) none of these

Ans. A

Sol. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^{1/a}\left(n^{a-\frac{1}{a}} + k^{a-\frac{1}{a}}\right)}{n^{a+1}} = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\left(\left(\frac{k}{n}\right)^{\frac{1}{a}} + \left(\frac{k}{n}\right)^{a}\right) = \int_0^1 \left(x^{1/a} + x^a\right) dx$

20. Let F_1, F_2 be the foci of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and F_3, F_4 the foci of its conjugate hyperbola. If e_H and e_C are their eccentricities respectively, then pick the correct statement.

(A) The equations of their asymptotes are different.

(B) $e_H > e_C$

(C) Area of the quadrilateral formed by their foci is 50 sq. units.

(D) Their auxiliary circles have the same equation.

Ans. C

Sol. $e_H = \sqrt{1 + \frac{9}{16}} = \frac{5}{4} \Rightarrow F_1 \equiv (5, 0); F_2 \equiv (-5, 0)$

$\frac{1}{e_H^{\,2}} + \frac{1}{e_C^{\,2}} = 1$

$\Rightarrow e_C = \frac{5}{3}$

$\therefore F_3 \equiv (0, 5); F_4 \equiv (0, -5)$

PHYSICS

21. A stone thrown down with a speed u takes a time t_1 to reach the ground, while another stone, thrown upwards from the same point with the same speed, takes time t_2. The maximum height the second stone reaches from the ground is

(A) ½ $gt_1 t_2$ (B) $g/8(t_1 + t_2)^2$
(C) $g/8(t_1 - t_2)^2$ (D) ½ gt_2^2

Ans. B

Sol.

$$-h = -Ut_1 + \frac{-1}{2} g t_1^2$$

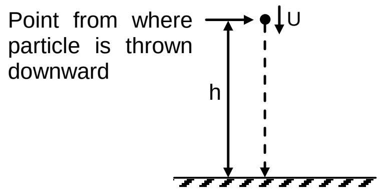
$$h = Ut_1 + \frac{1}{2} g t_1^2 \qquad \ldots(1)$$

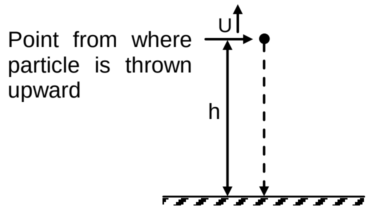
$$-h = Ut_2 - \frac{1}{2} g t_2^2 \qquad \ldots(2)$$

$$Ut_1 + Ut_2 = \frac{1}{2} g t_2^2 - \frac{1}{2} g t_1^2 \qquad \text{(from 1 \& 2)}$$

$$U(t_1 + t_2) = \frac{g}{2}(t_2 - t_1) \times (t_1 + t_2)$$

$$U = \frac{g}{2}(t_2 - t_1)$$





22. The river 'A' and river 'B' merge to form a river 'C' with their speeds in the ratio 1 : 1.5. The cross-sectional areas of the river 'A', river 'B' and the river 'C' are in the ratio 1 : 2 : 3. Assuming streamline flow, the ratio of the speed of river 'C' to that of the river 'B'.

(A) 7 : 9 (B) 4 : 3
(C) 8 : 9 (D) 5 : 3

Ans. C

Sol. Let
Area of river 'A' ⇒ A
Area of river 'B' ⇒ 2A
Area of river 'C' ⇒ 3A

$$V_A : V_B : V_C \quad \Rightarrow \quad V : \frac{3}{2}V : V_1$$

By equation of continuity

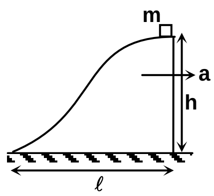
$$AV + \frac{3}{2}A \cdot 2V \quad \Rightarrow \quad 3A\,V_1$$

$$V_{\text{river 'C'}} = \frac{4}{3}V$$

$$\frac{V_{\text{river 'B'}}}{V_{\text{river 'C'}}} \Rightarrow \frac{\frac{3}{2}V}{\frac{4}{3}V} = \frac{9}{8}$$

$$V_{\text{river 'C'}} : V_{\text{river 'B'}} = 8 : 9.$$

23. A block of mass m is placed on the top of a wedge having undefined smooth curved surface. If the wedge is now accelerated horizontally with acceleration a, then the speed of block with respect to wedge when it reaches the bottom of wedge is



(A) $\sqrt{2gh}$ (B) $\sqrt{2(a+g)h}$
(C) $\sqrt{2(a\ell-gh)}$ (D) $\sqrt{2(a\ell+gh)}$

Ans. D

Sol. $ma\ell + mgh = \frac{1}{2}mv^2$

$\therefore\ v = \sqrt{2(a\ell+gh)}$

24. A block of mass 2.0 kg is sliding on a frictionless horizontal floor with a velocity of 5.0 m/s due east. It is subjected to a variable net force for 10 seconds resulting in an impulse of 6 N-s in the North and 18.0 N-s in the west. The total work done by this force is
(A) 0.0 J (B) 90 J
(C) 100 J (D) 200 J

Ans. A

Sol. $v_{final} = \sqrt{v_{North}^2 + v_{West}^2}$ $V_{North} = \frac{6}{2}$ = 3 m/s

$= \sqrt{(3)^2+(4)^2}$ $V_{West} = \frac{18-10}{2}$ = 4 m/s

= 5

$W = \frac{1}{2}m(v_f^2 - v_i^2) = 0.$

25. A ball dropped from a high altitude acquires a terminal velocity before hitting the ground, where it bounces off elastically. If air resistance depends on the speed of the ball, what will it acceleration be just after the first bounce?
(A) zero (B) g downwards
(C) 2g downwards (D) 3g downwards

Ans. C

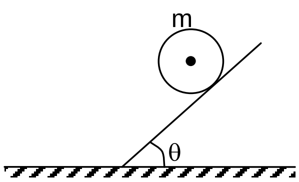
Sol. Let F_{air} = kv then mg = kv

S_Q $v = \frac{mg}{k}$

But just after the bounce F_{net} = mg + kv

$$a = g + \frac{kv}{m} = 2g$$

26. A sphere of mass m has to purely roll on a rough inclined plane of coefficient of friction 'μ'. The friction force acting on the sphere is



(A) μmg cosθ
(B) $\frac{2mg\sin\theta}{7}$ downward
(C) $\frac{2mg\sin\theta}{7}$ upward
(D) $\frac{5mg\sin\theta}{7}$ downward

Ans. C

Sol. $f = \dfrac{mg\sin\theta}{1+\dfrac{R^2}{K^2}}$

27. Two particles move parallel to x axis about the origin with same amplitude a and frequency ω. At a certain instant they are found at a distance a/3 from the origin on opposite sides but their velocities are in the same direction. What is the phase difference between the two.

