KVPY – CLASS-XI
PART TEST – 2
(OLTS-1819-T2-PT-2-KVPY-XI)

PART – I

MATHEMATICS

1. If $S_n = \sum_{r=1}^{n} t_r = \frac{1}{6}n\left(2n^2 + 9n + 13\right)$, then $\sum_{r=1}^{n} \sqrt{t_r}$ equals

(A) $\frac{1}{2}n(n+1)$ (B) $\frac{1}{2}n(n+2)$

(C) $\frac{1}{2}n(n+3)$ (D) $\frac{1}{2}n(n+5)$

Ans. C

Sol. We have $t_n = S_n - S_{n-1}\ \forall\, n \geq 2$

$\therefore\ t_n = \frac{1}{6}\left[2\left(n^3 - (n-1)^3\right) + 9\left(n^2 - (n-1)^2\right) + 13(n - n + 1)\right]$

$= \frac{1}{6}\left[6n^2 - 6n + 2 + 9(2n-1) + 13\right]$

$= \frac{1}{6}\left(6n^2 + 12n + 6\right) = (n+1)^2$

Also, $t_1 = S_1 = 4 = (1+1)^2$

$\therefore \sum_{r=1}^{n} \sqrt{t_r} = \sum_{r=1}^{n} (r+1) = \frac{1}{2}(n+1)(n+2) - 1$

$= \frac{1}{2}n(n+3)$.

2. A 3 digit number 4a3 is added to another 3 – digit number 984 to give a 4 – digit number 13b7, which is divisible by 11. Then (a + b) =

(A) 10 (B) 11

(C) 12 (D) 15

Ans. A

Sol.
$$\left.\begin{matrix} 4 & a & 3 \\ 9 & 8 & 4 \\ 13 & b & 7 \end{matrix}\right\} \Rightarrow a + 8 = b \Rightarrow b - a = 8$$

Also, 13 b7 is divisible by $11 \Rightarrow (7+3) - (b+1) = (9-b) \Rightarrow (9-b) = 0 \Rightarrow b = 9$

$\therefore (b = 9 \text{ and } a = 1) \Rightarrow (a + b) = 10$.

3. What is the value of digit A, if 1A927B1 is divisible by 11, also difference between A and B is 1?

(A) 0 (B) 1

(C) 2 (D) 2 or 3

Ans. D

Sol. Given that 1A927B1 is divisible by 11
$\Rightarrow$ [Sum of digits at odd places] – [Sum of digits at even places]
Must be divisible by 11.
$\therefore\left[(1+7+9+1)-(A+B+2)\right]$ is divisible by 11.
$\Rightarrow(16-A-B)$ can be 0, 11, 22,.....
Also, $A+B\le 18$
$\therefore|16-A-B|=0$ or $|16-A-B|=11$
Given that $|A-B|=1$ ……..(X)
If $|16-A-B|=0,$ then A and B become non – integers
$\therefore|16-A-B|=11$ ……(Y)
Solving (X) and (Y) simultaneously,
A = 2 or 3

4. Find the sum of all the factors of 720
(A) 2340 (B) 78
(C) 2440 (D) 2418

Ans. D

Sol. Given that $N=720=2^4\times3^2\times5^1$
Sum of all the factors
$=\left(2^0+2^1+2^2+2^3+2^4\right)\left(3^0+3^1+3^2\right)\left(5^0+5^1\right)$
$=\frac{2^5-1}{2-1}\times\frac{3^3-1}{3-1}\times\frac{5^2-1}{5-1}$
$=31\times13\times6=2418$

5. The sum of the series $\frac{7}{1^3.2^3}+\frac{19}{2^3.3^3}+\frac{37}{3^3.4^3}+.....\text{upto }\infty$ is
(A) 3 (B) 2
(C) 1 (D) 1.5

Ans. C

Sol. Use $t_r=\frac{1}{r^3}-\frac{1}{(r+1)^3}$

6. Sum of the series $\frac{2}{5}+\frac{3}{5^2}+\frac{4}{5^3}+\frac{2}{5^4}+\frac{3}{5^5}+\frac{4}{5^6}+....$ is
(A) $\frac{69}{124}$ (B) $\frac{71}{124}$
(C) $\frac{35}{124}$ (D) $\frac{75}{124}$

Ans. A

Sol. $\frac{\frac{2}{5}}{1-\frac{1}{5^3}}+\frac{\frac{3}{5^2}}{1-\frac{1}{5^3}}+\frac{\frac{4}{5^3}}{1-\frac{1}{5^3}}=\frac{69}{124}$

7. Let x and y be two digit numbers such that y is obtained by reversing the digits of x. Suppose they also satisfy $x^2 - y^2 = m^2$ for some positive integer m. The value of $x + y + m$ is

(A) 88 (B) 112
(C) 144 (D) 154

Ans. D

Sol. Let $x = 10a + b$ $y = 10b + a$

$x^2 - y^2 = m^2$

$\therefore (10a + b)^2 - (10b + a)^2 = m^2$

$99(a + b)(a - b) = m^2$

$9 \times 11 \times (a - b)(a + b) = m^2$

If $(a - b).(a + b) = 11$ L.H.S. is perfect Square i.e. $a = 6, b = 5$

$\therefore m = \sqrt{99 \times 11} = 33$

$\therefore x + y + m = 10a + b + 10b + a + 33$

$= 11a + 11b + 33$

$= 121 + 33 = 154$

8. The house on one side of a road are numbered using consecutive even numbers. The sum of the numbers of all the houses in that row is 170. If there are at least 6 houses in that row and a is the number of the sixth house, then

(A) $2 \le a \le 6$ (B) $8 \le a \le 12$
(C) $14 \le a \le 20$ (D) $22 \le a \le 30$

Ans. C

Sol. House number starting from $2m + 2$ to $2n$

Then, $2\frac{(n)(n+1)}{2} - 2\frac{(m)(m+1)}{2} = 170$

$(n - m)(n + m + 1) = 170$

$\Rightarrow n - m = 10$

$n + m + 1 = 17$

$\Rightarrow n = 13, m = 3$

House number starting from 8, 26

$\Rightarrow 14 \le a \le 20$

9. The value of $\sum_{n=0}^{1947} \frac{1}{2^n + \sqrt{2^{1947}}}$ is equal to

(A) $\frac{487}{\sqrt{2^{1945}}}$ (B) $\frac{1946}{\sqrt{2^{1947}}}$
(C) $\frac{1947}{\sqrt{2^{1947}}}$ (D) $\frac{1948}{\sqrt{2^{1947}}}$

