KVPY – CLASS-XII
PART TEST – 3
(OLTS-1819-T3-PT-3-KVPY-XII)

PART – I

MATHEMATICS

1. The number of positive integers that are divisors of at least one of $10^{10}, 15^{7}$ and 18^{11} is

(A) $22\times30\times18$
(B) $12\times23\times11$
(C) $23\times13\times12$
(D) $23\times39\times17$

Ans. B

Sol. L.C.M of $2^{10}.5^{10}, 3^{7}.5^{7}, 2^{11}, 3^{22}$ is $2^{11}.3^{22}.5^{10}$ and its number of divisors is $12\times23\times11$.

2. The values of θ and λ for which the following system of equations

$(\sin\theta)x-(\cos\theta)y+(\lambda+1)z=0;$

$(\cos\theta)x+(\sin\theta)y-\lambda z=0$ and

$\lambda x+(\lambda+1)y+(\cos\theta)z=0$ has non trivial solution

(A) $\theta=n\pi, \lambda\in R-\{0\}$
(B) $\theta=2n\pi$, λ is any rational number
(C) $\theta=(2n+1)\pi, \lambda\in R^{+}$, $n\in I$
(D) $\theta=(2n+1)\frac{\pi}{2}, \lambda\in R, n\in I$

Ans. D

Sol. $\begin{vmatrix} \sin\theta & -\cos\theta & \lambda+1 \\ \cos\theta & \sin\theta & -\lambda \\ \lambda & \lambda+1 & \cos\theta \end{vmatrix}=0$

$\cos\theta\left(2\lambda^{2}+2\lambda+\sin^{2}\theta+1+\cos^{2}\theta\right)=0$

If $\cos\theta=0$ then $\lambda\in R$

Hence D

3. The lines $x+2y+z-4=2x+y+z-4=0$ and $3x-y-z-1=0=x+7y+4z-k$ are coplanar for k =

(A) 10
(B) 12
(C) 14
(D) 16

Ans. B

Sol. Lines will be coplanar if point of intersection of any three plane lie on the fourth. Indeed first three planes meet at (1, 1, 1) by inspection. If it lies on fourth plane we must have k = 12

4. If $S_n=\frac{1}{2n}-\frac{1}{n+1}+\frac{1}{2(n+2)}$ and $\sum_{n=1}^{14}S_n=\frac{135}{k}$, then the k must be ______

(A) 545
(B) 544
(C) 546
(D) 548

Ans. B

Sol. Indeed $S_n = \frac{1}{n(n+1)(n+2)}$

$= \frac{1}{2}\left[\frac{n+2-n}{n(n+1)(n+2)}\right]$

$= \frac{1}{2n(n+1)} - \frac{1}{2(n+1)(n+2)}$

$= p_n - p_{n+1}$

$\Rightarrow \sum_{n=1}^{14} S_n = p_1 - p_{15}$

$= \frac{1}{2\times1\times2} - \frac{1}{2\times16\times17} = \frac{135}{544}$

5. In triangle ABC, $\angle A = 30^o$, H is the orthocentre and D is the midpoint of BC. Segment HD is produced to T such that HD = DT then the value of $\frac{\left|\overrightarrow{AT}\right|}{BC}$ is

(A) 1 (B) 2

(C) 3 (D) 4

Ans. B

Sol. Consider origin as the circumcentre of triangle ABC.

Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}$ and $\overrightarrow{OT}$

Here, $|\vec{a}| = |\vec{b}| = |\vec{c}| = R$ (circum radius)

Point $D = \frac{\overrightarrow{OH} + \overrightarrow{OT}}{2} = \frac{\vec{b} + \vec{c}}{2} \Rightarrow \overrightarrow{OT} = \vec{b} + \vec{c} - \overrightarrow{OH}$

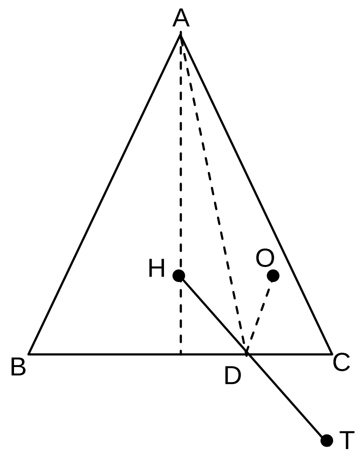
$\overrightarrow{OT} = -\vec{a} \Rightarrow \left|\overrightarrow{OT}\right| = |\vec{a}| = R$

$\overrightarrow{AT} = \overrightarrow{OT} - \overrightarrow{OA} = -2\vec{a}$

$\Rightarrow \frac{BC}{\sin A} = 2R$

As $BC = 2R\sin A = 2R\sin 30^o = R$

$\frac{\left|\overrightarrow{AT}\right|}{BC} = 2$



6. The shortest distance between the line $x = y = z$ and the line $2x + y + z - 1 = 0 = 3x + y + 2z - 2$ is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$

(C) $\frac{3}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{2}$

Ans. A

Sol. Any plane through the second line is $2x + y + z - 1 + k(3x + y + 2z - 2) = 0$

If this is parallel to the first line, then $(2+3k) + (1+k) + (1+2k) = 0 \Rightarrow k = -\frac{2}{3}$

$\Rightarrow$ Plane is $2x+y+z-1-\frac{2}{3}(3x+y+2z-2)=0$

or $y-z+1=0$. The required SD must be distance of this plane from any point on the line $x=y=z$ say (1, 1, 1)

$$\Rightarrow \quad SD=\frac{1-1+1}{\sqrt{0^2+1^2+(-1)^2}}=\frac{1}{\sqrt{2}}$$

7. Let A, B, C be three events in a probability space. Suppose that $P(A)=0.5, P(B)=0.3, P(C)=0.2, P(A\cap B)=0.15,\ P(A\cap C)=0.10$ and $P(B\cap C)=0.06$. Then the smallest possible value of $P\left(A^c\cap B^c\cap C^c\right)$ is:

[Here A^c denotes compliment of A.]

(A) 0.31 (B) 0.25
(C) 0 (D) 0.26

Ans. B

Sol. $P(A'\cap B'\cap C')=P(A\cup B\cup C)'=1-P(A\cup B\cup C)$

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)$

$-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

So, putting all the given values, we get

$P(A'\cap B'\cap C')=0.31-P(A\cap B\cap C)$

Not maximum value of $P(A\cap B\cap C)=P(A)$ or $P(B)$ or $P(C)$

or $P(B\cap C)$ or $P(A\cap C)$ or $P(A\cap B)$

For least value of $P(A\cap B\cap C)=P(B\cap C)$

So, $P(A'\cap B'\cap C')=0.31-0.06=0.25$

8. If $b-c,\ 2b-\alpha,\ b-a$ are in H.P., then $a-\frac{\alpha}{2},\ b-\frac{\alpha}{2},\ c-\frac{\alpha}{2}$ are in

(A) A.P. (B) G.P.
(C) H.P. (D) none of these

Ans. B

Sol. $2b-\alpha=\frac{2(b-c)(b-a)}{2b-(a+c)}\Rightarrow(2b-\alpha)\left[2b-(a+c)\right]-2\left[b^2-b(a+c)+ac\right]=0$

$\Rightarrow b^2-b\alpha+\frac{\alpha}{2}(a+c)-ac=0$

$\Rightarrow\left(b-\frac{\alpha}{2}\right)^2=\frac{\alpha^2}{4}-\frac{\alpha}{2}(a+c)+ac=\left(a-\frac{\alpha}{2}\right)\left(c-\frac{\alpha}{2}\right).$

9. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral, is

(A) $\frac{1}{2}$ (B) $\frac{1}{5}$

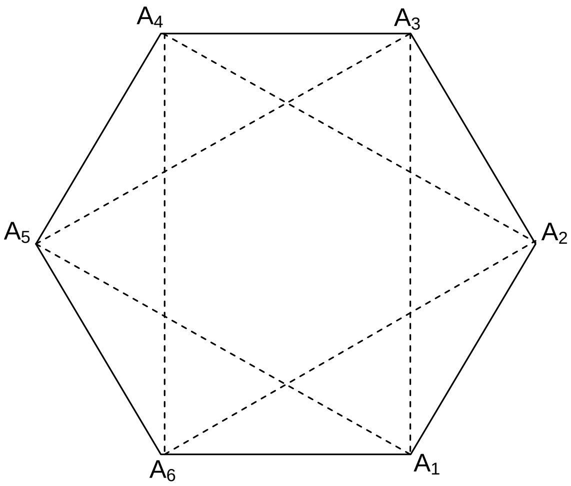
(C) $\frac{1}{10}$ (D) $\frac{1}{20}$

Ans. C

Sol. Total = 6 points

Selection of 3 points = ${}^{6}C_3 = 20$

Now, there are two possibilities of equilateral triangle $P(E) = \frac{2}{20} = \frac{1}{10}$



10. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^2 + k - 1}{(k+1)!}$ is equal to

(A) 1
(B) 2
(C) 3
(D) 4

Ans. B

Sol. $\frac{k^2 + k - 1}{(k+1)!} = \frac{k(k+1)}{(k+1)k!} - \frac{1}{(k+1)!}$

$= \frac{k}{k(k-1)!} - \frac{1}{(k+1)!} = \frac{1}{(k-1)!} - \frac{1}{(k+1)!}$

$t_1 = 1 - \frac{1}{2!}$

$t_2 = \frac{1}{1!} - \frac{1}{3!}$

$t_3 = \frac{1}{2!} - \frac{1}{5!}$

:

$t_n = \frac{1}{(n-1)!} - \frac{1}{(n+1)!}$ thus $\text{Sum}_{\infty} = 2$

11. In a certain test, a_i students gave wrong answers to at least i questions, where $i = 1, 2, ..., k$. No students gave more than k wrong answers. The total number of wrong answers given is:

(A) $\sum_{i=1}^{K} a_i$
(B) $\sum_{i=1}^{k} a_{2i}$
(C) $\left(\sum_{i=1}^{K} a_i\right) - 1$
(D) none of these

Ans. A

Sol. Number of students who gave wrong answers to exactly one question is $(a_1 - a_2)$.