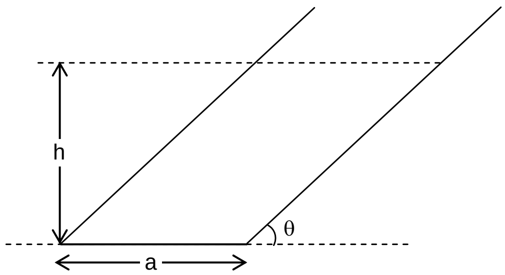
(A) $\cos^{-1}\left(\dfrac{7}{9}\right)$ (B) $\cos^{-1}\left(\dfrac{5}{9}\right)$

(C) $\cos^{-1}\left(\dfrac{4}{9}\right)$ (D) $\cos^{-1}\left(\dfrac{1}{9}\right)$

Ans. A

Sol. $\cos\phi = 1 - 2\times\left(\dfrac{1}{3}\right)^2 = \dfrac{7}{9}$

28. A hollow tilted cylindrical vessel of negligible mass rest on horizontal plane as shown in figure. The diameter of the base is a and the side of the cylinder makes an angle θ with the horizontal. Water is then slowly poured into the cylinder. The cylinder topple's over when the water reaches a certain height h, given by



(A) $h = 2a\tan\theta$
(B) $h = 2a\tan^2\theta$
(C) $h = a\tan\theta$
(D) $h = \dfrac{a}{2}\tan\theta$

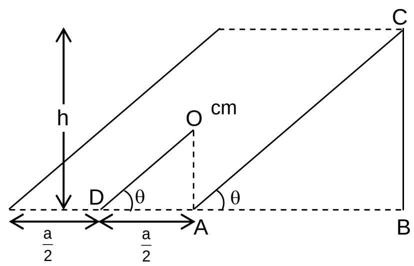
Ans. C

Sol. $\because \quad AC = \dfrac{h}{\sin\theta}$

$\therefore \quad OD = \dfrac{h}{2\sin\theta}$; But $OD = \dfrac{a}{2\cos\theta}$

$\therefore \quad \dfrac{h}{2\sin\theta} = \dfrac{a}{2\cos\theta}$

$h = a\tan\theta$



29. The sun orbits the centre of the galaxy (Milky-way) in almost circular path of radius 'R' in a period 'T' and earth also orbits the sun in an almost circular path of radius 'r' in a period 't'. Assume whole mass of the galaxy concentrated at its centre and find an expression for the ratio of mass of galaxy to that of sun.

(A) $\left(\dfrac{R}{r}\right)^2\left(\dfrac{t}{T}\right)^2$ (B) $\left(\dfrac{R}{r}\right)^3\left(\dfrac{T}{t}\right)^2$

(C) $\left(\dfrac{R}{r}\right)^2\left(\dfrac{t}{T}\right)^3$ (D) $\left(\dfrac{R}{r}\right)^2\left(\dfrac{T}{t}\right)^3$

Ans. A

Sol. $T = 2\pi\sqrt{\frac{R^3}{GM}}$

$M = \frac{4\pi^2 R^3}{GT^2}$

30. Two rubber balloons filled with same ideal gas when held at the bottom of the lake in thermal equilibrium with surrounding water occupy equal volumes. Rubber of the first balloon is a good conductor of heat while that of the second balloon is a good insulator. Both the balloons are set free simultaneously. If temperature of water in the lake is uniform, which balloon will occupy more volume when it comes near surface of lake?
(A) The first balloon.
(B) The second balloon
(C) Both the balloons will occupy equal volumes.
(D) Decision depends on the adiabatic exponent of the gas.

Ans. A

Sol. For 1st balloon
$(P_0 + h\rho g) \times V_1 = P_o V_1'$
For 2nd balloon
$(P_0 + h\rho g) \times V_2' = P_o V_1'$

31. A point source is emitting sound in all directions. The ratio of distance of two points from the point source where the difference in loudness levels is 3 dB is: ($\log_e 2 = 0.3$)

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{4}$ (D) $\frac{2}{3}$

Ans. B

Sol. $3 = 10 \log\left(\frac{r_2}{r_1}\right)^2$

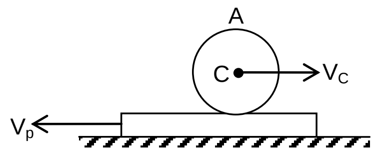
32. A certain sample of monoatomic ideal gas is subject to a thermodynamic process in which V and T are related as $V^2 = kT$ (k is constant). The molar specific heat of the gas in this process is

(A) $\frac{3R}{2}$ (B) 2R
(C) $\frac{5R}{2}$ (D) $\frac{7R}{2}$

Ans. B

Sol. $v^2 = kT$
or PV^{-1} = constant
$C = C_V + \frac{R}{2} = 2R$

33. The velocities are in ground frame and the cylinder is performing pure rolling on the plank. Velocity of point 'A' would be



(A) $2V_c$
(B) $2V_c + V_p$
(C) $2V_c - V_p$
(D) $2(V_c + V_p)$

Ans. C

Sol. $V_{Ap} = 2V_c$
$V_{Ag} = V_{Ap} + V_{Pg} = 2V_c - V_p$

34. A superconducting rigid planer loop of area 'A' and self inductance 'L' carrying a current is held motionless in a region of free space. Now a uniform magnetic field of induction 'B' pointing everywhere parallel to the magnetic moment '$\vec{m}$' of the loop s switched on. The current in the loop after magnetic field is switched on is given by