Ans. A

Sol. $\sum_{n=0}^{1947} \frac{1}{2^n + 2^{1947/2}}$

Pair terms equidistant from both ends

$= \left(\frac{1}{2^0 + 2^{1947/2}} + \frac{1}{2^{1947} + 2^{1947/2}} \right) + \left(\frac{1}{2^1 + 2^{1947/2}} + \frac{1}{2^{1946} + 2^{1947/2}} \right)$

$$+\ldots\ldots\ldots\ldots\ldots\left(\frac{1}{2^{974}+2^{1947/2}}+\frac{1}{2^{973}+2^{1947/2}}\right)$$

On solving each bracket becomes

$$=\underbrace{\frac{1}{2^{1947/2}}+\frac{1}{2^{1947/2}}+\ldots\ldots\ldots\ldots+\frac{1}{2^{1947/2}}}_{974\text{ terms}}$$

$$=\frac{974}{2^{1947/2}}=\frac{487}{\sqrt{2^{1945}}}$$

10. A train running at 54 kmph takes 20 seconds to pass a platform. Next it taken 12 seconds to pass a man walking at 6 kmph in the same direction in which the train is going. Find the length of the train and the length of the platform.

(A) 120 (B) 130

(C) 140 (D) 150

Ans. C

Sol. Let the length of train be x metres and length of platform be y metres.

Speed of the train relative to man $=(54-6)$ kmph = 48 kmph

$$=\left(48\times\frac{5}{18}\right)\text{m/sec}=\frac{40}{3}\text{m/sec}$$

In passing a man, the train covers its own length with relative speed.

$\therefore$ Length of train = (Relative speed $\times$ Time) = $\left(\frac{40}{3}\times12\right)$ m = 160 m

Also, speed of the train $=\left(54\times\frac{5}{18}\right)\text{m/sec}=15\,\text{m/sec}$

$$\therefore\ \frac{x+y}{15}=20\Leftrightarrow x+y=300\Leftrightarrow y=(300-160)\text{m}=140\text{ m}$$

11. The value of $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots.+\sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$ is equal to

(A) $2008-\frac{1}{2008}$ (B) $2009-\frac{1}{2008}$

(C) $2008-\frac{1}{2009}$ (D) $2009-\frac{1}{2009}$

Ans. A

Sol. $t_n=1+\frac{1}{n}-\frac{1}{n+1}\Rightarrow\sum_{n=1}^{2007}t_n=2007+1-\frac{1}{2008}=2008-\frac{1}{2008}$

12. A watch which gains uniformly is 2 minutes low at noon on Monday and is 4 min. 48 sec fast at 2 p.m. on the following Monday. When was it correct?

(A) 2 p.m. on Tuesday (B) 2 p.m. on Wednesday

(C) 3 p.m. on Thursday (D) 1 p.m. on Friday

Ans. B

Sol. Time from 12 pm on Monday to 2 pm on the following Monday = 7 days 2 hours = 170 hours

$\therefore$ The watch gains $\left(2+4\frac{4}{5}\right)$ min or $\frac{34}{5}$ min in 170 hrs.

Now, $\frac{34}{5}$ min. are gained in 170 hrs.

$\therefore$ 2 min are gained in $\left(170\times\frac{5}{34}\times 2\right)\text{hrs}=50$ hrs.

$\therefore$ Watch is correct 2 days 2 hrs. after 12 p.m. on Monday i.e. it will be correct at 2 pm on Wednesday

13. If x,y and z are positive real numbers, then the minimum value of the expression $x^{\ln\frac{y}{z}}+y^{\ln\frac{z}{x}}+z^{\ln\frac{x}{y}}$ is equal to

(A) 3 (B) 1
(C) 2 (D) none of these

Ans. A

Sol. let $A=x^{\ln\frac{y}{z}}+y^{\ln\frac{z}{x}}+z^{\ln\frac{x}{y}}$

And $A_1=x^{\ln\frac{y}{z}},A_2=y^{\ln\frac{z}{x}}$ and $A_3=z^{\ln\frac{x}{y}}$

Also, $\ln A_1+\ln A_2+\ln A_3=(\ln y-\ln z)\ln x+(\ln z-\ln x)\ln y+(\ln x-\ln y)\ln z=0$

Or $A_1A_2A_3=e^0=1$

Using $A.M\geq G.M$

$\frac{A_1+A_2+A_3}{3}\geq(A_1A_2A_3)^{1/3}=1$

Or $A_1+A_2+A_3=A\geq 3$

So that minimum value of A is 3

14. The average speed of a train in the onward journey is 25% more than that in the return journey. The train halts for one hour on reaching the destination. The total time taken for the complete to and fro journey is 17 hours, covering a distance of 800 km. The speed of the train in the onward journey is:

(A) 45 km/hr (B) 47.5 km/hr
(C) 52 km/hr (D) 56.25 km/hr

Ans. D

Sol. Let the speed in return journey be x km/hr

Then, speed in onward journey $\frac{125}{100}x=\left(\frac{5}{4}x\right)$km/hr.

Average speed $=\left(\frac{2\times\frac{5}{4}x\times x}{\frac{5}{4}x+x}\right)\text{km/hr}=\frac{10x}{9}\text{km/hr.}$

$\therefore\left(800\times\frac{9}{10x}\right)=16\Leftrightarrow x=\left(\frac{800\times 9}{16\times 10}\right)=45$

So, speed in onward journey $=\left(\frac{5}{4}\times 45\right)\text{km/hr}=56.25\text{ km/hr}$

15. a,b,c are positive integers forming an increasing G.P. and b-a is a perfect cube and $\log_6 a+\log_6 b+\log_6 c=6$, then a+b+c=

(A) 100 (B) 111
(C) 122 (D) 189

Ans. D

Sol. $\log_6(abc) = 6$

$\Rightarrow (abc) = 6^6$

Let $a = \frac{b}{r}$ and c = br

$\Rightarrow$ b = 36 and a= $\frac{36}{r} \Rightarrow r = 2,3,4,6,9,12,18$

Also, $36\left(1-\frac{1}{r}\right)$ is a perfect cube

$\Rightarrow 36\left(1-\frac{1}{r}\right)$

$\Rightarrow$ a + b + c = 36 + 9 + 144 = 189

PHYSICS

16. If a wheel is rolling without slipping on a horizontal surface at a constant linear speed then the point of contact has
(A) backward acceleration (B) zero acceleration
(C) an upward acceleration (D) a downward acceleration

Ans. C

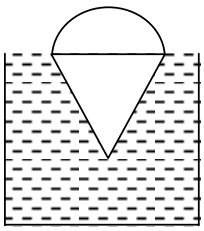
Sol. $v = v_c - \omega R = 0$

In the frame of centre of mass, which is moving with constant velocity V_C.

