KVPY – CLASS-XII

PART TEST – 2

(OLTS-1819-T2-PT-2-KVPY-XII)

PART – I

MATHEMATICS

1. The differential equation of $x = c_1y^2 + c_2y + c_3$, where c_1, c_2, c_3 are real arbitrary constants is

(A) $\left(\frac{dy}{dx}\right)\left(\frac{d^3y}{dx^3}\right) = 3\left(\frac{d^2y}{dx^2}\right)^2$

(B) $3\left(\frac{d^3y}{dx^3}\right) = \left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)^2$

(C) $\frac{d^3y}{dx^3} = 0$

(D) None of these

Ans. A

Sol. Differentiating we get $\frac{1}{y_1} = 2c_1y + c_2$

$\Rightarrow \quad -\frac{1}{y_1^2}y_2 = 2c_1y_1 \Rightarrow 2c_1 = \frac{-y_2}{y_1^3}$

Differentiating again $y_1y_3 = 3y_2^2$

2. Line $y = mx$ bisects two distinct chords drawn from (4, 4) on $y^2 = 4x$ if

(A) $m = \frac{-1}{2}$

(B) $m = -\frac{3}{8}$

(C) $m = -\frac{5}{8}$

(D) $m = 1$

Ans. D

Sol. Any point on $y = mx$ is (t, mt)

Equation of the chord of the parabola $y^2 = 4x$ having (t, mt) as the mid point is

$y(2mt) - 2(x + t) = m^2t^2 - 4t$

It passes through (4, 4)

$\Rightarrow \quad m^2t^2 - 2(4m+1)t + 8 = 0$

For two distinct chords, $(D > 0)$

$4(4m+1)^2 - 32m^2 > 0$

$\Rightarrow 8m^2 + 8m + 1 > 0$

$\Rightarrow \left(m + \frac{1}{2}\right)^2 > \frac{1}{8}$

3. The sum of all real roots of the equation $x^6 + 4x^4 - 10x^3 + 4x^2 + 1 = 0$ is

(A) 2

(B) 0

(C) 1

(D) 4

Ans. A

Sol. Clearly $x = 0$ is not a solution.

$\therefore$ Dividing by x^3

$$\left(x^3 + \frac{1}{x^3}\right) + 4\left(x + \frac{1}{x}\right) - 10 = 0$$

Let $x + \frac{1}{x} = z$

$\Rightarrow z^3 + z - 10 = 0 \Rightarrow (z-2)(z^2 + 2z + 5) = 0$

For $z^2 + 2z + 5 = 0$ Discriminant < 0

$\therefore$ Real roots of $x + \frac{1}{x} = 2$

$\therefore (x-1)^2 = 0 \Rightarrow x = 1, 1$

Sum = 2

4. Let w be a cube root of 1 other than 1. Then number of ordered pairs (a, b), where $a, b \in Z$ (set of all integers) such that $(a\omega + b)(a\omega^2 + b) = 1$ is

(A) 4
(B) 5
(C) 6
(D) 8

Ans. C

Sol. On multiplying $a^2 + b^2 - ab - 1 = 0$ $\quad (\omega^3 = 1, 1 + \omega + \omega^2 = 0)$

It is Quadratic in a $\therefore$ discriminate ≥ 0

$\Rightarrow b^2 \geq 3 \Rightarrow b \in \{-1, 0, 1\}$

If $b = -1 \Rightarrow a \in \{0, -1\}$

If $b = 0 \Rightarrow a \in \{-1, 1\}$

If $b = 1 \Rightarrow a \in \{0, 1\}$

5. Let $a_1, a_2, a_3, \text{..........} a_{2016}$ are the roots of the equation $x^{1008} - 7x - 2016 = 0,$ then the value of $\sqrt{\frac{a_1^{2016} + a_2^{2016} + \text{.................} + a_{2016}^{2016}}{2016}}$ is

(A) 2016
(B) $(2016)^{2016}$
(C) $(2016)^2$
(D) $\sqrt{(2016)}$

Ans. A

Sol. Given $a_1, a_2 \text{..........} a_{2016}$ are roots of $x^{1008} - 7x - 2016 = 0$

$\Rightarrow (a_i)^{1008} = 7a_i + 2016 \qquad i = 1, 2 \text{........} 2016$

$\Rightarrow (a_i)^{2016} = (7a_i)^2 + (2016)^2 + 14(a_i)(2016)$

$$\therefore \sum_{i=1}^{2016} (a_i)^{2016} = 49\sum_{i=1}^{2016} (a_i)^2 + (2016)^3 + (14)(2016)\sum_{i=1}^{2016} a_i$$

but $\sum_{i=1}^{2016} a_i = 0 = \sum_{i=1}^{2016} (a_i)^2$

Given expression $= \sqrt{\frac{(2016)^3}{(2016)}} = 2016$

6. Find the coefficient of x^{2009} in the expansion of $(1-x)^{2008}(1+x+x^2)^{2007}$

(A) –1 (B) –2
(C) 0 (D) 1

Ans. C

Sol. $(1-x)^{2008}(1+x+x^2)^{2007}$

$=(1-x)\left[(1-x)(1+x+x^2)\right]^{2007}$

$=(1-x)(1-x^3)^{2007}$

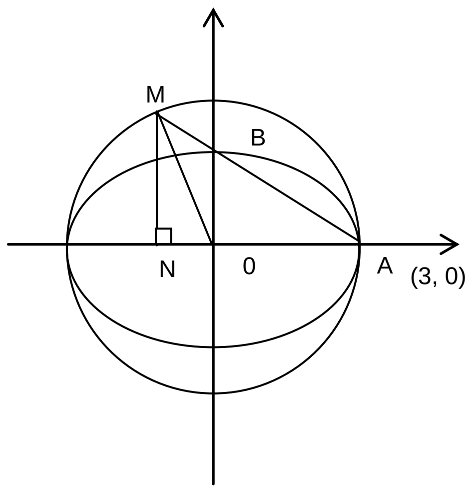
$=(1-x^3)^{2007}-x(1-x^3)^{2007}$

7. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse. $x^2+9y^2=9$, meets the auxillary circle at the point M, then the area of the triangle with vertices at A, M and the origin O is

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$

(C) $\frac{21}{10}$ (D) $\frac{27}{10}$

Ans. D



Sol. Equation of the ellipse is $\frac{x^2}{9}+\frac{y^2}{1}=1$ and of the auxillary circle is $x^2+y^2=9$.

Equation of AB is $\frac{x}{3}+\frac{y}{1}=1$

y – coordinate of M is given by $\left[3(1-y)\right]^2+y^2=9$

$\Rightarrow\ 10y^2-18y=0$

$\Rightarrow\ y=0$ or $y=\frac{9}{5}$

Area of $\Delta OAM=\frac{1}{2}(OA)(MN)$

$=\frac{1}{2}\times 3\times\frac{9}{5}=\frac{27}{10}$

8. The range of a for which the expression $y=\frac{(x-1)(x-5)}{(x-a)}$ is capable of taking all real values for $x\in R$ (the set of all real numbers), is

(A) $(-\infty, 1)$ (B) $a\in(5, \infty)$

(C) $a\in(1, 5)$ (D) $a\in\phi$

Ans. C

Sol. $x^2-(y+6)x+5+ay=0$

If $x\in R$, then $(y+6)^2-4(5+ay)\ge 0\Rightarrow y^2+4(3-a)y+16\ge 0$

$\Rightarrow$ Disc. $\le 0 \Rightarrow 16(3-a)^2 - 64 \le 0 \Rightarrow 1 \le a \le 5$

If $a = 1,5$ then x cannot take values 1 and 5

$\Rightarrow a \in (1, 5)$

9. If equations $x^2 - 3x + 4 = 0$ and $4x^2 - 2[3a+b]x + b = 0 (a,b \in R)$ have a common root then the complete set of values of a, is (where [.] denotes the Greatest Integer function)

(A) $\left[\frac{-11}{3}, \frac{-10}{3}\right)$ (B) $\left(\frac{-10}{3}, -3\right]$

(C) $\left[\frac{-10}{3}, -3\right)$ (D) $\left(3, \frac{10}{3}\right]$

Ans. **C**

Sol. Since roots of $x^2 - 3x + 4 = 0$ are imaginary, therefore both roots will be common

$\Rightarrow \frac{4}{1} = \frac{2[3a+b]}{3} = \frac{b}{4} \Rightarrow b = 16 \Rightarrow [3a+b] = 6 \Rightarrow [3a] + 16 = 6 \Rightarrow [3a] = -10$

$\Rightarrow -10 \le 3a < -9 \Rightarrow \frac{10}{3} \le a < -3 \Rightarrow a \in \left[-\frac{10}{3}, -3\right)$

10. A curve passes through the point $\left(1, \frac{\pi}{4}\right)$ and its slope at any point is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$. Then the curve has the equation

(A) $y = x\tan^{-1}\left(\ell n\frac{x}{e}\right)$ (B) $y = x\tan^{-1}(\ell n(ex))$

(C) $y = x\tan^{-1}\left(\ell n\frac{e}{x}\right)$ (D) none

Ans. A

Sol. $\frac{dy}{dx} = \frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$

$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx} \Rightarrow v + x\frac{dv}{dx} = v - \cos^2 v \Rightarrow \sec^2 v\, dv + \frac{dx}{x} = 0$

$\Rightarrow \tan v + \ln x = c$

$\Rightarrow \tan\left(\frac{y}{x}\right) + \ln x = c \Rightarrow c = 1$

$\Rightarrow t\left(\frac{y}{x}\right) = v - \ln x = \ln\left(\frac{e}{x}\right)$

$\frac{y}{x} = \tan^{-1}\left(\ell n\left(\frac{e}{x}\right)\right)$

11. A circle with radius unity has its centre on the positive y – axis. If this circle touches the parabola y= $2x^2$ tangentially at the points P and Q then the sum of the ordinates of P and Q is

(A) $\frac{15}{4}$ (B) $\frac{15}{8}$

(C) $2\sqrt{15}$ (D) 5

Ans. A

Sol. Let centre of circle be (0, k) i.e. circle is $x^2+(y-k)^2=1$. Solving with $x^2=\frac{y}{2}$, we get

$\frac{y}{2}+(y-k)^2=1$

i.e. $y^2-\left(2k-\frac{1}{2}\right)y+k^2-1=0$

Since the two curves touch each other

Disc. $=0 \Rightarrow \left(2k-\frac{1}{2}\right)^2-4k^2+4=0 \Rightarrow k=\frac{17}{8}$