Number of students who gave wrong answers to exactly two questions is $(a_2 - a_3)$

Number of students who gave wrong answers to exactly three questions is $(a_3 - a_4)$

Number of students who gave wrong answers to exactly k questions is $a_{k-1} - a_k$, Therefore, total number of wrong answers, is

$1(a_1 - a_2) + 2(a_2 - a_3) + 3(a_3 - a_4) + ...k(a_{k-1} - a_k)$

12. Let P be the plane containing the line $x + y - z - 1 = 0 = x + 4y + 3z$ and parallel to the line $6(x-1) = 3y = 2(z+1)$. The perpendicular distance of the plane P from origin is:

(A) 1

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{1}{\sqrt{6}}$

Ans. C

Sol. Equation of plane $(x + y - z - 1) + \lambda(x + 4y + 3z) = 0$

$(1+\lambda)x + (1+4\lambda)y + (3\lambda - 1)z - 1 = 0$

$\frac{x-1}{1/6} = \frac{y}{1/3} = \frac{z+1}{1/2}$

Lines is parallel to plane,

$\frac{1}{6}(1+\lambda) + \frac{1}{3}(1+4\lambda) + \frac{1}{2}(3\lambda - 1) = 0$

$(1+\lambda) + 2(1+4\lambda) + 3(3\lambda - 1) = 0 \ \lambda = 0$

Thus, equation of plane is $x + y - z - 1 = 0$

distance from origin $= \frac{1}{\sqrt{3}}$

13. In a triangle ABC let O, G and H be circumcentre, centroid and orthocentre of the triangle ABC then $\vec{O}A + \vec{O}B + \vec{O}C$ must be

(A) $\vec{O}G$

(B) $3\vec{O}G$

(C) $\vec{O}H$

(D) $2\vec{O}H$

Ans. C

Sol. We know from geometry that O, G ahd H are collinear with $\frac{OG}{GH} = \frac{1}{2}$

Let O be origin let position vector of orthocentre be $\vec{h}$ then $\frac{\vec{O}A + \vec{O}B + \vec{O}C}{3} = \frac{1 \times \vec{h} + 2 \times 0}{3}$

$\Rightarrow \vec{h} = \vec{O}A + \vec{O}B + \vec{O}C = \vec{O}H = \vec{O}A + \vec{O}B + \vec{O}C$

$= a_1 + a_2 + a_3 + ... + a_k$

14. The number of 6 digit numbers of the form abcdef if the digits satisfy the condition $a + b + c + d + e + f = a^2 + b^2 + c^2 + d^2 + e^2 + f^2$ is

(A) 32

(B) 64

(C) 128

(D) none of these

Ans. A

Sol. $a + b + c + d + e + f = a^2 + b^2 + c^2 + d^2 + e^2 + f^2$

a b c d e f

↑ ↑ ↑ ↑ ↑ ↑ then digits will be 0 or 1.

2 2 2 2 2

1 way only
So, total number of possible way $= 2^5 = 32$

15. Let $g(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$ then the equation of normal drawn to g(x) at x = 0, is

(A) x = 0
(B) y = 0
(C) x = 1
(D) y = 1

Ans. A

Sol. Given, $g(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

$$= \cos x\left(x^2 - 2x^2\right) - x\left(2\sin x - 2x\tan x\right) + 1\left(2x\sin x - x^2 \cdot \tan x\right)$$

$$= -x^2 \cdot \cos x + 2x^2 \cdot \tan x - x^2 \cdot \tan x$$

$$= -x^2 \cdot \cos x + x^2 \tan x$$

$\therefore$ $g(x) = x^2\left(\tan x - \cos x\right)$

Also, $g(0) = 0$ and $g'(x) = x^2\left(\sec^2 x + \sin x\right) + 2x \cdot \left(\tan x - \cos x\right)$

$\therefore$ $g'(0) = 0$

So, equation for normal drawn to g(x) at x = 0, is x = 0.

16. Given a parallelogram ABCD. If $\left|\overrightarrow{AB}\right| = a, \left|\overrightarrow{AD}\right| = b$ and $\left|\overrightarrow{AC}\right| = c,$ then the value of $\frac{1}{2}\left(\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{AC}\right|^2 + 2\overrightarrow{DB}.\overrightarrow{AB}\right)$ is

(A) $2a^2 + b^2$
(B) $a^2 + 2b^2$
(C) $a^2 + b^2 + c^2$
(D) $a^2 + b^2$

Ans. A

Sol. $\overrightarrow{DB}.\overrightarrow{AB} = \left(\vec{a} - \vec{b}\right).\vec{a} = \left|\vec{a}\right|^2 - \vec{a}.\vec{b}$

$\vec{a} + \vec{b} = \vec{c}$

$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a}.\vec{b} = \left|\vec{c}\right|^2$

$2\overrightarrow{DB}.\overrightarrow{AB} = 2\left|\vec{a}\right|^2 - 2\vec{a}.\vec{b}$

$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a}.\vec{b} = \left|\vec{c}\right|^2$

$\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{AC}\right|^2 + 2\overrightarrow{DB}.\overrightarrow{AB}$

$= a^2 + b^2 + c^2 + 2a^2 + a^2 + b^2 - c^2 = 4a^2 + 2b^2$

17. The value of the sum $\sum_{k=1}^{\infty}\sum_{n=2}^{\infty}\frac{k}{2^{n+k}}$ is equal to:

(A) 5
(B) 4
(C) 3
(D) 2

Ans. D

Sol. $\sum_{n=1}^{\infty}\frac{k}{2^{n+k}}=\sum_{n=1}^{\infty}\frac{k}{2^n2^k}=\frac{k}{2^k}\sum_{n=1}^{\infty}\frac{1}{2^n}=\frac{k}{2^k}.\frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)}=\frac{k}{2^k}$

$\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{k}{2^{n+k}}=\sum_{2^k}^{k}=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+......\infty$

Let $S=\frac{1}{2}+\frac{2}{2^3}+\frac{3}{2^3}=......\infty$

$\frac{1}{2}S=\frac{1}{2^2}+\frac{2}{2^3}+..........\infty$

$\frac{S}{2}=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.......\infty=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1$

$S=2$

18. A fair coin is tossed 15 times. The probability of getting at least 9 consecutive heads is

(A) $\frac{1}{2^7}$ (B) $\frac{3}{2^9}$

(C) $\frac{5}{2^{10}}$ (D) $\frac{1}{2^8}$

Ans. A

Sol. The 9 consecutive heads may start with 1st, 2nd, 3rd…..,7th toss.

HH …….. H …… for 1st toss

TH …….. HH ……. for 2nd toss

$\therefore$ The probability $=\frac{1}{2^9}+\frac{6}{2^{10}}=\frac{1}{2^7}$

19. The number of positive integer pairs (x, y) such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{2007}, x<y$, is

(A) 5 (B) 6

(C) 7 (D) 8

Ans. C

Sol. $2007(x+y)=xy \Rightarrow (x-2007)(y-2007)=2007^2$

The number of pairs is the number of divisors of $2007^2=3^4.223^2$ which is $5\times3=15$. The number of pairs (x, y) with $x<y$ is 7.

20. Let P_1 : 2x – y + z – 2 = 0 and P_2 : x + 2y – z = 3. Then the equation of plane passing through the point(–1, 3, 2) and perpendicular to the planes P_1 and P_2 is

(A) x + 3y – 5z + 2 = 0 (B) x + 3y + 5z – 18 = 0

(C) x – 3y – 5z + 20 = 0 (D) x – 3y + 5z = 0

Ans. C

Sol. Normal vector of required plane is parallel to vector $=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 2 & -1\end{vmatrix}$

$=\hat{i}(1-2)-\hat{j}(-2-1)+\hat{k}(4+1)=-\hat{i}+3\hat{j}+5\hat{k}$

$\therefore$ Equation of required plane is
$-1(x + 1) + 3(y - 3) + 5(z - 2) = 0$ or $x - 3y - 5z + 20 = 0$.