Centripetal acceleration = $\frac{V_C^2}{R}$ upward.

$\therefore$ Required acceleration = $\frac{V_C^2}{R}$ upward.

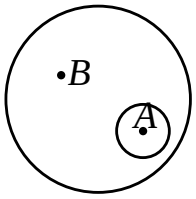
17. A solid toy of uniform density floats in water in equilibrium such that its hemispherical portion is completely outside the water and the conical portion is completely sub merged. The toy must be in
(A) neutral equilibrium
(B) stable equilibrium
(C) unstable equilibrium
(D) information is insufficient of equilibrium



Ans. C

Sol. System will be in unstable equilibrium.

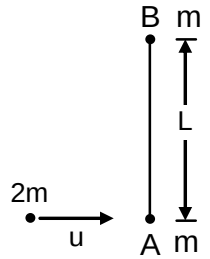
18. There is an air bubble of radius R inside a drop of water of radius 3R. Find the ratio of gauge pressure at point B to that at point A.
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$
(C) $\frac{1}{3}$ (D) 1



Ans. B

Sol. $P_B - P_0 = \frac{2T}{3R}$, $P_A = P_B + \frac{2T}{R}$, $P_A - P_0 = \frac{2T}{3R} + \frac{2T}{R} = 2T\left[\frac{4}{3R}\right]$, $\frac{P_B - P_0}{P_A - P_0} = \frac{1}{4}$

19. Two small balls A and B, each of mass m, are joined rigidly at the ends of a light rod of length L. They are placed on a frictionless horizontal surface. Another ball of mass 2 m moving with speed u towards one of the ball and perpendicular to the length of the rod on the horizontal frictionless surface as shown in the figure. If the coefficient of restitution is 1/2 then the angular speed of the rod after the collision will be



(A) $\frac{4}{3}\frac{u}{L}$ (B) $\frac{u}{L}$

(C) $\frac{2}{3}\frac{u}{L}$ (D) None of these

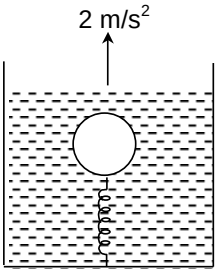
Ans. C

Sol. Velocity of ball 'A' just after the collision $V=\left(1+\frac{1}{2}\right)\cdot\frac{2}{3}u=u$

Velocity of ball 'B' just after the collision = 0.

$\therefore \quad \omega=\frac{u-0}{L}=\frac{u}{L}$

20. A ball of mass 10 kg and density 1 gm/cm^3 is attached to the base of a container having a liquid of density 1.1 gm/cm^3, with the help of a spring as shown in the figure. The container is going up with an acceleration 2 m/s^2. If the spring constant of the spring is 200 N/m, the elongation in the spring is



(A) 2 cm (B) 4 cm

(C) 6 cm (D) 8 cm

Ans. C

Sol. Writing the equation of motion

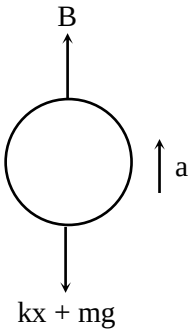
$B-kx-mg=ma$

$kx=B-m(g+a)$

$kx=\left(\frac{10}{10^3}\right)\times\left(1.1\times10^3\right)\times(10+2)-10\times(10+2)$

$=120\times0.1$

$x=\frac{12}{200}m=6cm$



21. A rubber ball of mass 'm' and radius 'r' is submerged in water to a depth 'h' and released. What height will the ball jump up to above the surface of the water? Neglect the resistance of water and air. Density of water is double of density of ball.

(A) $\frac{h}{2}$ (B) 2h

(C) h (D) 0

Ans. C

Sol. Let the ball go up by x. Applying conservation of energy between initial and final positions.

$\therefore$ -mgh + $\left[\frac{4}{3}\pi r^3\rho g\right]H$ = mg. x

$$\Rightarrow \qquad x = \frac{(4/3)\pi r^3 \rho - m}{m} \times h = h$$

22. Which of the following are correct?
(A) For a very small deformation of a material, the ratio of stress and strain is constant.
(B) For a large deformation of a material, the ratio of stress and strain is constant.
(C) Two wires made of different materials, having the same diameter and length are connected end to end. A force is applied which stretches their combined length by 2 mm. Now the strain is same in both the wire but stress is different.
(D) None of these

Ans. A

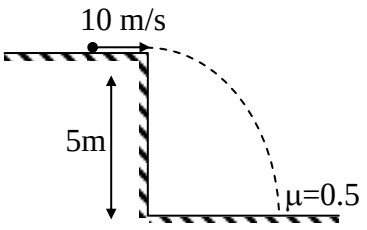
Sol. Within elastic limit, $\frac{\text{Stress}}{\text{Strain}}$ = constant

Beyond elastic limit $\frac{\text{Stress}}{\text{Strain}}$ is not constant

$\because$ Stress is same end different Y.

$\therefore$ Strain will different.