Putting $k=\frac{17}{8}$ in above equation, we get $y^2-\frac{15}{4}y+\frac{225}{4}=0$

$\Rightarrow y_1+y_2=\frac{15}{4}$

12. AB is a double ordinate of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔAOB (where 'O' is the origin) is an equilateral triangle, then the eccentricity of the hyperbola satisfies

(A) $e>\sqrt{3}$
(B) $1<e<\frac{2}{\sqrt{3}}$
(C) $e=\frac{2}{\sqrt{3}}$
(D) $e>\frac{2}{\sqrt{3}}$

Ans. D

Sol. If $A=(x_1,y_1)$, then $\sqrt{x_1^2+y_1^2}=2y_1 \Rightarrow x_1\sqrt{3}\,y_1 \Rightarrow a\sec\theta=\sqrt{3}\,b\tan\theta$

$\Rightarrow \frac{b}{a}=\frac{1}{\sqrt{3}}\frac{\sec\theta}{\tan\theta} \Rightarrow e^2=1+\frac{1}{3}\frac{\sec^2\theta}{\tan^2\theta}=1+\frac{1}{3}\left(1+\cot^2\theta\right)=\frac{4}{3}+\frac{1}{3}\cot^2\theta$

$\Rightarrow e>\frac{2}{\sqrt{3}}$

13. If all the real solutions of the equation $4^x-(a-3)2^x+(a-4)=0$ are non positive, then

(A) $4<a\le 5$
(B) $0<a<4$
(C) $a>4$
(D) $a<3$

Ans. A

Sol. $4^x-(a-3)2^x+(a-4)=0$

$x\le 0$, Let $y=2^x$

$y^2-(a-3)y+(a-4)=0$

The roots of Quadratic must lie b/w (0, 1]

(a) $(a-3)^2-4(a-4)\ge 0$

$a^2+9-6a-4a+16\ge 0$

$a^2-10a+25\ge 0$

$a\in R$

(b) $0<\frac{a-3}{2}\le 1$

$0 < a - 3 \le 2$
$0 < a \le 5$
(c) f(0) = a – 4 > 0 a > 4
(d) f(1) = 1 – a + 3 + a – 4 ≥ 0 $a \in (4, 5]$

14. If $p(x) = ax^2 + bx$ and $q(x) = lx^2 + mx + n$ with p(1) = q(1), p(2) – q(2) = 1 and p(3) – q(3) = 4, then p(4) – q(4) is
(A) 0 (B) 5
(C) 6 (D) 9

Ans. D

Sol. We have, p(1) – q(1) = 0 ⇒ (a + b) – (l + m + n) = 0 …… (i)
p(2) – q(2) = 1 ⇒ (4a + 2b) – (4l + 2m + n) = 1 …… (ii)
p(3) – q(3) = 4 ⇒ (9a + 3b) – (9l + 3m + n) = 4 …… (iii)
now use the operation 3 × (3) + 1 × (1) – 3 × (2),
we get (16a + 4b) – (16l + 4m + n) = 9 ⇒ p(4) – q(4) = 9.

15. Circles are drawn on chords of the rectangular hyperbola xy=4 parallel to the line y=x as diameters. All such circles pass through two fixed points whose coordinates are
(A) (2,2) and (-2,-2) (B) (2,-2) and (-2,-2)
(C) (-2,2) and (-2,-2) (D) (2,-2) and (2,2)

Ans. A

Sol. Circle with points $P\left(2t_1, \frac{2}{t_1}\right)$ and $Q\left(2t_2, \frac{2}{t_2}\right)$ as diameter is given by

$$\left(x - 2t_1\right)\left(x - 2t_2\right) + \left(y - \frac{2}{t_1}\right)\left(y - \frac{2}{t_2}\right) = 1$$

Also, slope of PQ is given by

$$-\frac{1}{t_1 t_2} = 1 \Rightarrow t_1 t_2 = -1$$

hence, from (1), circle is

$$\left(x^2 + y^2 - 8\right) - \left(t_1 + t_2\right)\left(x - y\right) = 0$$

which is of the form $S + \lambda L = 0$.
Hence, circles passé through the points of intersection of the circle $x^2 + y^2 - 8 = 0$ and the line x=y.
The points of intersection are (2,2) and (-2,-2)

16. Sum of all the possible real values of x for which the sixth term of the expansion of

$$E = \left(3^{\log_3 \sqrt{9^{|x-2|}}} + 7^{(1/5)\log_7\left[(4).3^{|x-2|} - 9\right]}\right)^7$$ is 567, is

(A) 1 (B) 2
(C) 3 (D) 4

Ans. D

Sol. Put $y = 3^{\log_3 \sqrt{9^{|x-2|}}} \Rightarrow \log_3 y = \log_3 \sqrt{9^{|x-2|}}$

$$\Rightarrow\ y = \sqrt{9^{|x-2|}} = 3^{|x-2|}$$

Next, put $z = \left(7^{(1/5)\log_7\left[(4).3^{[x-2]} - 9\right]}\right)$

$\Rightarrow \log_7 z = \frac{1}{5}\log_7\left[(4)3^{|x-2|} - 9\right]$

$= \log_7\left[(4)3^{|x-2|} - 9\right]^{1/5}$

$\Rightarrow z = \left[(4)3^{|x-2|} - 9\right]^{1/5}$

Now, $E = (y+z)^7$ and 6th term is given by

$t_6 = {}^7C_5\, y^{7-5} z^5 = 21\left(3^{|x-2|}\right)^2 \left\{(4)3^{|x-2|} - 9\right\}$

$\Rightarrow 567 = 21\left\{3^{2|x-2|}\right\}\left\{(4)3^{|x-2|} - 9\right\}$

$\Rightarrow 27 = \left\{(4)3^{|x-2|} - (9)\left(3^{2|x|-2}\right)\right\}$

$\Rightarrow 27 = (4)3^{3|x-2|} - (9)^{2|x-2|}$

$\Rightarrow 4u^3 - 9u^2 - 27 = 0$ where $u = 3^{|x-2|}$

Now that $u = 3$ satisfies this equation

$\therefore 3^{|x-2|} = 3 \Rightarrow |x-2| = 1 \Rightarrow x - 2 = \pm 1$

$x = 2 \pm 1 = 3$ or 1.

17. Let a,b,c are complex quantities and roots of $z^3 + az^2 + bz + c = 0$ are unimodular then $|a| - |b|$ equal to

(A) 0 (B) 1

(C) –1 (D) 2

Ans. A

Sol. $|\alpha + \beta + \gamma| = |a|$

$|\alpha\beta + \beta\gamma + \gamma \propto| = |b|$

$|\alpha\beta\gamma|\left|\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right| = |b|$

$|\bar{\alpha} + \bar{\beta} + \bar{\gamma}| = |b| = |a|$

18. If y_1 and y_2 are two different solutions of the equation $\frac{dy}{dx} + P(x)y = Q(x)$ and $y = 3\alpha y_1 + 2\beta y_2$ is also the solution of the equation, (where $\alpha - \beta = 4$), then which of the following is not correct. ($\alpha, \beta \in R$)

(A) $4\alpha + \beta = 5$ (B) $\alpha + 4\beta = 7$

(C) $5\alpha + 5\beta + 2 = 0$ (D) $5\alpha + 10\beta + 13 = 0$

Ans. B

Sol. $\frac{dy_1}{dx} + P(x)y_1 = Q(x)$, $\frac{dy_2}{dx} + P(x)y_2 = Q(x)$

$\frac{d(3\alpha y_1 + \beta y_2)}{dx} + P(x)\left(3\alpha y_1 + 2\beta y_2\right) = Qx$

$3\alpha Q(x) + 2\beta(Q(x)) = Q(x) \Rightarrow 3\alpha + 2\beta = 1$ and $\alpha - \beta = 4$

$\Rightarrow \alpha = \frac{9}{5}$, $\beta = \frac{-11}{5}$

19. If α, β be the roots of $ax^2 + bx + c = 0$ (a, b, c ε R), $\frac{c}{a}<1$ and $b^2-4ac<0$, f(n) = $\sum_{r=1}^{n}\left(|\alpha|^r+|\beta|^r\right)$, then $\underset{n\to\infty}{\text{Lt}}$ f(n) is equal to

(A) $\frac{1}{\sqrt{\frac{a}{c}}-1}$

(B) $\frac{1}{\sqrt{\frac{a}{c}-1}}$

(C) $\frac{\sqrt{c}}{-\sqrt{a}+\sqrt{c}}$

(D) $\frac{2}{\sqrt{\frac{a}{c}}-1}$

Ans. D

Sol. $|z|^2=\frac{c}{a} \Rightarrow |\alpha|=|\beta|=\sqrt{\frac{c}{a}}$

20. Let $f(n)=\sum_{k=0}^{n}(n-2k)^2 C_k^2$ f find f (6).

(A) 3021

(B) 3022

(C) 3023

(D) 3024

Ans. D

Sol. We have $(n-2k)^2=(n-k)^2+k^2-2k(n-k)$

Thus, $\sum_{k=0}^{n}(n-2k)^2 C_k^2=\sum_{k=0}^{n}(n-k)^2 C_{n-k}^2$

$+\sum_{k=0}^{n}k^2C_k^2-2\sum_{k=0}^{n}k(n-k)C_k^2$ $\quad [\because C_k=C_{n-k}]$

$=2\sum_{k=0}^{n}k^2 C_k^2-2\sum_{k=0}^{n}k(n-k)C_k^2$

But $k(n-k)C_k^2=(k.C_k)((n-k)C_{n-k})$

$=n\left({}^{n-1}C_{k-1}\right)\left(n^{n-1}C_{n-k-1}\right)$

$=n^2\left({}^{n-1}C_{k-1}\right)\left({}^{n-1}C_k\right)$

Thus, $\sum_{k=0}^{n}k(n-k)C_k^2$

$=n^2\sum_{k=1}^{n-1}\left({}^{n-1}C_{k-1}\right)\left({}^{n-1}C_k\right)=n^2\left({}^{2n-2}C_n\right)$

Hence,

$\sum_{k=0}^{n}(n-2k)^2 C_k^2=2n^2\left({}^{2n-2}C_{n-1}\right)-2n^2\left({}^{2n-2}C_n\right)$

$=\left(2n^2\right)\left[\frac{(2n-2)!}{(n-1)!^2}-\frac{(2n-2)!}{n!(n-2)!}\right]$

$=\frac{\left(2n^2\right)(2n-2)!}{n!(n-1)!}\left[n-(n-1)\right]$

$=(2n)\left({}^{2n-2}C_{n-1}\right)$

PHYSICS

21. A transverse wave is described by the equation $y = y_0 \sin 2\pi (ft - x/a)$. The maximum particle velocity is equal to four times the wave velocity if a is equal to
(A) $\pi y_0 / 4$
(B) $\pi y_0/2$
(C) πy_0
(D) $2\pi y_0$

Ans. B

Sol. The maximum particle velocity of a SHM of amplitude Y_0 and frequency f is $2\pi f Y_0$. The wave velocity is $f\lambda$. For $2\pi fY_0$ to be equal to $4f\lambda$, λ has to be $\pi Y_0 / 2$. (Here λ = a).

