KVPY – CLASS-XII
PART TEST – 4
(OLTS-1819-T4-PT-4-KVPY-XII)

PART – I

MATHEMATICS

1. If A and B are two matrices such that $AB = B$ and $BA = A,$ then $A^2 + B^2$ is equal to
(A) 2AB (B) 2BA
(C) $A+B$ (D) AB

Ans. C

Sol. We have $A^2 + B^2 = (BA)^2 + (AB)^2$
$= (BA)(BA) + (AB)(AB)$
$= B(AB)A + A(BA)B$
$= B(BA) + A(AB) = BA + AB = A + B.$

2. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix}$ is equal to
(A) 0 (B) $12\cos^2 - 10\sin^2 x$
(C) $12\sin^2 x - 10\cos^2 x - 2$ (D) $10\sin^2 x$

Ans. A

Sol. Apply $R_1 \to R_1 + R_2 - R_3,$ then the given determinant is equal to $\begin{vmatrix} 0 & \cos^2 x & 1 \\ 0 & \sin^2 x & 1 \\ 0 & 12 & 2 \end{vmatrix} = 0$.

3. If the function $f(x) = \dfrac{cx+d}{(x-1)(x-4)}$ has a turning point at the point (2, –1) then
(A) $c = 2, d = 0$ (B) $c = 1, d = 0$
(C) $c = 1, d = -1$ (D) $c = 1, d = 1$

Ans. B

Sol. $f'(x) = \dfrac{c(x-1)(x-4) - (cx+d)(2x-5)}{(x-1)^2(x-4)^2}$
So, $0 = f'(2) = \dfrac{-2c + (2c+d)}{4} = \dfrac{d}{4} \Rightarrow d = 0$
Also $-1 = f(2) = \dfrac{2c+d}{-2} = -c \Rightarrow c = 1$

4. For the parabola $y^2 = 16x$, the ratio of the length of the subtangent to the abscissa is
(A) $2:1$ (B) $1:1$
(C) $x : y$ (D) $x^2 : y$

Ans. A

Sol. Differentiating, $2y\frac{dy}{dx}=16$ So $\frac{dy}{dx}=\frac{8}{y}$.

Thus the length of the subtangent is $y.\frac{dx}{dy}=\frac{y^2}{8}=\frac{16x}{8}=2x$.

Hence length of the subtangent : abscissa $=2x:x=2:1$.

5. The trigonometric equation $\sin^{-1}x=2\sin^{-1}a$ has a solution for

(A) all real values of a

(B) $|a|\le\frac{1}{\sqrt{2}}$

(C) $|a|\ge\frac{1}{\sqrt{2}}$

(D) $\frac{1}{2}<|a|<\frac{1}{\sqrt{2}}$

Ans. B

Sol. $\sin^{-1}x=\sin^{-1}2a\sqrt{1-a^2}$ if $|a|\le\frac{1}{\sqrt{2}}$

$\Rightarrow$ $x=2a\sqrt{1-a^2}$ which is possible

If $x^2=4a^2\left(1-a^2\right)\le 1$ $\left[\because\ -1\le\sin^{-1}x\le 1\right]$

or if $4a^4-4a^2+1\ge 0$ if $\left(2a^2-1\right)^2\ge 0$

Which is true, so $|a|\le\frac{1}{\sqrt{2}}$

6. Let $\Delta=\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$ and $\Delta_1=\begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$ then

(A) $\Delta=2\Delta_1$

(B) $\Delta=-2\Delta_1$

(C) $\Delta=4\Delta_1$

(D) $\Delta=-4\Delta_1$

Ans. A

Sol. Using $R_1\to R_1+R_2+R_3$, we get $\Delta=\begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$

Taking 2 common from R_1 and applying $R_2\to R_2-R_1$, $R_3\to R_3-R_1$, we obtain

$$\Delta=2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$$

Applying $R_1\to R_1+R_2+R_3$, we get $\Delta=2(-1)(-1)\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}=-2\begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix}$

$$=2\begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}=2\Delta_1$$

7. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$, then

(A) $\Delta_1 = 3(\Delta_2)^2$

(B) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$

(C) $\frac{d}{dx}(\Delta_1) = 3\Delta_2{}^2$

(D) $\Delta_1 = 3\Delta_2{}^{3/2}$

Ans. B

Sol. We have $\frac{d\Delta_1}{dx} = \begin{vmatrix} 1 & b & b \\ 0 & x & b \\ 0 & a & x \end{vmatrix} + \begin{vmatrix} x & 0 & b \\ a & 1 & b \\ a & 0 & x \end{vmatrix} + \begin{vmatrix} x & b & 0 \\ a & x & 0 \\ a & a & 1 \end{vmatrix}$

$= \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix}$

$= 3\Delta_2$

8. If $\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$ and $\sin 2x = a - b\sqrt{7}$, then ordered pair (a, b) can be:

(A) (6, 2)

(B) (8, 3)

(C) (22, 8)

(D) (11, 4)

Ans. C

Sol. $\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$

$\Rightarrow (\sin x + \cos x) + \left(\frac{1}{\sin x \cos x}\right) + \frac{(\sin x + \cos x)}{\sin x . \cos x} = 7$

$\Rightarrow (\sin x + \cos x)\left(1 + \frac{2}{\sin 2x}\right) = \left(7 - \frac{2}{\sin 2x}\right)$

$\Rightarrow (1 + \sin 2x)\left(1 + \frac{4}{\sin^2 2x} + \frac{4}{\sin 2x}\right)$

$= 49 + \frac{4}{\sin^2 2x} - \frac{28}{\sin 2x}$ (squaring both sides)

$\Rightarrow \sin^3 2x - 44\sin^2 2x + 36\sin 2x = 0$

$\Rightarrow \sin 2x = 22 - 8\sqrt{7}$

9. Which of the following is the solution set of the equation $2\cos^{-1}(x) = \cot^{-1}\left(\frac{2x^2 - 1}{2x\sqrt{1-x^2}}\right)$?

(A) (0, 1)

(B) $(-1, 1) - \{0\}$

(C) $(-1, 0)$

(D) $[-1, 1]$

Ans. A

Sol. $2\cos^{-1} x = \cot^{-1}\left(\frac{2x^2 - 1}{2x\sqrt{1-x^2}}\right)$

Put $x = \cos\theta$; LHS $= 2\theta$; $0 \le \theta \le \pi$ and $-1 \le x \le 1$ ……(i)

RHS = $\cot^{-1}\left(\frac{\cos 2\theta}{2\cos\theta|\sin\theta|}\right) = \cot^{-1}(\cot 2\theta) = 2\theta$ ……(ii)

If $0 < 2\theta < \pi$ or $0 < \theta < \frac{\pi}{2}$

From equation (i) and (ii), $0 < \theta < \frac{\pi}{2}$

$\therefore\ x \in (0, 1)$

10. There is an equilateral triangle with side 4 and a circle with the centre on one of the vertex of that triangle. The arc of that circle divides the triangle into two parts of equal area. How long is the radius of the circle?

(A) $\sqrt{\frac{12\sqrt{3}}{\pi}}$ (B) $\sqrt{\frac{24\sqrt{3}}{\pi}}$

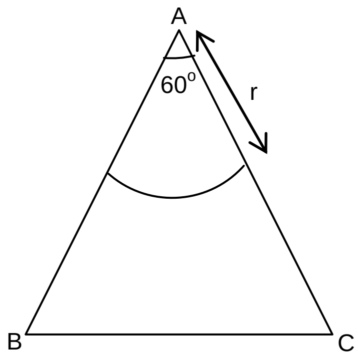
(C) $\sqrt{\frac{30\sqrt{3}}{\pi}}$ (D) $\frac{6\sqrt{3}}{\pi}$

Ans. A

Sol. $\frac{\sqrt{3}}{4}.16 = 2.\frac{1}{2}r^2.\frac{\pi}{3}$

$\pi r^2 = 12\sqrt{3}$

$\Rightarrow\ r = \sqrt{\frac{12\sqrt{3}}{\pi}}$



11. Let f be a twice differentiable function such that $f(1) = 3,\ f(2) = 2,\ f(3) = 1$, then identify the correct statement:

(A) $f'(x) = -1$ for atleast two $x \in (1, 2)$ (B) $f''(x) = 0$ for atleast one $x \in (1, 3)$

(C) $f''(x) = 0$ for atleast one $x \in (1, 2)$ (D) $f''(x) = 0$ for atleast two $x \in (2, 3)$

