OLTS – 1920 – JEEM 2020

FULL TEST – 6

PART – A : PHYSICS

SECTION - A
Single Correct Choice Type (1-20)

1. A boat having length of 3 metres and breadth 2 metres is floating on a lake. The boat sinks by one cm when a man gets on it. The mass of the man is
(A) 60 kg (B) 62 kg
(C) 72 kg (D) 128 kg

Ans. **A**

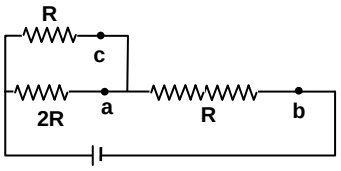
Sol. $mg = V\rho g$
$= 3 \times 2 \times (0.01\ m) \times (10^3\ kg/m3) \times 10$
$Mg = 600$
$\Rightarrow$ $m = 60\ kg$

2. A body of mass m rises to a height h = R/5 from the surface of earth, where R is the radius of earth. If g is the acceleration due to gravity at the surface of earth the increase in potential energy is
(A) (1/6) mgh (B) (5/6) mgh
(C) (6/7) mgh (D) mgh

Ans. **B**

Sol. $\Delta U = -\dfrac{GmM}{(6R/5)} - \left(\dfrac{-GmM}{R}\right)$

3. Referring to the shown circuit, the current will be minimum in
(A) a
(B) b
(C) c
(D) same in all the branches



Ans. **A**

Sol. $I_a = \dfrac{3V}{15R}, I_b = \dfrac{3V}{5R}$
$I_c = \dfrac{6V}{15R}.$

4. Two electrons are moving with the same speed V. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field. After sometime if the de Broglie wavelength of the two are λ_1 and λ_2, then (select the best alternative)
(A) $\lambda_1 = \lambda_2$ (B) $\lambda_1 > \lambda_2$
(C) $\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$ (D) $\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$ or $\lambda_1 = \lambda_2$

Ans. **D**

Sol. For electron in electric field, magnitude of its momentum may increase, decrease or remain constant w.r.t. initial value.

5. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger is sitting in a moving train. A records a frequency of 5.5 kHz when the train approaches the siren. During his return journey in a different train B he records the frequency of 6 kHz while approaching the same siren. The ratio of velocity of train B to train A is

(A) $\frac{242}{252}$ (B) $\frac{5}{6}$

(C) 2 (D) $\frac{11}{6}$

Ans. **C**

Sol. Using doppler's effect to calculate apparent frequency.

6. What change in surface energy will be noticed when a drop of radius R splits up into 1000 droplets of radius r, surface tension of T?

(A) $4\pi R^2T$ (B) $32\pi R^2T$

(C) $3\pi R^2T$ (D) $36\pi R^2T$

Ans. **D**

Sol. $\Delta U = U_f - U_i = (1000)\left[4\pi\left(\frac{R}{10}\right)^2 T\right] - 4\pi R^2T = 36\pi R^2T$

7. A hydrogen atom and a Li^{++} ions are both in the second excited state. If ℓ_H and ℓ_{Li} are their respective electronic angular momenta, and E_H and E_{Li} their respective energies, then

(A) $\ell_H > \ell_{Li}$ and $|E_H| > |E_{Li}|$ (B) $\ell_H = \ell_{Li}$ and $|E_H| < |E_{Li}|$

(C) $\ell_H = \ell_{Li}$ and $|E_H| > |E_{Li}|$ (D) $\ell_H < \ell_{Li}$ and $|E_H| < |E_{Li}|$

Ans. **B**

Sol. According to Bohr's atomic model.

8. Unit vector perpendicular to vectors $\vec{A} = -3\hat{i} - 2\hat{j} - 3\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} + 6\hat{k}$ both is

(A) $\frac{3\hat{j} - 2\hat{k}}{\sqrt{13}}$ (B) $\frac{3\hat{j} - 2\hat{j}}{\sqrt{13}}$

(C) $\frac{-3\hat{j} + \hat{k}}{\sqrt{10}}$ (D) $\frac{\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{13}}$

Ans. **A**

Sol. $\hat{C} = \frac{\vec{A} \times \vec{B}}{\left|\vec{A} \times \vec{B}\right|}$

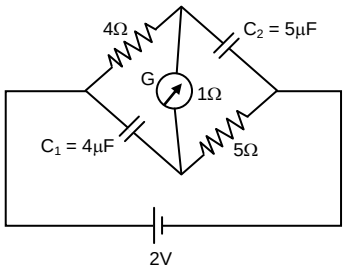
9. In the circuit shown, the cell is ideal with emf = 2V. The resistance of the coil of the galvanometer G = 1 Ω. Then

(A) No current flows in G.

(B) 0.2 A current flow in G.

(C) Potential difference across C_1 = 1.2

(D) Potential difference across C_2 = 2 V.

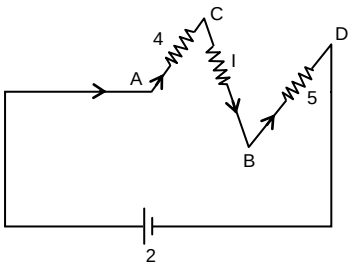


Ans. **B**

Sol. No current will flow through the capacitor.

$V_{C_1} = V_{AB} = (I)\,(5\Omega) = 1$ V

$V_{C_2} = V_{CD} = (I)\,(6) = 1.2$ V.



10. Which of the following phenomenon is not common to sound and light waves?

(A) interference (B) diffraction

(C) polarization (D) reflection

Ans. **C**

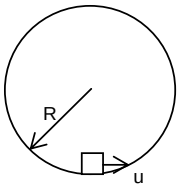
Sol. Polarization occurs for transverse waves and not for longitudinal waves.