22. If the time of revolution of a satellite is T, then Kinetic energy is proportional to
(A) 1/T
(B) $1/T^2$
(C) $1/T^3$
(D) $1/T^{2/3}$

Ans. D

Sol. Orbital velocity $v = \frac{2\pi r}{T}$

Hence, KE $\propto \frac{r^2}{T^2} = \frac{T^{4/3}}{T^2} = \frac{1}{T^{2/3}}$

23. A rubber rod of length 10 m density 1.3×10^3 kg/m^3 and Young's modulus 6×10^6 N/m^2 hangs from the ceiling of a room. The elongation produced in the rod due to its own weight is
(A 2 cm
(B) 2.3 cm
(C) 6.6 cm
(D) 10.6 cm

Ans. D

Sol. Mass of the rod = $AL\rho$ if A is its cross sectional area

Weight acts at the mid point $\therefore$ Y = $\frac{mg}{A} \times \frac{(L/2)}{\Delta L}$ if L is the original length

$\Rightarrow \Delta L = \frac{mgL}{2AY} = \frac{g\rho L^2}{2Y} = \frac{9.8 \times 1.3}{120}$ = 10.6 cm

24. A stationary observer receives sound waves from two tuning forks, one of which approaches and the other recedes with the same velocity. The observer hears beats of frequency ν = 2 Hz. What is the velocity of each turning fork if their actual frequency ν_0 = 680 Hz, and velocity of sound in air is V_s = 340 m/s.
(A) 0.1 m/s
(B) 0.2 m/s
(C) 0.3 m/s
(D) 0.5 m/s

Ans. D

Sol. $n' = \frac{C - v_0}{C - v_s} \times n$

$n' = \frac{340 - 0}{340 - v} \times 680 = \frac{340 \times 680}{340}\left[1 - \frac{v}{340}\right]^{-1} = 680\left(1 + \frac{v}{340}\right)$

$n'' = \frac{340}{340 + V} \times 680 = 680\left(1 - \frac{v}{340}\right)$

$n' - n'' = 2$

$\Rightarrow \quad 680\left(1+\frac{v}{340}-1+\frac{v}{340}\right)=2$

$\Rightarrow$ 4v = 2

$\Rightarrow$ v = 1/2.

Putting all the values v = 0.5 m/s

25. An open pipe is closed at one end. As a result the frequency of the third harmonic of the closed pipe is found to be higher than the fundamental frequency of the open pipe by 100 Hz. What is the fundamental frequency of the open pipe?

(A) 50 Hz (B) 100 Hz

(C) 200 Hz (D) 300 Hz

Ans. C

Sol. $\frac{3}{4L}V=\frac{V}{2L}+100 \quad \text{Hz}$

$\Rightarrow \frac{1}{4}\frac{V}{L}=100 \quad \text{Hz}$

$\Rightarrow \frac{V}{2L}=200 \quad \text{Hz}$

26. An ideal gas (γ = 1.5) is expanded adiabatically. How many times has the gas to be expanded to reduce the root mean square velocity of molecules 2.0 times

(A) 4 times (B) 16 times

(C) 8 times (D) 2 times

Ans. B

Sol. $V_{rms}=\sqrt{\frac{3RT}{M}}$

$\therefore \; V_{rms} \propto \sqrt{T}$

V_{rms} is to reduce two times i.e, temperature of the gas will have to reduce four times or

$\frac{T'}{T}=\frac{1}{4}$

During adiabatic process

$TV^{\gamma-1}=T'V'^{\gamma-1}$

or, $\frac{V'}{V}=\left(\frac{T}{T'}\right)^{\frac{1}{\gamma-1}}=(4)^{\frac{1}{1.5-1}}=4^2=16$

$\therefore \; V'=16V$

27. A small ball of mass m and density ρ is dropped in a long tube filled with a liquid of density $\frac{\rho}{3}$. The viscous force acting on the ball after a long time will be

(A) $\frac{mg}{3}$ (B) $\frac{2mg}{3}$

(C) mg (D) cannot be determined

Ans. B

Sol. F = $6\pi\eta rv$

$= 6\pi\eta r \;\; v_f\,(\text{after long}) \qquad \frac{4}{3}\pi r^3.\rho=m$

$= 6\ \pi\eta r.\frac{2}{9}\frac{(\rho-\sigma).gr^2}{\eta}$

$= \frac{4}{3}.\pi r^3\left(\rho-\frac{\rho}{3}\right).g$

$= mg.\frac{2}{3}$

$= \frac{2}{3}mg$

28. A particle of mass 2 kg is performing SHM, given by equation x = 10 sin $2\pi t$ (x is in m and t in s). The work done on the particle in time 0.25 sec to 0.75 sec, will be

(A) $200\pi^2$J (B) $100\pi^2$J
(C) 50πJ (D) zero

Ans. D

Sol. $F = m\frac{d^2x}{dt^2} = 2x-(10\times\ 4\pi^2\ \sin\ 2\pi t)$

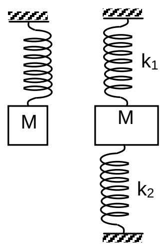
$= -\ 80\ \pi^2 \sin 2\pi t$

$dx = 20\ \pi\cos 2\ \pi t\ dt$

$W = \int_{0.25}^{0.75} F\ dx = -80\pi^2\times 20\quad \pi\int_{0.25}^{0.75} \sin 2\pi t\ \cos 2\pi t\ dt$

$= -\ 40\ \pi^2\times 20\pi\int_{0.25}^{0.75} \sin\ 4\pi t\ dt = \frac{800\ \ \pi^3}{4\pi}\ [\cos\ 4\pi t]_{0.25}^{0.75} = 0$

29. A mass M is oscillating with frequency f_o if hung with a spring of stiffness k. Now it is cut in two parts in the ratio of 2 : 1 and connected as shown. The new frequency is now



(A) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{3k}{M}}$
(C) $\frac{1}{2k}\sqrt{\frac{3k}{2M}}$ (D) $\frac{3}{2\pi}\sqrt{\frac{k}{2M}}$

Ans. D

Sol. $k_1 = \frac{3}{2}k$; $k_2 = 3k$

$k_{eff} = k_1 + k_2\ = \frac{1}{2\pi}\sqrt{\frac{9k}{2m}} = \frac{9}{2}k$

30. A simple pendulum oscillating with small amplitude has time period T. What will be the percentage change in its time period if its amplitude is decreased by 5 %?

(A) 6 % (B) 3 %
(C) 1.5 % (D) 0 %

Ans. D

Sol. For small amplitude the time period is independent of the amplitude therefore there will be no change in its time period.

31. A string of length 1 m and mass 10 gm is tightly clamped at its ends. The tension in the string is 4 N. Identical wave pulses are produced at one end at equal intervals of time t. What is the minimum value of t which allows a constructive interference between successive pulses?

(A) 0.05 s (B) 0.1 s
(C) 0.15 s (D) 0.2 s

Ans. B

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{4 \times 1}{10 \times 10^{-3}}} = 20 \quad m/s$

$t = \frac{2\ell}{v} = 0.1 sec$

32. A liquid is kept in a vertical cylindrical vessel which is being rotated about its axis. The liquid rises at the sides. If the radius of the vessel is 0.05 m and the speed of rotation is 2 rev/s, find the difference in the heights of the liquid at the centre of the vessel and at its sides.

(A) 10 cm (B) 12 cm
(C) 6 cm (D) 2 cm

Ans. D

Sol. According to Bernoulli's theorem

$P + \frac{1}{2}\rho v^2 = \text{constant}$

or $P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$

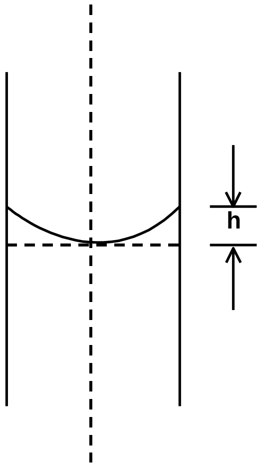
or $\Delta P = \frac{1}{2}\rho \Delta v^2$

or $h\rho g = \frac{1}{2}\rho \Delta v^2$

or $h = \frac{1}{2}\frac{v^2}{g}$

But $v = \omega r = 2 \times 2\pi \times 0.05 = 0.2\pi$ m/s

$h = \frac{0.04 \quad \pi^2}{2 \times 9.8} = 0.02$ m



33. A cubical block of wood of specific gravity 0.5 and a chunk of concrete of specific gravity 2.5 are fastened together. The ratio of the mass of wood to the mass of concrete which makes the combination to float with its entire volume submerged under water is

(A) $\frac{1}{5}$ (B) $\frac{1}{3}$
(C) $\frac{3}{5}$ (D) $\frac{2}{5}$

Ans. C

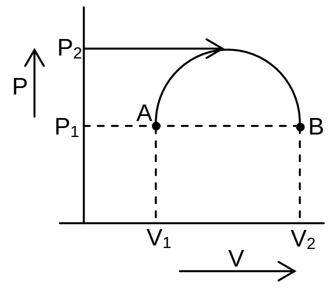
Sol. let mass of wood = m_1
And mass of concrete = m_2

$\therefore \left(\frac{m_1}{0.5\rho} + \frac{m_2}{2.5\rho}\right).\rho.g = (m_1 + m_2)g$

$\Rightarrow m_1 = \frac{1.5}{2.5} m_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{3}{5}$

34. Find work done on 2 moles of an ideal gas in the process $A \to B$ shown in the P-V diagram. Path A to B is semicircular.