23. A small ball moving with a velocity 10 m/s, horizontally (as shown in figure) strikes a rough horizontal surface having $\mu = 0.5$. If the coefficient of restitution is e = 0.4. Horizontal component of velocity of ball after first impact will be (g = 10 m/s^2)



(A) 10 m/s (B) 8 m/s
(C) 3 m/s (D) 4 m/s

Ans. C

Sol. $\int N dt = mv_y - mu_y, e = \frac{v_y}{u_y} \Rightarrow v_y = 0.4 \times 10 = 4\ \text{m/s}$

$\int N dt = m \times 4 - (-10m) = 14\ m$

$-\int \mu N dt = mv_x - mu_x$

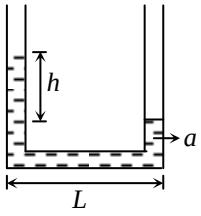
$-0.5 \times 14m = mv_x - m \times 10 \Rightarrow v_x = 3\,\text{m/s}$

24. In a one dimensional collision between two identical particles. A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A.
(A) The total momentum of the 'A plus B ' system is p before and after the impact, and (p-J) during the impact
(B) During the impact, A gives impulse J to B
(C) The coefficient of restitution is $\frac{J}{p} - 1$
(D) The coefficient of restitution is $\frac{J}{p} + 1$

Ans. B

Sol. Factual

25. At rest, a liquid stands at the same level in the tubes. As the system is given an acceleration a towards the right, a height difference h occurs as shown in the figure. The value of h is:



(A) $\frac{aL}{2g}$ (B) $\frac{gL}{2a}$

(C) $\frac{gL}{a}$ (D) $\frac{aL}{g}$

Ans. D

Sol. Newton's equations are :

$A\Delta P\sin\theta = ma$ …(i)

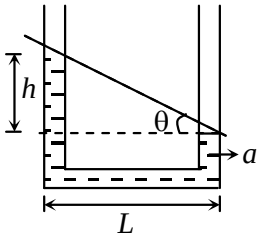
and

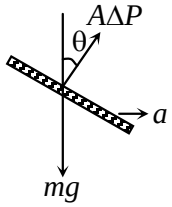
$A\Delta P\cos\theta = mg$ …(ii)

By (i) and (ii)

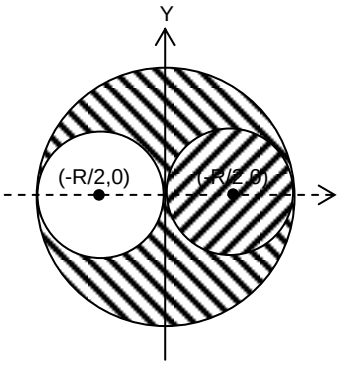
$\tan\theta = \frac{a}{g} = \frac{h}{L}$

or $h = \frac{aL}{g}$





26. Figure shows a uniform disc of radius R from which a hole of radius R/2 has been cut out from left of the centre and is placed on right of the centre of disc. The distance of centre of mass of resulting disc from centre of uniform disc will be



(A) $\frac{R}{2}$ (B) $\frac{R}{4}$

(C) $\frac{R}{6}$ (D) R

Ans. B

Sol. $X_{cm} = \frac{\sigma\pi R^2(O) + \sigma\pi\left(\frac{R}{2}\right)^2\frac{R}{2} + (-\sigma)\pi\left(\frac{R}{2}\right)^2\left(-\frac{R}{2}\right)}{\sigma\pi R^2 + \sigma\pi\left(\frac{R}{2}\right)^2 + (-\sigma)\pi\left(\frac{R}{2}\right)^2} = \frac{R}{4}$

27. A cylinder containing water stands on a table of height H. A small hole is punched in the side of cylinder at its base. The stream of water strikes the ground at a horizontal distance R from the table. Then the depth of water in the cylinder is

(A) H (B) R

(C) $\sqrt{(RH)}$ (D) $R^2/4H$

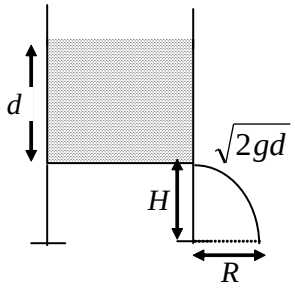
Ans. D

Sol. $t = \sqrt{\frac{2H}{g}}$

$R = \sqrt{2gd}\sqrt{\frac{2H}{g}}$

$R = \sqrt{4dH}$

$d = \frac{R^2}{4H}$



28. A body of mass 30 kg stands at the edge of a circular platform of radius 1 m rotating at an angular velocity of 2 rad/s about its axis. The moment of inertia of platform is 60 kg-m^2. The body catches a ball of mass 2 kg moving horizontally with a velocity of 25 m/s along the tangent to the edge of the platform, which is thrown by his friend. The final angular velocity of the platform is

(A) 2 rad/s (B) 3.5 rad/s

(C) 2.5 rad/s (D) 3 rad/s

Ans. C

Sol. The initial angular momentum of the system is

$L_i = mvR + (MR^2 + I)\omega_o$, where, M = mass of the boy and m = mass of the ball

$= 2 \times 25 \times 1 + [30(1)^2 + 60]\times 2 = 230$ kg-m^2/s

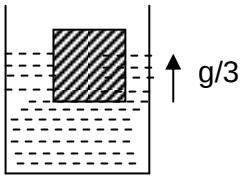
and final angular momentum of the system is

$L_f = [(30 + 2)(1)^2 + 60]\omega = 92\omega$ where ω = final angular velocity)

Now using conservation of angular momentum

$L_i = L_f \Rightarrow 230 = 92\omega \Rightarrow \omega = 2.5$ rad/s

29. A cubical block is floating in a liquid with half of its volume immersed in the liquid. When the whole system accelerates upwards with a net acceleration of g/3. The fraction of volume immersed in the liquid will be



(A) 1/2 (B) 3/8

(C) 2/3 (D) 3/4

Ans. A

Sol. The apparent weight of block and apparent weight of fluid are changed by same amount.

30. A disc of radius R rolls with slipping on a horizontal surface with linear velocity $v\hat{i}$ and angular velocity $\omega(-k)$. There is a particle p on the circumference of the disc which has velocity in vertical direction. The height of that particle from the ground will be

(A) $R + \frac{V}{\omega}$ (B) $R - \frac{V}{\omega}$

(C) $R + \frac{V}{2\omega}$ (D) $R - \frac{V}{2\omega}$

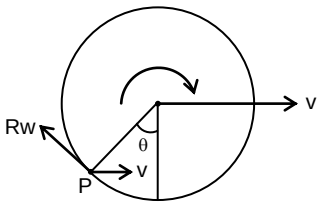
Ans. B

Sol. If velocity of point P in vertical direction, then

$v = R\omega \cos\theta$ …(i)

Height of P from ground

$h = R - R\cos\theta$ …(ii)



CHEMISTRY

31. The equilibrium constant K_C of the following reaction $A(g)+2B(g) \rightleftharpoons 3C(g)+D(g)$ is 1.4 mol L^{-1}. If a one litre vessel contains two moles each of A, B, C and D gases, along which direction will the reaction proceed at constant temperature?
(A) Forward direction (B) Backward direction
(C) Both direction (D) Unpredictable