11. The image formed by an objective of a compound microscope is
(A) virtual and diminished (B) real and diminished
(C) real and enlarged (D) virtual and enlarged

Ans. **C**

Sol. Factual based.

12. A particle is given an initial speed 'u' inside a smooth fixed spherical shell of radius R = 1 m such that it is just able to complete the circle. Acceleration of the particle when its velocity is vertical, is



(A) $g\sqrt{10}$ (B) g
(C) $g\sqrt{2}$ (D) 3 g

Ans. **A**

Sol. $a = \sqrt{a_t^2 + a_c^2}$

13. The conductivity of a semiconductor increases with increase in temperature because
(A) number density of free current carries increases
(B) relaxation time increases
(C) both number density of carriers and relaxation time increases
(D) number density of current carries increases, relaxation time decreases but effect of decrease in relaxation time is much less than increase in number density

Ans. **C**

Sol. Fact based.

14. A train of mass M is moving on a circular track of radius R with constant speed v. The length of train is half the perimeter of track. The linear momentum of the train will be
(A) 0 (B) $2Mv/\pi$
(C) MvR (D) Mv

Ans. **B**

Sol. Total momentum = $\int_{\theta=-90}^{+90} (dm)v\cos\theta$

$dm = \frac{M}{\pi R} \times Rd\theta$

Total momentum = $\int \frac{M}{\pi R} vRd\theta\cos\theta = \frac{2Mv}{\pi}$

15. A ball falls from a height h_0. There are n collisions with the earth. If after n collisions the ball rises to a height h_n, then coefficient of restitution e is given by
(A) $e^n = \sqrt{\frac{h_n}{h_0}}$ (B) $e^n = \sqrt{\frac{h_0}{h_n}}$
(C) $ne = \sqrt{\frac{h_n}{h_0}}$ (D) $\sqrt{n}\,e = \sqrt{\frac{h_n}{h_0}}$

Ans. **A**

Sol. After first impact

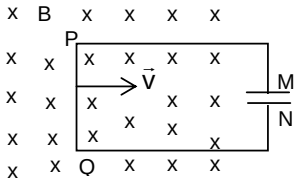
$(0)^2 - \left(e\sqrt{2gh}\right)^2 = 2(-g)h_1$

$\Rightarrow h_1 = e^2h$

$\Rightarrow h_2 = e^2h_1 = e^4h$

$\Rightarrow h_n = e^{2n}h$

16. A rod PQ is connected to the capacitor plates. The rod is placed in a magnetic field (B) directed downward perpendicular to the plane of the paper. If the rod is pulled out of magnetic field with velocity v as shown in figure.



(A) plate M will be positively charged
(B) plate N will be positively charged
(C) both plates will be similarly charged
(D) no charge will be collected on plates

Ans. **A**

Sol. $\varepsilon = B\ell v$ develops across rod PQ.

By Lenz's law, plate M gets positively charged.

17. In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5×10^{-2} m towards the slits the change in fringe width is 3×10^{-5} m. If the distance between the slits is 10^{-3} m, the wavelength of light used is

(A) 4000 Å
(B) 5550 A^o
(C) 6000 Å
(D) 6500 A^o

Ans. **C**

Sol. $W = \dfrac{\lambda D}{d}$; $\Delta W = \dfrac{\lambda \Delta D}{d}$

18. A tank has a hole made at its bottom. The time needed to empty the tank from level h_1 to h_2 will be proportional to

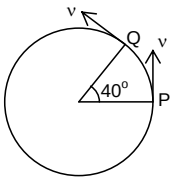
(A) $h_1 + h_2$
(B) $\sqrt{h_1} + \sqrt{h_2}$
(C) $h_1 - h_2$
(D) $\sqrt{h_1} - \sqrt{h_2}$

Ans. **D**

Sol. $A_1V_1 = A_2V_2 \Rightarrow a\sqrt{2gh} = A\left(-\dfrac{dh}{dt}\right)$

$$\Rightarrow \quad \frac{a}{A}\int_0^t dt = -A\int_{h_1}^{h_2}\frac{dh}{\sqrt{2gh}}$$

19. A particle is moving in a circle of radius r with constant v. The change in velocity in moving from P to Q is



(A) 2v cos 20°
(B) 2v sin 20°
(C) 2v cos 40°
(D) 2v sin 40°

Ans. **B**

Sol. $\left|\vec{v}_Q - \vec{v}_P\right| = \sqrt{v^2 + v^2 - 2v^2\cos 40}$

20. A simple pendulum has time period T = 2s in air. If the whole arrangement is placed in a nonviscous liquid whose density is $\dfrac{1}{2}$ times the density of bob. The time period of the simple pendulum in the liquid will be

(A) $\dfrac{2}{\sqrt{2}}$ s
(B) 4 s
(C) $2\sqrt{2}$ s
(D) $4\sqrt{2}$ s

Ans. **C**

Sol. $T = 2\pi\sqrt{\dfrac{\ell}{g_{eff}}}$

SECTION – B

Numerical Based Questions 0 to 9 Answer Type (21-22)

21. A lens of power 16 D is used as a simple microscope. In order to obtain maximum magnification at find the distance (in cm) from the lens should a small object be placed.

Ans. **5**

Sol. $-\frac{1}{25}-\frac{1}{-x}=\frac{16}{100}$

$\Rightarrow$ x = 5 cm

22. The power radiated by a black body is P, and it radiates maximum energy around the wavelength λ_0. If the temperature of black body is now changed so that it radiates maximum energy around a wavelength $\frac{3\lambda_0}{4}$, the new power radiated by it is $\frac{64\,P}{81}\times n$, then find the value of 'n'.