(A) $\frac{AB}{L}$ (B) $\frac{m}{A}$
(C) $\frac{m}{A} - \frac{AB}{L}$ (D) $\frac{m}{A} + \frac{AB}{L}$

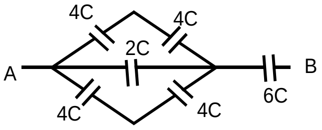
Ans. C

Sol. Initial current $i = \frac{m}{A}$ $(\because\ m = iA)$

Induced current $i = \frac{AB}{L}$ $(\because\ \phi = Li)$

Net current = $\frac{m}{A} - \frac{AB}{L}$

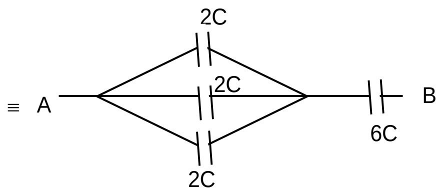
35. The equivalent capacitance between points A and B of the circuit will be

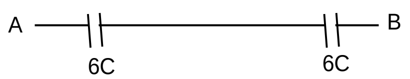


(A) 12C (B) 6 C
(C) 3 C (D) 24 C

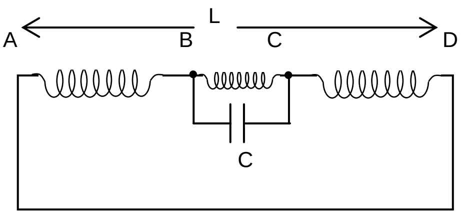
Ans. C

Sol. Equivalent circuit of the above figure can be drawn as
$C_{AB} = 3C$





36. An inductance L is split into three equal parts AB, BC and CD and a capacitor C is connected across its two centre terminals (figure). The outer terminals are short circuited. The frequency of oscillation is



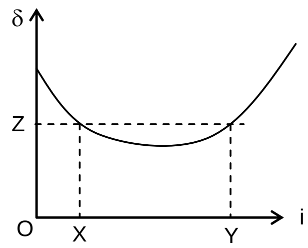
(A) $\frac{3}{2\sqrt{2}p}$ ´ $\frac{1}{\sqrt{LC}}$ (B) $\frac{1}{2p}$ ´ $\frac{1}{\sqrt{LC}}$

(C) $\frac{\sqrt{3}}{2p}$ ´ $\frac{1}{\sqrt{LC}}$ (D) $\frac{3}{2p}$ ´ $\frac{1}{\sqrt{LC}}$

Ans. A

Sol. $L' = \frac{(2/3)L.(1/3)L}{(2/3)L + (1/3)L} = \frac{2}{9}L, w = \frac{1}{\sqrt{(2/9)L \times C}}$

37. Graph shown is between deviation (δ) and angle of incidence I then angle of prism is



(A) –x + y + z
(B) x + y – z
(C) x + z – y
(D) x – y + z

Ans. B

Sol. $\delta = x + y - z$

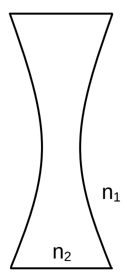
38. A hollow lens is made of thin glass and in the shape of a double concave lens. It can be filled with air, water of refractive index 1.33 or CS_2 of refractive index 1.6. It will act as a diverging lens if it is
(A) filled with air and immersed in water.
(B) filled with water and immersed in CS_2.
(C) filled with air and immersed in CS_2.
(D) filled with CS_2 and immersed in water.

Ans. D

Sol. μ_{air} = I.

$\mu_{water} = 1.33$

$\mu_{CS_2} = 1.6$



$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left[-\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f} = -\left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

For diverging lens f must be –ve.

$\therefore$ for this $\frac{n_2}{n_1} > 1$

$n_2 > n_2$

$\therefore$ Answer (D) is the correct option as

$\mu_{CS_2} > \mu_{water}$

39. When the electron in a hydrogen atom jumps from the second orbit to the first orbit, the wavelength of the radiation emitted is λ. When the electron jumps from the third to the first orbit, the wavelength of the radiation emitted as

(A) $\frac{9}{4}\lambda$ (B) $\frac{4}{9}\lambda$

(C) $\frac{27}{32}\lambda$ (D) $\frac{32}{27}\lambda$

Ans. C

Sol. $\frac{hc}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R$ ...(i)

$\frac{hc}{\lambda'} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8}{9}R$ ...(ii)

$\frac{\lambda'}{\lambda} = \frac{3}{4} \times \frac{9}{8}$, $\lambda' = \frac{27}{32}\lambda$

40. If photoelectron emitted by a metal on irradiating it by a light of wavelength λ cannot move away farther than a distance 'd' in presence of a retarding electric field 'E'. The threshold wavelength of the metal is

(A) $\frac{hc}{eEd}$ (B) $\lambda - \frac{hc}{eEd}$

(C) $\lambda + \frac{hc}{eEd}$ (D) $\left(\frac{1}{\lambda} - \frac{eEd}{hc}\right)^{-1}$

Ans. D

Sol. $\frac{hc}{\lambda} = \frac{hc}{\lambda_o} + Eed$

CHEMISTRY

41. Which of the following two substances can be distinguished by heating?

(A) $CaCO_3$ and $MgCO_3$ (B) $LiNO_3$ and $NaNO_3$

(C) $Ba(NO_3)_2$ and $Ca(NO_3)_2$ (D) $NaHCO_3$ and $MgCO_3$

Ans. B

Sol. $LiNO_3$ will evolve a coloured gas, $NaNO_3 \rightarrow$ Colourless gas.

42. Which of the following can react with hydrogen in presence of light?

(A) N_2 (B) Cl_2

(C) O_2 (D) S_8

Ans. B

Sol. $H_2 + Cl_2 \xrightarrow{\text{Light}} 2HCl$

43. The energy of first orbit of hydrogen atom is -13.6 eV. How much energy is required to excite the electron of hydrogen from ground state to first excited state?