(A) $P_1(V_2 - V_1) + \frac{1}{4}\pi(V_2 - V_1)(P_2 - P_1)$

(B) $P_1(V_2 + V_1) + \frac{1}{2}\pi(V_2 + V_1)(P_2 + P_1)$

(C) $P_1(V_2 - V_1) + \frac{1}{2}\pi(V_2 - V_1)(P_2 + P_1)$

(D) $P_1(V_2 + V_1) + \frac{1}{2}\pi(V_2 + V_1)(P_2 - P_1)$

Ans. A

Sol. work done = area enclosed from V_1 to V_2

$= P_1(V_2 - V_1) + \frac{1}{4}\pi(V_2 - V_1)(P_2 - P_1)$

35. A black body is at a temperature of 2880K. The energy of radiation emitted by this object with wave length between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. Wien's constant b=2.88 x 10^6 nmK. Then

(A) $U_1 = 0$
(B) $U_3 = 0$
(C) $U_1 = U_2$
(D) $U_2 > U_1$

Ans. D

Sol. From Wien's Law, $\lambda_m T =$ constant, where T is temperature of black body and λ_m is wavelength corresponding to maximum energy of emission.
Energy distribution of blackbody radiation is given below :
1. U_1 and U_3 are not zero because a blackbody emits nearly radiation's of all wavelengths.
2. Since U_1 corresponds lower wavelength and U_3 corresponds higher wavelength and U_2 corresponds medium wavelength. Hence $U_2 > U_1$.

36. Infinite numbers of particles each of mass 2kg each are placed on the positive x-axis at a distance 1m, 2m, 4 m and 8 m respectively from the origin. The net intensity of gravitational field at the origin will be

(A) 8G/3
(B) ∞
(C) zero
(D) data insufficient

Ans. A

Sol. $|E| = GM/r^2$

$E_{net} = G \times 2\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \ . \ . \ .\right]$ (as $a + ar + ar^2 + \ldots = a/1-r$)

$= G \times 2\left[\frac{1}{1 - 1/4}\right] = 8G/3$

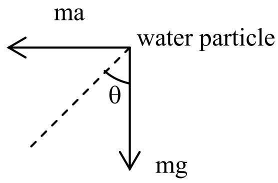
37. An open vessel containing water is moving with a constant acceleration a in the horizontal direction. The free surface of water gets shaped with the horizontal at an angle θ

(A) $\theta = \cos^{-1} g/a$ (B) $\theta = \tan^{-1} a/g$

(C) $\theta = \sin^{-1} a/g$ (D) $\theta = \tan^{-1} g/a$

Ans. B

Sol. The free surface becomes perpendicular to the resultant force



$\tan\theta = \dfrac{ma}{mg}$

$\theta = \tan^{-1}(a/g)$

38. An open pipe of length 47cm has fundamental frequency 340 Hz. What is the diameter of the tube if velocity of sound in air is 340 m/sec?

(A) 2 cm (B) 5 cm

(C) 8 cm (D) 10 cm

Ans. B

Sol. Antinodes are formed at a distance 0.6r from the end of the tube, where r is radius of the tube.

$47 + 2 \times .6r = \lambda/2$

$\lambda = v/n = 100$ cm

$47 + .6D = 100/2$

D = 5 cm

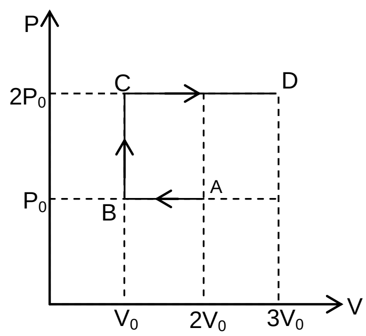
39. P –V diagram of an ideal gas is as shown, work done by the gas in the process ABCD is

(A) $4\,P_0V_0$

(B) $2\,P_0V_0$

(C) $3\,P_0V_0$

(D) P_0V_0



Ans. C

Sol. $W_{AB} = -P_0V_0$

$W_{CD} = +4P_0V_0$

$W_{ABCD} = +3P_0V_0$

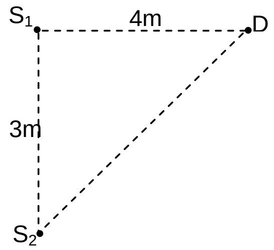
40. In the figure the intensity of waves arriving at D from two coherent sources S_1 and S_2 is I_0. The wave length of the wave is $\lambda = 4$m. Resultant intensity at D will be

(A) $4I_0$

(B) I_0

(C) $2I_0$

(D) zero



Ans. C

Sol. Path difference = 5 – 4 = 1 m

Phase difference = $\Delta\phi = \frac{2\pi}{\lambda}.\Delta x = \frac{2\pi}{4} = \frac{\pi}{2}$

$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi)$

$I_R = I_0 + I_0 + 0$

$I_R = 2I_0$

CHEMISTRY

41. Which monomer will give radical polymerisation most readily?

(A) $H_2C = CH_2$ (B) $C_6H_5 - CH = CH_2$

(C) $CH_3 - CH = CH_2$ (D) $CH_3-\underset{\underset{CH_3}{|}}{C}=CH_2$

Ans. B

Sol. $C_6H_5\dot{C}HCH_3$ is most stable free radical.

42. Ziegler-Natta catalyst is:

(A) R_3Al (B) $TiCl_4$

(C) $R_3Al/TiCl_4$ (D) $R_3B/TiCl_2$

Ans. C

Sol. Fact based

43. $\sim\!\left[NH(CH_2)_6 NHCO(CH_2)_4 CO\right]_n\!\sim$ is a :

(A) homopolymer (B) copolymer

(C) addition polymer (D) thermosetting polymer

Ans. B

Sol. Fact based

44. Which of the following is the first member of aldose monosaccharides?

(A) $CH_2OH-\overset{\overset{O}{||}}{C}-CH_2OH$ (B) $CH_2OH - CHOH - CHO$

(C) $CH_2OH - CHOH - CHOH - CHO$ (D) $CH_2OH-CHOH-\overset{\overset{O}{||}}{C}-CH_2OH$

Ans. B

Sol. Glyceraldehyde is first member of monosaccharides.

45. Which of the following hexoses will form the same osazone when treated with excess phenyl hydrazine?

(A) D-glucose, D-fructose and D-galactose

(B) D-glucose, D-fructose and D-mannose

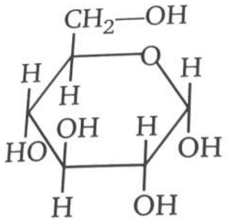
(C) D-glucose, D-mannose and D-galactose

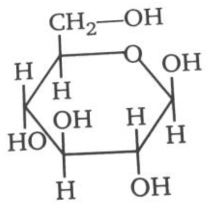
(D) D-fructose, D-mannose and D-galactose

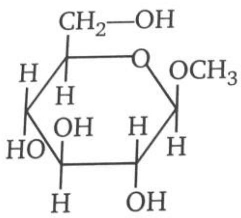
Ans. B

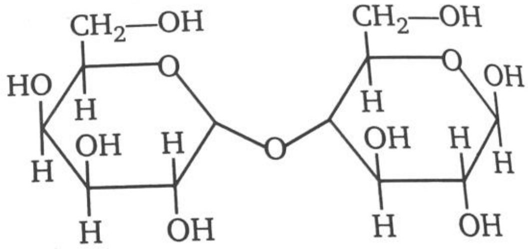
Sol. Fact based

46. Which of the following is NOT a reducing sugar?

(A) 

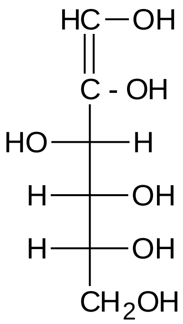
(B) 

(C) 

(D) 

Ans. C

Sol. Acetal sugars are non-reducing sugar.

47. 
, the given structural formula is enol form of:

(A) D-glucose
(B) D-mannose
(C) D-fructose
(D) All of these

Ans. D

Sol. Due to Van Ekenstein transformation.

48. If Pd vs P(Where P denotes pressure in atom and d denotes density in gm/L) is plotted for He gas(assume ideal) at a particular temperature, and $\left[\frac{d}{dP}(Pd)\right]_{P=8.21\text{ atm}} = 5$, then the temperature will be

(A) 160 K
(B) 320 K
(C) 80 K
(D) none of these

Ans. A

Sol. PM = dRT

$$Pd = p^2\left(\frac{M}{2T}\right)$$

$$\frac{d(Pd)}{dP} = \frac{2PM}{RT} = 5$$

$$\frac{0.2 \times 8.21 \times 4}{0.0821 \times T} = 5$$

T = 160 K

49. Two closed vessels A and B of equal volume containing air at pressure P_1 and temperature T_1 are connected to each other through a narrow open tube. If the temperature of one is now maintained at T_1 and other at T_2 (Where $T_1 > T_2$) then that will be the final pressure?

(A) $\frac{T_1}{2P_1T_2}$
(B) $\frac{2P_1T_2}{T_1 + T_2}$
(C) $\frac{2P_1T_1}{T_1 - T_2}$
(D) $\frac{2P_1}{T_1 + T_2}$

Ans. B

Sol. Let $T_1 > T_2$: final pressure will be same, Let x mole transfer from A to B vessel

$\therefore P_AV = (n - x)RT$

and $P_AV = (n + x)RT_2$

$$\therefore x = \frac{n(T_1 - T_2)}{T_1 + T_2}$$

$$V = \frac{nRT_1}{P_1}$$

$$\therefore P_A \times \frac{nRT_1}{P_1} = \left(n - \frac{n(T_1 - T_2)}{(T_1 + T_2)}\right)RT_1$$

$$P_A = \frac{2P_1T_2}{T_1 + T_2}$$

50. A given volume of ozonised oxygen (containing 60% oxygen by volume) required 220 sec to effuse while an equal volume of oxygen took 200 sec only under identical conditions. If density of O_2 is 1.6 g/L then find density of O_3.

(A) 1.936 g/L
(B) 2.16 g/L
(C) 3.28 g/L
(D) 2.24 g/L

Ans. D

Sol. Let V mL of gas effused

$$\frac{V/220}{V/200} = \sqrt{\frac{d_{O_2}}{d_{mix}}} \Rightarrow d_{mix} 1.6 \times (1.1)^2 = 1.936\,g/L$$

Let density of ozone is d; In 100 volume ozonised oxygen, 60% O_2 and 40% by volume O_3 is present.

$\therefore$ Mass of mixture = mass of ozone + mass of oxygen

$100 \times 1.936 = 40 \times d + 60 \times 1.6$

Density of O_3 is 2.44 g/L

51. Fe_2O_3(s) may be converted to Fe by the reaction

$$Fe_2O_3(s) + 3H_2(g) \rightleftharpoons 2Fe(s) + 3H_2O(g)$$

for which $K_C = 8$ at temp 720°C.

What percentage of the H_2 remains unreacted after the reaction has come to equilibrium?