Ans. **4**

Sol. $T\times\frac{1}{\lambda}$ and $P \times T^4$

SECTION – C

Numerical Answer Type with Answer XXXXX.XX (23 – 25)

23. A steel wire of cross-sectional area $2mm^2$ and Young's modulus $2\times10^{11}N/m^2$ is stretched longitudinally by a force of 200 N. The elastic potential energy stored per unit volume in the string is $n \times 10^4$. Find the value of 'n'.

Ans. **00002.50**

Sol. EPE per unit volume = $\frac{1}{2}\times\text{stress}\times\text{strain}$

24. A 10 m wide spacecraft moves through the interstellar space at a speed 3×10^6 m/s. A magnetic field $B = 3 \times 10^{-9}$ T exists in the space in a direction perpendicular to the plane of motion. Treating the spacecraft as a conductor, calculate the emf induced across its width. (in volt)

Ans. **00000.09**

Sol. $\varepsilon = vB\ell = 3 \times 10^6 \times 10 \times 3 \times 10^{-9} = 9 \times 10^{-2} \approx 0.09$ V

25. The refractive index for water w.r.t. air for a sound wave if the velocities of sound in water and air are 1440 ms^{-1} and 340ms^{-1}, is approximately,

Ans. **00000.23**

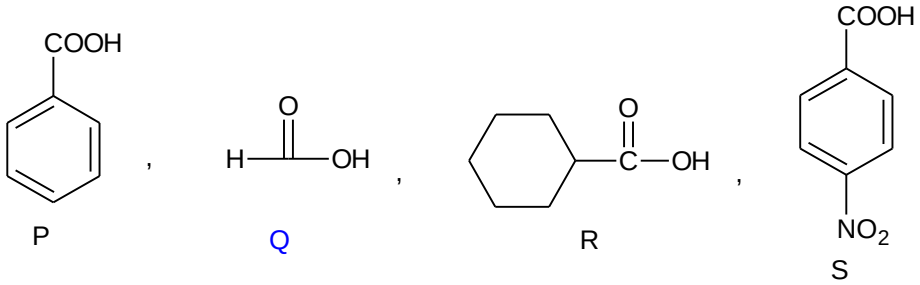
Sol. $\mu=\frac{\text{velocity in air}}{\text{velocity in medium}}$

PART – B : CHEMISTRY

SECTION – A

Single Correct Choice Type (26-45)

26. Arrange the given acids in the increasing order of acidic strength



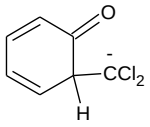
P , Q , R , S

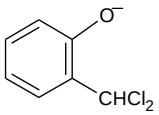
(A) Q < R < P < S
(B) R < Q < P < S
(C) R < P < Q < S
(D) R < P < S < Q

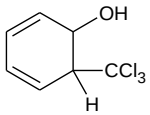
Ans. **C**

Sol. Conceptual

27. When phenol is treated with $CHCl_3$ and KOH, then the intermediate(s) or species formed during the reaction

(A) 

(B) 

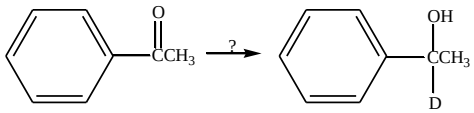
(C) 

(D) Both (A) and (B)

Ans. **D**

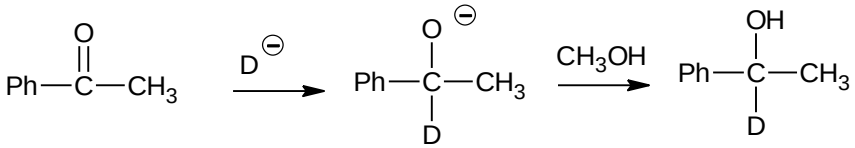
Sol. By mechanism

28. Which of the following reagents would carry out the following transformation? (D = $^{2}_{1}H$)



(A) $NaBD_4$ in CH_3OH
(B) $LiAlH_4$, then D_2O
(C) $NaBD_4$ in CH_3OD
(D) $LiAlD_4$, then D_2O

Ans. **A**

Sol. 

29. Which of the following pairs of reagents will not form ether ?

(A) $C_2H_5Br + CH_3COONa$
(B) $C_2H_5Br + C_2H_5ONa$
(C) $C_2H_5Br + (CH_3)_2CHONa$
(D) $(CH_3)_2CHBr + CH_3ONa$

Ans. **D**

Sol. $(CH_3)_2CHBr + CH_3ONa \longrightarrow CH_3 - HC = CH_2 + NaBr + CH_3OH$

CH_3ONa at as a base elimination reaction takes place and resulting product is alkene

30. In the following sequence of reactions, the end product is :

$CaC_2 \xrightarrow{H_2O} (A) \xrightarrow{Hg^{2+}/H_2SO_4} (B) \xrightarrow{[O]} (C) \xrightarrow{Ca(OH)_2} (D) \xrightarrow{Heat} (E)$

(A) acetaldehyde (B) formaldehyde
(C) acetic acid (D) acetone

Ans. **D**

Sol. (D) $CaC_2 \xrightarrow{H_2O} C_2H_2 \xrightarrow{Hg^{2+}/H_2SO_4} CH_3CHO \xrightarrow{(O)} CH_3COOH \xrightarrow[(ii)\ heat]{(i)\ Ca(OH)_2} CH_3-COCH_3$

31. The correct order of reactivity of PhMgBr with

$Ph-\overset{O}{\overset{\|}{C}}-Ph$ $CH_3-\overset{O}{\overset{\|}{C}}-H$ $CH_3-\overset{O}{\overset{\|}{C}}-CH_3$ is :