(A) 3.4 eV (B) 10.2 eV

(C) 13.6 eV (D) 12.1 eV

Ans. B

Sol. $\Delta E = E_2 - E_1 = -3.4-(-13.6) = 10.2$ eV

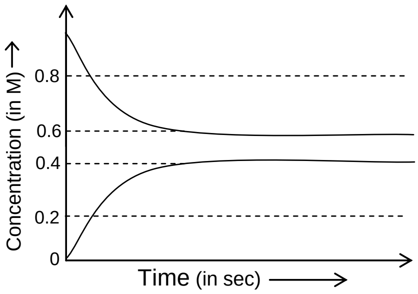
44. In which of the following molecules/ions both the central atoms have same type of hybridization?

(A) CO_2 and SO_2
(B) NF_3 and BF_3
(C) H_2O and OF_2
(D) $BeCl_2$ and SCl_2

Ans. C

Sol. 'O' undergoes sp^3 hybridization in both compounds.

45.



Above graph is given for the following reversible reaction.

$X(g) \rightleftharpoons Y(g)$

The equilibrium constant K_C of above reaction is:

(A) $\frac{1}{4}$
(B) $\frac{3}{2}$
(C) $\frac{2}{3}$
(D) 4

Ans. C

Sol. $K_C = \frac{[Y]}{[X]} = \frac{0.4}{0.6} = \frac{2}{3}$

46. $CN^- + H_2O \rightleftharpoons HCN + OH^-$

What will be the ionization constant (K_a) of HCN if the hydrolysis constant (K_h) of above reaction is 10^{-8}?

(A) 10^{-4}
(B) 10^{-6}
(C) 10^{6}
(D) 10^{-10}

Ans. B

Sol. $K_h = \frac{K_w}{K_a} = \frac{10^{-14}}{10^{-8}} = 10^{-6}$

47. Which of the following solution does NOT display common ion effect?

(A) H_2S + NaHS
(B) HCN + KH
(C) NaHS + HCl
(D) H_2CO_3 + $KHCO_3$

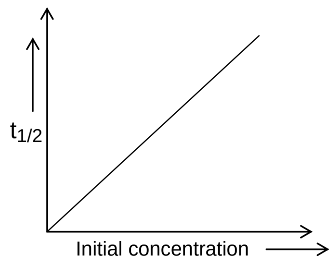
Ans. B

Sol. No common ion present between HCN and KH.

48. What is the order of the reaction for which the above graph is given?

(A) Zero (B) First

(C) Second (D) Third



Ans. A

Sol. $t_{½}\ \alpha$ a for zero-order reaction.

49. The solubility product of PbI_2 is 32×10^{-9} $mol^3\ L^{-3}$ at a certain temperature. What is the molarity of the saturated solution of PbI_2?

(A) 0.0002 M (B) 0.002 M

(C) 0.2 M (D) 0.02 M

Ans. B

Sol. $4s^3 = 32 \times 10^{-9} \Rightarrow s = 2 \times 10^{-3}$ M.

50. The most acidic compound out of the following is:

(A) $CH_3CH(F)CH_2COOH$ (B) $CH_3CH_2CH(F)COOH$

(C) $CH_3CH_2CH(Cl)COOH$ (D) $CH_2(Cl)CH_2CH_2COOH$

Ans. B

Sol. Electron withdrawing atom increases acidic strength.

51. Which isomer of $CH_3CH_2CH_2CH_2NH_2$ is most basic in nature?

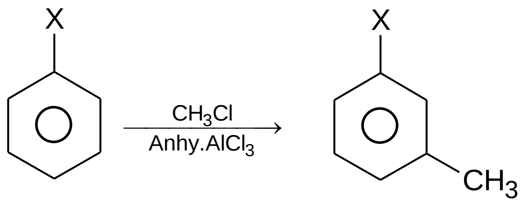
(A) Chain isomer (B) Position isomer

(C) Functional isomer (D) Metamer

Ans. C

Sol. 2° and 3° amines are the functional isomers of 2°-amine.

52.



In the above reaction (X) should be

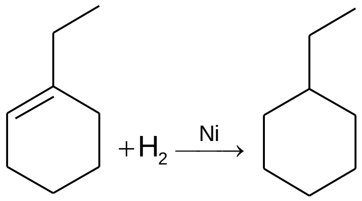
(A) OH (B) Cl

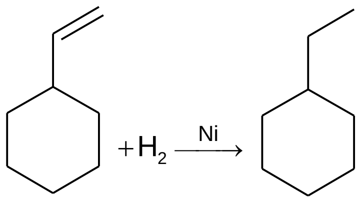
(C) CH_2OH (D) CHO

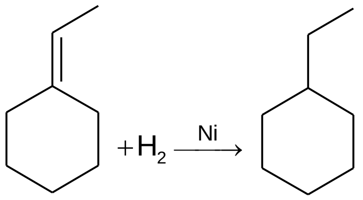
Ans. D

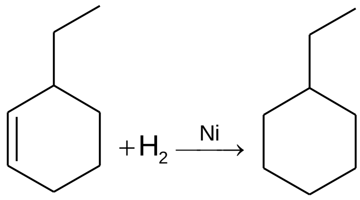
Sol. X(CHO)is a meta directing group.

53. Which of the following reaction releases maximum heat energy?

(A) 

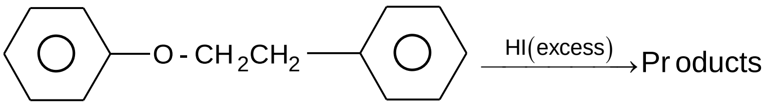
(B) 

(C) 

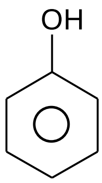
(D) 

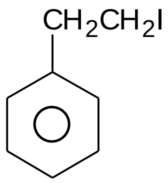
Ans. B

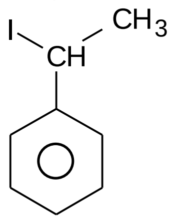
Sol. Unsubstituted or less substituted alkenes are more reactive towards H_2/Ni.

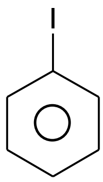
54. 

Which of the following is NOT a product of above reaction?

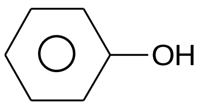
(A) 

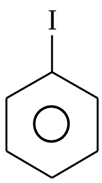
(B) 

(C) 

(D) 

Ans. D

Sol. 

will not react with HI to form 

55. Which of the following contains (3c-2e) bond?

(A) H_3BO_3
(B) B_2H_6
(C) $NaBH_4$
(D) B_2O_3

Ans. B

Sol. B_2H_6 contains 2c – 3e bond.