Ans. B

Sol. Consider $H(x) = f(x) + x - 4$

$H(x)$ is continuous and differentiability in (1, 3)

$H(1) = H(2) \Rightarrow H'(x) = 0$ for atleast one

$x \in (1, 2)$

$H(2) = H(3) \Rightarrow H'(x) = 0$ for atleast one

$x \in (2, 3)$

$H''(x) = 0$ for atleast one $x \in (1, 3)$

12. The exact value of $\frac{96\sin 80^o \sin 65^o \sin 35^o}{\sin 20^o + \sin 50^o + \sin 110^o}$ is equal to:

(A) 12 (B) 24

(C) –12 (D) 48

Ans. B

Sol. $\sum \sin A = 4\prod \cos\frac{A}{2}$ in Dr. as A + B + C = π

13. Consider a rectangular sheet of perimeter 6 meters. A circular sector of radius equal to smaller side of sheet and centre at one corner is removed such that the area of remaining portion is maximum, then area of removed portion is:

(A) $\frac{\pi}{(4+\pi)^2}$
(B) $\frac{9}{(4+\pi)^2}$
(C) $\frac{36\pi}{(4+\pi)^2}$
(D) $\frac{9\pi}{(4+\pi)^2}$

Ans. D

Sol. Area of remaining portion $A = ab - \frac{1}{4}\pi a^2$

$= a(3-a) - \frac{1}{4}\pi a^2$

$\Rightarrow \frac{dA}{da} = 3 - 2a - \frac{\pi a}{2} = 0 \Rightarrow a = \frac{6}{4+\pi}$

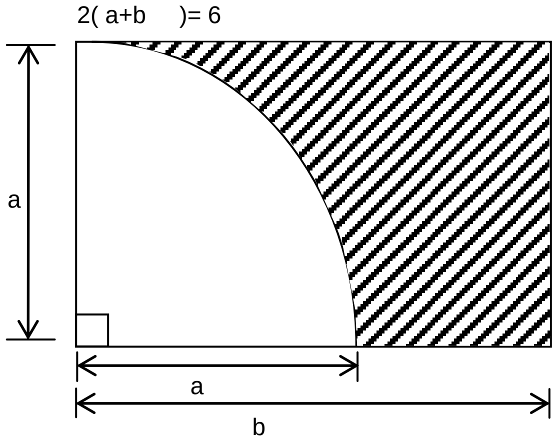
and $\frac{d^2A}{da^2} = -2 - \frac{\pi}{2} < 0$

$\Rightarrow \quad a = \frac{6}{4+\pi}$

at $a = \frac{6}{4+\pi}$, A is maximum

$\Rightarrow$ Removed portion $= \frac{1}{4}\pi . a^2$

$\Rightarrow \frac{1}{4}\pi . \frac{36}{(4+\pi)^2} = \frac{9\pi}{(4+\pi)^2}$



14. A car is to be driven 200kms on a highway at an uniform speed of x km/hr (speed rules of the highway require $40 \le x \le 70$). The cost of diesel is Rs 30/ litre and is consumed at the rate of $100 + \frac{x^2}{60}$ litres per hour. If the wage of the driver is Rs200 per hour then the most economical speed to drive the car is

(A) 55.5
(B) 70
(C) 40
(D) 80

Ans. B

Sol. Let cost incurred to travel 200kms be c(x)

$\Rightarrow c(x) = \left(100 + \frac{x^2}{60}\right)\frac{200}{x} \times 30 + 200 \times \frac{200}{x} = \frac{640000}{x} + 100x$

$\Rightarrow c'(x) < 0$ in $x \in [40,70]$

$\Rightarrow$ c(x) is minimum for x=70 in $x \in [40,70]$

15. If the functions $f(x)$ and $g(x)$ are defined on $R \to R$ such that

$$f(x) = \begin{cases} x+3, & x \in \text{rational} \\ 4x, & x \in \text{irrational} \end{cases} \text{ and } g(x) = \begin{cases} x+\sqrt{5}, & x \in \text{irrational} \\ -x, & x \in \text{rational} \end{cases}$$

Then $(f-g)(x)$ is

(A) one-one and onto
(B) neither one-one nor onto
(C) one-one but not onto
(D) onto but not one-one

Ans. B

Sol. We have, $(f-g)(x) = (f(x) - g(x))$

$$= \begin{cases} 2x+3, & x \in \text{rational} \\ 3x-\sqrt{5}, & x \in \text{irrational} \end{cases}$$

As, $f\left(\frac{-3}{2}\right) = 0 = f\left(\frac{\sqrt{5}}{3}\right)$

or $f(-1) = 1 = f\left(\frac{\sqrt{5}+1}{3}\right)$ and so on

$\Rightarrow$ $f(x)$ is many one function

Also, $\sqrt{5}$ does not belong to the range of $f(x)$, because of $3x - \sqrt{5} = -\sqrt{5}$

$\therefore$ $x = 0 \notin Q$

$\Rightarrow f(x)$ is into function.

16. If f(x) = $\sin^{-1}$ cosec($\sin^{-1}$ x) + $\cos^{-1}$(sec($\cos^{-1}$ x)) then range of f(x) has

(A) exactly two values
(B) one value
(C) undefined
(D) infinite values

Ans. B

Sol. f(x) is defined only for x = $\pm$ 1

and $f(1) = f(-1) = \frac{\pi}{2}$

17. If A and B are two square matrices of order 3 x 3 which satisfy AB = A, BA = B then $(A+B)^7$ is

(A) $64(A+B)$
(B) $16(A+B)$
(C) $4(A+B)$
(D) none of these

Ans. A

Sol. $A = AB$

$A^2 = ABA = AB = A$

$\Rightarrow A^2 = A \Rightarrow A^3 = A^2 = A$

$\Rightarrow A^2 = A^3 = A^4 = A^5 = A^6 = A^7 = A$

Similarly $B^2 = B^3 = B^4 = B^5 + B^6 + B^7 = B$

and $(A+B)^2 = A^2 + AB + BA + B^2 = 2(A+B)$

$(A+B)^4 = 4(A+B)^2 = 8(A+B)$

$(A+B)^6 = 16(A+B)^2 = 32(A+B)$

$(A+B)^7 = 64(A+B)$

18. Vertices of a variable triangle are (3,4), $(5\cos\theta, 5\sin\theta)$ and $(5\sin\theta, -5\cos\theta)$, where $\theta \in R$, . Locus of its orthocenter is

(A) $(x+y-1)^2 + (x-y-7)^2 = 100$ (B) $(x+y-7)^2 + (x-y-1)^2 = 100$

(C) $(x+y-7)^2 + (x+y-1)^2 = 100$ (D) $(x+y-7)^2 + (x-y+1)^2 = 100$

Ans. D

Sol. Origin is circumcentre centre of the variable triangle.

19. Let $f(x) = \ln\left(\log_{1/3}\left(\log_7(\sin x + a)\right)\right)$ be defined for every real value of x, then the possible value of a is

(A) 2 (B) 4

(C) 8 (D) 6

Ans. B

Sol. 1 < sinx + a < 7

20. a, b, c are the length of sides BC, CA, AB respectively of ΔABC satisfying

$$\log\left(1+\frac{c}{a}\right) + \log a - \log b = \log 2$$

Also, the quadratic equation $a(1-x^2) + 2bx + c(1+x^2) = 0$ has two equal roots.

The value of $(\sin A + \sin B + \sin C)$ is equal to

(A) $\frac{5}{2}$ (B) $\frac{12}{5}$

(C) $\frac{8}{3}$ (D) 2

Ans. B

Sol. Equal roots $\Rightarrow C = 90^o$

and $\sin A + 1 = 2\sin B \Rightarrow \cos B = 2\sin B - 1$ $\quad (\because A + B = 90^o)$

$\Rightarrow 1 - \sin^2 B = 4\sin^2 B - 4\sin B + 1$

$\Rightarrow \sin B = \frac{4}{5}$

$\Rightarrow \sin A + \sin B + \sin C = 3\sin B = \frac{12}{5}$

PHYSICS

21. In a photodiode the conductivity increases when light of wavelength less than 620 nm is incident. The bandgap is

(A) 1.12 eV (B) 1.8 eV

(C) 2.0 eV (D) 1.62 eV

Ans. C

Sol. $E_g(eV) = \frac{1242}{\lambda(nm)} = \frac{1242}{620} = 2eV$

22. If x and y be the distances of the object and image formed by a concave mirror from its focus and f be the focal length then

(A) $xf = y^2$ (B) $xy = f^2$

(C) $x/y = f$ (D) $x/y = f^2$

Ans. B

Sol. According to Newton's formula $xy = f^2$.