(A) I > II > III (B) III > II > I
(C) II > III > I (D) I > III > II

Ans. **C**

Sol. Conceptual

32. For $Zn^{2+}/zn, E^0 = -0.76V$, for $Ag^+/Ag\ E^0 = 0.799V$. The correct statement is

(A) the reaction Zn getting reduced Ag getting oxidized in spontaneous
(B) Zn undergoes reduction and Ag is oxidized
(C) Zn undergoes oxidation Ag^+ gets reduced
(D) No suitable answer

Ans. **C**

Sol. Due to high reduction potential of Ag $\rightarrow$ It gets reduced

33. Which has maximum conductivity?

(A) $\left[Cr(NH_3)_3Cl_3\right]$ (B) $\left[Cr(NH_3)_4Cl_2\right]Cl$
(C) $\left[Cr(NH_3)_5Cl\right]Cl_2$ (D) $\left[Cr(NH_3)_6\right]Cl_3$

Ans. **D**

Sol. The compound $\left[Cr(NH_3)_6\right]Cl_3$ produce more ions in the solution.

34. Borax bead test is responded by

(A) divalent metal (B) heavy metals
(C) light metals (D) metals which form coloured metaborates.

Ans. **D**

Sol. Coloured metaborates will give borax bead test.

35. The dissolution of $Al(OH)_3$ by a solution of NaOH results in the formation of

(A) $\left[Al(H_2O)_4(OH)\right]^{2+}$ (B) $\left[Al(H_2O)_2[OH]_4\right]^-$
(C) $\left[Al(H_2O)_3(OH)_3\right]$ (D) $\left[Al(H_2O)_6(OH)_3\right]$

Ans. **B**

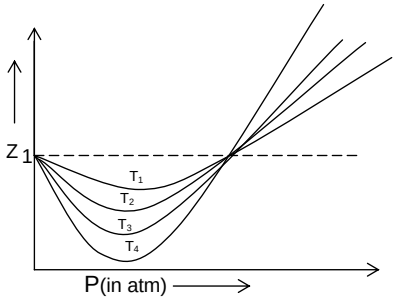
Sol. Al can accommodate $4OH^-$ ions around the metal

36. Phosphine can be prepared by the reaction of water with

(A) calcium phosphide (B) calcium hydride
(C) calcium dihydrogen phosphate (D) calcium phosphate

Ans. **A**

Sol. $Ca_3P_2 + 6H_2O \rightarrow 3Ca(OH)_2 + 2PH_3$

37. 

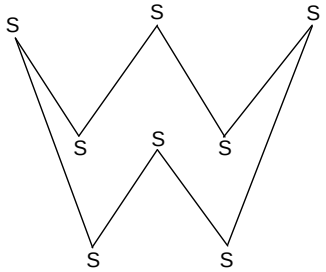
Above curves are given for N_2 gas at different temperature. The highest temperature observed in the above figure is:

(A) T_1 (B) T_2

(C) T_3. (D) T_4

Ans. **A**

Sol. As temperature increases, deviation from ideal behaviour decreases.

38. 

What is the hybridization of sulphur in the above crown ring structure?

(A) sp (B) sp^2

(C) sp^3 (D) sp^3d

Ans. **C**

Sol. Fact based.

39. Which of the following does not illustrate the anomalous properties of lithium?

(A) The melting point and boiling point of Li are comparatively high

(B) Li is much softer than the other group I metals

(C) Li forms a nitride Li_3N unlike group I metal

(D) The ion of Li and its compounds are more heavily hydrated than those of the rest of the group

Ans. **B**

Sol. Actually Li is harder than other alkali metals.

40. For which of the following, the orbital angular momentum for the last filled electron (as per Aufbau principle) is zero?

(A) Mg^{2+} (B) Fe

(C) K (D) Cs^+

Ans. **C**

Sol. $K = [Ar]4s^1$

$$\mu = \sqrt{\ell(\ell+1)} = \frac{h}{2\pi} = 0$$

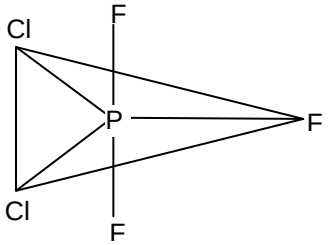
41. Which of the following has non-zero dipole moment?

(A) PCl_5 (B) PF_5

(C) PCl_3F_2 (D) PCl_2F_3

Ans. **D**

Sol. The structure of $PCl_2F_3$is



The resultant dipole moment is not zero.

42. From the following list of atoms, choose the number of pairs of isotopes, isobars and isotones respectively

$^{16}_{8}O$, $^{39}_{19}K$, $^{235}_{92}U$, $^{40}_{19}K$, $^{14}_{7}N$, $^{18}_{8}O$, $^{14}_{6}C$, $^{40}_{20}Ca$, $^{238}_{92}U$

(A) 3, 2, 2 (B) 2, 3, 2

(C) 2, 2, 3 (D) 2, 2, 2

Ans. **A**

Sol. Isotopes : $\left(^{16}_{8}O, ^{18}_{8}O\right), \left(^{39}_{19}K, ^{40}_{19}K\right), \left(^{235}_{92}U, ^{238}_{92}U\right)$

Isobars : $\left(^{40}_{19}K, ^{40}_{20}Ca\right), \left(^{14}_{7}N, ^{14}_{6}c\right)$

Isotones : $\left(^{39}_{19}K, ^{40}_{20}Ca\right), \left(^{14}_{6}C, ^{16}_{8}O\right)$