56. Which of the following contains metal ions?

(A) Silica gel
(B) Silicones
(C) Silanes
(D) Silicates

Ans. D

Sol. Silicates are metal salts.

57. In a primitive cubic unit cell of a metal, the metal atoms are present at the
(A) corners
(B) corners and face centres
(C) corners and body centre
(D) face centre and body centre

Ans. A

Sol. In primitive cubic unit cell, only corners are occupied.

58. Which transition metal ions contains a half-filled electron configuration?
(A) Ti^{2+}
(B) Mn^{2+}
(C) Cr^{2+}
(D) Fe^{2+}

Ans. B

Sol. Mn^{2+} is $[Ar]_{18}3d^5$.

59. Which of the following electrolyte(molten salt) on electrolysis deposits the maximum mass of metal by passing same amount of electricity?
(A) NaCl
(B) $MgCl_2$
(C) $AlCl_3$
(D) $CaCl_2$

Ans. A

Sol. Maximum amount of Na is formed.

60. ΔH = Enthalpy change of a thermodynamic process
ΔE = Internal energy change of a thermodynamic process
$\therefore$ The quantity ($\Delta H - \Delta E$) represents
(A) entropy change
(B) work done
(C) temperature change
(D) none of these

Ans. B

Sol. $\Delta H - \Delta E = P\Delta V$(Work done).

PART – II

MATHEMATICS

61. If $a_n = \sum_{k=1}^{n} \frac{1}{k(n+1-k)}$, then for $n \geq 2$,
(A) $a_{n+1} > a_n$
(B) $a_{n+1} < a_n$
(C) $a_{n+1} = a_n$
(D) $a_{n+1} - a_n = \frac{1}{n}$

Ans. B

Sol. We have $a_n = \frac{1}{n+1}\sum_{k=1}^{n}\left(\frac{1}{k} + \frac{1}{n+1-k}\right)$

$= \frac{2}{n+1}\sum_{k=1}^{n}\frac{1}{k}$

For $n \geq 2$,

$\frac{1}{2}(a_n - a_{n+1}) = \frac{1}{n+1}\sum_{k=1}^{n}\frac{1}{k} - \frac{1}{n+2}\sum_{k=1}^{n+1}\frac{1}{k}$

$$=\left(\frac{1}{n+1}-\frac{1}{n+2}\right)\sum_{k=1}^{n}\frac{1}{k}-\frac{1}{(n+1)(n+2)}$$

$$=\frac{1}{(n+1)(n+2)}\sum_{k=2}^{n}\frac{1}{k}>0$$

$$\Rightarrow a_n > a_{n+1}$$

62. For real x, the expression $2(k-x)\left(x+\sqrt{x^2+k^2}\right)$ cannot exceed

(A) k^2
(B) $2k^2$
(C) $3k^2$
(D) none of these

Ans. B

Sol. Let $y=2(k-x)\left(x+\sqrt{x^2+k^2}\right)$.

Put $\theta=\tan^{-1}\frac{x}{k},\ \theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

$$y=2k^2(1-\tan\theta)(\tan\theta+\sec\theta)=\frac{2k^2(\cos\theta-\sin\theta)(1+\sin\theta)}{\cos^2\theta}$$

$$\le\frac{2k^2(1-\sin\theta)(1+\sin\theta)}{\cos^2\theta}=2k^2$$

63. If z_1,z_2,z_3 are 3 distinct complex numbers such that $\frac{3}{|z_2-z_3|}=\frac{4}{|z_3-z_1|}=\frac{5}{|z_1-z_2|}$, then the value of $\frac{9}{z_2-z_3}+\frac{16}{z_3-z_1}+\frac{25}{z_1-z_2}$ equals

(A) 0
(B) 3
(C) 4
(D) 5

Ans. A

Sol. Let $\frac{3}{|z_2-z_3|}=\frac{4}{|z_3-z_1|}=\frac{5}{|z_1-z_2|}=k$

$$\Rightarrow|z_2-z_3|^2=\frac{9}{k^2}\Rightarrow(z_2-z_3)\left(\bar{z}_2-\bar{z}_3\right)=\frac{9}{k^2}\Rightarrow\frac{9}{z_2-z_3}=k^2\left(\bar{z}_2-\bar{z}_3\right)$$

Similarly, $\frac{16}{z_3-z_1}=k^2\left(\bar{z}_3-\bar{z}_1\right)$; $\frac{25}{z_1-z_2}=k^2\left(\bar{z}_1-\bar{z}_2\right)$

64. Let $k=\lim_{n\to\infty} n^2\int_{-1/n}^{1/n}(2012\sin x+2013\cos x)\ |x|\ dx$. The value of $k-2012$ is equal to

(A) –1
(B) 0
(C) 1
(D) 2012

Ans. C

Sol. $$\int_{-1/n}^{1/n}(2012\sin x+2013\cos x)\ |x|\ dx=4026\left(\frac{1}{n}\sin\frac{1}{n}+\cos\frac{1}{n}-1\right)=4026\left(\frac{1}{n}\sin\frac{1}{n}-2\sin^2\frac{1}{2n}\right)$$

Hence, $k = 4026 \lim_{n\to\infty} \left(\frac{\sin\frac{1}{n}}{\frac{1}{n}} - \frac{\sin^2 \frac{1}{2n}}{2\left(\frac{1}{2n}\right)^2} \right) = 2013$

65. Let $f(x) = \sqrt{\frac{x^2 + ax + 4}{x^2 + bx + 16}}$ be defined for all real x. The number of possible ordered pairs (a, b) (where $a, b \in I$) is equal to

(A) 89 (B) 85
(C) 135 (D) 150

Ans. C

Sol. $x^2 + bx + 16$ should be positive for all real x and $x^2 + ax + 4$ should be non-negative for all real x.