Note that $m = \frac{f}{f-u} = \frac{f-v}{f}$

or $\frac{f}{x} = \frac{y}{x}$ $\Rightarrow xy = f^2$

23. An electron (mass = m_e) and proton (mass = 1836 m_e) are moving with the same speed. The ratio of their de-Broglie wavelength $\frac{\lambda_{electron}}{\lambda_{proton}}$ will be

(A) 1 (B) 1836

(C) 1/1836 (D) 918

Ans. B

Sol. $\lambda = \frac{h}{p} = \frac{h}{mv}$

When speed is same

$\lambda \; \alpha \; \frac{1}{m}$

$\frac{\lambda_{electron}}{\lambda_{proton}} = \frac{1836 \; m_e}{m_e} = 1836$

24. A real image of magnification m_1 is formed on a screen by a convex lens. If the lens is moved through a distance x and the object is the moved until a new image of magnification m_2 is formed on the screen. The focal length of the lens is

(A) $\frac{x}{m_2 - m_1}$ (B) $\frac{x}{m_1 - m_2}$

(C) $\frac{x}{\sqrt{m_1 m_2}}$ (D) None of these

Ans. A

Sol. In first case, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ and $\frac{q}{p} = m_1$

$\Rightarrow \quad 1 + m_1 = \frac{q}{f}$ …(1)

In the second case $\frac{1}{q+x} + \frac{1}{p'} = \frac{1}{f}$

And $\frac{q+x}{p'} = m_2$ $\Rightarrow$ $m_2 = \frac{q+x}{f}$ …(2)

(1) and (2)

$\Rightarrow$ $m_2 - m_1 = x/f$

$\Rightarrow$ $f = \frac{x}{m_2 - m_1}$

25. Imagine a Young's double slit interference experiment performed with wave associated with fast moving electrons produced from an electron gun. The distance between successive maxima will decrease maximum if
(A) the accelerating potential in the electron gun is decreased.
(B) the accelerating potential is increased and the distance of screen from slit is decreased.
(C) the distance of the screen from the slit is increased.
(D) the distance between the slits is decreased.

Ans. B

Sol. If ΔV is increased λ will decrease and hence fringe width will decrease.

26. The dimensions of Stefan-Boltzmann constant σ can be written in terms of Planck's constant h, Boltzmann constant k_B and the speed of light c as $\sigma = h^{\alpha}k_B^{\beta}c^{\gamma}$. Here
(A) $\alpha = 3, \beta = 4$ and $\gamma = -3$ (B) $\alpha = 3, \beta = -4$ and $\gamma = 2$
(C) $\alpha = -3, \beta = 4$ and $\gamma = -2$ (D) $\alpha = 2, \beta = -3$ and $\gamma = -1$

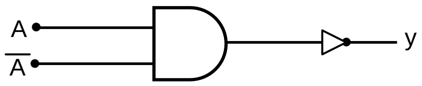
Ans. C

Sol. $\sigma = h^{\alpha}k_{\beta}^{\beta}c^{\gamma}$

$\Rightarrow MT^{-3}K^{-4} = \left(ML^2T^{-1}\right)^{\alpha}\left(ML^2T^{-2}K^{-1}\right)^{\beta}\left(LT^{-1}\right)^{\gamma}$

$$\left.\begin{aligned} &\Rightarrow \alpha + \beta = 1 \\ &2\alpha + 2\beta + \gamma = 0 \\ &-\beta = -4 \end{aligned}\right\} \begin{aligned} \beta &= 4 \\ \alpha &= -3 \\ \gamma &= -2 \end{aligned}$$

27. Write the Boolean expression of the block diagram represented in figure



(A) 1 (B) 0
(C) $A + \overline{A}$ (D) $\left(A + \overline{A}\right)$

Ans. A

Sol. Fact based

28. A spectral line results from the transition n =2 to n =1, in the atoms/species given below which one of these will produce the shortest wavelength emission?
(A) Hydrogen atom (B) Singly ionised Helium
(C) Doubly ionized Lithium (D) Deuterium atoms

Ans. C

Sol. According to Bohr theory

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Thus $\lambda \propto \frac{1}{Z^2}$

More the atomic number, smaller is the wavelength obtained in the transition of electron (for identical transition).

29. When an electron jumps from an orbit with n = 1 to n = 2, the change in orbital angular momentum is

(A) $\frac{h}{2\pi}$ (B) $\frac{h}{4\pi}$

(C) $\frac{3h}{4\pi}$ (D) Zero

Ans. A

Sol. According to Bohr postulates for hydrogen like atoms, electron can orbit in those orbits in which it's orbital angular momentum is integral multiple of $h/2\pi$.
Thus change in orbital angular momentum

$= \frac{2h}{2\pi} - \frac{1\ h}{2\pi} = \frac{h}{2\pi}$

30. When ultraviolet radiation of a certain frequency falls on a potassium target, the photoelectrons released can be stopped completely by a retarding potential of 0.6 V. If the frequency of the radiation is increased by 10%, this stopping potential rises to 0.9 V. The work function of potassium is

(A) 2.0 eV (B) 2.4 eV

(C) 3.0 eV (D) 2.8 eV

Ans. B

Sol. $h\nu - 0 = 0.6$ ev. $\Rightarrow$ $1.1\ h\nu - 1.1\phi = 0.66$
$1.1\ h\nu - \phi = 0.9$ ev. $-1.1h\nu \underset{+}{-} \phi = \_0.9$

$0.1\phi = 0.24$

$\phi = 2.4$ ev.

31. A transistor having β equal to 80 has a change in base current of 250 μA, then the change in collector current is

(A) $(250 + 8)\mu A$ (B) 250/80 μA

(C) $(250 - 80)\mu A$ (D) $80 \times 250\mu A$

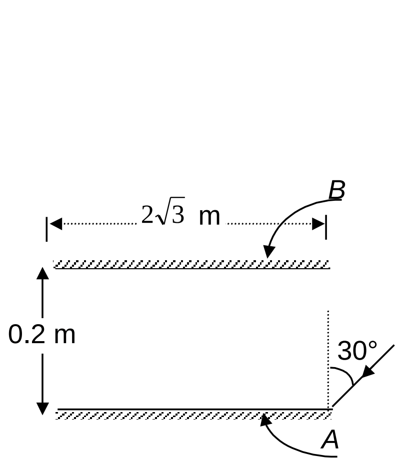
Ans. D

Sol. $I_c = \beta I_B$

$= 80 \times 250\ \mu A$

Transistor M, A

32. Two plane mirrors A and B are aligned parallel to each other as shown in figure. A light ray is incident at an angle of 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is



(A) 28 (B) 30

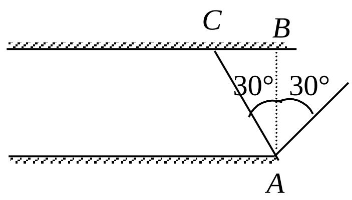
(C) 32 (D) 34

Ans. B

Sol. From the law of refection

$\tan 30° = \frac{BC}{AB} = \frac{BC}{0.2}$; $BC = 0.2 \times \frac{1}{\sqrt{3}} = 0.115$

Total no. of reflection = 30



33. In a pure semiconductor the number of conduction electrons is 6×10^{19} /m^3. The number of holes in a sample of 1 cm × 1 cm × 1 mm are

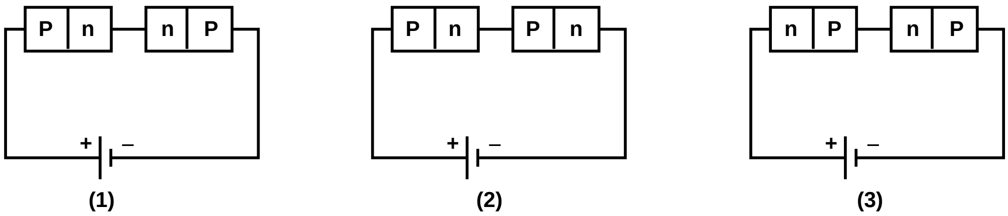
(A) 6×10^{13}
(B) 6×10^{12}
(C) 6×10^{15}
(D) 6×10^{16}

Ans. B

Sol. Since $h_i = n_i = 6 \times 10^{19} \times 10^{-2} \times 10^{-2} \times 10^{-3}$
$= 6 \times 10^{12}$

34. Two identical P-N junctions may be connected in series with a battery in three ways, as shown in figure. The potential drops across the P-N junctions are equal in

(A) circuit 1 and circuit 2.
(B) circuit 2 and circuit 3.
(C) circuit 3 and circuit 1.
(D) circuit 1 only.