(A) $\simeq$ 22%
(B) $\simeq$ 34%
(C) $\simeq$ 66%
(D) $\simeq$ 78%

Ans. B

Sol. Let initial moles of H_2(g) is 1

$$Fe_2O_3(s) + 3H_2(g) \rightleftharpoons 2Fe(s) + 3H_2O(g)$$

At equ 1 – 3x 3x

$$K_c = \frac{\left(\frac{3x}{V}\right)^3}{\left(\frac{1-3x}{V}\right)^3}$$

$$\Rightarrow 8 = \left(\frac{3x}{1-3x}\right)$$

$\Rightarrow$ x = 0.22

% of H_2 unreacted

$$= \frac{1-3\times 0.22}{1} \times 100 = 34\%$$

52. For the reaction (1) and (2)

$$A(g) \rightleftharpoons B(g) + C(g) \quad .....(1)$$

$$X(g) \rightleftharpoons 2Y(g) \quad .....(2)$$

Given, $K_{P_1} : K_{P_2} = 9 : 1$

If the degree of dissociation of A(g) and X(g) be same then the total pressure at equilibrium (1) and (2) are in the ratio

(A) 3 : 1
(B) 36 : 1
(C) 1 : 1
(D) 0.5 : 1

Ans. B

Sol.

$$A(g) \rightleftharpoons B(g) + C(g)$$

t = 0 a mole

t_{eq} (a - aα) (aα) (aα)

if $P_{Total} = P_1$

Then

$$K_{p_1} = \frac{\left(\frac{\alpha}{1+\alpha}P_1\right)\left(\frac{\alpha}{1+\alpha}P_1\right)}{\left(\frac{1-\alpha}{1+\alpha}.P_1\right)}$$

Also

$$X(g) \rightleftharpoons 2Y(g)$$

t = 0 b mole

t_{eq} (b - bα) (2bα)

if $P_{Total} = P_2$

Then

$$K_{p_2} = \frac{\left(\frac{2\alpha}{1+\alpha}.P_2\right)^2}{\left(\frac{1-\alpha}{1+\alpha}.P_2\right)^1}$$

$$\text{Given}: \frac{K_{P_1}}{K_{P_2}} = \frac{1}{9}$$

53. Le-Chatelier principle is NOT applicable to

(A) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ (B) $Fe(s) + S(s) \rightleftharpoons FeS(s)$

(C) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$ (D) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

Ans. B

Sol. Fact based.

54. Two solid compounds X and Y dissociates at a certain temperature as follows

$X(s) \rightleftharpoons A(g) + 2B(g); K_{P_1} = 9 \times 10^{-3}\ atm^3$

$Y(s) \rightleftharpoons 2B(g) + C(g); K_{P_2} = 4.5 \times 10^{-3}\ atm^3$

The total pressure of gases over a mixture of X and Y is

(A) 4.5 atm (B) 0.45 atm

(C) 0.6 atm (D) None of these

Ans. B

Sol. Let x is partial pressure of A and y is partial pressure of C when both equilibrium simultaneously established in a vessel.

$X(s) \rightleftharpoons \underset{x}{A(g)} + \underset{(2x+2y)}{2B(g)}$ $\quad K_{P_1} = P_A . P^2_{B(total)}$

$Y(s) \rightleftharpoons \underset{y}{C(g)} + \underset{(2y+2x)}{2B(g)}$ $\quad K_{P_2} = P_C . P^2_{B(total)}$

$\frac{K_{P1}}{K_{P2}} = \frac{x}{y} \Rightarrow x = 2y$

$K_{P1} = x(2x + 2y)^2$

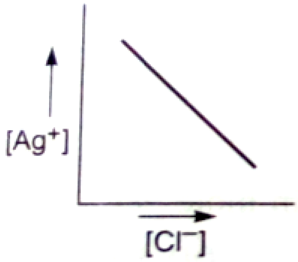
$\Rightarrow x = 0.1$ atm

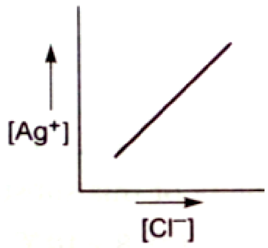
$\therefore y = 0.05$ atm

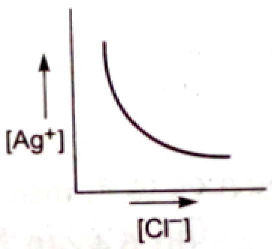
Total pressure of gases = $P_A + P_B + P_C$

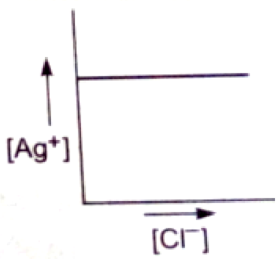
= 3(x + y) = 0.45 atm

55. In a saturated solution of AgCl, NaCl is added gradually. The concentration of Ag^+ is plotted against the concentration of Cl^-. The graph appears as:

(A)


(B)


(C)


(D)


Ans. C

Sol. For AgCl

$K_{sp} = [Ag^+][Cl^-]$

y x

$\Rightarrow$ xy = constant

i.e., Rectangular hyperbolic.

56. Which of the following is most soluble in water?

(A) $Ba_3(PO_4)_2$ ($K_{sp} = 6 \times 10^{-39}$)
(B) ZnS ($K_{sp} = 7 \times 10^{-19}$)
(C) $Fe(OH)_3$($K_{sp} = 6 \times 10^{-38}$)
(D) Ag_3PO_4($K_{sp} = 1.8 \times 10^{-18}$)

Ans. D

Sol. For most soluble salt, solubility should be maximum.

57. At 25°C, K_{sp} for $PbBr_2$ is equal to 8×10^{-5}. If the salt is 80% dissociated, what is the solubility of $PbBr_2$ in mol/litre?

(A) $\left[\frac{10^{-4}}{1.6\times1.6}\right]^{1/3}$
(B) $\left[\frac{10^{-5}}{1.6\times1.6}\right]^{1/3}$
(C) $\left[\frac{10^{-4}}{0.8\times0.8}\right]^{1/3}$
(D) $\left[\frac{10^{-5}}{1.6\times1.6}\right]^{1/2}$

Ans. A

Sol. $PbBr_2(s) \rightleftharpoons \underset{0.8\,S}{Pb^{2+}(aq)} + \underset{2\times0.8\times S}{2Br^-(aq)}$

$K_{sp} = [Pb^{2+}][Br^-]^2$

$\Rightarrow 8 \times 10^{-5} = (0.8\ S)\ (1.6\ S)^2$

$\Rightarrow S = \left[\frac{10^{-4}}{1.6\times1.6}\right]^{1/3}$

58. The simultaneous solubility of AgCN($K_{sp} = 2.5 \times 10^{-16}$) and AgCl($K_{sp} = 1.6 \times 10^{-10}$) in 1.0 M NH_3(aq) are respectively: $\left[\text{Given}: K_f\left[Ag(NH_3)_2^+\right] = 10^7\right]$

(A) 0.037, 5.78×10^{-8}
(B) 5.78×10^{-8}, 0.037
(C) 0.04, 6.25×10^{-8}
(D) 1.58×10^{-3}, 1.26×10^{-5}

Ans. A

Sol. $AgCl(s) + 2NH_3(aq) \rightleftharpoons Ag(NH_3)_2^+(aq) + Cl^-(aq)$

$K_1 = 1.6 \times 10^{-10} \times 10^7 = 1.6 \times 10^{-3}$

$= \frac{\left[Ag(NH_3)_2^+\right]\left[Cl^-\right]}{[NH_3]^2}$

$AgCN(s) + 2NH_3(aq) \rightleftharpoons Ag(NH_3)_2^+(aq) + CN^-(aq)$

$K_2 = 2.5 \times 10^{-16} \times 10^7 = 2.5 \times 10^{-9}$

$= \frac{\left[Ag(NH_3)_2^+\right]\left[CN^-\right]}{[NH_3]^2}$

$$\frac{[Cl^-]}{[CN^-]} = \frac{1.6\times10^{-3}}{2.5\times10^{-9}} = 6.4\times10^{5}$$

$$K_1 = \frac{x^2}{(1-2x)^2} \Rightarrow \frac{x}{1-2x} = 0.04$$

$$x = 0.037$$

$$[CN^-] = \frac{0.037}{6.4\times10^{5}} = 5.78\times10^{-8}$$

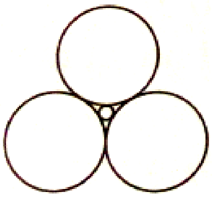
59. In the closest packing of atoms
(A) the size of tetrahedral void is greater than that of octahedral void.
(B) the size of tetrahedral void is smaller than that of octahedral void.
(C) the size of tetrahedral void is equal to that of octahedral void.
(D) the size of tetrahedral void may be greater or smaller or equal to that of octahedral void depending upon the size of atoms.

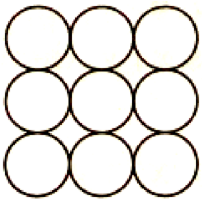
Ans. B

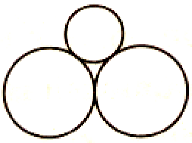
Sol. $\frac{r_c}{r_a}$ = 0.225 (for tetrahedral void)

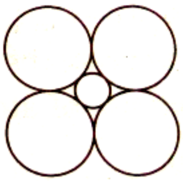
$\frac{r_c}{r_a}$ = 0.414 (for octahedral void)

60. Which of the following figures represents the cross-section of an octahedral site?

(A) 

(B) 

(C) 

(D) 

Ans. D

Sol. Fact based

PART – II

MATHEMATICS

61. A particle P starts from the point $z_0 = 1+2i$, where $i = \sqrt{-1}$. It moves first horizontally away from the origin by 5 units and then vertically away from the origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of $\hat{i}+\hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in the anticlockwise direction on a circle with centre at origin, to reach point z_2. The point z_2 is given by

(A) $6+7i$ (B) $-7+6i$
(C) $7+6i$ (D) $-6+7i$

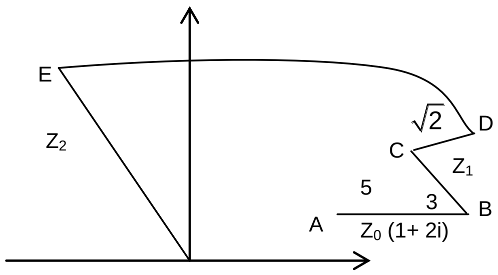
Ans. D

Sol. Affix of B is $(1+2i)+5 = 6+2i$

Affix of C is $(6+2i)+3i = 6+5i$

Affix of D is $(6+5i)+\sqrt{2}\left(\frac{1+i}{\sqrt{2}}\right) = 7+6i$

Affix of E is $(7+6i)e^{\pi i/2} = (7+6i)i = -6+7i$



62. If α, β, γ are such that $\alpha+\beta+\gamma = 2, \alpha^2+\beta^2+\gamma^2 = 6, \alpha^3+\beta^3+\gamma^3 = 8,$ then $\alpha^4+\beta^4+\gamma^4$ is