66. $\int \frac{\ln\left(x + \sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx =$

(A) $\frac{1}{2}\ln\left(x + \sqrt{1+x^2}\right) + C$ (B) $\frac{1}{2}\left(\ln\left(x + \sqrt{1+x^2}\right)\right)^2 + C$
(C) $\ln\left(x + \sqrt{1+x^2}\right) + C$ (D) $x\ln\left(x + \sqrt{1+x^2}\right) + C$

Ans. B

Sol. Integrating by parts taking $\frac{1}{\sqrt{1+x^2}}$ as 2nd function, we get

$I = \left(\ln\left(x + \sqrt{1+x^2}\right)\right)^2 - \int \frac{\ln\left(x + \sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx$

$\Rightarrow 2I = \left(\ln\left(x + \sqrt{1+x^2}\right)\right)^2 + 2C$

67. The number of functions f from set A = $\{0, 1, 2\}$ to set $B = \{0,1,2,3,4,5,6,7\}$ such that $f(i) \le f(j)$ for $i < j$ and, $i, j \in A$ is

(A) ${}^8C_3$ (B) ${}^8C_3 + 2\left({}^8C_2\right)$
(C) ${}^{10}C_3$ (D) none of these

Ans. C

Sol. A function $f : A \to B$ such that $f(0) \le f(1) \le f(2)$ falls in one of the following four categories:

Case 1 $f(0) < f(1) < f(2)$

There are ${}^8C_3$ functions in this category

Case 2 $f(0) = f(1) < f(2)$

There are ${}^8C_2$ functions in this category

Case 3 $f(0) < f(1) = f(2)$

There are ${}^{8}C_{2}$ functions in this category

Case 4 $f(0) = f(1) = f(2)$

There are ${}^{8}C_{1}$ functions in this category

Thus, the number of desired functions is

${}^{8}C_{3} + {}^{8}C_{2} + {}^{8}C_{2} + {}^{8}C_{1} = {}^{9}C_{3} + {}^{9}C_{2} = {}^{10}C_{3}$

68. If the equation $x^3 + bx^2 + cx + 1 = 0$ $(b < c)$ has only one real root α, then the value of $2\tan^{-1}(\operatorname{cosec}\alpha) + \tan^{-1}(2\sin\alpha\sec^2\alpha)$ is equal to

(A) $-\pi$ (B) $-\frac{\pi}{2}$

(C) $\frac{\pi}{2}$ (D) π

Ans. A

Sol. $\alpha \in (-1, 0)$. Hence, $\tan^{-1}\left(\frac{2\sin\alpha}{1-\sin^2\alpha}\right) = 2\tan^{-1}(\sin\alpha)$ and

$\tan^{-1}\left(\frac{1}{\sin\alpha}\right) = -\pi + \cot^{-1}(\sin\alpha)$.

69. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. The value of $\frac{a_7}{a_{13}}$ is equal to

(A) $\frac{1}{8}$ (B) $\frac{1}{2}$

(C) 2 (D) 8

Ans. D

Sol. Replace x by $\frac{2}{x}$.

70. The number of triplets of integers for which $2a^2 + b^2 - 8c = 7$ is equal to

(A) 0 (B) 1

(C) 2 (D) infinite

Ans. A

Sol. $2a^2 + b^2 = 8c + 7 \quad \Rightarrow b$ must be odd.

Let $b = 2m + 1$.

$\therefore\ a^2 + 2m^2 + 2m = 4c + 3 \quad \Rightarrow a$ must be odd.

Let $a = 2n + 1$.

$\therefore\ 2n^2 + 2n + m(m+1) = 2c + 1$, which is not possible.

PHYSICS

71. A particle is projected with initial speed $u = \sqrt{1500}$ m/s such that the radius of the path at highest point is equal to maximum height of the projectile. The horizontal range of the projectile is

(A) 150 m
(B) $100\sqrt{2}$ m
(C) $100\sqrt{3}$ m
(D) $150\sqrt{2}$ m

Ans. B

Sol. Radius at highest point $r = \frac{u^2\cos^2\theta}{g} = H$

$\frac{u^2\cos^2\theta}{g} = \frac{u^2\sin^2\theta}{2g} \Rightarrow \tan\theta = \sqrt{2}$

i.e. $\sin\theta = \frac{\sqrt{2}}{\sqrt{3}}$ and $\cos\theta = \frac{1}{\sqrt{3}}$

Range = $\frac{u^2}{g} \times 2\sin\theta\cos\theta = \frac{1500}{10} \times \frac{2\times\sqrt{2}}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = 100\sqrt{2}$ m

72. A particle moves along x-axis. The position of the particle at time t is given as
$x = t^3 - 9t^2 + 24t + 1$
The distance traveled in first 5 seconds is

(A) 20 m
(B) 10 m
(C) 18 m
(D) 28 m

Ans. D

Sol. Distance Travelled = $\int_0^5 |\vec{v}|\,dt = \int_0^5 |3t^2 - 18t + 24|\,dt = 28$

73. A small object of mass m is suspended by a thread of length ℓ and released when thread is horizontal. Find out the velocity of the object when its vertical component is maximum

(A) $\sqrt{g\ell}$
(B) $\sqrt{\frac{2}{\sqrt{3}}g\ell}$
(C) $\sqrt{\frac{\sqrt{3}}{2}g\ell}$
(D) $\sqrt{\frac{\sqrt{2}}{3}g\ell}$

Ans. B

Sol. Vertical velocity will be maximum when vertical acceleration is zero.