Ans. B

Sol. Factual

35. A thin prism P_1 with angle 4^o and made from glass (μ = 1.54) is combined with another prism P_2 made of another glass of μ = 1.72 to produce dispersion without deviation. The angle of prism P_2 is

(A) 53.3^o
(B) 4^o
(C) 3^o
(D) 2.6^o

Ans. C

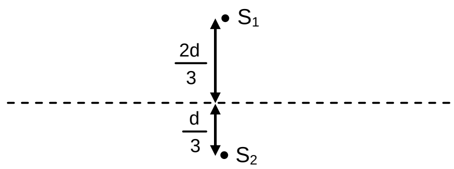
Sol. For no deviation

$(\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$

$4^o(1.54 - 1) = (1.72 - 1)A_2$

$A_2 = \frac{4 \times 0.54}{0.72} = 3°$.

36. Two coherent point sources of frequency $(f = \frac{10v}{d}$ where v is speed of light) are placed at a distance d apart as shown in figure. The receiver is free to move along the dotted line shown in the figure. Find total number of maxima observed by receiver.



(A) 6
(B) 7
(C) 5
(D) 8

Ans. A

Sol. $f\lambda = v = \frac{10\lambda v}{d} \Rightarrow \lambda = \frac{d}{10}$.

Maximum path diff. can be $\frac{d}{3}$. Hence, number of maximum will be 6.

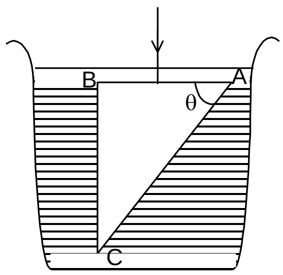
37. Critical angle of light passing from glass to air is maximum for

(A) red colour.
(B) green colour
(C) yellow colour.
(D) blue colour.

Ans. A

Sol. $C = \text{Sin}^{-1}\frac{1}{u}$ and $u_v > u_r$

38. A glass prism of refractive index 1.5 is immersed in water (R.I. = 4/3). The beam of light incident normally on the face AB is totally reflected to reach the face BC, if



(A) sin $\theta \geq 8/9$
(B) sin $\theta < 2/3$
(C) $2/3 <$ sin $\theta < 8/9$
(D) None of these

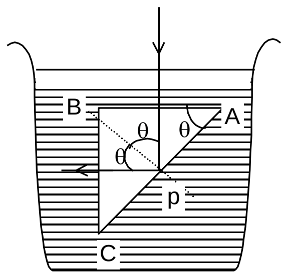
Ans. A

Sol. The light ray is totally reflected internally at the point P (say).

Therefore the critical angle $\theta_C \leq \theta$. Also,

$\frac{\sin\ \theta_c}{\sin\ 90°} = \frac{n_w}{n_g} \Rightarrow$ $\sin \theta_c = \frac{4/3}{3/2} = \frac{8}{9}$

$\Rightarrow \sin\theta \geq 8/9$



39. The angles of incidence and refraction of a monochromatic ray of light of wavelength λ at an air-glass interface are I and r, respectively. A parallel beam of light with a small spread $\delta\lambda$ in wavelength about a mean wavelength λ is refracted at the same air-glass interface. The refractive index μ of glass depends on the wavelength λ as $\mu(\lambda) = a + b/\lambda^2$ where a and b are constants. Then the angular spread in the angle of refraction of the beam is

(A) $\left|\frac{\sin i}{\lambda^3 \cos r}\delta\lambda\right|$
(B) $\left|\frac{2b}{\lambda^3}\delta\lambda\right|$
(C) $\left|\frac{2b \tan r}{a\lambda^3 + b\lambda}\delta\lambda\right|$
(D) $\left|\frac{2b(a + b/\lambda^2)\sin i}{\lambda^3}\delta\lambda\right|$

Ans. C

Sol. $\sin i = \mu \sin r$

$\Rightarrow \frac{d\mu}{dr} = \frac{\mu}{\tan r}$ ...(i)

$\mu = \frac{a+b}{x^2}$ $\frac{d\mu}{d\lambda} = \frac{-2b}{\lambda^3}$ ...(ii)

From (i) and (ii)

$$\Rightarrow \quad dr = \left|\frac{2b\tan r}{a\lambda^3 + b\lambda} d\lambda\right|$$

40. In Young's Double slit Experiment, the wavelength of the red light is 7800 Å and that of blue light is 5200 Å. The value of n for which n^{th} bright band due to red light coincides with $(n + 1)^{th}$ bright band due to blue light, is

(A) 1 (B) 2
(C) 3 (D) 4

Ans. B

Sol. $y_{n(red)} = \frac{n\lambda_1 D}{d}$

$y_{nt(blue)} = \frac{(n+1)\lambda_2 D}{d}$

Apply $y_{n\ (red)} = y_{n+1\ (blue)}$

$n(\lambda_r) = (n + 1)\lambda_b$

$n(7800) = (n + 1)(5200)$

$\Rightarrow \quad n = 2$

CHEMISTRY

41. ZnS reacts with dilute H_2SO_4 to produce a colorless gas, which on reaction with Cl_2 yields

(A) HCl (B) SCl_4
(C) HOCl (D) S_2Cl_2

Ans. A

Sol. The colorless gas is H_2S.

42. When copper ore is mixed with silica, in a reverberatory furnace, copper matte is produced. The copper matte contains ____________.

(A) sulphides of copper (II) and iron (II) (B) sulphides of copper (II) and iron (III)
(C) sulphides of copper (I) and iron (II) (D) sulphides of copper (I) and iron (III)

Ans. C

Sol. Cu_2S and FeS

43. Which of the following are peroxoacids of sulphur?

(A) H_2SO_5 and $H_2S_2O_8$ (B) H_2SO_5 and $H_2S_2O_7$
(C) $H_2S_2O_7$ and $H_2S_2O_8$ (D) $H_2S_2O_6$ and $H_2S_2O_7$

Ans. A

Sol. H_2SO_5 (Peroxomonosulphuric acid) and $H_2S_2O_8$ (Peroxodisulphuric acid)

44. On addition of small amount of $KMnO_4$ to concentrated H_2SO_4, a green oily compound is obtained which is highly explosive in nature. Identify the compound from the following.

(A) Mn_2O_7 (B) MnO_2
(C) $MnSO_4$ (D) Mn_2O_3

Ans. A

Sol. $2HMnO_4 - H_2O \rightarrow Mn_2O_7$ (covalent green oil)

45. Which of the following complex does NOT exist at room temperature?

(A) $[BiI_4]^-$ (B) $[CuI_4]^{-2}$

(C) $[PbI_4]^{-2}$ (D) $[HgI_4]^{-2}$

Ans. B

Sol. Cu(II) oxidizes I^- to I_2.

46. Which of the following compounds liberate CO_2 on heating?

(A) Li_2CO_3 (B) Na_2CO_3

(C) K_2CO_3 (D) All of these

Ans. A

Sol. $Li_2CO_3 \xrightarrow{\Delta} Li_2O + CO_2$

47. Which of the following is NOT a carbonate ore?

(A) Siderite (B) Malachite

(C) Zincite (D) Calamine

Ans. C

Sol. Siderite-$FeCO_3$, Malachite-$CuCO_3.Cu(OH)_2$, Zincite-ZnO, Calamine-$ZnCO_3$.

48. Among the given properties, which is correct for both borazine and benzene?

(A) They are aromatic compounds

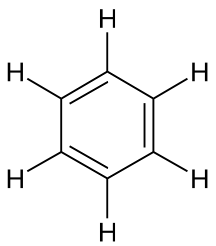
(B) They are isoelectronic having total 42 electrons each

(C) The B and N-atoms in borazine and C-atoms in benzene are sp^2-hybridized

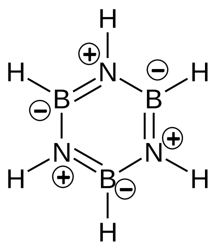
(D) All of these

Ans. D

Sol.