(A) 5 (B) 18
(C) 12 (D) 36

Ans. B

Sol. We have $(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2+2(\beta\gamma+\gamma\alpha+\alpha\beta)$

$\Rightarrow \quad 4 = 6+2(\beta\gamma+\gamma\alpha+\alpha\beta)$

$\Rightarrow \quad \beta\gamma+\gamma\alpha+\alpha\beta = -1$

Also, $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma$

$= (\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\beta\gamma-\gamma\alpha-\alpha\beta)$

$\Rightarrow \quad 8-3\,\alpha\beta\gamma = 2(6+1)$

$\Rightarrow \quad 3\alpha\beta\gamma = 8-14 = -6$ or $\alpha\beta\gamma = -2$

Now, $(\alpha^2+\beta^2+\gamma^2)^2 = \sum\alpha^4 + 2\sum\beta^2\gamma^2$

$\quad = \sum\alpha^2 + 2\left[\left(\sum\beta\gamma\right)^2 - 2\alpha\beta\gamma\left(\sum\alpha\right)\right]$

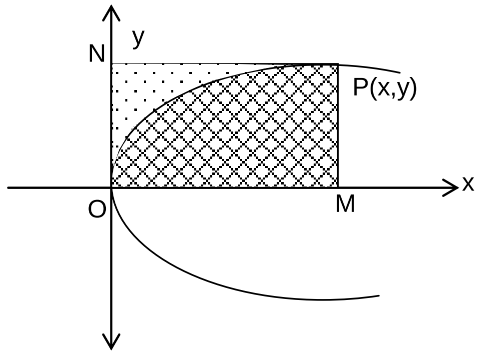
$\Rightarrow \quad \sum\alpha^4 = 36-2\left[(-1)^2-2(-2)(2)\right] = 18$

63. Through any point (x, y) of a curve which passes through the origin, lines are drawn parallel to the coordinate axes. The curve, given that it divides the rectangle formed by the two lines and the axes into two areas, one of which is twice the other, represents a family of

(A) circles (B) parabolas
(C) hyperbolas (D) straight lines

Ans. B

Sol. Let $P(x,y)$ be the point on the curve passing through the origin O (0, 0), and let PN and PM be the lines parallel to the x – and y – axes, respectively (Figure). If the equation of the curve is $y = y(x)$, the area POM equals $\int_0^x y\,dx$ and the area PON equals $xy - \int_0^x y\,dx$. Assuming that $2(POM) = PON$, we therefore have $2\int_0^x y\,dx = xy - \int_0^x y\,dx \Rightarrow 3\int_0^x y\,dx = xy$.



Differentiating both sides of this gives

$3y = x\dfrac{dy}{dx} + y \Rightarrow 2y = x\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{y} = 2\dfrac{dx}{x}$

$\Rightarrow \log|y| = 2\log|x| + C \qquad \Rightarrow y = Cx^2$, with C being a constant.

This solution represents a parabola. We will get a similar result if we had started instead with 2 (PON) = POM.

64. The orthogonal trajectories for $y = Cx^2, x \neq 0$ where C is arbitrary constant represent

(A) a family of circles
(B) a family of straight lines
(C) a family of ellipses
(D) a family of hyperbola

Ans. C

Sol. We have $f(x,y,c) = y - cx^2 = 0$

$\Rightarrow \dfrac{y}{x^2} = c$. Differentiating $\dfrac{x^2y' - 2xy}{x^4} = 0$

$\Rightarrow y' = \dfrac{2y}{x}$

Replacing $\dfrac{dy}{dx}$ by $-\dfrac{dx}{dy}$, we have $y' = \dfrac{-x}{2y}$

$\Rightarrow 2y\ dy + x\ dx =$ constant

$\Rightarrow y^2 + \dfrac{1}{2}x^2 =$ constant which represent a family of ellipses

65. Equation of the curve obtained by reflecting the parabola P; $y^2 = 4x$ in the tangent to the parabola P at the point (1, 2) is

(A) $x^2 = 4y$
(B) $(x-1)^2 = 4(y-2)$
(C) $(x+1)^2 = 4(y-1)$
(D) $(x-1)^2 = 4(y+1)$

Ans. C

Sol. Equation of tangent to the parabola at (1, 2) is $x - y + 1 = 0$

Any point P on the parabola is $(t^2, 2t)$ whose reflection Q (h, k) in the line $x - y + 1 = 0$ is given by $\dfrac{h - t^2}{1} = \dfrac{k - 2t}{-1} = \dfrac{-2(t^2 - 2t + 1)}{2}$

$\Rightarrow h = 2t - 1$ and $k = t^2 + 1$

Now eliminate t.

66. A monic quadratic trinomial P(x) is such that $P(x)=0$ and $P(P(P(x)))=0$ have a common root, then (monic polynomial has its leading coefficient equal to 1)

(A) $P(0).P(1)>0$ (B) $P(0).P(1)<0$

(C) $P(0).P(1)=0$ (D) none

Ans. C

Sol. Let $x=\alpha$ be root of both $P(x)=0$ as well as of $P(P(P(x)))=0$

$\Rightarrow P(\alpha)=0$

$\Rightarrow P(P(P(\alpha)))=0 \Rightarrow P(P(0))=0$

If $P(x)=x^2+ax+b$, then $P(0)=b \Rightarrow P(P(0))=P(b) \Rightarrow P(b)=0$

$\Rightarrow b^2+ab+b=0 \Rightarrow b(1+a+b)=0 \Rightarrow P(0)P(1)=O$

67. Let z_1, z_2, z_3 be three distinct complex numbers lying on a circle with centre at the origin such that $z_1+z_2z_3, z_2+z_3z_1$ and $z_3+z_1z_2$ are real numbers, then $z_1z_2z_3$ equals

(A) – 1 (B) 0

(C) 1 (D) none of these

Ans. C

Sol. As z_1, z_2, z_3 lie on a circle with centre at the origin, $|z_1|=|z_2|=|z_3|=r$ (say)

As $z_1+z_2z_3 \in R$. $z_1+z_2z_3=\bar{z}_1+\bar{z}_2\bar{z}_3$

$=\frac{r^2}{z_1}+\frac{r^4}{z_2z_3}$

$=\frac{r^2(z_2z_3+z^2z_1)}{z_1z_2z_3}$

$\Rightarrow \quad \frac{r^2}{z_1z_2z_3}=\frac{z_1+z_2z_3}{z_2z_3+r^2z_1} \quad (1)$

Similarly, $\frac{r^2}{z_1z_2z_3}=\frac{z_1+z_2+z_3}{z_3z_1+r^2z_3} \quad (2)$

and $\frac{r^2}{z_1z_2z_3}=\frac{z_3+z_1z_2}{z_1z_2+r^2z_3} \quad (3)$

From (1), (2) and (3), we get

$$\frac{r^2}{z_1z_2z_3}=\frac{z_1+z_2z_3}{z_2z_3+r^2z_1}=\frac{z_2+z_3z_1}{z_3z_1+z^2z_2}=\frac{z_3+z_1z_2}{z_1z_2+r^2z_3}$$

$$=\frac{z_1+z_3z_3-(z_2+z_3z_1)}{z_2z_3+r^2z_1-(z_3+z_1r^2z_2)}$$

$$=\frac{(z_1-z_2)(z_3-1)}{(z_1-z_2)(z_3+r^2)}=\frac{z_3-1}{z_3-r^2}$$

$$\therefore \quad \frac{r^2}{z_1z_2z_3}=\frac{z_3-1}{z_3-r^2}=\frac{z_2-1}{z_2-r^2}=\frac{z_1-1}{z_1-r^2}$$

$$=\frac{(z_2-1)-(z_1-1)}{(z_2-r^2)-(z_1-r^2)}$$

$$= \frac{z_2 - z_1}{z_2 - z_1} = 1$$

$\Rightarrow \quad z_3 - 1 = z_3 - r^2 \Rightarrow \quad r^2 = 1$

Hence, $z_1 z_2 z_3 = 1$

68. If coefficient of x^{20} in $\left(1 - x + x^2\right)^{20}$ and in $\left(1 + x - x^2\right)^{20}$ are respectively a and b, then

(A) $a = b$
(B) $a > b$
(C) $a < b$
(D) $a + b = 0$

Ans. B

Sol. Let $\left(1 - x + x^2\right)^{20} = a_0 + a_1 x + ... + a_{40} x^{40}$

$\Rightarrow \quad \left(1 + x + x^2\right)^{20} = a_0 - a_1 x + a_2 x^2 - ..... + a_{40} x^{40}$

Thus, coefficient of x^{20} in $\left(1 - x + x^2\right)^{20}$

= coefficient of x^{20} in $\left(1 + x + x^2\right)^{20}$

Therefore, we consider the expansion

$\left(1 + x + px^2\right)^{20} = b_0 + b_1 x + b_2 x^2 + ... + b_{40} x^{40}$

Dividing both the sides by x^{20}, we get

$b_{20} =$ coefficient of constant term in $\left(\frac{1}{x} + 1 + px\right)^{20}$

But $\left(\frac{1}{x} + 1 + px\right)^{20} = 1 + {}^{20}C_1\left(\frac{1}{x} + px\right) + {}^{20}C_2\left(\frac{1}{x} + px\right)^2 + ... + {}^{20}C_{20}\left(\frac{1}{x} + px\right)^{20}$

$\therefore \quad b_{20} = 1 + \left({}^{20}C_2\right)\left({}^{2}C_1\right)p + \left({}^{20}C_4\right)\left({}^{4}C_2\right)p^2 + ... + \left({}^{20}C_{20}\right)\left({}^{20}C_{10}\right)p^{10}$

Thus, $a = b_{20}\big]_{p=1}$ and $b = b_{20}\big]_{p=-1}$

Hence, $a > b$

69. If P is any point on the ellipse with foci, S_1 and S_2 and eccentricity $\frac{1}{2}$ such that $\angle PS_1S_2 = \alpha, \angle PS_2S_1 = \beta$ and $\angle S_1PS_2 = \gamma$, then $\cot\frac{\alpha}{2}, \cot\frac{\gamma}{2}, \cot\frac{\beta}{2}$ are in