PHYSICS

21. Electric charge q, q and – 2q are placed at the corners of an equilateral triangle ABC of side L. The magnitude of electric dipole moment of the system is:
(A) qL (B) 2qL
(C) $\sqrt{3}$ qL (D) 4qL

Ans. C

Sol. Two dipoles will be formed at angle 60^o to each other.
$\therefore P_{net} = \sqrt{(qL)^2 + (qL)^2 + 2(qL)^2 \cos 60^o} = \sqrt{3}qL$

22. Potential in the x-y plane is given as $V = 5(x^2 + xy)$ volts. The electric field at the point (1, –2) will be
(A) 3J V/m (B) –5 J V/m
(C) 5J V/m (D) –3J V/m

Ans. B

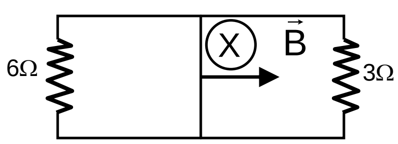
Sol. $E_x = -\frac{\partial V}{\partial x} = -(10x + 5y) = -10 + 10 = 0$
$E_y = -\frac{\partial V}{\partial x} = -5x = -5$
$\therefore \vec{E} = -5\hat{j}$ V/m.

23. A uniform magnetic field of 30 mT exists in the + X direction. A particle of charge + e and mass 1.67×10^{-27} kg is projected through the field in the + Y direction with a speed of 4.8×16^6 m/s. Radius of the circular path followed by the particle is
(A) 6.17 m (B) 1.67 m
(C) 1.76 m (D) 1.77 m

Ans. B

Sol. $F = qVB \sin\theta$
$= (1.6 \times 10^{-19})(4.8 \times 10^6)(30 \times 10^{-3}) \sin 90^0$
$= 230.4 \times 10^{-6}$ N.
The direction of the force is in the (-z) direction.
(b) If the particle were negatively charged, the magnitude of the force will be the same but the direction will be along (+z) direction.
(c) As $V \perp B$, the path described is a circle
$R = mV/qB$
$= (1.67 \times 10^{-27}) \cdot (4.8 \times 10^6)/(1.6 \times 10^{-19}) \cdot (30 \times 10^{-3}) = 1.67$ m.

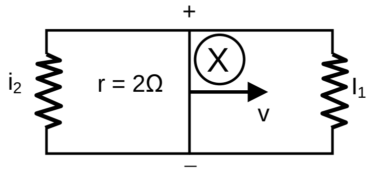
24. A rectangular loop with a sliding connector of length L = 1.0 m is situated in a uniform magnetic field B = 2 T perpendicular to the plane of loop. Resistance of connector is r = 2 Ω. Two resistances of 6 Ω and 3 Ω are connected as shown in figure. The external force required to keep the connector moving with a constant velocity v = 2 m/s is:

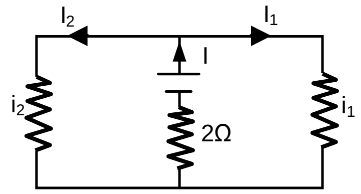


(A) 6 N (B) 4 N
(C) 2 N (D) 1 N

Ans. C

Sol.





e = BVl
= 2 × 2 × 1
= 4 volt

$I = I_1 + I_2$ …(i)

$+4 + 3I_1 - 2I = 0$

$$I_1 = \frac{4-2I}{3}$$

$4 - 6I_2 - 2I = 0$

$$I_2 = \frac{4-2I}{3}$$

From (i)

$$I = (4-2I)\left(\frac{1}{3}+\frac{1}{6}\right)$$

2I = 4 – 2I

I = 1A

Force, F = BIl = 2 × 1 × 1 = 2 N

25. A magnetised wire of moment M is bent into an arc of a circle subtending an angle of 60° at the centre, the new magnetic moment is

(A) $\frac{2M}{\pi}$ (B) $\frac{M}{\pi}$

(C) $\frac{3\sqrt{3}M}{\pi}$ (D) $\frac{3M}{\pi}$

Ans. D

Sol. New magnetic moment

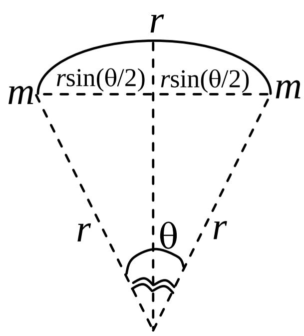
$M' = m \times$ Distance between Poles

$= m \times 2r \sin(\theta/2)$

$= m \times 2 \times \frac{\text{arc}}{\text{angle}} \times \sin\frac{\theta}{2}$

$= \frac{2 \times (m \times \text{arc}) \times \sin(\theta/2)}{\theta}$

$M' = \frac{2M\sin(\theta/2)}{\theta} = \frac{2M\sin(\pi/6)}{\pi/3} = \frac{3M}{\pi}$



26. An aeroplane with wings span of 50m flies at 540 km/hr. The component of the earth's magnetic field perpendicular to the velocity of the plane is 0.2 gauss. The potential difference between is

(A) 0.15V (B) 1. 5V

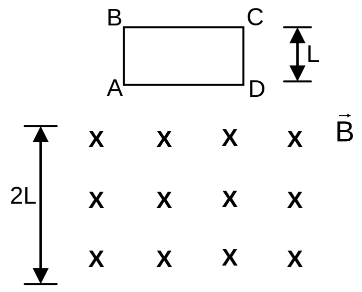
(C) 15V (D) 150V

Ans. A

Sol. $\varepsilon = B\ell V$

$= 0.2\times10^{-4} \times 50\times150$ Volts {$V=\frac{5}{18} \times 540$ m/s, 1 gauss=10^{-4} wb/m^2 = 0.15 V

27. A square coil ACDE with its plane vertical is released from rest in a horizontal uniform magnetic field $\vec{B}$ of length 2 L (see figure). The acceleration of the coil is:

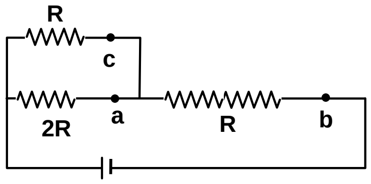


(A) less than g for all the time till the loop crosses the magnetic field completely
(B) less than g when it enters the field and greater than g when it comes out of the field
(C) g all the time
(D) less than g when it enters and comes out of the field but equal to g when it is within the filed

Ans. D

Sol. Lenz law.

28. Referring to the given circuit, the current will be minimum in



(A) a
(B) b
(C) c
(D) same in all the branches

Ans. A

Sol. $I_a = \frac{3V}{15R}, I_b = \frac{3V}{5R}$

$I_c = \frac{6V}{15R}$.

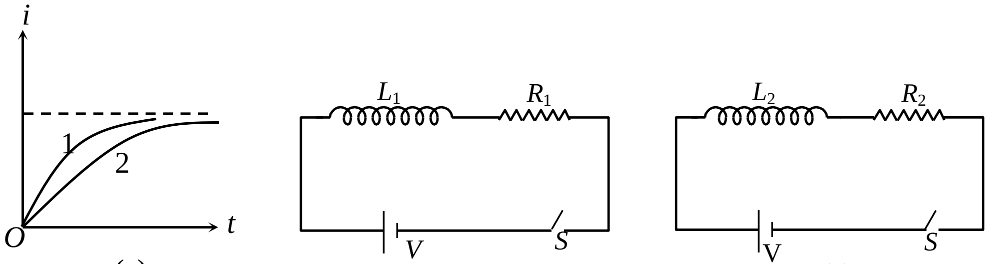
29. The current in a wire varies with time according to the equation I = 4 + 2t, where I is in ampere and t is in sec. The quantity of charge which has passed through a cross-section of the wire during the time t = 2 sec to t = 6 sec will be
(A) 60 coulomb (B) 24 coulomb
(C) 48 coulomb (D) 30 coulomb

Ans. C

Sol. Let dq be the charge which has passed in a small interval of time dt, then dq = idt = (4+2t)dt
Hence total charge passed between interval t = 2 sec and t = 6 sec

$q = \int_2^6 (4+2t)dt = 48$ coulomb

30. Current grows into L-R circuits (b) and (c) as shown in figure (a). Let L_1, L_2, R_1 and R_2 be the corresponding values of inductance and resistance in two circuits, then



(a) (b) (c)

(A) $R_1 > R_2$ (B) $R_1 < R_2$
(C) $L_1 > L_2$ (D) $L_1 < L_2$

Ans. D

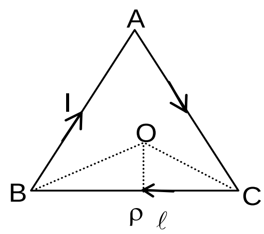
Sol. Steady state current for both the circuits is same.

Therefore $\frac{V}{R_1} = \frac{V}{R_2}$ or $R_1 = R_2$

Further $\tau_{L_1} < \tau_{L_2}$ (τ_L = time constant)

$\therefore$ $\frac{L_1}{R_1} < \frac{L_2}{R_2}$ or $L_1 < L_2$ ($R_1 = R_2$)

31. Current I is stabilised in a closed equilateral triangular loop ABC of side ℓ. The magnetic field at the centroid 'O'.