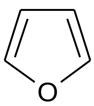
Benzene



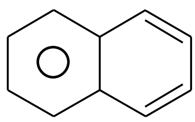
Borazine

49. Which of the following molecules is NOT aromatic?

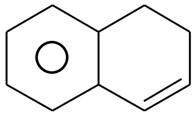
(A)



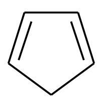
(B)



(C)

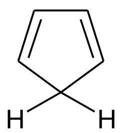


(D)



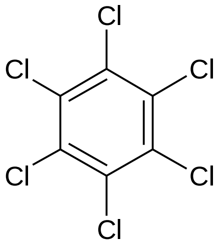
Ans. D

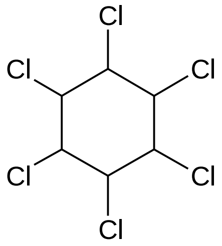
Sol.

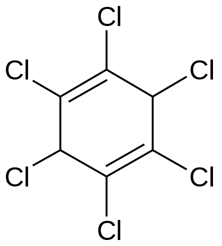


contains sp^3 – carbon and molecule does not obey Huckel's rule.

50. The correct structure for Gammaxane is

(A) 

(B) 

(C) 

(D) None of these

Ans. B

Sol. Gammaxane is benzene hexachloride $C_6H_6Cl_6$.

51. Which of the following reagents can be used to distinguish But-1-yne and But-2-yne?

(A) H_2/Ni
(B) dil.$H_2SO_4/HgSO_4(aq)$
(C) HBr/acetone
(D) $NaNH_2$

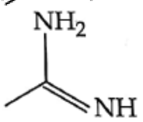
Ans. D

Sol. $CH_3-CH_2-C\equiv CH \xrightarrow{Na^+NH_2^-} CH_3-CH_2-C\equiv C^-Na^+ + NH_3\uparrow$

52. The increasing order of basicity of the following compounds is:

(a) 

(b) 

(c) 

(d) 

(A) (a) < (b) < (c) < (d)
(B) (b) < (a) < (c) < (d)
(C) (b) < (a) < (d) < (c)
(D) (d) < (b) < (a) < (c)

Ans. C

Sol. Higher the electron density on N, more its basic strength.

53. In graphite and diamond, the percentage of p-characters of the hybrid orbitals in hybridization are respectively:

(A) 33 and 25
(B) 33 and 75
(C) 67 and 75
(D) 50 and 75

Ans. C

Sol. Graphite (sp^2), Diamond (sp^3)

54. Xenon hexafluoride on partial hydrolysis produces X and Y. Compounds X and Y and oxidation state of Xe, respectively are:

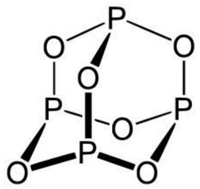
(A) $XeOF_4$ (+6) and XeO_3 (+6)
(B) $XeOF_4$ (+6) and XeO_2F_2 (+6)
(C) XeO_2F_2 (+6) and XeO_2 (+4)
(D) XeO_2 (+4) and XeO_3 (+6)

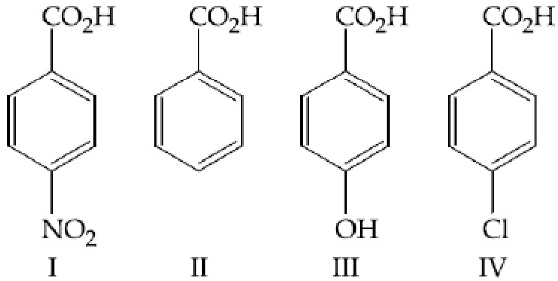
Ans. B

Sol. $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$
$XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$

55. The number of P – O bonds in P_4O_6 is
(A) 9 (B) 18
(C) 12 (D) 6

Ans. C

Sol.


56. The increasing order of the acidity of the following carboxylic acids is :


(A) III < II < IV < I (B) I < III < II < IV
(C) IV < II < III < I (D) II < IV < III < I

Ans. A

Sol. Acidity of the acid (HA) depends on stability of its conjugate anion (A^-).

57. In KO_2, the nature of oxygen species and oxidation state of oxygen atom are respectively
(A) Superoxide and -1/2 (B) Peroxide and -1/2
(C) Oxide and -2 (D) Superoxide and -1

Ans. A

Sol. Potassium superoxide, $K^+ O_2^-$

58. In Wilkinson's catalyst, the hybridization of central metal ion and its shape are respectively:
(A) d^2sp^3, octahedral (B) sp^3d, trigonal bipyramidal
(C) sp^3, tetrahedral (D) dsp^2, square planar

Ans. D

Sol. Wilkinson's catalyst, $[(Ph_3P)_3RhCl]$

59. $Mg_3N_2 + H_2O \rightarrow X(solution) + Y(gas)$
Gas (Y) in the above reaction is
(A) N_2 (B) NH_3
(C) NO (D) NO_2

Ans. B

Sol. $Mg_3N_2 + H_2O \rightarrow Mg(OH)_2 + NH_3$

60. What is the oxidation number of hydrogen in CaH_2?
(A) +1 (B) – 1
(C) – 2 (D) + 2

Ans. B

Sol. The oxidation numbers of calcium and hydrogen in CaH_2 are, respectively, +2 and -1

PART – II

MATHEMATICS

61. If $3\sin\beta = \sin(2\alpha+\beta)$, then $\tan(\alpha+\beta) - 2\tan\alpha$ is
(A) independent of α (B) independent of β
(C) independent of both α and β (D) independent of none of these

Ans. C

Sol. $\sin(2\alpha+\beta) = 3\sin\beta$

$$\Rightarrow \frac{\sin(2\alpha+\beta)+\sin\beta}{\sin(2\alpha+\beta)-\sin\beta} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{2\sin(\alpha+\beta)\cos\alpha}{2\cos(\alpha+\beta)\sin\alpha} = 2 \Rightarrow \tan(\alpha+\beta) - 2\tan\alpha = 0$$

62. Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2015} P$ is

(A) $\begin{bmatrix} 1 & -2015 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 2015 & 2015 \\ 0 & 2015 \end{bmatrix}$

Ans. C

Sol. $$P = \begin{bmatrix} \cos\left(\frac{\pi}{6}\right) & \sin\left(\frac{\pi}{6}\right) \\ -\sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) \end{bmatrix}$$

$$\Rightarrow \quad P^T = \begin{bmatrix} \cos\left(\frac{\pi}{6}\right) & -\sin\left(\frac{\pi}{6}\right) \\ \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) \end{bmatrix}$$

Since $PP^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \quad P^T = P^{-1}$

We have $Q = PAP^T = PAP^{-1}$

$\Rightarrow \quad Q^{2015} = \left(PAP^{-1}\right)^{2015} = PA^{2015}P^{-1}$

Thus, $P^TQ^{2015}P = P^{-1}\left(PA^{2015}P^{-1}\right)P$

$= \left(P^{-1}P\right)A^{2015}\left(P^{-1}P\right)$

Now, $A = I + B$ where $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Since, $B^2 = O$, we get $B^r = O \,\forall r \geq 2$.

Thus, $A^{2015} = I + 2015B = \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

63. If the tangent to the curve $x^3 - y^2 = 0$ at $\left(m^2, -m^3\right)$ is parallel to $y = -\frac{1}{m}x - 2m^3$, then the value of m^2 is

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{2}{3}$ (D) $\frac{-2}{3}$

Ans. C

Sol. Differentiating $x^3 - y^2 = 0$, we have $\frac{dy}{dx} = \frac{3x^2}{2y} \Rightarrow \left.\frac{dy}{dx}\right|_{\left(m^2, -m^3\right)} = -\frac{3m^4}{2m^3} = -\frac{3m}{2}$

According to the given condition $-\frac{3}{2}m = -\frac{1}{m} \Rightarrow m^2 = \frac{2}{3}$

64. The value of $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}\left(\cot A\right) + \tan^{-1}\left(\cot^3 A\right)$ for $0 < A < \left(\frac{\pi}{4}\right)$ is:

(A) $4\tan^{-1}(1)$ (B) $2\tan^{-1}(2)$

(C) 0 (D) none of these

Ans. A

Sol. $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}\left(\cot A\right) + \tan^{-1}\left(\cot^3 A\right)$

$= \tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}\left(\frac{\cot A + \cot^3 A}{1 - \cot^4 A}\right) + \pi$

$\left(0 < A < \frac{\pi}{4} \Rightarrow \cot A > 1\right)$

$= \tan^{-1}\left(\frac{\tan A}{1 - \tan^2 A}\right) + \pi + \tan^{-1}\frac{\cot A\left(1 + \cot^2 A\right)}{\left(1 - \cot^2 A\right)\left(1 + \cot^2 A\right)}$

$= \pi + \tan^{-1}\left(\frac{\tan A}{1 - \tan^2 A}\right) + \tan^{-1}\left(\frac{\cot A}{1 - \cot^2 A}\right)$