43. In which of the following pairs, both the species have the same hybridization?

(I) SF_4, XeF_4 (II) I_3^-, XeF_2

(III) $ICl_4^+, SiCl_4$ (IV) ClO_3^-, PO_4^{3-}

(A) I, II (B) II, III

(C) II, IV (D) I, II, III

Ans. **C**

Sol. $I_3^- \rightarrow sp^3d$ $SF_4 \rightarrow sp^3d$

$XeF_2 \rightarrow sp^3d$ $XeF_4 \rightarrow sp^3d^2$

$ICl_4^+ \rightarrow sp^3d$ $ClO_3^- \rightarrow sp^3$

$SiCl_4 \rightarrow sp^3$ $PO_4^{3-} \rightarrow sp^3$

44. Dimethyl ether decomposes as

$$CH_3OCH_3(g) \rightarrow CH_4(g) + CO(g) + H_2(g)$$

When CH_3OCH_3 decomposes to 20% extent under certain conditions, what is the ratio of diffusion of pure CH_3OCH_3 with methane?

(A) 0.59:1 (B) 1.18:1

(C) 2.36:1 (D) 1.77:1

Ans. **C**

Sol. $$\frac{r_{CH_3OCH_3}}{r_{CH_4}} = \sqrt{\frac{M_{CH_4}}{M_{CH_3OCH_3}}} \times \frac{P_{CH_3OCH_3}}{P_{CH_4}}$$

$$\sqrt{\frac{16}{46}} \times \frac{0.8}{0.2} = 2.36:1$$

45. In which of the following the oxidation number of oxygen has been arranged in increasing order:

(A) $OF_2 < KO_2 < BaO_2 < O_3$ (B) $BaO_2 < KO_2 < O_3 < OF_2$

(C) $BaO_2 < O_3 < OF_2 < KO_2$ (D) $KO_2 < OF_2 < O_3 < BaO_2$

Ans. **A**

Sol.

-1	-1/2	0	+2
$\underline{Ba}O_2$ <	$\underline{K}O_2$ <	$\underline{O}_2$ <	$\underline{O}F_2$

SECTION – B
Numerical Based Questions 0 to 9 Answer Type (46 – 47)

46. The difference in number of angular nodes and number of radial nodes in the orbital to which last electron of chromium present

Ans. **2**

Sol. No. of angular nodes in 3d = 2

Radial nodes in 3d = 0

Difference = 2

47. On dissolving 2.0 g of metal in sulphuric acid 4.51 g of metal sulphate was formed. The specific heat of the metal is 0.057 cal/g/^oC. The valency of meal is

Ans. **3**

Sol. Let metal sulphate is $M_2(SO_4)_x$

x = valency of metal

Specific heat $\times$ molar mass = 6.4

Molar mass = $\frac{6.4}{0.057} = 112.2\ g/mol$

POAC

$\frac{2}{112.2} = \frac{4.51}{2\times112.2+96x}\times 2$ $\Rightarrow$ Solving x = 3

SECTION – C
Numerical Answer Type with Answer XXXXX.XX (48 – 50)

48. In diamond lattice carbon atom occupy fcc lattice points as well as alternate tetrahedral voids. If edge length of the unit cel is 356 pm then radius of carbon atom in pm is

Ans. **00077.07**

Sol. $2r = \frac{\sqrt{3}a}{4}$

$r = \frac{\sqrt{3}\times 356}{8} = 77.07$

49. What is the percentage hydrolysis of NaCN in $\frac{N}{80}$ solution when the dissociation constant for HCN is 1.3×10^{-9} and K_w is 1.0×10^{-14}

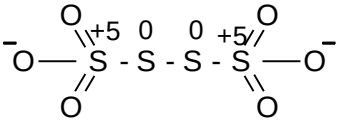
Ans. **00002.48**

Sol. $h = \sqrt{\frac{K_w}{K_a C}} = \sqrt{\frac{1\times10^{-14}\times80}{1.3\times10^{-9}\times1}} = 2.48\%$

50. Oxidation state of sulphur in $S_4O_6^{2-}$ is

Ans. **00002.50**

Sol.



Oxidation state of S = $\frac{10}{4} = 2.5$

PART – C : MATHEMATICS

SECTION – A

Single Correct Choice Type (51-70)

51. If $\left(a, a^2\right)$ falls inside the angle made by the lines $y = \frac{x}{2}, x > 0$ and $y = 3x, x > 0$, then a belong to

(A) $(3, \infty)$ (B) $\left(\frac{1}{2}, 3\right)$

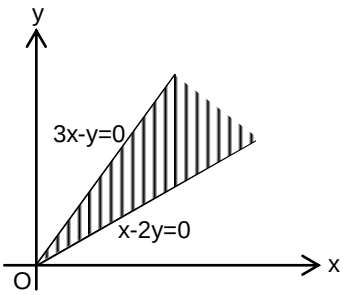
(C) $\left(-3, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{2}\right)$

Ans. B

Sol. The graph of equations $x - 2y = 0$ and $3x - y = 0$ is as shown in the figure. Since, given point $\left(a, a^2\right)$ lies in the shaded region

$\therefore \left(a - 2a^2\right)\left(3a - a^2\right) < 0$

$\Rightarrow a^2(2a-1)(a-3) < 0 \Rightarrow \frac{1}{2} < a < 3$



52. The locus of the middle points of the chords of the circle $x^2 + y^2 = a^2$ which subtend a right angle at the centre, is

(A) $\left(x^2 + y^2\right) - a^2 = 0$ (B) $2\left(x^2 + y^2\right) - 2a^2 = 0$

(C) $2\left(x^2 - y^2\right) + a^2 = 0$ (D) $2\left(x^2 + y^2\right) - a^2 = 0$

Ans. D

Sol. Let (h, k) be the mid point of a chord AB of the circle $x^2 + y^2 = a^2$ Then, the equation of AB is