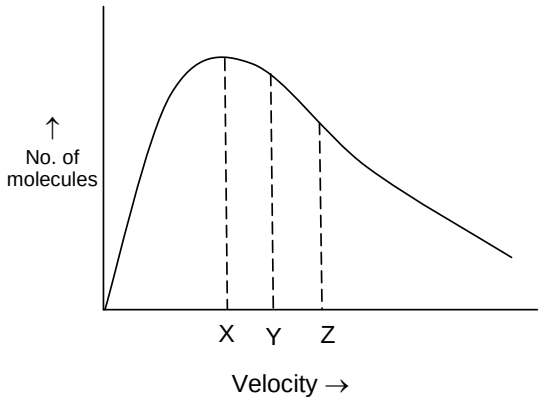
Ans. B

Sol. $A(g)+2B(g) \rightleftharpoons 3C(g)+D(g)$, $K_C = 1.4$

$$Q=\frac{[C]^3[D]}{[A][B]^2}=\frac{(2)^3\times 2}{2\times(2)^2}=2$$

Since $Q > K_C \rightarrow$ reaction proceeds towards backward direction.

32. The velocity occupied by the gas molecules at point 'Z' can be represented as
(A) $\sqrt{\frac{2RT}{M}}$
(B) $\sqrt{\frac{3RT}{M}}$
(C) $\sqrt{\frac{8RT}{\pi M}}$
(D) $\sqrt{\frac{2RT}{\pi M}}$



Ans. B

Sol. The largest velocity occupied by the molecules of a gas is r.m.s velocity which is represented by $\sqrt{\frac{3RT}{M}}$.

33. The pH of the aqueous solution of NaCl is maximum at?
(A) 25°C (B) 40°C
(C) 60°C (D) 85°C

Ans. A

Sol. pH of NaCl = pH of water at 25°C because NaCl is a neutral salt. On heating ionic product of water increase and pH of water decreases.

34. For which of the following reaction the ratio of K_P and K_C is one?
(A) $2NO(g)+O_2(g) \rightleftharpoons 2NO_2(g)$ (B) $N_2(g)+3H_2(g) \rightleftharpoons 2NH_3(g)$
(C) $S(s)+O_2(g) \rightleftharpoons SO_2(g)$ (D) $PCl_5(g) \rightleftharpoons PCl_3(g)+Cl_2(g)$

Ans. C

Sol. $K_P = K_C(RT)^{\Delta n}$
$K_P = K_C$ if $\Delta n = 0$

35. A container contains 2 moles of He and one mole of CH_4 gases at a certain temperature. If the partial pressure of He is 2 atm, then what will the total pressure of both gases in the container?
(A) 2 atm (B) 4 atm
(C) 3 atm (D) 2.5 atm

Ans. C

Sol. $p_{He} = \chi_{He} P_T$ or $2 = \frac{2}{3} \times P_T \Rightarrow P_T = 3$ atm.

36. $Al(OH)_3(s) \rightleftharpoons Al^{3+}(aq) + 3OH^-(aq)$
Addition of which of the following substance increases the degree of dissociation of $Al(OH)_3$ by maximum extent?
(A) NH_4OH (B) NH_4Cl
(C) NaCl (D) KNO_3

Ans. B

Sol. Degree of dissociation increases in acidic medium.

37. Under which of the following conditions the compressibility factor of a gas will be 1?
(A) PV > nRT (B) PV < nRT
(C) PV = nRT (D) P α V

Ans. C

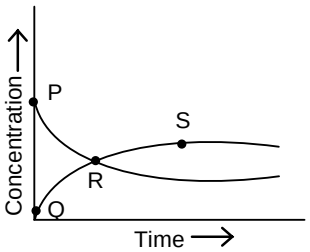
Sol. For ideal gas, Z = 1.

38. NH_3 can be easily liquefied than CH_4 gas. Which of the following characteristics of NH_3 will be greater than that of CH_4?
(A) Compressibility factor(Z) (B) van der Waal's constant(a)
(C) No. of moles(n) (D) Rate of effusion(r)

Ans. B

Sol. Liquification increases with increase in intermolecular force of attraction which depends on 'a' (Vander Waal's Constant)

39. The figure is given for the following reaction.
$X(g) \rightleftharpoons Y(g)$
At what point on the curve, Q = K_{eq}?(Q = Reaction quotient)
(A) P (B) Q
(C) R (D) S



Ans. D

Sol. At equilibrium, Q = K_{eq}.

40. The ionic product of a solution of $Be(OH)_2$ is 4×10^{-12}. What is the pH of the solution?
(A) 8 - log2 (B) 10 - log2
(C) 10 + log2 (D) 8 + log2

Ans. C

Sol. $I.P = [Be^{2+}][OH^-]^2 = 4 \times 10^{-12}$
$s \quad 2s$
or, $4s^3 = 4 \times 10^{-12} \Rightarrow s = 10^{-4}$
$[OH^-] = 2s = 2 \times 10^{-4}$ or $pOH = -\log(2 \times 10^{-4}) = 4 - \log 2$
$pH = 14 - pOH = 14 - (4 - \log 2) = 10 + \log 2$

41. What is the kinetic energy of 1.7 g of NH_3 gas at 1000 K?
(A) 50 R (B) 100 R
(C) 150 R (D) 200 R

Ans. C

Sol. $K.E = \frac{3}{2}nRT = \frac{3}{2} \times \frac{1.7}{17} \times R \times 1000 = 150\ R$

42. What is the pH of a solution which is made by mixing 500 mL of 0.2 M NaOH and 500 mL of 0.1 M HCN.
(K_a of HCN = 10^{-10})
(A) 10 - log2 (B) 12 - log5
(C) 10 + log2 (D) 12 + log5

Ans. D

Sol. $NaOH + HCN \longrightarrow NaCN + H_2O$
Initial $500 \times 0.2 \quad 500 \times 0.1 \quad 0 \quad 0$
$= 100 \quad = 50$
At equ^m $50 \quad 0 \quad 50 \quad 50$
$[NaOH] = \frac{50}{1000} = 0.05$
$pOH = -\log(0.05) = -\log 5 + 2 = 2 - \log 5$
$pH = 14 - (2 - \log 5) = 12 + \log 5$

43. The molecules of a gas move with the following velocities in cm s^{-1}
4, 5, 5, 7, 2, 1, 4, 3, 4, 8, 9
The most probable velocity of the gas is
(A) 2 cms^{-1} (B) 7 cms^{-1}
(C) 4 cms^{-1} (D) 9 cms^{-1}

Ans. C

Sol. The most probable velocity is the velocity which is possessed by maximum no. of gas molecules.