(A) A.P.
(B) G.P.
(C) H.P.
(D) none of these

Ans. A

Sol. $e = \frac{1}{2}$

we know for a triangle $\cot\left(\frac{\alpha}{2}\right) + \cot\left(\frac{\beta}{2}\right) + \cot\left(\frac{\alpha}{2}\right) = \cot\left(\frac{\alpha}{2}\right).\cot\left(\frac{\beta}{2}\right).\cot\left(\frac{\gamma}{2}\right)$ _____(1)

$$\cot\left(\frac{\alpha}{2}\right) = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$$

$$\cot\left(\frac{\beta}{2}\right) = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$$

$$\Rightarrow \cot\left(\frac{\alpha}{2}\right).\cot\left(\frac{\beta}{2}\right) = \frac{s}{s-a} = \frac{PS_1 + S_1S_2 + PS_2}{PS_1 + PS_2 - S_1S_2}$$

$\Rightarrow \cot\left(\frac{\alpha}{2}\right).\cot\left(\frac{\beta}{2}\right) = 3$ put in equation (1)

$PS_1 + PS_2 = 2AA'$ and $S_1S_2 = 2AA'e$

70. Sum of the series $S = \sum_{r=0}^{n} \frac{3^{r+4}\left({}^{n}C_r\right)}{{}^{r+4}C_4} + \sum_{r=0}^{3} \frac{{}^{n+4}C_r 3^r}{{}^{n+4}C_4}$ is

(A) 4^{n+4} (B) $4^{n+4}\left({}^{2n}C_4\right)$

(C) $\frac{4^{n+4}}{{}^{n+4}C_4}$ (D) $\frac{3^{n+4} + 2^{n+4}}{{}^{n+4}C_4}$

Ans. C

Sol. $\frac{{}^{n}C_r}{{}^{r+4}C_4} = \frac{n!}{r!(n-r)!}\frac{4!r!}{(r+4)!}$

$= \frac{4!}{(n+1)(n+2)(n+3)(n+4)}\left({}^{n+4}C_{r+4}\right)$

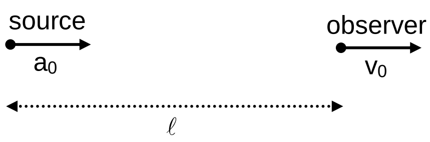
$= \frac{{}^{n+4}C_{r+4}}{{}^{n+4}C_4}$

Thus, $S = \frac{1}{{}^{n+4}C_4}\left[\sum_{r=0}^{n} 3^{r+4}\left({}^{n+4}C_{r+4}\right) + \sum_{r=0}^{3} {}^{n+4}C_r\, 3^r\right]$

$= \frac{1}{{}^{n+4}C_4}\sum_{k=0}^{n+4} {}^{n+4}C_k\, 3^k = \frac{4^{n+4}}{{}^{n+4}C_4}$

PHYSICS

71. At t = 0, source starts accelerating with an acceleration a and observer starts moving with constant velocity v_0 as shown in the figure simultaneously. Source emits a frequency f_0 and velocity of sound in the air is v. The frequency detected by the observer initially is

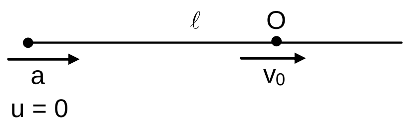


(A) $\frac{(v - v_o)f^2}{(2vf - a)}$ (B) $\frac{2(v - v_o)f^2}{(2vf - a)}$

(C) $\frac{(v - v_o)f^2}{2(2vf - a)}$ (D) $\frac{2(v - v_o)f^2}{(vf - a)}$

Ans. B

Sol. Let first pulse be released at t = 0.



Time when first pulse reaches O = $t_1 = \frac{\ell}{v - v_o}$

Time when second pulse reaches O =

$t_2 = T + \frac{\ell + v_oT - \frac{1}{2}aT^2}{v - v_o}$

$T' = t_2 - t_1 = \frac{vT}{v - v_o} - \frac{aT^2}{2(v - v_o)}$

$\therefore\ f' = \frac{2f^2(v - v_o)}{2fv - a}$

72. A planet of mass m moves along an ellipse around the sun so that its maximum and minimum distances from the sun are equal to R and r respectively. The angular momentum of this planet relative to the centre of the sun is

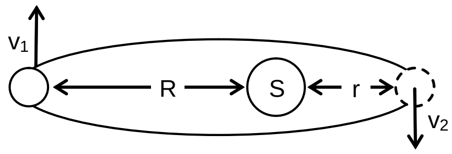
(A) $2\sqrt{\frac{GMmrR}{r+R}}$ (B) $2R\sqrt{\frac{GMmR}{r+R}}$

(C) $2m\sqrt{\frac{GMrR}{r+R}}$ (D) $m\sqrt{\frac{2GMrR}{r+R}}$

Ans. D

Sol. According to Kepler's Second Law the angular momentum of the planet is constant, we have $mv_1R = mv_2r$, $v_1R = v_2r$

If the mass of the Sun is M conserving total mechanical energy of the system at two given positions we have,



$$-\frac{GMm}{R} + \frac{1}{2}mv_1^2 = -\frac{GMm}{r} + \frac{1}{2}mv_2^2$$

$$\therefore \quad -GM\left[\frac{1}{R} - \frac{1}{r}\right] = \frac{v_2^2}{2} - \frac{v_1^2}{2}$$

Or $-GM\left[\frac{r-R}{Rr}\right] = \frac{v_1^2R^2}{2r^2} - \frac{v_1^2}{2}$

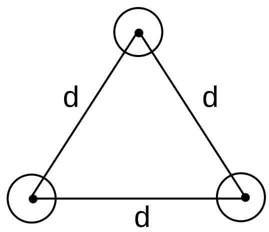
Or $-GM\left[\frac{r-R}{Rr}\right] = \frac{v_1^2}{2}\left(\frac{R^2}{r^2} - 1\right) = \frac{v_1^2}{2}\left(\frac{R^2-r^2}{r^2}\right)$

$$\therefore \quad v_1^2 = \frac{2GM(R-r)r^2}{Rr(R^2-r^2)} = \frac{2GMr}{R(R+r)}$$

Or $\sqrt{\frac{2GMr}{R(R+r)}}$

Now angular momentum = $mv_1R = m\sqrt{\frac{2GMrR}{(R+r)}}$.

73. Three solid spheres each of mass m and radius R are released from the position shown in figure. The speed of any one sphere at the time of collision would be

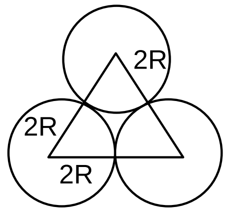


(A) $\sqrt{Gm\left(\frac{1}{d}-\frac{3}{R}\right)}$ (B) $\sqrt{Gm\left(\frac{3}{d}-\frac{1}{R}\right)}$

(C) $\sqrt{Gm\left(\frac{2}{R}-\frac{1}{d}\right)}$ (D) $\sqrt{Gm\left(\frac{1}{R}-\frac{2}{d}\right)}$

Ans. D

Sol. From conservation of mechanical energy



$$3\left\{\frac{1}{2}mv^2\right\} = 3\left\{\frac{Gm^2}{2R} - \frac{Gm^2}{d}\right\}$$

$$v^2 = Gm\left\{\frac{1}{R} - \frac{2}{d}\right\}$$

$$\therefore \quad v = \sqrt{Gm\left\{\frac{1}{R} - \frac{2}{d}\right\}}$$

74. In a room where temperature is 30ºC a body cools from 61ºC to 59ºC is 4 minutes. The time taken by the body to cool from 51ºC to 49ºC will be:

(A) 4 minutes
(B) 6 minutes
(C) 5 minutes
(D) 8 minutes

Ans. B

Sol. Rate of cooling α difference in temperature

$-\frac{dT}{dt} \alpha \Delta\theta$

$\frac{dT}{dt} = -K\Delta\theta$

$dT = -K\Delta\theta . dt$

In First Case

$dT = 61 - 59 = 2$

$\Delta\theta = 60 - 30 = 30$

$dt = 4$ minutes

$\therefore K = -\frac{dT}{\Delta\theta dt} = -\frac{2}{30 \times 4} = -\frac{1}{60}$

For second case

$dT = 2$

$\Delta\theta = 50 - 30 = 20$

$\therefore dt = \frac{dT}{K\Delta\theta} = \frac{2}{\frac{1}{60} \times 20} = 6$ min.

75. Two rods of length l_1 and l_2 are made of materials whose coefficients of linear expansion are α_1 and α_2. If the difference between two lengths is independent of temperature then,

(A) $\frac{l_1}{l_2} = \frac{\alpha_1}{\alpha_2}$
(B) $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$
(C) $l_2^2 \alpha_1 = l_1^2 \alpha_2$
(D) $\frac{\alpha_1^2}{l_1} = \frac{\alpha_2^2}{l_2}$

Ans. B

Sol. If change in length is Δl

Then $\Delta l_1 = L_{01} \alpha_1 \Delta T$

$\Delta l_2 = L_{02} \alpha_2 \Delta T$

difference in length is $l_2 - l_1 = (L_{02} + \Delta l_2) - (L_{01} + \Delta l_1)$

$= (L_{02} - L_{01}) + (\Delta l_2 - \Delta l_1)$

$L_{02} - L_{01}$ is independent of temperature for $l_2 - l_1$ to be independent of temperature.

$\Delta l_2 - \Delta l_1$ must be equal to zero

i.e. $L_1 \alpha_1 \Delta T = L_2 \alpha_2 \Delta T$

i.e. $L_1 \alpha_1 = L_2 \alpha_2$

i.e. $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$.