(A) 0 (B) $\frac{9\mu_o I}{2\pi\ell}$ units.
(C) $\frac{3\mu_o I}{2\pi\ell}$ units. (D) $\frac{5\mu_o I}{2\pi\ell}$ units.

Ans. B

Sol. From geometry

$OP = \frac{\ell}{2\sqrt{3}}$

Magnetic fields due to current in all three sides are equal in magnitude and directed into the plane of the paper.

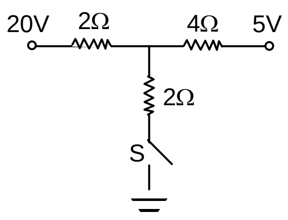
Hence net $B = 3\frac{\mu_o I}{4\pi r}[\sin\theta]_{-\pi/3}^{+\pi/3}$

where $r = \frac{\ell}{2\sqrt{3}}$

$= \frac{3\mu_o I}{4\pi r} \times 2\sin\frac{\pi}{3}$

$B = \frac{9\mu_o I}{2\pi\ell}$ units.

32. As the switch S is closed in the circuit shown in figure, current passed through it is



(A) 4.5 A (B) 6.0 A
(C) 3.0 A (D) zero

Ans. A

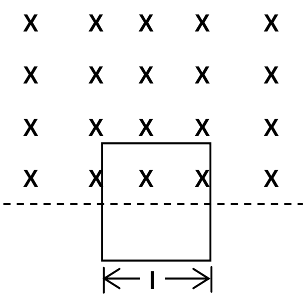
Sol. $\frac{20-V}{2}+\frac{5-V}{4}=\frac{V-0}{2}$

$V \rightarrow$ Potential of junction

$V = 9$

$I=\frac{9}{2}=4.5\,A$

33. Figure shows a horizontal magnetic field which is uniform above the dotted line and is zero below it. A long, rectangular, conducting loop of width l mass m, resistance R is placed partly above and partly below the dotted line with the lower edge parallel to it. The velocity with which the loop be pushed downward so that it may continue to fall without any acceleration is :



(A) $\frac{mgR}{B^2l^2}$ (B) $\frac{mg}{B^2R}$

(C) $\frac{B^2l^2}{mgR}$ (D) $\frac{B^2l^2}{mg}$

Ans. A

Sol. $mg = IB\ell = \frac{B^2\ell^2 v}{R}$

34. The deflection in moving coil galvanometer falls from 40 to 10 division when a shunt of 10Ω is connected across it. The resistance of the galvanometer is

(A) 40Ω (B) 30Ω

(C) 20Ω (D) 10Ω

Ans. B

Sol. Let 40 diV = I

$\therefore$ 10 diV = I_g

$S(I - I_g) = R_g (I_g)$

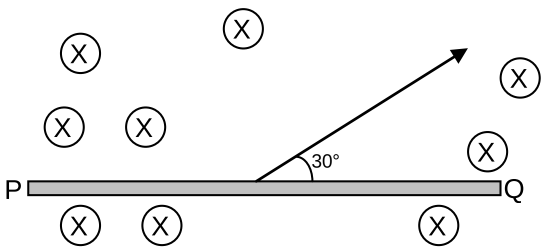
$\therefore R_g = \frac{S(I-I_g)}{I_g}$

$= S\left(\frac{I}{I_g}-1\right)$

$= 10\left(\frac{40}{10}-1\right)$

$R_g = 30\Omega$

35. A conducting rod PQ of length l = 2 m is moving at a speed of 2 ms^{-1} making an angle of 30° with its length. A uniform magnetic field B = 2 T exists in a direction perpendicular to the plane of motion. Then



(A) $V_P - V_Q = 8$ V

(B) $V_P - V_Q = 4$ V

(C) $V_Q - V_P = 8$ V

(D) $V_Q - V_P = 4$ V

Ans. B

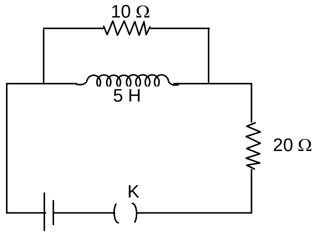
Sol. $V_P - V_Q = BV_\perp l$

$= 2 \times V \sin 30° \times 2$

$2 \times 2 \times \frac{1}{2} \times 2$

$V_P - V_Q = 4$ volt.

36. Two resistances of 10 Ω and 20 Ω and an ideal inductor of inductance 5 H are connected to a 2 V battery through a key k, as shown in figure. The key is closed at t = 0. What is the final value of current in the 10 Ω resistor



(A) $\frac{2}{3}$ A
(B) $\frac{1}{3}$ A
(C) $\frac{1}{6}$ A
(D) zero

Ans. D

Sol. At $t = \infty$ the resistance of inductor be zero.

37. The mass of the three wires of copper are in the ratio 1 : 3 : 5. and their lengths are in ratio 5 : 3 : 1 . The ratio of their electrical resistance is
(A) 1 : 3 : 5
(B) 5 : 3 : 1
(C) 1 : 15 : 125
(D) 125 : 15 : 1

Ans. D

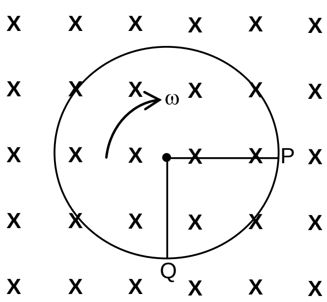
Sol. $R = \frac{\rho\ell}{A} = \frac{\rho\ell^2}{A\ell} = \frac{\rho\ell^2}{V} = \frac{\rho\ell^2}{m/d}$

$R = \frac{\rho d\ell^2}{m}$ or $R \propto \frac{\ell^2}{m}$

$R_1 : R_2 : R_3 = \frac{\ell_1^2}{m_1} : \frac{\ell_2^2}{m_2} : \frac{\ell_3^2}{m_3}$

$= \frac{25}{1} : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$

38. A conducting ring of radius r is rolling without slipping with a constant angular velocity ω (given figure). If the magnetic field strength is B and is directed into the page then the emf induced across PQ is:



(A) $B\omega r^2$
(B) $\frac{B\omega r^2}{2}$
(C) $4B\omega r^2$
(D) $\frac{\pi^2 r^2 B\omega}{8}$

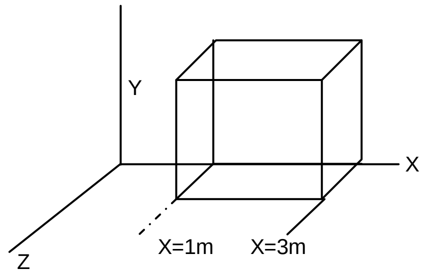
Ans. B

Sol. $e = \frac{B\ell^2\omega}{2}$

as, $l = r$

$$e = \frac{Br^2\omega}{2}$$

39. A non – uniform electric field given by $\vec{E} = 3x\hat{i} + 4\hat{j}$ (N/C) pierces the Gaussian cube shown in figure. What is the electric flux through the right face



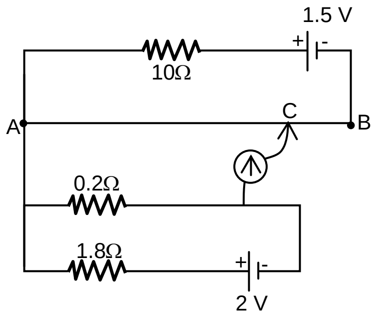
(A) 12 Nm^2/c
(B) 36 Nm^2/c
(C) 6 Nm^2/c
(D) zero

Ans. B

Sol. Electric field at right face $\vec{E} = 3 \times 3\hat{i} + 4\hat{j}$

$\phi = \vec{E}.\vec{A}$ = (9 i + 4j) . (4 i) =36 Nm^2/C

40. In the figure, AB is a uniform wire of length 100 cm whose resistance is 5 Ω. If galvanometer reads zero, then AC =



(A) 10cm
(B) 20cm
(C) 40cm
(D) 80cm

Ans. C

Sol. $V_{AC} = 2 \times \left(\frac{0.2}{1.8 + 0.2}\right)$ …(i)

Also, V_{AC} = $(1.5)\left(\frac{5}{15}\right)\left(\frac{\ell}{100}\right)$ …(ii)

a = $\frac{V^2}{2s}$ = $\frac{g}{4}\left(\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right)$

CHEMISTRY

41. The half-life period of a reaction involving one reactant is expressed as: $t_{1/2} = k\sqrt{C_o}$
Where k = Rate constant of the reaction
C_o = Initial concentration of the reaction
What is the order of the reaction?
(A) 0.5 (B) 1.5
(C) -0.5 (D) -1.5

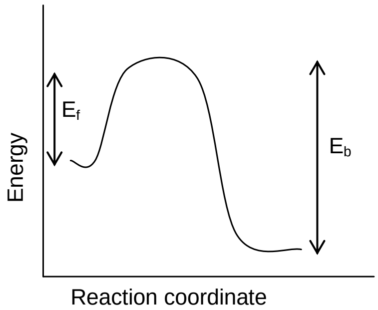
Ans. A

Sol. $t_{1/2} \alpha \frac{1}{C_o^{n-1}}$
On comparing n – 1 = -½
n = ½

42. If E_f and E_b are the activation energies of the forward and reverse reactions and the reaction is known to be exothermic then
(A) $E_f > E_b$ (B) $E_f < E_b$
(C) $E_f = E_b$ (D) $E_f \approx E_b$

Ans. B

Sol.