$hx + ky - a^2 = h^2 + k^2 - a^2$ [using T = S]

$\Rightarrow \quad hx + ky = h^2 + k^2$ ……..(i)

The combined equation of OA and OB is

$$x^2 + y^2 = a^2\left(\frac{hx + ky}{h^2 + k^2}\right)^2$$

$\Rightarrow \left(h^2 + k^2\right)^2\left(x^2 + y^2\right) - a^2(hx + ky)^2 = 0$

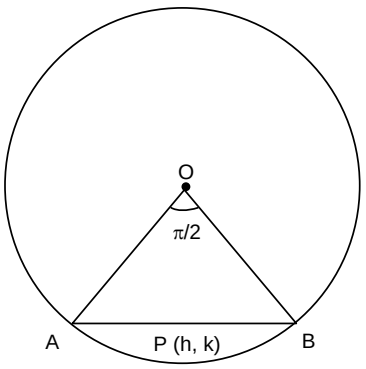
OA and OB will be perpendicular, if

Coefficient of x^2 + Coefficient of $y^2 = 0$

$\Rightarrow \quad \left(h^2 + k^2\right) - a^2h^2 + \left(h^2 + k^2\right)^2 - a^2k^2 = 0$

$\Rightarrow \quad 2\left(h^2 + k^2\right)^2 - a^2\left(h^2 + k^2\right) = 0$

$\Rightarrow \quad 2\left(h^2 + k^2\right) - a^2 = 0$



So, locus of (h, k) is $2\left(x^2 + y^2\right) - a^2 = 0$

53. If a, b are complex numbers and one of the roots of the equation $x^2 + ax + b = 0$ is purely real whereas the other is purely imaginary, and $a^2 - \bar{a}^2 = kb$, then k is

(A) 2 (B) 1
(C) 0 (D) 4

Ans. D

Sol. Let us consider α as the real and $i\beta$ as the imaginary root then

$\alpha + i\beta = -a \Rightarrow \alpha - i\beta = -\bar{a}$

$\Rightarrow 2\alpha = -(a + \bar{a})$ and $2i\beta = -(a - \bar{a})$

$\Rightarrow 4i\alpha\beta = a^2 - \bar{a}^2 \Rightarrow a^2 - \bar{a}^2 = 4b.$

54. Two tangents drawn to parabola at points (3,9) and (9,7) intersect at (2,2) then slope of directrix is

(A) $\frac{3}{2}$ (B) $-\frac{2}{3}$
(C) $-\frac{5}{2}$ (D) $\frac{2}{5}$

Ans. B

Sol. Use the property : "The line segment joining the point of intersection tangents drawn at the points A and B on the parabola to the mid point of the corresponding chord of contact is always parallel to the axis of the parabola". Here, If $A = (3, 9)$, $B = (9, 7)$ and $C = (2, 2)$ and also let M $(6, 8)$ is the mid point of AB. Then slope of CM = $\frac{8-2}{6-2} = \frac{6}{4} = \frac{3}{2}$

$\therefore$ Slope of directrix = $\frac{-2}{3}$

55. If z is a complex number then $|z + 1| = \sqrt{3}|z - 2|$ represents

(A) y-axis (B) a circle
(C) x-axis (D) a line parallel to y-axis

Ans. B

Sol. $|z + 1|^2 = 3|z - 2|^2$
solve it.

56. The number of ways in which 15 boys and 2 girls can sit in a row such that between the girls at the most 2 boys sit is

(A) 17! – (12! × 3!) (B) 17! – ($^{12}C_3 \times 3!$)
(C) 17! – ($^{12}C_3 \times 15!$) (D) 17! – (91× 2! × 15!)

Ans. **D**

Sol. Required no, of ways = the total no of ways – no of ways in which between the girls at least 3 boys sit.
= 17! – coeff. of x^{12} in $(1 - x)^{-3} \times 2! \times 15!$
= 17! – $^{12+3-1}C_2 \times 2! \times 15!$
= 17! – 91× 2! × 15!.

57. The distance of the center of the ellipse $x^2+2y^2 - 2 = 0$ to those tangents of the ellipse which are equally inclined to both the axes is

(A) $\frac{3}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$
(C) $\frac{\sqrt{2}}{3}$ (D) $\sqrt{\frac{3}{2}}$

Ans. **D**

Sol. Equation of ellipse is $\frac{x^2}{2}+\frac{y^2}{1}=1$. General tangent to the ellipse of slope m is y = mx $\pm$ $\sqrt{2m^2+1}$, since this is equally inclined to axes, so m = $\pm$ 1. Thus tangents are
$y = \pm x \pm \sqrt{2+1} = \pm x \pm \sqrt{3}$

Distance of any tangent from origin is then equal to $\sqrt{\frac{3}{2}}$.