76. A particle executes SHM between x = -A and x = +A. The time taken for it to go from 0 to A/2 is T_1 and to go from A/2 to A is T_2. Then

(A) $T_1 < T_2$
(B) $T_1 > T_2$
(C) $T_1 = T_2$
(D) $T_1 = 2T_2$

Ans. A

Sol. Let $x = A \sin \omega t$

From 0 – A/2

$\therefore A/2 = A \sin \omega T_1$

or $\sin \omega T_1 = \frac{1}{2} = \sin \frac{\pi}{6}$

$\therefore \frac{2\pi}{T}.T_1 = \frac{\pi}{6}$ or $T_1 = \frac{T}{12}$

From 0 – A

$A = A \sin \omega T'_1$

or, $\sin \omega T'_1 = 1 = \sin (\pi / 2)$

or, $\frac{2\pi}{T}.T'_1 = \frac{\pi}{2}$ or $T'_1 = \frac{T}{4}$

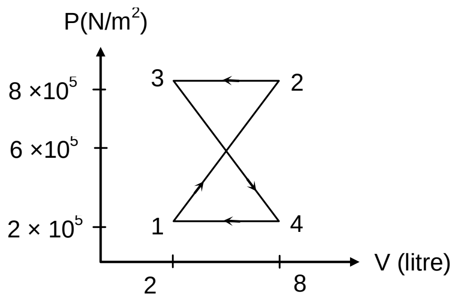
$\therefore$ From A/2 – A :

$T_2 = T'_1 - T_1 = \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$

$\because T_1 = \frac{T}{12}$ and $T_2 = \frac{T}{6}$

$\therefore T_1 < T_2$

77. The work done in the process 1 – 2– 3–4 shown on P-V diagram is

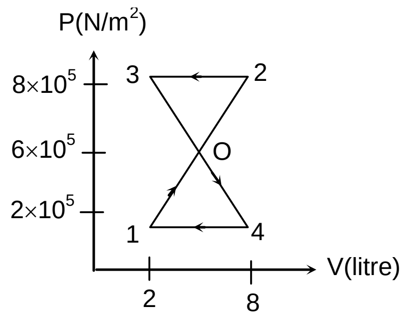


(A) 300 J
(B) 600 J
(C) 900 J
(D) 1200J

Ans. C

Sol. $V_4 - V_1 = 6\text{litre}$

From geometry $V_2 - V_3 = 3\text{litre}$



$W_{104} = \frac{1}{2} \times 4 \times 10^5 \times 6 \times 10^{-3}$

$= 1200\text{J}$

$W_{230} = -\frac{1}{2} \times 2 \times 10^5 \times 3 \times 10^{-3}$

$= -300\text{J}$

$W = 900$ J

78. The speed of sound through oxygen at T K is (300 m/s). When the temperature is increased to 3T, the molecule dissociates into oxygen atom, now the speed of sound will be

(A) 520 m/s (B) 801 m/s
(C) 600 m/s (D) 580 m/s

Ans. B

Sol. $v_T = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{7}{5}RT \Big/ 32 \times 10^{-3}} = 300$ m/s

$v_{3T} = \sqrt{\frac{5}{3}R3T \Big/ 16 \times 10^{-3}}$

$\frac{v_{3T}}{v_T} = \sqrt{\frac{5}{3}\frac{R3T}{16 \times 10^{-3}} \times \frac{32 \times 10^{-3} \times 5}{7RT}} = \sqrt{\frac{50}{7}} = 2.67$

$v_{3T} = 300 \times 2.67 = 801$ m/s

79. A bullet of mass of 10 gm moving with a speed of 400 m/s hits an ice block of mass 990 gm kept on a frictionless floor and gets stuck in it. The mass of ice melted, if 50% of the lost Kinetic energy goes to ice will be (Temperature of ice block = 0°C).

(A) 1.2 gm
(B) 2.4 gm
(C) 0.6 gm
(D) 3 gm

Ans. A

Sol. Velocity of bullet + ice block, V = $\frac{(10)x(400)}{1000}$ m/s

V =4 m/s

Loss of K.E. = $\frac{1}{2}mv^2 - \frac{1}{2}(m+M)V^2$

= $\frac{1}{2}[0.01\times(400)^2 - 1\times(4)^2]$ = $\frac{1}{2}[1600 - 16]$= (1584/2) J = 792 J

$\therefore$ Heat generated = $\frac{1}{2}(1584/2)\times 4.2$ = 95 Cal

$\therefore$ Mass of ice melted = $\frac{95 \ \ cal}{80 \ \ Cal/gm}$ = 1.2 gm.

80. A cubical block of wood of specific gravity 0.5 and a chunk of concrete of specific gravity 2.5 are fastened together. The ratio of the mass of wood to the mass of concrete which makes the combination to float with its entire volume submerged under water is

(A) $\frac{1}{5}$
(B) $\frac{1}{3}$
(C) $\frac{3}{5}$
(D) $\frac{2}{5}$

Ans. C

Sol. let mass of wood = m_1
And mass of concrete = m_2

$\therefore \left(\frac{m_1}{0.5\rho} + \frac{m_2}{2.5\rho}\right).\rho.g = (m_1 + m_2)g$

$\Rightarrow m_1 = \frac{1.5}{2.5}m_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{3}{5}$

CHEMISTRY

81. How many effective Na^+ and Cl^- ions are present respectively in a unit cell of NaCl solid (Rock salt structure) if all ions along line connecting opposite face centres are absent?

(A) 3, 3
(B) $\frac{7}{2}, 4$
(C) $\frac{7}{2}, \frac{7}{2}$
(D) $4, \frac{7}{2}$

Ans. A

Sol. No. of Na^+ = 4 – 1 = 3

No. of Cl^- = $4 - \left(2 \times \frac{1}{2}\right) = 3$

82. The pH of which of the following salt solution does depend on concentration of solution?

(A) CH_3COONH_4 (B) NaCl

(C) NH_4CN (D) $FeCl_3$

Ans. D

Sol. CH_3COONH_4 and NH_4CN are salts of weak acids and weak bases so their pH only depends on their p^{K_a} and p^{K_b} values. NaCl is a neutral salts of strong acid and strong base. It's pH = 7 at room temperature. Fe^{3+} undergoes hydrolysis. Hence its pH depends on concentration.

83. Which of the following compound(s) form(s) primary amine(s) when treated with Br_2 and KOH?

(A) $CH_3\overset{O}{\overset{||}{C}}NH_2$

(B) $CH_3 - \overset{O}{\overset{||}{C}} - NHCH_3$

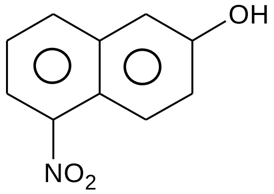
(C) $CH_3 - \overset{O}{\overset{||}{C}} - NHC_2H_5$

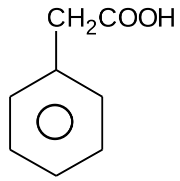
(D) $CH_3NHCH_2CH_3$

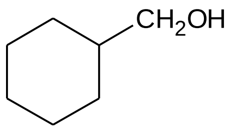
Ans. A

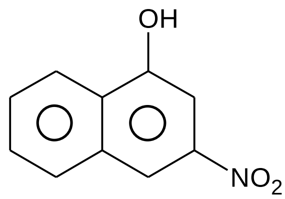
Sol. Unsubstituted amides responds to this reaction(Hoffmann's bromamide reaction)

84. Which of the following substance(s) do NOT react with NaOH to form sodium salts?

(A) 

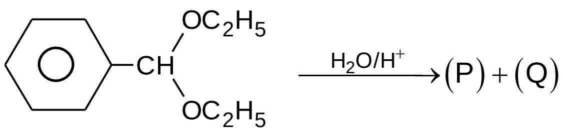
(B) 

(C) 

(D) 

Ans. C

Sol. Phenols and acids react with NaOH whereas alcohols do not react with it.

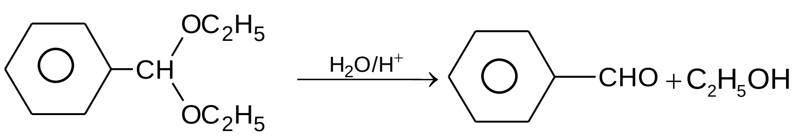
85. 

In the above reaction, the products (P) and (Q) can be distinguished by using

(A) NaOH (B) Tollen's reagent

(C) Fehling's solution (D) $FeCl_3$

Ans. B

Sol. 

Benzaldehdye does not react with Fehling's solution. So, Tollen's reagent is used.

86. Which of the following reagent converts CH_3CH_2COOH to a chiral compound?

(A) $SOCl_2$ (B) PCl_5

(C) Cl_2/red P (D) PCl_3

Ans. C

Sol. $$(\text{achiral})\,CH_3CH_2COOH \xrightarrow{Cl_2/P} CH_3\underset{\underset{Cl}{|}}{C}HCOOH\,(\text{Chiral})$$

87. Which of the following can absorb the highest number of CH_3I molecules? (in presence of moist Ag_2O)

(A) $NH_2-CH_2-\underset{\underset{NH_2}{|}}{C}H-CH_2-CH_3$ (B) $CH_3-CH_2-NH-CH_2-CH_2-NH_2$

(C) $CH_3-NH-CH_2-\underset{\underset{CH_3}{|}}{N}-CH_2-NH_2$ (D) $CH_3-NH-CH_2-NH-CH_2-CH_2\,NH_2$

Ans. D

Sol. No. of CH_3I molecules can be absorbed by

1^o – amine $\rightarrow$ 3, 2^o – amine $\rightarrow$ 2, 3^o – amine $\rightarrow$ 1

88. $$\begin{array}{l}CHO\\ |\\ (CHOH)_4 \xrightarrow{HI/red\,P} \text{Products}\\ |\\ CH_2OH\end{array}$$

The product of above reaction confirms that

(A) glucose contains an aldehyde group

(B) glucose contains five OH-groups

(C) glucose contains a straight carbon chain

(D) glucose contains four secondary OH groups

Ans. C

Sol. $$\begin{array}{l}CHO\\ |\\ (CHOH)_4 \xrightarrow{HI/red\,P} 2-\text{iodohexane}+n-\text{hexane}\\ |\\ CH_2OH\end{array}$$

89. $$(p^{K_a}=4)\ HOOC-CH_2-\underset{\underset{H}{|}}{C}H-\underset{\underset{NH_3^+\ (p^{K_a}=8)}{|}}{C}H-CH_2COOH\,(p^{K_a}=2)$$

What is the isoelectric point of the above amino acid?

(A) 4 (B) 6

(C) 3 (D) 8

Ans. C

Sol. $$I.P=\frac{p^{K_{a_1}}+p^{K_{a_2}}}{2}=\frac{4+2}{2}=3$$

90. Which of the following on heating does NOT form cyclic ester?

(A) $HO-\overset{\overset{O}{||}}{C}-(CH_2)_4-OH$

(B) $HO-\overset{\overset{O}{||}}{C}-(CH_2)_3-OH$

(C) $HO-\overset{\overset{O}{||}}{C}-CH_2CH_2OH$

(D) $HO-\overset{\overset{O}{||}}{C}-CH_2OH$

Ans. C

Sol. β-hydroxy carboxylic acid gives H_2O on heating.