$= \pi = 4\tan^{-1}(1) \Rightarrow A$

65. If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points on curve $y = e^{2|x|+x}$ $(x_1 > 0 \text{ and } x_2 < 0)$ such that tangents at points A and B are perpendicular, then number of such pair of points A and B, is:

(A) 0
(B) 1
(C) 2
(D) more than 2

Ans. A

Sol. If $x > 0,\ y = e^{3x} \Rightarrow \frac{dy}{dx} = 3e^{3x_1}$

$x < 0,\ y = e^{-x} \Rightarrow \frac{dy}{dx} = -e^{-x_2}$

$\Rightarrow\ 3e^{3x_1}.e^{-x_2} = 1 \Rightarrow e^{3x_1 - x_2} = \frac{1}{3} < 1$

$\Rightarrow 3x_1 - x_2 < 0$ which is impossible

66. A spherical balloon is expanding. If at any instant rate of increase of its volume is 16 times of rate of increase of its radius, then its radius at that instant, is

(A) $\frac{1}{\sqrt{\pi}}$
(B) $\frac{2}{\sqrt{\pi}}$
(C) $\frac{2}{\pi}$
(D) $\frac{4}{3\sqrt{\pi}}$

Ans. B

Sol. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow 16 = 4\pi r^2 \Rightarrow r = \frac{2}{\sqrt{\pi}}$

67. If $\mu(x)$ be a differentiable function such that $\mu(5) \neq 0$ and $\mu(-3) \neq 0$ and Rolle's theorem is not applicable to $f(x) = \left(\frac{x^2 - 2x - 15}{\mu(x)}\right)$ in $[-3, 5]$, then:

(A) $\mu(x)$ has atleast one root in $[-3, 5]$
(B) $\mu(x)$ has exactly one root in $[-3, 5]$
(C) $\mu(x)$ has exactly two root in $[-3, 5]$
(D) $\mu(x)$ has no root in $[-3, 5]$

Ans. A

Sol. $f(-3) = f(5) = 0$

As rolle's theorem is not applicable for $f(x)$ in $[-3, 5]$ so either $f(x)$ is not continuous or not differentiable. So, $\mu(c) = 0$ for at least one $c \in (-3, 5)$.

68. Let A_n and B_n be square matrices of order 3, which are defined as:

$A_n = [a_{ij}]$ and $B_n = [b_{ij}]$ where $a_{ij} = \frac{2i + j}{3^{2n}}$ and $b_{ij} = \frac{3i - j}{2^{2n}}$ for all i and j, $1 \leq i,\ j \leq 3$.

If $I = \lim_{n\to\infty} Tr.\left(3A_1 + 3^2A_2 + 3^3A_3 + ... + 3^nA_n\right)$ and

$m = \lim_{n\to\infty} Tr.\left(2B_1 + 2^2B_2 + 2^3B_3 + ... + 2^nB_n\right)$, then find value of $\frac{(I+m)}{3}$

[Note : Tr. (P) denotes the trace of matrix P.]

(A) 3
(B) 5
(C) 7
(D) none of these

Ans. C

Sol. $T_r\left(A_1\right) = a_{11} + a_{22} + a_{33}$

$= \frac{1}{9}(3+6+9) = 2$

$T_r\left(A_2\right) = \frac{1}{3^4}(3+6+9) = \frac{18}{3^4}$

$\Rightarrow I = 9$

$T_r\left(B_1\right) = b_{11} + b_{22} + b_{33} = \frac{1}{4}(2+4+6)$

$T_r\left(B_2\right) = \frac{1}{2^4}(2+4+6)$

$\Rightarrow$ m = 12

69. Let $f(x) = 100 - 99x - x^3$ then set of values of x satisfying the inequality

$100 - 99f(x) - \left(f(x)\right)^3 > f\left(100(1-x)\right)$ is

(A) (–99, 0) ∪ (99, ∞)
(B) (– ∞, –1) ∪ (0, 1)
(C) (–1, 0) ∪ (1, ∞)
(D) (–100, 0) ∪ (100, ∞)

Ans. C

Sol. f(x) is everywhere decreasing. $\therefore$f(f(x))>f(100(1–x)) $\Rightarrow$f(x)<100(1–x).

70. Vertices of a variable acute angle triangle ABC lies on a fixed circle. If a, b, c are length of sides and angles of triangle ABC and x_1,x_2, x_3 are distances of orthocenter from A, B, C respectively then find the maximum value of $\sqrt{3}\left(\frac{dx_1}{da} + \frac{dx_2}{db} + \frac{dx_3}{dc}\right) + 11$

(A) 2
(B) 3
(C) 4
(D) 5

Ans. A

Sol. $x_1 = 2R\cos A,\ x_2 = 2R\cos B, x_3 = 2R\cos C$

$\frac{dx_1}{dA} = -2R\sin A$ also $a = 2R\sin A \Rightarrow \frac{da}{dA} = 2R\cos A$

so, $\frac{dx_1}{da} = -\tan A, \frac{dx_2}{db} = -\tan B, \frac{dx_3}{dc} = -\tan C$

$\tan A + \tan B + \tan C \geq 3\sqrt{3}$

So, $\sqrt{3}\left(\frac{dx_1}{da} + \frac{dx_2}{db} + \frac{dx_3}{dc}\right) + 11 \leq 2$

PHYSICS

71. If I = 0.1 (mA) [$e^{V/V}$T – 1] is valid for a pn junction. Then find the resistance when V = 0.5 volt and V_T = 0.025 volt.

(A) 50 Ω (B) 25 Ω

(C) 10 Ω (D) zero

Ans. D

Sol. $\frac{dI}{dV} = \frac{0.1}{V_T} e^{V/V}T = \frac{0.1 \times 10^{-3} \times e^{20}}{0.025}$

$R = \frac{dV}{dI} = \frac{250}{5 \times 10^8} \to 0$

72. A nuclear reactor is operating at 600 M Watt per hour. If energy released per fission is 3×10^{-11} J, the number of nuclei consumed per minute will be

(A) 2×10^{19} (B) 1.2×10^{21}

(C) 2×10^{13} (D) 3.3×10^{17}

Ans. D

Sol. Energy per Hour = 600×10^6 Joules

Energy per minute = $\frac{600 \times 10^6}{60} = 10^7$ Joule

Number of nuclei consumed per minute

$\frac{10^7}{3 \times 10^{-11}} = 3.3 \times 10^{17}$

73. Two coherent monochromatic light beams of intensities I and 4I are superimposed. The maximum and minimum possible intensities in the resulting beam are:

(A) 5 I and I (B) 5 I and 3I

(C) 9 I and I (D) 9 I and 3 I

Ans. C

Sol. Intensity α (Amplitude)^2

⇒ $I \;\alpha\; A^2$

When two waves (beams) of amplitude A_1 and A_2 superimpose, at maxima and minima, the amplitude of the resulting wave are ($A_1 + A_2$) and ($A_1 - A_2$) respectively. If the maximum and minimum possible intensities are I_{max} and I_{min} respectively, then

$I_{max} \;\alpha\; (A_1 + A_2)^2$

And $I_{min} \;\alpha\; (A_1 - A_2)^2$

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left\{\frac{\frac{A_1}{A_2} + 1}{\frac{A_1}{A_2} - 1}\right\}^2$$

where $\frac{A_1}{A_2} = \frac{\sqrt{I}}{\sqrt{4I}} = \frac{1}{2}$

$\Rightarrow \frac{I_{max}}{I_{min}} = \frac{9}{1}$ $\quad \Rightarrow I_{max} = 9I,\ I_{min} = I$

74. A thin rod of length f/3 is placed along the optic axis of a concave mirror of focal length f such that its image, which is real and elongated, just touches the rod. The magnification is

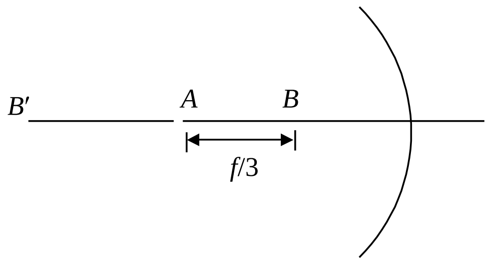
(A) 2 (B) 4

(C) 2.4 (D) 1.5

Ans. D

Sol. For B

$\frac{1}{V}+\frac{1}{-(2f-\frac{f}{3})}=-\frac{1}{f}$, $\frac{1}{V}=\frac{3}{5f}-\frac{1}{f}=\frac{3-5}{5f}$