44. $A(g) \rightleftharpoons 3B(g) + C(g)$
One mole of A was taken in a one litre vessel. At equilibrium the concentration of A and B are same. What will be the degree of dissociation of A?
(A) 0.5 (B) 0.25
(C) 0.125 (D) 0.6

Ans. B

Sol. $A(g) \rightleftharpoons 3B(g) + C(g)$
$1 - x \quad 3x \quad x$
At equ^m, $3x = 1 - x$ or $4x = 1$, $x = \frac{1}{4} = 0.25$

45. The strongest acidic solution out of the following is:
(A) KCN (B) $FeCl_3$
(C) NaCl (D) CH_3COONa

Ans. B

Sol. KCN $\rightarrow$ basic, $FeCl_3 \rightarrow$ acidic, NaCl $\rightarrow$ neutral, $CH_3COONa \rightarrow$ basic

BIOLOGY

46. Ivan Pavlov performed experiments on
(A) Simple reflexes (B) Conditioned reflexes
(C) Cardiac reflexes (D) Origin of life

Ans. B

Sol. Ivan Pavlov performed experiments on conditioned reflexes.

47. The thermoregulatory centre in the body is
(A) Spinal cord (B) Hypothalamus
(C) Cerebellum (D) Pituitary

Ans. B

Sol. Thermoregulatory centre in body is hypothalamus.

48. Which of the following is a living fossil?
(A) *Cycas* (B) Moss
(C) Yeast (D) *Spirogyra*

Ans. A

Sol. *Cycas* is a living fossil.

49. Hypothalamus
(A) Is helpful in sleep (B) Has centres for thirst and hunger
(C) Controls body temperature (D) All the above

Ans. D

Sol. Functions of hypothalamus includes:
(i) Aids in sleep
(ii) Regulate body temperature
(iii) Has centre for thirst and hunger

50. The 1^{st} organisms were:
(A) Chemoautotroph (B) Chemoheterotroph
(C) Eukaryote (D) Autotroph

Ans. B

Sol. Chemoheterotrophs were first organisms.

51. Venom of cobra affects
(A) Circulatory system (B) Digestive system
(C) Nervous system (D) Respiratory system

Ans. C

Sol. Venom of cobra affects nervous system. Hence, it is neurotoxin.

52. Which one is not a reflex action?
(A) Yawing (B) Weeping
(C) Coughing (D) Sneezing

Ans. A

Sol. Yawning is caused by simultaneous inhalation of air and stretching of eardrums, followed by exhalation.

53. Excessive stimulation of vagus nerve in humans may lead to
(A) Hoarse voice (B) Efficient digestion of proteins
(C) Peptic ulcers (D) Irregular contraction of diaphragm

Ans. C

Sol. Excessive stimulation of vagus nerve in human leads to peptic ulcers.

54. Middle piece of mammalian sperm contains ___
(A) Nucleus (B) Centriole
(C) Mitochondria (D) Vacuole

Ans. C

Sol. Middle piece of mammalian sperm contains mitochondria.

55. The egg of rabbit and humans is
(A) Alecithal (B) Microlecithal
(C) Macrolecithal (D) Telolecithal

Ans. A

Sol. Egg of rabbit and human is alecithal.

56. Which of the following amino acids was not found to be synthesized in MILLER'S Experiment?
(A) Alanine (B) Glycine
(C) Asparatic acid (D) Glutamic acid

Ans. D

Sol. Glutamic acid was not synthesized in MILLER'S experiment.

57. Animals living in the bottom of the sea are known as
(A) Lentic (B) Pelagic
(C) Benthic (D) Lotic

Ans. C

Sol. Animals living in the bottom of the sea are called benthic.

58. Mr. X is eating curd / yoghurt. For this food intake in a food chain he should be considered as occupying
(A) First trophic level (B) Second trophic level
(C) Third trophic level (D) Fourth trophic level

Ans. C

Sol. Mr. X belongs to third trophic level.

59. Annual carbon fixation by photosynthesis is
(A) 7 x 10^{13} kg (B) 7 x 10^{10} kg
(C) 7 x 10^{12} kg (D) 7 x 10^{20} kg

Ans. A

Sol. Annual carbon fixation by photosynthesis is 7 x 10^{13} kg.

60. Annual world precipitation is
(A) $7\times10^{10}G$ (B) $7\times10^{20}G$
(C) $4.46\times10^{10}G$ (D) $4.46\times10^{20}G$

Ans. D

Sol. Annual world precipitation is $4.46\times10^{20}G$

PART – II

MATHEMATICS

61. A can do a piece of work in 10 days; B in 15 days. They work for 5 days. The rest of the work was finished by C in 2 days. If they get Rs. 1500 for the whole work, the daily wages of B and C are:
(A) Rs. 150 (B) Rs. 225
(C) Rs. 250 (D) Rs. 300

Ans. B

Sol. Part of the work done by $A=\left(\frac{1}{10}\times5\right)=\frac{1}{2}$.

Part of the work done by $B=\left(\frac{1}{15}\times5\right)=\frac{1}{3}$

Part of the work done by $C=1-\left(\frac{1}{2}+\frac{1}{3}\right)=\frac{1}{6}$.

So, (As's share): (B's Share) : (C's share) $=\frac{1}{2}:\frac{1}{3}:\frac{1}{6}=3:2:1$

$\therefore$ A's share $=Rs.\left(\frac{3}{6}\times1500\right)=$ Rs. 750, B's share = Rs. $\left(\frac{2}{6}\times1500\right)$ = Rs. 500,

C's share $=Rs.\left(\frac{1}{6}\times1500\right)=Rs.250$.