$mg = T\cos\theta$

$= \left(mg\cos\theta + \frac{mv^2}{r}\right)\cos\theta$

$v^2 = \frac{gr\sin^2\theta}{\cos\theta}$

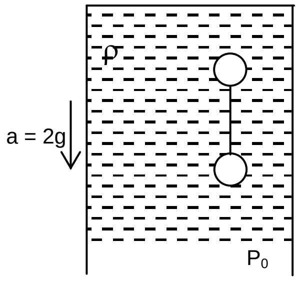
also mgr $\cos\theta = \frac{1}{2}mv^2$

$v^2 = 2gr\cos\theta$

$\tan^2\theta = 2$

$V = \sqrt{\frac{2gr}{\sqrt{3}}}$

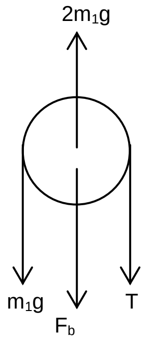
74. Two identical shaped spheres of specific gravity 0.8 and 1.6 are connected through string and placed in liquid container as shown in figure. If spheres are in equilibrium with respect to container then tension in the string will (take volume of each sphere v_0 = 250 CC)



(A) 1N (B) 2N
(C) 3N (D) none of these

Ans. A

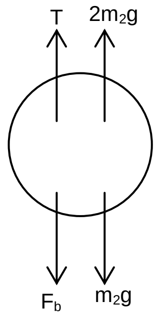
Sol. $F_b + m_1g + T = 2m_1g$



$F_b + T = m_1g$

$F_b + m_2g = T + 2m_2g$

$F_b - T = m_2g$



$$\frac{F_b + T}{F_b - T} = \frac{m_1g}{m_2g} = \frac{\rho_1}{\rho_2} = \frac{1.6}{0.8}$$

$F_b + T = 2F_b - 2T$

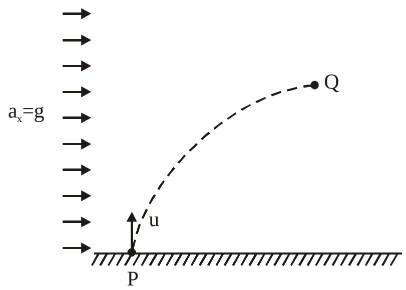
$3T = F_b$

$$T = \frac{F_b}{3} = \frac{V\rho g}{3}$$

$$\rho = \frac{\rho_1 + \rho_2}{2} = \frac{1.6\rho_\omega + 0.8\rho_\omega}{2} = 1.2\rho_\omega$$

T = 1N

75. Air is blowing and is providing a constant horizontal acceleration a_x = g to the particle as shown in the figure. Particle is projected from point P with a velocity u in upward direction. Let Q be the highest point of particle. Speed of the particle at highest point Q is



(A) $\sqrt{2}u$ (B) u
(C) $u/\sqrt{2}$ (D) None

Ans. B

Sol. $v_x = u_x + a_x t$

$v_x = 0 + gt$ .....(1)

$v_y = u_y + a_y t$

$0 = u - gt$

$t = \frac{u}{g}$ .....(2)

By (1) and (2) we get

$v_x = u$ and $v_y = 0$

Hence net velocity = u

76. A balloon filled with of 1.0 g of hydrogen is in thermal equilibrium with air in a room. Root mean square speed of air molecules at the room temperature is 500 m/s, the average molecular mass of air molecules is 29 g/mol. If hydrogen in the balloon is considered as an ideal gas its total internal energy is near to

(A) 1813 J (B) 3021 J

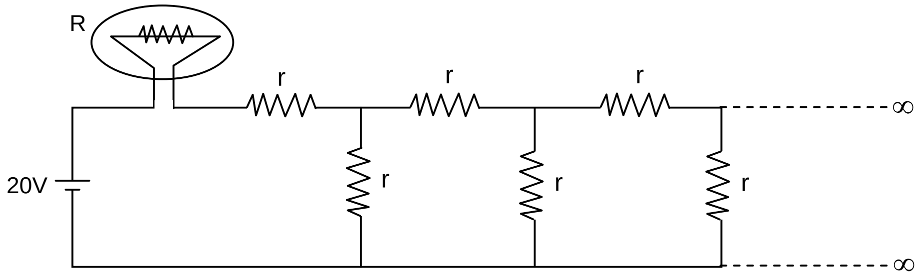
(C) 4223 J (D) 6042 J

Ans. B

Sol. $v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow RT = \frac{mv_{rms}^2}{3}$ ...(i)

$U = \frac{nRT}{\nu - 1} = \frac{nMv_{rms}^2}{3(\nu - 1)}$ = 3021 J

77. A light bulb of resistance $R = 8(\sqrt{5}+1)\Omega$ is attached in series with an infinite resistor network with identical resistances 'r' as shown in figure. A 20V battery drives current in the circuit. What should be the value of r such that the bulb dissipates maximum power?

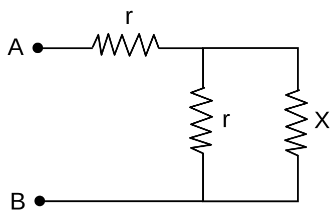


(A) 16 Ω (B) 8 Ω

(C) 4 Ω (D) $8(\sqrt{5}+1)\Omega$

Ans. A

Sol. Let 'x' be the equivalent resistance of infinite network.



$R_{AB} = \frac{rx}{r+x} + r = x$

rx + r^2 + rx = rx + x^2

x^2 – rx – r^2 = 0

$x = \frac{r + \sqrt{r^2 + 4r^2}}{2} = \left(\frac{\sqrt{5}+1}{2}\right)r$

According to maximum power transfer theorem, for maximum power in R.

x = R

$\left(\frac{\sqrt{5}+1}{2}\right)r = 8\left(\sqrt{5}+1\right)$

r = 16 Ω

78. A parallel plate capacitor consists of square plates of edge length 'a' separated by a distance d << a. It is charged to a potential difference 'V' and made to move with a constant velocity 'v' directed along one of its edges. The magnetic field exists inside the capacitor will be

(A) $\frac{\mu_0\varepsilon_0 vV}{d}$ (B) $\frac{\mu_0\varepsilon_0 vV}{a}$

(C) $\frac{\mu_0\varepsilon_0 dvV}{a^2}$ (D) none of these

Ans. A

Sol. $\frac{qd}{\varepsilon_0 A} = V \quad \Rightarrow \quad q = \frac{\varepsilon_0 AV}{d} = \frac{\varepsilon_0 Va^2}{d}$

$i = \frac{dq}{dt} = \frac{2\varepsilon_0 Va}{d} \times \frac{da}{dt} = \frac{2\varepsilon_0 Vv\,a}{d}$

$B = \frac{1}{2}\mu_0\left(\frac{i}{a}\right) = \frac{\mu_0\varepsilon_0 vV}{d}$ (For infinite sheet)

79. What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil decreases down to zero uniformly during a time interval Δt?