43. For a reaction 2A +B → C. the rate expression is given as r = K[A]^2[B]. If the concentration of B kept constant and A is tripled then the new rate is:
(A) 9r (B) 3r
(C) r (D) r/9

Ans. A

Sol. Since order with respect to A is 2 so rate will increase by 9 times on tripling the concentration.

44. Which of the following is NOT an intensive property?
(A) density (B) molarity
(C) enthalpy (D) specific heat

Ans. C

Sol. Enthalpy is an extensive property.

45. Which of the following statements is NOT correct for chemisorption?
(A) It is caused by chemical bond formation.
(B) It increases with increase in surface area of adsorbate.
(C) It is reversible in nature.
(D) It is highly specific in nature as compared to physisorption.

Ans. C

Sol. Chemisorption is irreversible in nature.

46. The freezing point of water decreases to -3^oC if a non-volatile solute is dissolved in it. What is the molality of the resulting solution?
[K_f of H_2O = 1.8 Kg K mol^{-1}]
(A) 2.116
(B) 1.66
(C) 1.126
(D) 2.612

Ans. B

Sol. $\Delta T_f = K_f \times m$
or, $0 - (-3) = K_f \times m$
or, $3 = 1.8 \times m \Rightarrow m = \frac{3}{1.8} = 1.66$

47. Which of the following difference between dispersed phase and dispersion medium of a colloid makes it display more pronounced Tyndall effect?
(A) Refractive index
(B) Number of particles
(C) Charge on particles
(D) Size of particles

Ans. A

Sol. Tyndall effect will be more pronounced if there is maximum difference of refractive index between dispersed phase and dispersion medium.

48. Which of the following solution is isotonic with 0.4 M $CaCl_2$ solution?
[Assume 100% dissociation of the solutes]
(A) 0.4 M $AlCl_3$
(B) 0.6 M NaBr
(C) 0.3 M $Al_2(SO_4)_3$
(D) 0.6 M $K_4[Fe(CN)_6]$

Ans. B

Sol. Isotonic solutions have same osmotic pressure.

49. The gold number of four protective colloids P, Q, R and S are 0.02, 0.002, 0.4 and 0.9 respectively. Protective power of these colloids will be in the order:
(A) Q > P > R > S
(B) P > Q > R > S
(C) R > Q >S > P
(D) S > P > R > Q

Ans. A

Sol. Lesser the gold number, higher is the protective power of colloids.

50. $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$
If the rate of formation of CO_2 in the above reaction is 0.1 mol L^{-1} s^{-1}, what will be the rate of consumption of H^+ in mol L^{-1} s^{-1} unit?
(A) 16
(B) 0.16
(C) 1.6
(D) 0.016

Ans. B

Sol. Rate = $-\frac{1}{16}\frac{\Delta[H^+]}{\Delta t} = \frac{1}{10}\frac{\Delta[CO_2]}{\Delta t} = \frac{1}{10} \times 0.1$

$\therefore \frac{\Delta[H^+]}{\Delta t} = \frac{16}{100}$ = 0.16 mol L^{-1} s^{-1}

51. Which of the following is correct for a reversible reaction at equilibrium?
(A) ΔS(system) = 0
(B) ΔS(surrounding) = 0
(C) ΔS(total) = 0
(D) unpredictable

Ans. C

Sol. At equilibrium $\Delta S_{Total} = 0$

52. The specific conductance of an electrolytic solution of concentration x M is y S cm^{-1}. The molar conductance of the solution in S cm^2 mol^{-1} unit is:
(A) $\frac{1000 \times x}{y}$
(B) $\frac{1000 \times y}{x}$
(C) $\frac{x}{1000\,y}$
(D) $\frac{y}{1000x}$

Ans. B

Sol. $\Lambda_m = \kappa \times \frac{1000}{M} = y \times \frac{1000}{x}$

53. For a reaction, the time needed for 75% completion of the reaction is twice of its half-life period. What is the order of the reaction?
(A) Zero
(B) First
(C) Second
(D) Third

Ans. B

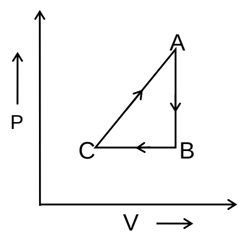
Sol. For 1st order reaction, $t_{75\%} = 2 \times t_{1/2}$.

54. Which of the following does NOT change by increasing the concentration of reactant(X) by maximum amount in the following reaction?
$2X(g) + Y(g) \longrightarrow X_2Y(g)$
(A) Order of reaction
(B) Rate of reaction
(C) Rate constant
(D) Rate of consumption of (Y)

Ans. C

Sol. Rate constant only changes with temperature.

55. Which point in the cyclic process (as given in figure) is at maximum temperature?
(A) A
(B) B
(C) C
(D) B and C are at maximum temperature



Ans. A

Sol. $T = \frac{PV}{nR}$
$\therefore$ T increases with increase in the value of P and V.

56. How much electricity should be passed through water in order to obtain 22.4 L of O_2 gas by electrolysis?

(A) (2×96500) Coulombs (B) (4×96500) Coulombs

(C) (8×96500) Coulombs (D) (6×96500) Coulombs

Ans. B

Sol. $H_2O \longrightarrow \frac{1}{2}O_2 + 2H^+ + 2e^-$

1 mole O_2 is formed by passing 4 F of electricity through water and 1 Faraday = 96500 coulombs

57. Which of the following 1 M solution has the highest molar conductance?

(A) CH_3COOH (B) HCN

(C) HCl (D) NH_4OH

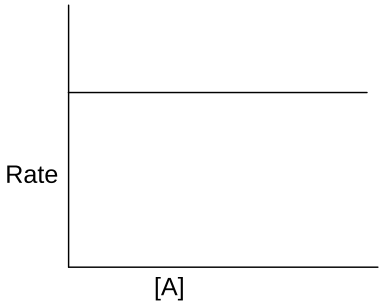
Ans. C

Sol. HCl is a strong acid. So, it produces more H^+ ion and provide higher conductance.

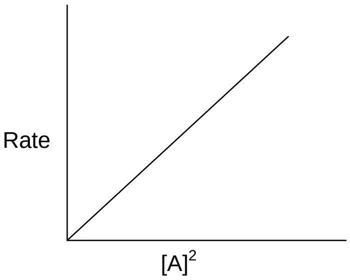
58. Which curve represents a zero order reaction?

(A)

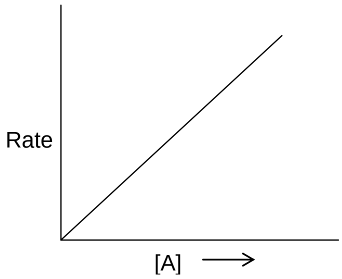
(B)



(C)



(D)



Ans. B

Sol. Rate of zero order reaction does not change with time or concentration.

59. In which of the following process, the P– V work done is zero?

(A) Isothermal process (B) Adiabatic process

(C) Isobaric process (D) Isochoric process

Ans. D

Sol. Volume does not change in isochoric processes.

$\therefore$ The work done is zero.

60. Which of the following statements is correct for an exothermic process?

(A) The entropy of the system increases

(B) Heat is absorbed by the system from the surrounding

(C) The entropy of the surrounding increases

(D) The enthalpy of the surrounding decreases

Ans. C

Sol. For exothermic process, randomness of the surrounding increases. So its entropy increases.

PART – II

MATHEMATICS

61. In a 3×3 matrix the entries a_{ij} are randomly selected from the digits $\{0,1,2,.....9\}$ with replacement. The probability that the 3 – digit numbers in each row will be divisible by 11 is

(A) $\frac{7^3}{10^6}$
(B) $\frac{13^3}{10^6}$
(C) $\frac{7^3.13^3}{5^6}$
(D) $\frac{7^3.13^3}{10^9}$

Ans. D

Sol. The number of multiples of 11 from 000 to 999 is 91

The required probability $=\left(\frac{91}{1000}\right)^3=\frac{7^3.13^3}{10^9}$.

62. Let $S_1,S_2,S_3..........$ and $t_1,t_2,t_3......$ are two arithmetic sequences such that $S_1=t_1\neq0;\ S_2=2t_2$ and $\sum_{i=1}^{10}S_i=\sum_{i=1}^{15}t_i$, then the value of $\frac{S_2-S_1}{t_2-t_1}$ is

(A) $\frac{8}{3}$
(B) $\frac{3}{2}$
(C) $\frac{19}{8}$
(D) 2

Ans. C

Sol. Let $S_1,S_2,S_3,........$ are in A.P. with common difference d_1 and t_1,t_2,t_3 are in A.P. with common difference d_2.