A's daily wages $=Rs.\left(\frac{750}{5}\right)=$ Rs. 150; B's daily wages = Rs. $\left(\frac{500}{5}\right)=$ Rs. 100;

C's daily wages $= \text{Rs.}\left(\frac{250}{2}\right) = \text{Rs.}125$

$\therefore$ Daily wages of B and C = Rs. $(100+125) = \text{Rs. } 225$

62. The sum of $\frac{1^2}{1^2+40} + \frac{2^2}{2^2+30} + \frac{3^2}{3^2+20} + ...$ up to 11 terms is

(A) $\frac{792}{62}$ (B) $\frac{791}{62}$

(C) $\frac{792}{61}$ (D) $\frac{791}{61}$

Ans. C

Sol. $\frac{n^2}{n^2-10x+50} = \frac{2n^2}{(n-10)^2+n^2}$

Putting n = 1 & n = 9, $t_1 + t_9 = 2$ etc.

63. Let P(x) be a polynomial with integer coefficients. It is known that P(x) takes the value 2013 for four distinct integers. Find the number of integral values of x for which P (x) equals 2020.

(A) 0 (B) 1

(C) 3 (D) 4

Ans. A

Sol. $P(x) - 2013 = q(x)(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

Let $P(\alpha) = 2020$

$\Rightarrow P(\alpha) - 2013 = q(\alpha)(\alpha-x_1)(\alpha-x_2)(\alpha-x_3)(\alpha-x_4)$

$\Rightarrow 7 = q(\alpha)(\alpha-x_1)(\alpha-\alpha_2)(\alpha-\alpha_3)(\alpha-\alpha_4)$

Impossible since 7 is prime.

64. If $a,b,c \in R$ and $225a^2 + 25b^2 + 9c^2 - 75ab - 15bc - 45ca = 0$ then a, b, c are in

(A) A.P. (B) G.P.

(C) H.P. (D) none of these

Ans. A

Sol. Multiply both the sides by 2 we get

$(15a-5b)^2 + (5b-3c)^2 + (3c-15a)^2 = 0$

$\Rightarrow 15a = 5b = 3c$

$\therefore$ abc are in A.P.

65. If the sixth term of an A.P is equal to 2. The value of the common difference of the A.P. which makes the product $a_1a_4a_5$ the greatest is (the i^{th} term is denoted by a_i)

(A) $\frac{8}{5}$ (B) 3

(C) 2 (D) $\frac{4}{5}$

Ans. A

Sol. $a_1 + 5x = 2$

$\Rightarrow a_1 = 2 - 5x$

now $a_1.a_4.a_5 = (2-5x)(2-5x+3x)(2-5x+4x)$

$= (2-5x)(2-2x)(2-x)$

$= 8 - 12x + 4x^2 - 20x + 30x^2 - 10x^3$

$p = 8 - 32x + 34x^2 - 10x^3$

$\Rightarrow \frac{dp}{dx} = -32 + 68x - 30x^2$

= 0

$\Rightarrow x = \frac{2}{3}, \frac{8}{5}$

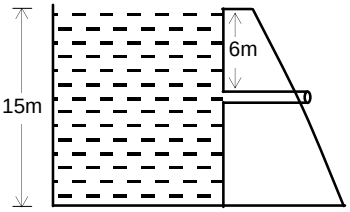
$\frac{d^2p}{dx^2} = 68 - 60x$

for $x = \frac{8}{5}, \frac{d^2p}{dx^2} < 0$

$\therefore$ common difference = $\frac{8}{5}$

PHYSICS

66. The fresh water behind a reservoir dam is 15m deep. A horizontal pipe 4 cm in diameter passes through the dam 6m below the water surface as shown in the figure. A pulley secures the pipe opening. The friction force between the plug and pipe walls will be nearly (take g = 9.8 m/s^2)



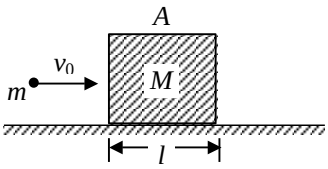
(A) 70 N (B) 72 N

(C) 74 N (D) 76 N

Ans. C

Sol. $f_r = \rho g h \pi r^2 = 10^3 \times 9.8 \times 6 \times \pi\,(2 \times 10^{-2})^2 = 74.$

67. A bullet of mass m moving with velocity v_0 hits a wooden block A of mass M placed on a smooth horizontal surface. The length of the plank is l. The bullet experiences a constant resistive force F inside the block. The minimum value of v_0 such that it is able to come out of the block is



(A) $\sqrt{\frac{Flm}{M^2}}$ (B) $\sqrt{\frac{2Fl(M+m)}{Mm}}$

(C) $\sqrt{\frac{2Flm}{M^2}}$ (D) $\sqrt{\frac{Fl(M+m)}{Mm}}$

Ans. B

Sol. From Newton's third law, force F will act on the block in forward direction

Acceleration of block $a_1 = \frac{F}{M}$

retardation of bullet $a_2 = \frac{F}{m}$

relative retardation of bullet

$a_r = a_1 + a_2 = \frac{F(M+m)}{Mm}$

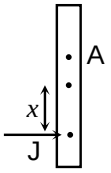
Applying $v^2 = u^2 - 2a_r l$

$$0 = v_0^2 - \frac{2F(M+m)}{Mm}.l \quad \text{or} \quad v_0 = \sqrt{\frac{2Fl(M+m)}{Mm}}$$

Therefore, minimum value of v_0 is

$$\sqrt{\frac{2Fl(M+m)}{Mm}}$$

68. A uniform rod of mass *m* and length *l* is placed in gravity free space and linear impulse *J* is given to the rod at a distance *x* = *l*/4 from centre and perpendicular to the rod. Point *A* is at a distance *l*/3 from centre as shown in the figure. Then



(A) Speed of centre of rod is $\frac{2J}{m}$

(B) Speed of point *A* is $\frac{5}{2}\frac{J}{m}$

(C) Speed of upper end of rod is $\frac{J}{m}$

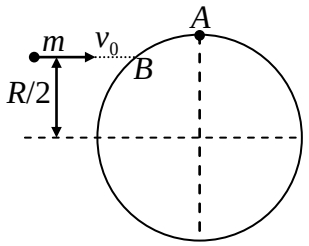
(D) Speed of lower end of rod is $\frac{5}{2}\frac{J}{m}$

Ans. D

Sol. $J\frac{l}{4} = \frac{ml^2}{12}\omega \quad \Rightarrow \omega = \frac{3J}{ml}, \quad V_{cm} = \frac{J}{m}$

$$V_A = V_{cm} - \frac{l}{3}\omega = 0, \quad V_{upper} = \left|V_{cm} - \frac{l}{2}\omega\right| = \frac{J}{2m} \quad ; \quad V_{lower} = \left|V_{cm} + \frac{l}{2}\omega\right| = \frac{5}{2}\frac{J}{m}$$