58. The sum of the infinite terms of the series $\frac{5}{3^2.7^2}+\frac{9}{7^2.11^2}+\frac{13}{11^{12}.15^2}+.....$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{36}$

(C) $\frac{1}{54}$ (D) $\frac{1}{72}$

Ans. **D**

Sol. $T_n=\frac{5+(n-1)4}{\left[3+(n-1)4\right]^2\left[7+(n-1)4\right]^2}$

$=\frac{1}{8}\left\{\frac{1}{(4n-1)^2}-\frac{1}{(4n+3)^2}\right\}$

$\therefore S_n = T_1+T_2+T_3+.....+T_n$

$=\frac{1}{8}\left\{\frac{1}{3^2}-\frac{1}{7^2}+\frac{1}{7^2}-\frac{1}{11^2}+\frac{1}{11^2}-\frac{1}{15^2}+.....+\frac{1}{(4n-1)^2}-\frac{1}{(4n+3)^2}\right\}=\frac{1}{8}\left[\frac{1}{3^2}-\frac{1}{(4n+3)^2}\right]$

$\therefore S_\infty=\frac{1}{8}\left\{\frac{1}{9}-0\right\}$

$=\frac{1}{72}$

59. A curve that passes through (2, 4) and having subnormal of constant length of 8 units can be;

(A) $y^2 = 16x - 16$ (B) $y^2 = 16x + 24$

(C) $x^2 = 16y - 60$ (D) $x^2 = -16y + 68$

Ans. **A**

Sol. Let the curve be y = f(x). Subnormal at any point = $\left|y\frac{dy}{dx}\right|$

$\Rightarrow y\frac{dy}{dx} = \pm 8 \Rightarrow y\,dy = \pm 8dx \Rightarrow \frac{y^2}{2} = \pm 8x + c$

$\Rightarrow y^2 = 16x + 2c_1, \Rightarrow c_1 = -8$ or $y^2 = -16x + 2c_2, \Rightarrow c_2 = 24$

Hence (A) are correct answers.

60. The degree of differential equation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x\log\frac{d^2y}{dx^2}$ is

(A) 1 (B) 2

(C) 3 (D) none of these

Ans. **D**

Sol. Since the equation is not a polynomial in all the differential coefficient so the degree of equation is not defined.

Hence (D) is the correct answer.

61. $\lim\limits_{x\to-\infty} \dfrac{x^4 \sin\left(\dfrac{1}{x}\right)+x^2}{1+|x|^3}$ equals

(A) –1 (B) 0

(C) 2 (D) 1

Ans. **A**

Sol. $\lim\limits_{x\to-\infty} \dfrac{\dfrac{\sin(1/x)}{1/x}+\dfrac{1}{x}}{\dfrac{1}{x^3}+\dfrac{|x|^3}{x^3}} = \dfrac{1+0}{0-1} = -1$

Hence (A) is the correct answer.

62. A pair of dice is thrown independenty three times. The probability of getting a score of 9 exactly twice is

(A) $\dfrac{1}{729}$ (B) $\dfrac{8}{9}$

(C) $\dfrac{8}{729}$ (D) $\dfrac{8}{243}$

Ans. **D**

Sol. Probability of getting score 9 in a single throw $= \dfrac{4}{36} = \dfrac{1}{9}$

Probability of getting score 9 exactly twice $= {}^3C_2 \times \left(\dfrac{1}{9}\right)^2 \times \dfrac{8}{9} = \dfrac{8}{243}$

63. The value of $\int\limits_0^1 |\sin 2\pi x|\, dx$ is equal to

(A)0 (B) $\dfrac{2}{\pi}$

(C) $\dfrac{1}{\pi}$ (D) 2

Ans. **B**

Sol. Since $|\sin 2\pi x|$ is periodic with period $\dfrac{1}{2}$,

$I = \int\limits_0^1 |\sin 2\pi x|\, dx = 2\int\limits_0^{\frac{1}{2}} \sin 2\pi x\, dx$

$= 2\left[-\dfrac{\cos 2\pi x}{2\pi}\right]_0^{\frac{1}{2}} = \dfrac{2}{\pi}$

Hence (B) is correct answer.

64. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for

(A) exactly two values of θ (B) more than two values of θ

(C) no value of θ (D) exactly one value of θ

Ans. **D**

Sol: $|2\hat{u}\times 3\hat{v}| = 1$

$6|\hat{u}||\hat{v}||\sin\theta| = 1$

$\sin\theta = \dfrac{1}{6}$

Hence there is exactly one value of θ for which $2\hat{u}\times 3\hat{v}$ is a unit vector.

65. The equation of the plane through the origin and parallel to the plane $3x-4y+5z-6=0$ is

(A) $3x-4y+5z+6=0$ (B) $3x+4y-5z+6=0$

(C) $3x-4y-5z-6=0$ (D) $3x-4y+5z=0$

Ans. **D**

Sol. Plane has normal's drs as (3, –4, 5) and pass from (0, 0, 0).

66. Let $A=\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}$. If $|A^2|=25$, then $|\alpha|$ equals

(A) 5^2 (B) 1

(C) $\frac{1}{5}$ (D) 5

Ans. **C**

Sol: $A^2=\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}$.

$A^2=\begin{bmatrix}25 & 25\alpha+5\alpha^2 & 5\alpha+25\alpha^2+5\alpha\\ 0 & \alpha^2 & 5\alpha^2+25\alpha\\ 0 & 0 & 25\end{bmatrix}$

$625\alpha^2=25$

$\Rightarrow|\alpha|=\frac{1}{5}$

67. The truth table of $p\wedge(\sim q)\Rightarrow p$ is

p	q	$p\wedge(\sim q)\Rightarrow p$
T	T	T
T	F	T
F	T	T
F	F	T

(A)

p	q	$p\wedge(\sim q)\Rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

(B)

p	q	$p\wedge(\sim q)\Rightarrow p$
T	T	T
T	F	T
F	T	T
F	F	F

(C)

p	q	$p\wedge(\sim q)\Rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	F

(D)

Ans. **A**

Sol. The truth table of $p\Rightarrow q$ is

p	q	$p\Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

68. The standard deviations of 25 number of 40. If each of the number of increased by 5, then the new standard deviation will be

(A) 40 (B) 25

(C) $\sqrt{40}$ (D) 1600

Ans. **A**

Sol. If each item of a data is increased or decreased by the same constant, the standard deviation of the data remains unchanged.