$\sum_{i=1}^{10}S_1=\frac{10}{2}\left[2S_1+(10-1)d_1\right]$

$\sum_{i=1}^{15}t_i=\frac{15}{2}\left[2t_1+(15-1)d_2\right]$

$10(2S_1+9d_1)=15(2t_1+14d_2)$

$2(2S_1+9d_1)=3(2t_1+14d_2)$

$2S_1+9d_1=3(t_1+7d_2)$

$2S_1+9d_1=3t_1+21d_2$

As, $S_1=t$

$2t_1+9d_1=3t_1+21d_2$

$9d_1=t_1+21d_2$

$d_1=S_2-S_1=2t_2-t_1$

$d_2=t_2-t_1$

$9(2t_2-t_1)=t_1+2t_2-21t_1$

$18t_2-9t_1=21t_2-20t_1$

$11t_1 = 3t_2$

Thus, $\frac{S_2 - S_1}{t_2 - t_1} = \frac{2t_2 - t_1}{t_2 - t_1} = \frac{2.\frac{11}{3} - 1}{\frac{11}{3} - 1} = \frac{22 - 3}{8} = \frac{19}{8}$

63. If α, β and γ are real numbers, then $\begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha - \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \gamma) & 1 \end{vmatrix} =$

(A) –1 (B) $\cos\alpha \cos\beta \cos\gamma$

(C) $\cos\alpha + \cos\beta + \cos\gamma$ (D) zero

Ans. D

Sol. $\Delta = \begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha - \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \gamma) & 1 \end{vmatrix}$

Writing 1 as $\sin^2\alpha + \cos^2\alpha$

We get, $\Delta = \begin{vmatrix} (\sin^2\alpha + \cos^2\alpha) & (\cos\beta\cos\alpha + \sin\beta\sin\alpha) & (\cos\gamma\cos\alpha + \sin\gamma\sin\alpha) \\ (\cos\alpha\cos\beta + \sin\alpha\sin\beta) & (\cos^2\beta + \sin^2\beta) & (\cos\gamma\cos\beta + \sin\gamma\sin\beta) \\ (\cos\alpha\cos\gamma + \sin\alpha\sin\gamma) & (\cos\beta\cos\gamma + \sin\beta\sin\gamma) & (\cos^2\gamma + \sin^2\gamma) \end{vmatrix}$

can be factorized into two determinant.

$\Delta = \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \cos\alpha & \sin\beta & 0 \\ \cos\gamma & \sin\gamma & 0 \end{vmatrix} \begin{vmatrix} \cos\alpha & \cos\beta & \cos\gamma \\ \sin\alpha & \sin\beta & \sin\gamma \\ 0 & 0 & 0 \end{vmatrix} = 0$

64 Let S = {1, 2, 3, 4,……..,n} and f_n be the number of those subsets of S which do not contain consecutive elements of S, then

(A) $f_6 = 22$ (B) $f_7 = 44$

(C) $f_4 = 9$ (D) $f_8 = 66$

Ans. B

Sol. $f_n =$ number of such subset in which n appears + number ofsuch subset in which n does not appears.

$S = \{1, 2, 3, ...(n-2), (n-1), n\}$ when n appears obviously (n – 1) will not appear and when n does not appear up to (n – 1) will appear

$\Rightarrow \quad f_n = f_{n-2} + f_{n-1}$

$f_1 = 2$

$f_2 = 3$

$\Rightarrow \quad f_3 = f_1 + f_2 = 2 + 3 = 5$

$f_4 = f_2 + f_3 = 3 + 5 = 8$

$f_5 = 13,\ f_6 = 21,\ f_7 = 44,\ f_8 = 65$

65. If all three planes

$P_1 : (a + 1)^3 x + (a + 2)^3 y + (a + 3)^3 z = 0$

$P_2 : (a + 1) x + (a + 2) y + (a + 3) z = 0$

$P_3 : x + y + z = 0$

pass through a point other than origin, then a equals

(A) –3 (B) 2

(C) –2 (D) 3

Ans. C

Sol. We must have $\Delta = 0$ i.e. $\begin{vmatrix} (a+1)^3 & (a+2)^3 & (a+3)^3 \\ (a+1) & (a+2) & (a+3) \\ 1 & 1 & 1 \end{vmatrix} = 0$

Put u = a + 1, v = a + 2, w = a + 3

then u – v = –1, v – w = –1, w – u = 2

Also, u + v + w = 3a + 6

Now, $\Delta = 0$

$$\Rightarrow \quad \begin{vmatrix} u^3 & v^3 & w^3 \\ u & v & w \\ 1 & 1 & 1 \end{vmatrix} = 0$$

i.e. (u – v) (v – w) (w – u) (u + v + w) = 0 $\Rightarrow$ (–1) (–1) (2) (3a + 6) = 0

or a = –2.

66. $\vec{a}, \vec{b}, \vec{c}$ are 3 unit vectors such that each is inclined at angle θ with the other. A unit vector $\vec{d}$ is equally inclined with these vectors at an angle α then $4\cos\theta - 3\cos 2\alpha =$

(A) 1 (B) –1

(C) $\sqrt{3}$ (D) $-\frac{1}{2}$

Ans. A

Sol. $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = \cos\theta$

$\vec{a}.\vec{d} = \vec{b}.\vec{d} = \vec{c}.\vec{d} = \cos\alpha$

$\vec{a}.\vec{b}.\vec{c}$ have to be non coplanar. If $\vec{d} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$, by symmetry,

$\alpha = \beta = \gamma = k$, say

$\therefore \vec{d} = k\left(\vec{a} + \vec{b} + \vec{c}\right)$

$\cos\alpha = \vec{d}.\vec{a} = k\left(1 + 2\cos\theta\right)$

$1 = \vec{d}.\vec{d} = 3k\cos\alpha$

Eliminating k, we get $3\cos^2\alpha = 1 + 2\cos\theta$

67. The determinant $\begin{vmatrix} 1+a+x & a+y & a+z \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+z \end{vmatrix} =$

(A) $(1+a+b+c)(1+x+y+z) - 3(ax+by+cz)$

(B) $a(x+y) + b(y+z) + c(z+x) - (xy+yz+zx)$

(C) $x(a+b) + y(b+c) + z(c+a) - (ab+bc+ca)$

(D) None of these

Ans. A

Sol. $\begin{vmatrix} 1+a+x & a+y & a+z \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+z \end{vmatrix}$

Using $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} (1+a+b+c+3x) & (1+a+b+c+3y) & (1+a+b+c+3z) \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+x \end{vmatrix}$$

$$= \begin{vmatrix} 1+a+b+c & 1+a+b+c & 1+a+b+c \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+z \end{vmatrix} + \begin{vmatrix} 3x & 3y & 3z \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+z \end{vmatrix}$$

$$= (1+a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+z \end{vmatrix} + 3 \begin{vmatrix} x & y & z \\ b+x & 1+b+y & b+z \\ c+x & c+y & 1+c+z \end{vmatrix}$$

$$= (1+a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ b+x & 1+y-x & z-x \\ c+x & y-x & 1+z-x \end{vmatrix} + 3 \begin{vmatrix} x & y & z \\ b & 1+b & b \\ c & c & 1+c \end{vmatrix}$$

$$= (1+a+b+c) \begin{vmatrix} 1+y-x & z-x \\ y-x & 1+z-x \end{vmatrix} + 3 \begin{vmatrix} x & y-z & z-x \\ b & 1 & 0 \\ c & 0 & 1 \end{vmatrix}$$

$$= (1+a+b+c)(1+x+y+z) - 3(ax+by+cz)$$

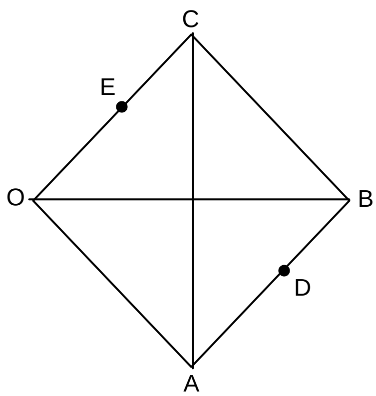
68. Given a regular tetrahedron OABC (where O is origin) with side length 1. Let D, E be the midpoint of the sides AB, OC respectively. The scalar product of two vectors $\vec{DE}$ and $\vec{AC}$ is equal to

(A) 0 (B) 1

(C) $\frac{1}{2}$ (D) 2

Ans. C

Sol. $\vec{OE} = \frac{\vec{OC}}{2}$



$\vec{OD} = \frac{\vec{OA} + \vec{OB}}{2}$

$\vec{DE} = \vec{OE} - \vec{OD}$

$= \frac{\vec{OC}}{2} - \frac{(\vec{OA} + \vec{OB})}{2}$

$= \frac{\vec{OC} - (\vec{OA} + \vec{OB})}{2}$

$\vec{AC} = \vec{OC} - \vec{OA}$

$\vec{DE}.\vec{AC} = \frac{(\vec{OC} - (\vec{OA} + \vec{OB}))}{2}.(\vec{OC} - \vec{OA})$

$$= \frac{\left|\vec{OC} - \vec{OA}\right|^2 + \vec{OB}.\left(\vec{OC} - \vec{OA}\right)}{2}$$

$$= \frac{\left|\vec{AC}\right|^2 + \vec{OB}.\vec{OC} - \vec{OB}.\vec{OA}}{2}$$

$$= \frac{1}{2} \quad \left[\left|\vec{AC}\right| = 1, \vec{OB}.\vec{OC} = \vec{OA}.\vec{OB}\right]$$