$V = -\frac{5f}{2}$, $|AB'| = \frac{5f}{2}-2f=\frac{f}{2}$

Magnification = $\frac{\frac{f}{2}}{\frac{f}{3}}=1.5$

75. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to

(A) $b\left(\frac{u-f}{f}\right)^{1/2}$ (B) $b\left(\frac{u-f}{f}\right)$

(C) $b\left(\frac{f}{u-f}\right)^{1/2}$ (D) $b\left(\frac{f}{u-f}\right)^{2}$

Ans. D

Sol. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ $\Rightarrow$ $\frac{1}{v}=-\frac{1}{f}+\frac{1}{u}$ $\Rightarrow$ $v = \frac{uf}{(f-u)}$

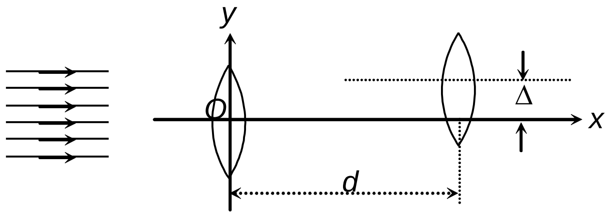
Differentiating $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ $\Rightarrow$ $-\frac{1}{u^2}du=-\frac{1}{v^2}dv$

$\because$ f is constant

$\therefore$ $\frac{dv}{du}=-\frac{v^2}{u^2}=-\frac{f^2}{(u-f)^2}$ $m = \frac{I}{O}=-\frac{dv}{du}=\frac{f^2}{(u-f)^2}$

$\therefore$ $I = \frac{bf^2}{(u-f)^2}$

76. Two thin convex lens of focal lengths f_1 and f_2 are separated by a horizontal distance d (where $d<f_1, d<f_2$), and their centres are displaced by a vertical separation Δ as shown in the figure. Taking the origin of coordinates, O at the centre of left lens, the x and y coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by



(A) $x=\frac{f_1f_2}{f_1+f_2}$ y = 0 (B) $x=\frac{f_1(f_2+d)}{f_1+f_2-d}$ $y=\frac{\Delta^2}{f_1+f_2}$

(C) $x=\frac{f_1f_2+d(f_1-d)}{f_1+f_2-d}$ $y=\frac{\Delta(f_1-d)}{f_1+f_2-d}$ (D) $x=\frac{f_1f_2+d(f_1-d)}{f_1+f_2-d}$ y = 0

Ans. C

Sol. For the second lens ; $u = (f_1 - d)$, $f = f_2$

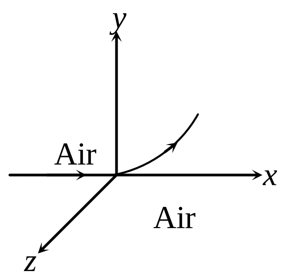
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{f_2(f_1 - f)}{(f_1 + f_2 - d)}$$

$$x = d + v \Rightarrow x = \frac{f_1 f_2 + f_1 d - d^2}{f_1 + f_2}$$

Height from principal axis = (y – Δ)

$$m = \frac{-(y - \Delta)}{\Delta} = \frac{v}{u} \Rightarrow y = \frac{\Delta(f_1 - d)}{(f_1 + f_2 - d)}$$

77. The refractive index n of a medium within a certain region x > 0, y >0 changes with y till it aquires a value n_{max}. After it aquires the value n_{max} it remains constant. A light ray travelling in air along the x-axis, strikes the medium at a grazing angle and moves through the medium along a circular arc as shown in the figure. If angular deviation of the ray before it starts moving on a straight line is 60^0, then n_{max} is



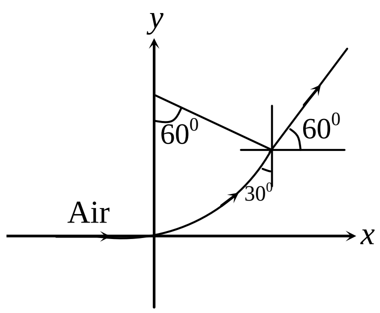
(A) 2 (B) $\sqrt{3}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{2}$

Ans. A

Sol. (1) sin 90^0 = (n_{max}) sin 30^0

n_{max} = 2



78. Choose the incorrect option regarding photoelectric effect supports quantum nature of light because

(A) these is a minimum frequency of light below which no photoelectron are emitted.

(B) the maximum kinetic energy of photo electrons depends only on the frequency of light and not on it’s intensity.

(C) even when the metal surface is faintly illuminated when the photoelectron leave the surface immediately.

(D) electric charge of the photoelectrons in quantized.

Ans. D

Sol. Factual

79. A prism (μ = 1.5) has a refracting angle of 30°. The deviation of a monochromatic ray incident normally on its one surface will be (sin48°36′ = 0.75)

(A) 18°36′ (B) 22°38′

(C) 18° (D) 22°1

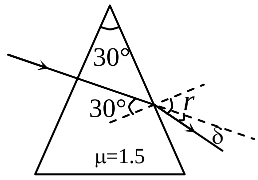
Ans. A

Sol. $1.5 \sin 30° = 1 \sin r$

$\Rightarrow \quad \sin r = \frac{3}{4} = 0.75 = \sin 48°36'$

$\therefore \quad r = 48°36'$

$\therefore \quad \delta = r - i = 48°36' - 30° = 18°36'$



80. A ray incident at a point as an angle of incidence of 60° enters a glass sphere of R.I. n = $\sqrt{3}$ and is reflected and refracted at the farther surface of the sphere. The angle between the reflected and refracted rays at this surface is

(A) 50° (B) 60°

(C) 90° (D) 40°

Ans. C

Sol. Refraction at P.

$\frac{\text{Sin } 60^0}{\text{Sin } r_1} = \sqrt{3} \quad \Rightarrow \text{Sin} r_1 = \frac{1}{2} \quad \Rightarrow \quad r_1 = 30°$

Since $r_2 = r_1$

$\therefore \quad r_2 = 30°$

Refraction at Q $\quad \frac{\text{Sin } r_2}{\text{Sin } i_2} = \frac{1}{\sqrt{3}}$

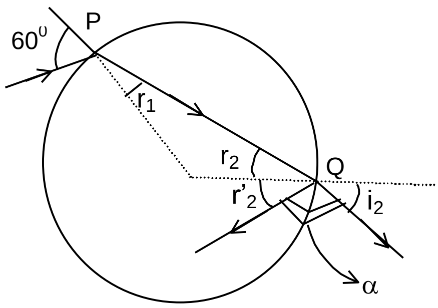
Putting R_2= 30° we obtain i_2 = 60°

Reflection at Q

$r_2' = r_2 = 30^o$

$\therefore \quad \alpha = 180^0 - (r_2' + i_2)$

$= 180° - (30° + 60°) = 90°$



CHEMISTRY

81. $NaCl + MnO_2 + H_2SO_4 \rightarrow MnCl_2 + NaHSO_4 + H_2O$ + [X]; Which of the following is/are correct about [X]?

(A) Its aqueous solution loses its color on standing.

(B) [X] on reaction with white phosphorus gives a compound which fumes in moisture.

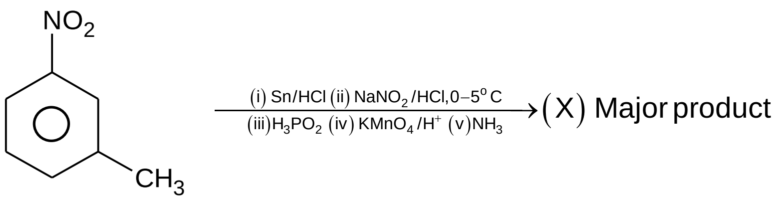
(C) It turns acidified light green solution of $FeSO_4$ into yellow

(D) All of these.

Ans. D

Sol. [X]: Cl_2

82.