69. If the function $f(x) = ax^3 + bx^2 + 11x - 6$ satisfies the Rolle's theorem in [1, 3] and $f'\left(2 + \frac{1}{\sqrt{3}}\right) = 0$, then the values of a and b are respectively

(A) –6, 1 (B) –2, 1

(C) 1, –2 (D) 1, –6

Ans. **D**

Sol. By Rolle' s theorem

$f(1) = f(3)$

$\Rightarrow \quad a + b + 11 - 6 = 27a + 9b + 33 - 6$

$\Rightarrow \quad 13a + 4b + 11 = 0$ ……(i)

Now, $f'(x) = 3ax^2 + 2bx + 11$

So, $f'\left(2 + \frac{1}{\sqrt{3}}\right) = 0$

$\Rightarrow 3a\left(2 + \frac{1}{\sqrt{3}}\right)^2 + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + 11 = 0$

Using equation (i)

$\Rightarrow \quad 6a + b = 0$ ……(ii)

$\therefore$ On solving (i) and (ii), we get $a = 1, b = -6$

70. AB is a vertical pole. The end A is on the level ground. C is the middle point of AB, P is a point on the level ground. The portion BC subtends an angle β at P. If AP = nAB, then the $\tan\beta$ =

(A) $\frac{n}{2n^2 + 1}$ (B) $\frac{n}{n^2 - 1}$

(C) $n\frac{n}{n^2 + 1}$ (D) None of these

Ans. **A**

Sol. Let $\angle APC = \alpha$. Then

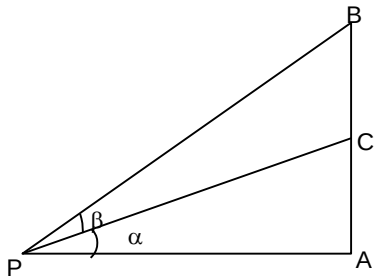
$\tan\alpha = \frac{AC}{AP} = \frac{AC}{nAB} = \frac{AB}{2nAB} = \frac{1}{2n}$

InΔ APB, $\tan(\alpha + \beta) = \frac{AB}{AP} = \frac{AB}{nAB} = \frac{1}{n}$

Now, $\beta = \alpha + \beta - \alpha$

$\Rightarrow \tan\beta = \frac{\tan(\alpha + \beta) - \tan\alpha}{1 + \tan(\alpha + \beta)\tan\alpha}$

=



$\frac{1/n - 1/2n}{1 + 1/n \cdot 1/2n} = \frac{n}{2n^2 + 1}$.

Hence (A) is the correct answer

SECTION – B
Numerical Based Questions 0 to 9 Answer Type (71 – 72)

71. If the sum of the slope of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value

Ans. **2**

Sol. The given pair of lines is $x^2 - 2cxy - 7y^2 = 0$

Given, $m_1 + m_2 = 4m_1 m_2$

$\Rightarrow \quad \frac{2c}{-7} = 4.\frac{1}{-7}$

$\Rightarrow \quad c = 2$

72. If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is equal to

Ans. **2**

Sol. $|9z_1z_2 + 4z_1z_2 + z_1z_3| = |z_3\bar{z}_3\, z_1z_2 + z_2\bar{z}_2z_1z_3 + z_1\bar{z}_1z_2z_3| = 12$

Or, $|z_1z_2z_3(\bar{z}_1 + \bar{z}_2 + \bar{z}_3)| = 12$ or, $|z_1||z_2||z_3|\left|\overline{z_1 + z_2 + z_3}\right| = 12$

Or, $\left|\overline{z_1 + z_2 + z_3}\right| = |z_1 + z_2 + z_3| = \frac{12}{3 \times 2} = 2$

SECTION – C
Numerical Answer Type with Answer XXXXX.XX (73 – 75)

73. The value of the expression $\log_2\left(1 + \frac{1}{2}\sum_{k=1}^{11} {}^{12}C_k\right)$

Ans. **00011.00**

Sol. $E = 1 + \frac{1}{2}\left[{}^{12}C_1 + {}^{12}C_2 + ... + {}^{12}C_{11}\right] = 1 + \frac{1}{2}\left[{}^{12}C_1 + ... + {}^{12}C_{10} + {}^{12}C_{12} - 1\right]$

$= 1 + \frac{1}{2}\left[2^{12} - 2\right] = 1 + 2^{11} - 1 = 2^{11}; \qquad \therefore \log_2 E = 11$

74. The area of the triangle inscribed in the parabola $y^2 = 4x$, the ordinates of whose vertices are 1, 2 and 4 is

Ans. **00000.75**

Sol. Area of the Δ inscribed in the parabola $= \frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$

75. If the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide then length of latus rectum of ellipse is

Ans. **00003.50**

Sol. Let eccentricity of ellipse and hyperbola be e_1 and e_2 respectively, then

$b^2 = 16\left(1 - e_1^2\right)$ and $81 = 144\left(e_2^2 - 1\right)$

$\Rightarrow e_1^2 = 1 - \frac{b^2}{16}$ and $e_2^2 = \frac{81}{144} + 1 = \frac{225}{144} \Rightarrow e_2 = \frac{15}{12}$

Now given $4e_1 = \frac{12}{5}e_2 \Rightarrow 4\sqrt{1 - \frac{b^2}{16}} = \frac{12}{5} \times \frac{15}{12} = 3$

$1 - \frac{b^2}{16} = \frac{9}{16} \Rightarrow b^2 = \frac{7}{16} \times 16 \Rightarrow b^2 = 7.$

Latus rectum of ellipse $= \frac{2b^2}{a} = \frac{2 \times 7}{4} = \frac{7}{2}$