69. The lines $L_1, L_2, L_3, \ldots L_{20}$ are distinct. All the lines L_4, L_8, L_{12}, L_{16} and L_{20} are parallel. All the lines $L_1, L_5, L_9, L_{13}, L_{17}$ pass through a given point A. The maximum number of points of intersection of these 20 lines is

(A) 171 (B) 161
(C) 201 (D) 151

Ans. A

Sol. Maximum number of points of intersection possible if there are no constraints $= {}^{20}C_2 = 190$.
But 5 of these 20 lines are parallel. There will not be any intersection point from these 5 lines.
$\therefore$ Maximum number of points of intersection $= 190 - {}^5C_2 = 180$. Also the times $L_1, L_5, L_9, L_{13}, L_{17}$ pass through one point.
$\therefore$ We need to replace ${}^5C_2 (= 10)$ points by 1.
$\therefore$ The maximum number of points of intersection $= 190 - 10 - 10 + 1 = 171$

70. The equation $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & (2x+1) & (x+1) \\ (1-x)^2 & 3x & 2x \\ 1+2x & 3x-2 & 2x-3 \end{vmatrix} = 0$:

(A) has no real solution
(B) has 4 real solutions
(C) has two real and two non – real solutions
(D) has infinite number of solutions, real or non – real

Ans. D

Sol. $$\Delta = \begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x)^2 \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & (2x+1) & (x+1) \\ (1-x)^2 & 3x & 2x \\ 1+2x & 3x-2 & 2x-3 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} (1+x)^2 & (1-x)^2 & -x^2-2x-1 \\ (2x+1) & 3x & -2x-1 \\ (x+1) & 2x & -x-1 \end{vmatrix} = (-1)\begin{vmatrix} (1+x)^2 & (1-x)^2 & (1+x)^2 \\ (2x+1) & 3x & (2x+1) \\ (x+1) & 2x & (x+1) \end{vmatrix} = 0$$

Thus, infinite many solutions.

PHYSICS

71. A electric current of 16 A exists in a metal wire of cross section 10^{-6} m^2 and length 1m. Assuming one free electrons per atom, the drift speed of the free electrons in the wire will be (Density of metal = 5×10^3 kg/m^3, atomic weight = 60)

(A) 5×10^{-3} m/s (B) 2×10^{-3} m/s

(C) 4×10^{-3} m/s (D) 7.5×10^{-3} m/s

Ans. B

Sol. According to Avogadro's hypothesis

$$\frac{N}{N_A} = \frac{m}{M}$$

so n = $\frac{N}{v} = N_A \frac{m}{VM} = \frac{N_A}{M}$

Hence total number of atoms n = $\frac{6 \times 10^{23} \times 5 \times 10^3}{60 \times 10^{-3}}$

= 5×10^{28} /m^3

As $I = n_e eA v_d$

Hence drift velocity $v_d = \frac{I}{n_e eA}$

$$v_d = \frac{16}{5 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-6}}$$

= 2×10^{-3} m/s

72. A uniform electric field of magnitude E = 100 kV/m is directed upward. Perpendicular to E and directed into the page there exists a uniform magnetic field of magnitude B = 0.5T. A beam of particles of charge +q enters this region. What should be the chosen speed of particles for which the particles will not be deflected by the electric and magnetic field?

(A) 2×10^{-5} m/s (B) 3×10^{-5} m/s

(C) 5×10^{-5} m/s (D) 6×10^{-5} m/s

Ans. A

Sol. $\vec{F} = q\vec{E} + q\left(\vec{V} \times \vec{B}\right)$

$\vec{F} = q\left(\vec{E} + \vec{V} \times \vec{B}\right)$

If there has to be no deflection of beam then

$\vec{F} = 0$

$\vec{E} + \left(\vec{V} \times \vec{B}\right) = 0$

$\vec{V} \times \vec{B} = \vec{E}$

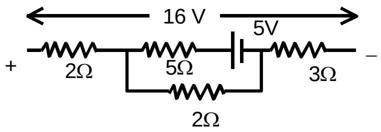
$\vec{V} \times \vec{B} = -\vec{E}$

$VB\sin 90^o = E$

$V = \frac{E}{B} = \frac{100 \times 10^3}{0.5}$

V = 2×10^5 m/sec

73. In the circuit shown below, determine the current through 5V cell

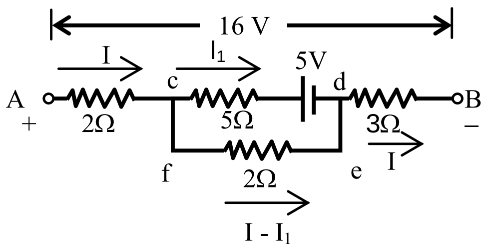


(A) $\frac{1}{15}$ Amp (B) $\frac{4}{15}$ Amp

(C) $\frac{1}{17}$ Amp (D) $\frac{4}{17}$ Amp

Ans. A

Sol. $\phi_A - \phi_B = 16$ ...(i)



Apply KVL on loop (cd efc)

$0 - 5I_1 - 5 + 2(I - I_1) = 0$

$\Rightarrow I_1 = \frac{1}{7}(2I - 5)$ ...(ii)

Apply KVL on path (AcdB)

$\phi_A - 2I - 5I_1 - 5 - 3I = \phi_B$

$\Rightarrow \phi_A - \phi_B = 5I + 5I_1 + 5$

$\Rightarrow 16 = 5I + 5\left(\frac{2I-5}{7}\right) + 5$

$\Rightarrow 16 - 5 + \frac{25}{7} = I\left(5 + \frac{10}{7}\right)$

$\Rightarrow I = \frac{34}{15}$ Amp

(i) and (ii) give

$I_1 = -\frac{1}{15}$ Amp

Current through battery (5V) = $\frac{1}{15}$ Amp

74. A conducting liquid bubble of radius a and thickness $t(t << a)$ is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet.

(A) $V\left(\frac{a}{3t}\right)^{1/3}$ (B) $Va^{1/3}$

(C) $V\left(\frac{a^2}{t^{2/3}}\right)$ (D) $\frac{Va^{1/3}}{3g^3}$

Ans. A

Sol. $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{a}$ (for bubble)

For droplet :- $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(a+t)^3 - \frac{4}{3}\pi a^3$

$\Rightarrow r^3 = 3a^2t \Rightarrow r = (3a^2t)^{1/3}$

$V_{droplet} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} = V\left[\frac{a}{3t}\right]^{1/3}$

75. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings +q and –q. The potential difference between the centres of the two rings is

(A) $\frac{qR}{4\pi\varepsilon_0 d^2}$ (B) $\frac{q}{2\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{\sqrt{R^2-d^2}}\right)$

(C) zero (D) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{\sqrt{R^2-d^2}}\right)$

Ans. B

Sol. $V_B = \frac{q}{4\pi\varepsilon_0\sqrt{R^2+d^2}} - \frac{q}{4\pi\varepsilon_0 R}$

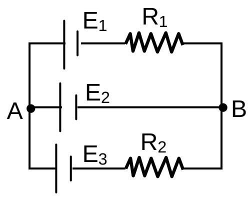
$V_A = \frac{q}{4\pi\varepsilon_0 R} - \frac{q}{4\pi\varepsilon_0\sqrt{R^2+d^2}}$

$V_A - V_B = \frac{q}{2\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{\sqrt{R^2-d^2}}\right)$

76. In the circuit shown here, $E_1 = E_2 = E_3 = 2$ V and $R_1 = R_2 = 4\ \Omega$. The current flowing between points A and B through battery E_2 is

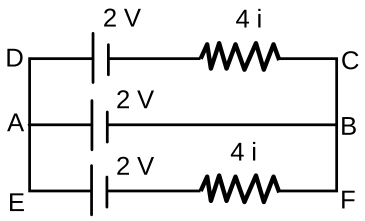
(A) zero (B) 2 A from a to B

(C) 2 A from B to A (D) none of the above



Ans. A

Sol.