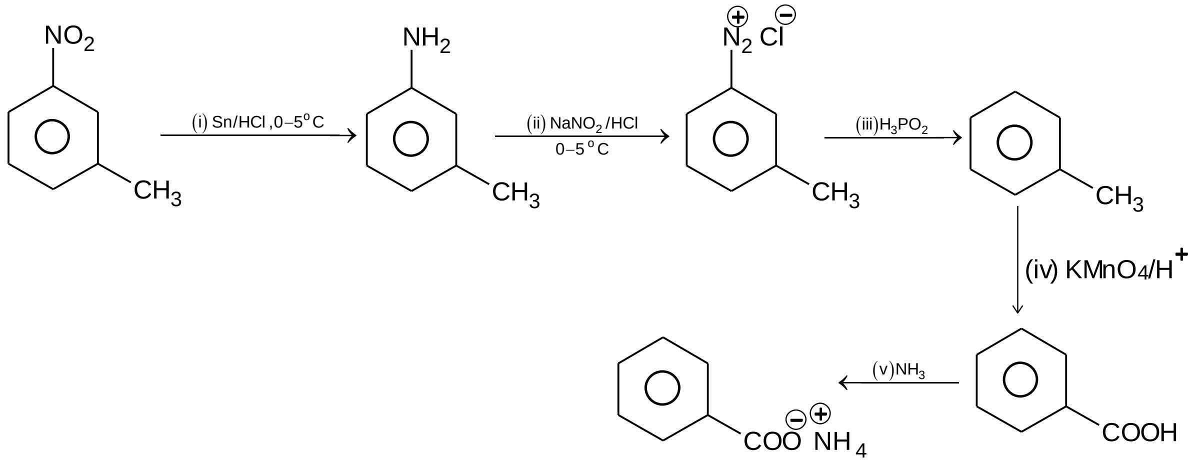
The molecular formula of (X) is

(A) $C_7H_6N_2O_3$ (B) $C_7H_9NO_2$

(C) $C_6H_6N_2O_2$ (D) $C_6H_5NO_3$

Ans. B

Sol



83. The process of producing 'syn-gas' from coal at 1270 K is called 'coal gasification'. Here, production to dihydrogen can be increased by
(A) carrying out the reaction at high pressures.
(B) adding carbon monoxide to the reaction mixture.
(C) reacting syn-gas mixture with carbon dioxide in presence of iron chromate as catalyst.
(D) reacting syn-gas mixture with steam.

Ans. D

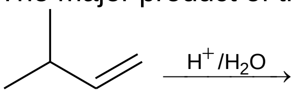
Sol. Coal gasification

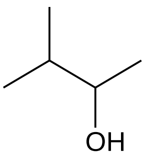
$$C(s)+H_2O(g) \rightleftharpoons \underbrace{CO(g)+H_2(g)}_{\text{Syn gas}}$$

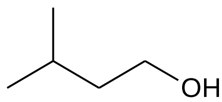
Water-gas shift reaction

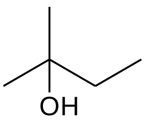
$$CO(g)+H_2O(g) \rightleftharpoons CO_2(g)+H_2(g)$$

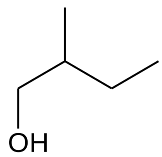
84. The major product of the reaction is



(A) 

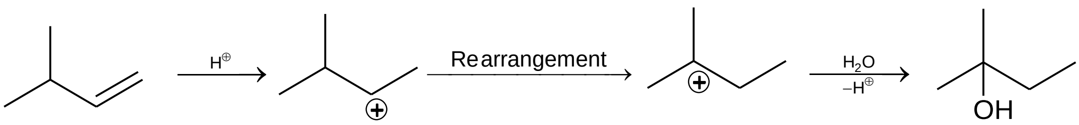
(B) 

(C) 

(D) 

Ans. C

Sol.



85. Which of the following species is paramagnetic?

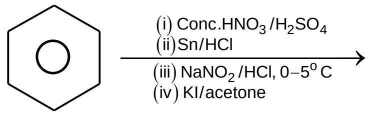
(A) BaO_2 (B) KO_2
(C) N_2O (D) Li_2O

Ans. B

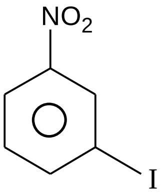
Sol. K^+ $1s^2 2s^2 2p^6 3s^2 3p^6$

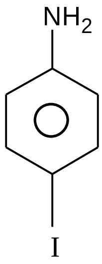
O_2^- $\sigma_{1s}^2 \sigma_{1s}^{*2} \sigma_{2s}^2 \sigma_{2s}^{*2} \sigma_{2P_z}^2 \begin{matrix} \pi_{2P_x}^2 \pi_{2P_x}^{*2} \\ \pi_{2P_y}^2 \pi_{2P_y}^{*1} \end{matrix}$

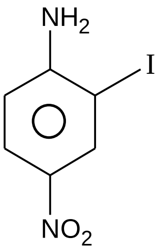
KO_2 contains one unpaired electron hence paramagnetic.

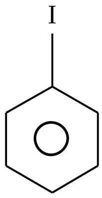
86. 

X(Major product); X is

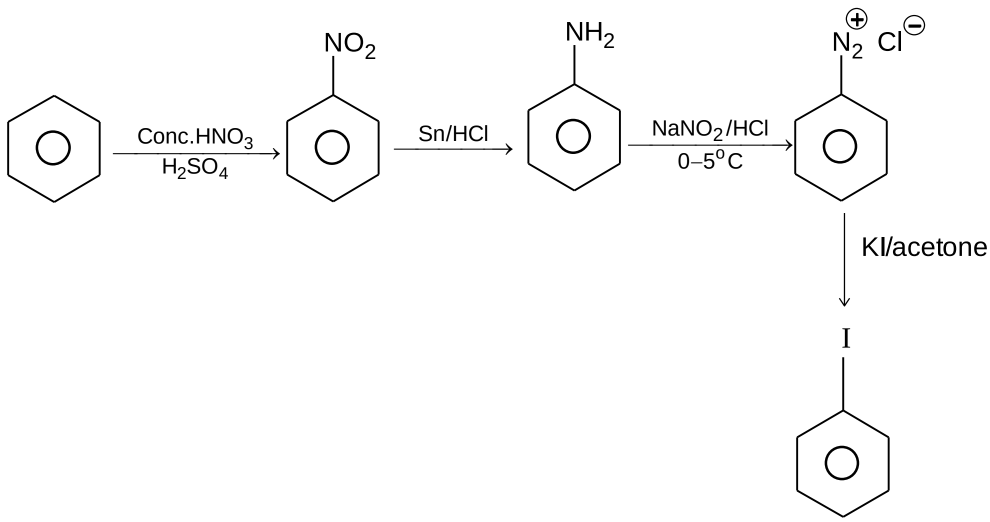
(A) 

(B) 

(C) 

(D) 

Ans. D

Sol. 

87. The correct combination is:
(A) $[NiCl_4]^{2-}$ – square-planar; $[Ni(CN)_4]^{2-}$ – paramagnetic
(B) $[NiCl_4]^{2-}$ – dimagnetic; $[Ni(CO)_4]$ – square-planar
(C) $Ni(CN)_4]^{2-}$ – tetrahedral; $[Ni(CO)_4]$ – paramagnetic
(D) $[NiCl_4]^{2-}$ - paramagnetic; $[Ni(CO)_4]$ – tetrahedral

Ans. D

Sol. As per Valance Bond theory, in $[NiCl_4]^{2-}$, Ni^{+2} has $3d^8$ configuration with 2 unpaired electrons.

88. When XO_2 is fused with an alkali metal hydroxide in presence of an oxidizing agent such as KNO_3; a dark green product is formed which disproportionates in acidic solution to afford a dark purple solution. X is

(A) Ti (B) Mn
(C) Cr (D) V

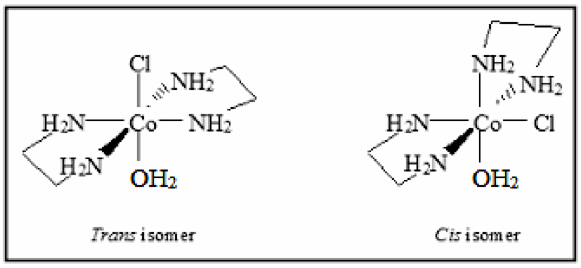
Ans. B

Sol. XO_2: MnO_2 ; Dark green product: MnO_4^{2-} ; Dark purple solution: MnO_4^-

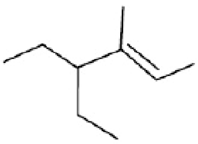
89. Which of the following complexes will show geometrical isomerism?
(A) potassium tris(oxalato)chromate(III)
(B) pentaaquachlorochromium(III) chloride
(C) potassium amminetrichloroplatinate(II)
(D) aquachlorobis(ethylenediamine)cobalt(II) chloride

Ans. D

Sol.



90. The IUPAC name of the following compound is:



(A) 3-ethyl-4-methylhex-4-ene (B) 4, 4-diethyl-3-methylbut-2-ene
(C) 4-methyl-3-ethylhex-4-ene (D) 4-ethyl-3-methylhex-2-ene

Ans. D

Sol. As per IUPAC Nomenclature system