(A) $\frac{4}{3}\frac{q^2R}{\Delta t}$ (B) $\frac{2}{3}\frac{q^2R}{\Delta t}$

(C) $\frac{3}{4}\frac{q^2R}{\Delta t}$ (D) $\frac{3}{2}\frac{q^2R}{\Delta t}$

Ans. A

Sol. Suppose initial current is i_0, then

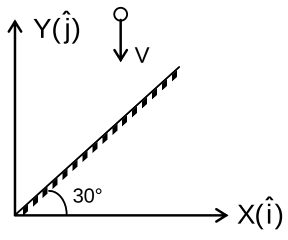
$i(t) = i_0\left(1 - \frac{t}{\Delta t}\right)$

$q = i_0\int_0^{\Delta t}\left(1 - \frac{t}{\Delta t}\right)dt$

So, $i_0 = \frac{2q}{\Delta t}$

$H = \int_0^{\Delta t}\left\{\frac{2q}{\Delta t}\left(1 - \frac{t}{\Delta t}\right)\right\}^2 R\,dt$

80. An object moves with velocity $V(-\hat{j})$ towards the inclined plane mirror as shown in figure. The velocity of the image is



(A) $V\,\hat{j}$ (B) $V\frac{\sqrt{3}}{2}(-\hat{i}) + \frac{V}{2}\hat{j}$

(C) $V\frac{\sqrt{3}}{2}\hat{i} + \frac{V}{2}\hat{j}$ (D) $\frac{V}{2}(-\hat{i}) + V\frac{\sqrt{3}}{2}(\hat{j})$

Ans. B

Sol. $\vec{v} = 2V\cos 30° \sin 30° (-\hat{i}) + V(\cos^2 30° - \sin^2 30°)\,\hat{j}$

CHEMISTRY

81. 100 mL of 0.2 M $CaCl_2$ solution is isotonic with [Assume complete dissociation of salts]
(A) 100 mL of 0.2 M $C_6H_{12}O_6$ solution
(B) 200 mL of 0.3 M NaCl solution
(C) 100 mL of 0.3 M KCl solution
(D) 200 mL of 0.1 M $Al_2(SO_4)_3$ solution

Ans. C

Sol. Isotonic solution have same osmotic pressure.

82. Which of the following complex displays ionization isomerism?
(A) $[Cr(H_2O)_3(NH_3)_3]^{3+}$
(B) $[Cu(NH_3)_3Cl]Br$
(C) $Na_3[FeF_6]$
(D) $[Cr(NH_3)_6]ClBr$

Ans. B

Sol. Ionization isomers occur in $[Cu(NH_3)_3Cl]Br$.

83. Which of the following forms a hydrocarbon, when reacts with C_2H_5MgBr?
(A) HCHO
(B) $CH_3COC_2H_5$
(C) CH_3COOH
(D) $CH_3COOC_2H_5$

Ans. C

Sol. $C_2H_5MgBr + CH_3COOH \longrightarrow C_2H_6 + Mg(Br)(CH_3COO)$

84. Glucose is converted to a black substance when reacts with
(A) $C_6H_5NHNH_2$
(B) Br_2/H_2O
(C) Conc.H_2SO_4
(D) NH_2OH

Ans. C

Sol. Conc. H_2SO_4 is a dehydrating agent. Hence, it removes water from glucose.

85. A 10 L vessel contains equal mass of the following gases at constant temperature. Which will produce the maximum partial pressure?
(A) CH_4
(B) CO
(C) CO_2
(D) SO_2

Ans. A

Sol. Number of moles of CH_4 is the highest, So, it forms maximum pressure.

86. The strongest reducing agent out of the following is:
(A) NH_3
(B) PH_3
(C) BiH_3
(D) AsH_3

Ans. C

Sol. Due to less bond energy.

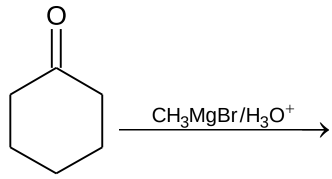
87. Which of the following reaction produces a substance that is used to purify water? (as a disinfectant)
(A) $Ca(OH)_2 + HCl$
(B) $Ca(OH)_2 + Cl_2$
(C) $Ca(OH)_2 + Cl_2O_7$
(D) $Ca(OH)_2 + HClO_2$

Ans. B

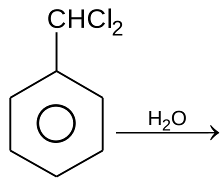
Sol. $Ca(OH)_2 + Cl_2 \longrightarrow Ca(OCl)Cl$ (Bleaching powder) $+ H_2O$

88. The product of which of the following reaction does NOT undergo oxidation reaction with acidified $KMnO_4$ solution?

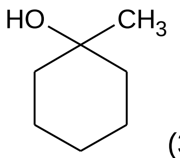
(A) $CH_3CH_2COCl \xrightarrow[Pd-BaSO_4/quinoline]{H_2}$

(B) 

(C) $CH_3COCH_3 \xrightarrow{LiAlH_4}$

(D) 

Ans. B

Sol. 
(3°-alcohol) will not be oxidized by MnO_4^-

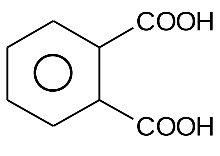
89. Which of the following substance will produce the heaviest product with NH_3?

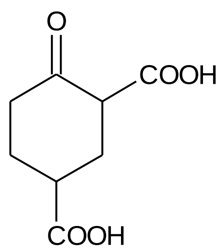
(A) HCHO
(B) HCOOH
(C) HCOCl
(D) HCOOCOH

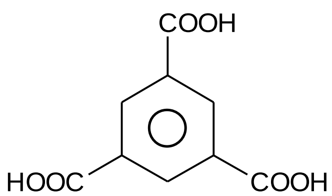
Ans. A

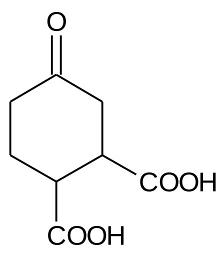
Sol. HCHO will form urotropene with NH_3.

90. Which of the following substance on heating undergoes decarboxylation?

(A) 

(B) 

(C) 

(D) 

Ans. B

Sol. β-keto acids undergo decarboxylation on heating.