ABCDA

$-2 - 4I_1 + 2 = 0$

$I_1 = 0$

ABEFA

$-2 - 4I_2 + 2 = 0$

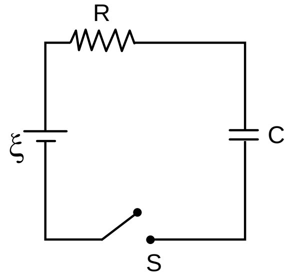
$I_2 = 0$

Hence, $I_1 + I_2 = 0$

77. As situation shown in figure the maximum value of rate of energy stored in the capacitor after the switch is closed

(A) $\frac{\xi^2}{2R}$ (B) $\frac{\xi^2}{4R}$

(C) $\frac{\xi^2}{8R}$ (D) none of these



Ans. B

Sol. Required value = $(V_C \cdot i_C)_{max} = \frac{\xi}{2}\cdot\frac{\xi}{2R} = \frac{\xi^2}{4R}$

78. A charge particle enters into a region containing uniform electric field (E) and uniform magnetic field (B) along x-axis and y-axis respectively. If it passes the region undeviated, the velocity of charge particle is given by

(A) $2\hat{i} + \frac{E}{B}\hat{k}$ (B) $2\hat{j} + \frac{E}{B}\hat{k}$

(C) $2\hat{i} - \frac{E}{B}\hat{k}$ (D) none of these

Ans. B

Sol. If $\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$, $\vec{E} = E\hat{i}$, $\vec{B} = B\hat{j}$

$\therefore \vec{F} = q\left[E\hat{i} + (V_x\hat{i} + V_y\hat{j} + V_z\hat{k}) \times B\hat{j}\right]$

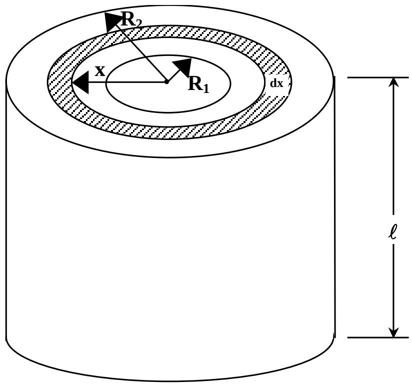
$= qE\hat{i} + qV_xB\hat{k} + 0 - qV_zB\hat{i}$

$= (E - V_zB)\ \hat{i} + V_xB\hat{k}$

For no deviation net force should either be zero or in the direction of velocity of particle.

$\therefore$ For $F = 0$, $V_Z = E/B$, $V_x = 0$, $V_y \rightarrow$ has any value

79. A cylindrical conductor of length ℓ and inner radius R_1 and outer radius R_2 has specific resistance ρ. A cell of emf ε is connected across the two lateral faces of the conductor. Find the current drawn from the cell.



(A) $\dfrac{2\pi\ell\varepsilon}{\rho \ln\frac{R_2}{R_1}}$

(B) $\dfrac{2\pi\ell\varepsilon}{\rho(R_2^2 - R_1^2)}$

(C) $\dfrac{2\ell\varepsilon}{\rho\pi(R_2^2 - R_1^2)}$

(D) None of thee

Ans. A

Sol. Consider the differential element of the cylinder as shown in the figure.

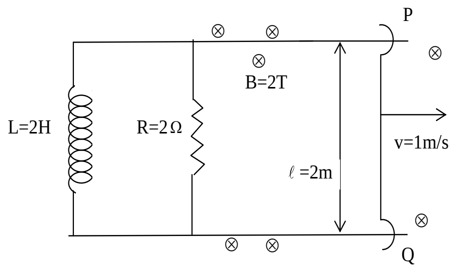
$\therefore \quad R = \int_{R_1}^{R_2} \rho\frac{dx}{2\pi x l} \qquad (\because R = \rho\frac{l}{a})$

$\Rightarrow R = \frac{\rho}{2\pi l}\ln\left(R_2/R_1\right)$

$I = \frac{\varepsilon}{R}$

$\Rightarrow I = \dfrac{2\pi l\varepsilon}{\rho\ln\left(\frac{R_2}{R_1}\right)}$

80. The given figure shows an inductor and resistance fixed on a conducting wire. A movable conducting wire PQ starts moving on the fixed rails from t = 0 with constant velocity 1m/s. The work done by the external force on the wire PQ in 2 seconds is



(A) 16J
(B) 32J
(C) 48J
(D) 64J

Ans. B

Sol. $W = B^2V^2\ell^2 t\left[\frac{1}{R}+\frac{t}{2L}\right]$ = 32 J

CHEMISTRY

81. The value of ΔS and ΔH of the chemical reaction given below are 60 J K^{-1} and 12 KJ mol^{-1} respectively. At what temperature the reaction becomes spontaneous?

$A_2(g) + B(g) \rightleftharpoons A_2B(g)$

(A) 160 K
(B) 200 K
(C) 240 K
(D) 180 K

Ans. C

Sol. $\Delta G = \Delta H - T\Delta S$

For spontaneous process

$\Delta G < 0 \Rightarrow \Delta H - T\Delta S < 0$

or, $\Delta H < T\Delta S$ or, $T > \frac{\Delta H}{\Delta S}$

or, $T > \frac{12,000}{60} = 200$

$\therefore$ Temperature must exceed 200 K for the process to be spontaneous.

82. The electrode potential of which of the following electrode is assumed to be zero?

(A) Pt, H_2(1 atm) | H^+(pH = 5)
(B) Pt, H_2(10 atm) | H^+(0.1 M)
(C) Pt, H_2(1 atm) | H^+(1 M)
(D) Pt, H_2(10 atm) | H^+(1 M)

Ans. C

Sol. The standard electrode potential of hydrogen electrode is zero.

83. The heat of combustion of monoclinic sulphur is –x kJ mol^{-1} and that of rhombic sulphur is –y kJ mol^{-1}. What is the heat of transition of monoclinic sulphur to rhombic sulphur in kJ mol^{-1} unit?

(A) x – y
(B) y – x
(C) x + y
(D) –(x + y)

Ans. B

Sol. $S(\text{monoclinic}) + O_2 \longrightarrow SO_2,\ \Delta H = -x\,\text{kJ mol}^{-1}$

$S(\text{rhombic}) + O_2 \longrightarrow SO_2,\ \Delta H = -y\,\text{kJ mol}^{-1}$

$S(\text{monoclinic}) \longrightarrow S(\text{rhombic});\ \Delta H = -x-(-y)$

$= (y-x)\,\text{kJ mol}^{-1}$

84. Which of the following statement is NOT correct?
(A) Emulsions are liquid-liquid colloidal system
(B) The range of diameter for colloidal particles lies between 1 and 1000 nm
(C) Lyophillic colloids are reversible sols
(D) The formation of micelles takes place below kraft temperature

Ans. D

Sol. Fact based.

85. Vapour pressure of pure A is 100 torr & that of pure B is 80 torr. If 2 moles of A are mixed with 3 moles of B, the total vapour pressure of the mixture is
(A) 440 torr (B) 460 torr
(C) 180 torr (D) 88 torr

Ans. D

Sol. $P = P_A^0 X_A + P_B^0 X_B = 100 \times \frac{2}{5} + 80 \times \frac{3}{5} = 88$

86. Which of the following will show positive deviation from ideal solution?
(A) Benzene and toluene (B) Acetone and ethyl alcohol
(C) Chloroform and acetone (D) None of the above

Ans. B

Sol. In pure ethanol molecules are hydrogen bonded, on adding acetone its molecules get in between the lost molecule and break the hydrogen bond.

87. Pt (H_2)| pH = 2 || pH = 3 | Pt (H_2). The cell reaction for the given cell is
1 atm 1 atm
(A) spontaneous (B) non – spontaneous
(C) in equilibrium (D) none of these

Ans. B

Sol. $[H^+]_A = 10^{-2}$ M;$[H^+]_C = 10^{-3}$ M
Since $[H^+]_{Anode} > [H^+]_{Cathode}$, Cell reaction will be non-spontaneous

88. If $E^0_{Fe^{2+}/Fe}$ is x_1, $E^0_{Fe^{3+}/Fe}$ is x_2, then $E^0_{Fe^{3+}/Fe^{2+}}$ will be :-
(A) $3x_2 - 2x_1$ (B) $x_2 - x_1$
(C) $x_2 + x_1$ (D) $2x_1 + 3x_2$

Ans. A

Sol. $Fe^{3+} \xrightarrow{x_3} Fe^{2+} \xrightarrow{x_1} Fe$

$\underbrace{Fe^{3+} \longrightarrow Fe}_{x_2}$

$X_3 + 2x_1 = 3x_2 \Rightarrow x_3 = 3x_2 - 2x_1$

89. Given $Fe^{+2} + 2e^- \rightleftharpoons Fe$ $E^0 = -0.44$ V

$Co^{+2} + 2e^- \rightleftharpoons Co$ $E^0 = -0.28$V

$Ca^{+2} + 2e^- \rightleftharpoons Ca$ $E^0 = -2.87$V

$Cu^{+2} + 2e^- \rightleftharpoons Cu$ $E^0 = +0.337$V

Which is the most electropositive metal?

(A) Fe (B) Co

(C) Ca (D) Cu

Ans. C

Sol. One which looses the electrons easily is most electropositive.

$$Ca \longrightarrow Ca^{2+} + 2e^-$$

So, $E^o = \underset{\text{Maximum}}{\{\underset{\downarrow}{2.87\,V}\}}$

90. For a dilute solution, Raoult's law states that

(A) the lowering of vapour pressure is equal to the mole fraction of the solute

(B) the relative lowering of vapour pressure is equal to the mole fraction of the solute

(C) the relative lowering of vapour pressure is proportional to the amount of the solute in solution

(D) the vapour pressure of the solution is equal to the mole fraction of the solvent.

Ans. B

Sol. Fact based.