69. A disc of mass m and radius R is lying on a smooth horizontal surface. A particle of mass m moving horizontally with a velocity v_0, collides with the disc at B and sticks to it. Speed of the point A on the disc just after impact will be



(A) $\frac{\sqrt{31}}{8}v_0$ (B) $\frac{\sqrt{5}}{16}v_0$

(C) $\frac{5v_0}{16}$ (D) $\frac{v_0}{2}$

Ans. A

Sol. COM, $mv_0\hat{i} = 2m\vec{v}_{cm} \Rightarrow \vec{v}_{cm} = \frac{v_0}{2}\hat{i}$

COAM about CM $\left(mv_0\frac{R}{2}\sin 30^o\right)(-\hat{k}) = \left(m\frac{R^2}{4} + \frac{1}{2}mR^2 + \frac{mR^2}{4}\right)\vec{\omega}$

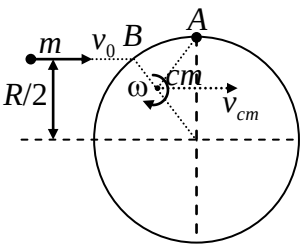
$$\vec{\omega} = \frac{v_0}{4R}(-\hat{k})$$

$$\vec{v}_A = \vec{v}_{cm} + \vec{\omega}\times\vec{r}$$

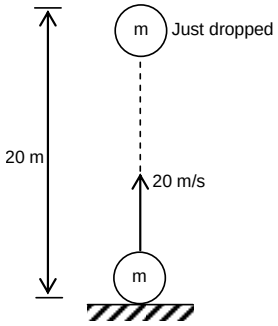
$$= \frac{v_0}{2}\hat{i} + \frac{v_0}{4R}(-\hat{k})\times\left(\frac{R}{2}.\frac{\sqrt{3}}{2}\hat{i} + \frac{3R}{4}\hat{j}\right)$$

$$= \frac{11v_0}{16}\hat{i} - \frac{\sqrt{3}v_0}{16}\hat{j}$$

$$|\vec{v}_A| = \frac{\sqrt{31}}{8}v_0$$



70. The mass collide in air stick together. After how much time combined mass will fall to the ground (calculate the time from the starting when the motion was started)



(A) $\left(1+\sqrt{2}\right)$s

(B) $2\sqrt{2}$s

(C) $\left(2+\sqrt{3}\right)$s

(D) $(\sqrt{3}+1)$ s

Ans. D

Sol. Collision will occur after 1 sec.
Just before collision velocity of each object will be 10 m/sec.
Just after collision velocity of combined system will be zero.
So, time taken to reach ground = $(\sqrt{3}+1)$ sec.

CHEMISTRY

71. $2KClO_3(s) \rightleftharpoons 2KCl(s)+3O_2(g)$

The equilibrium constant K_P of the above reaction is 8 atm^3 at a certain temperature. What will be the equilibrium partial pressure of O_2 gas if 2 moles of $KClO_3$ are added at equilibrium at constant temperature?

(A) 4 atm (B) 2 atm
(C) 8 atm (D) 6 atm

Ans. B

Sol. $K_P = \left(p_{O_2}\right)^3 = 8$

$\therefore p_{O_2}$ = 2 atm

72. $AgCN(s) \rightleftharpoons Ag^+(aq)+CN^-(aq)$

At what pH of the solution the solubility of AgCN will be maximum?

(A) 8 (B) 10
(C) 4 (D) 7

Ans. C

Sol. In acidic solution, the solubility will be maximum as H^+ will combine with CN^- and form weakly dissociated HCN.

73. What is the ratio of the relative rates of effusion of H_2 to D_2?

(A) 2 : 1 (B) $\sqrt{2}$: 1
(C) 4 : 1 (D) $\sqrt{2}$: 4

Ans. B

Sol $\frac{r_{H_2}}{r_{D_2}} = \sqrt{\frac{M_{D_2}}{M_{H_2}}} = \sqrt{\frac{4}{2}} = \sqrt{2}$

74. Which of the following mixture does NOT behave as buffer?

(A) H_3BO_3 + $Na[B(OH)_4]$ (B) $H_2B_4O_7$ + $Na_2B_4O_7$
(C) H_3PO_2 + NaH_2PO_2 (D) $NaHCO_3$ + Na_2CO_3

Ans. B

Sol. $H_2B_4O_7$ and $B_4O_7^{2-}$ are not conjugate acid-base pair.

75. Which of the following mixture display common ion effect?
(A) H_2S + NaH
(B) $NaHCO_3$ + HCl
(C) NaCl + HCl
(D) H_2S + Na_2SO_4

Ans. B

Sol. $HCO_3^- \rightleftharpoons H^+ + CO_3^{2-}$
$HCl \longrightarrow H^+ + Cl^-$

BIOLOGY

76. Major aerosol pollutant present in the jet plant emission is
(A) Carbon monoxide
(B) Carbon tetrachloride
(C) Chlorofluorocarbon
(D) Sulphur dioxide

Ans. C

Sol. Major aerosol pollutant present in the jet plane emission are chloroflurocarbons.

77. Acid rain is caused by increase in the atmospheric concentration of
(A) Ozone and dust
(B) SO_2 and NO_2
(C) SO_3 and CO
(D) CO_2 and CO

Ans. B

Sol. Oxides of Sulphur and nitrogen causes acid ran.

78. Of the following which is not normally a pollutant ?
(A) Hydrocarbons
(B) Carbon dioxide
(C) Carbon monoxide
(D) Sulphur dioxide

Ans. B

Sol. CO_2 (Carbon dioxide) is normally not a pollutant as it plays crucial role in sustainability of life on earth. But excessive concentration of CO_2 in atmosphere due to anthropogenic reasons make it a pollutant.

79. Contamination of water with sewage is indicated by cysts of
(A) *Escherichia*
(B) *Entamoeba*
(C) *Pseudomonas*
(D) *Leishmania*

Ans. A

Sol. Cyst of *Escherichia* species are indicators of contamination of sewage.

80. Supersonic jets cause pollution by thinning the layer of
(A) SO_2
(B) O_2
(C) CO_2
(D) O_3

Ans. D

Sol. Supersonic jet causes thinning of ozone.