# FIITJEE – JEE (Main)

## Physics, Chemistry & Mathematics

**QP Code:**

**Time Allotted: 3 Hours** **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

**BATCHES – 2022 A-Lot_Pt-2**

### Important Instructions

**Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.**

### A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & B** in the OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. No candidate is allowed to carry any textual material, printed or written, bits of papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices ext. except the Admit Card inside the examination hall / room.

### B. Filling of OMR Sheet:

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.
4. **Do not fold or make any stray marks on the Answer Sheet.**

### C. Marking Scheme for All Two Parts:

**(i)** **Part-A (01-20) –** Contains Twenty (20) multiple choice objective questions which have four (4) options each and only one correct option. Each question carries **+4 marks** which will be awarded for every correct answer and **-1 mark** will be deducted for every incorrect answer.

**(ii)** **Part-B (01-05)** contains five (05) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+4 marks** for correct answer and **there will be no negative marking.**

**Name of the Candidate :**_____

**Batch :**_____ **Date of Examination :**_____

**Enrolment Number :**_____

# SECTION – I : PHYSICS

## (PART – A)

### Single Correct Questions

This section contains 20 Single Correct Questions out of which only one option is correct

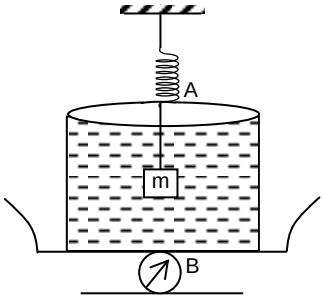
1. The linear velocity of a particle moving with angular velocity $\vec{\omega} = 2\hat{k}$ at position vector $\vec{r} = 2\hat{i} + 2\hat{j}$ is

(A) $4\left(\hat{i} + \hat{j}\right)$ (B) $4\left(\hat{j} - \hat{i}\right)$

(C) $4\left(-\hat{i} - \hat{j}\right)$ (D) $4\left(\hat{i} - \hat{j}\right)$

1. **B**

2. The spring balance A reads 2kg with a block of mass m suspended from it. A balances B reads 5kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in a beaker as shown in figure




(A) the balance A will read more than 2kg

(B) the balance B will read less than 5kg

(C) the balance A will read less than 2kg and B will read more than 5kg

(D) the balance A will read more than 2kg and B will read less than 5kg

2. **C**

3. A cylindrical object floats in water such that $\frac{3}{4}$th of its volume is immersed in water. Its density is _____ kg $m^{-3}$.

(A) 250 (B) 0.75

(C) 0.25 (D) 750

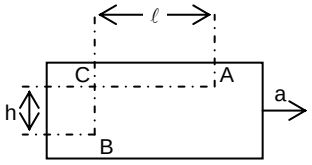
3. **D**

4. A cubical block of wood of specific gravity 0.5 and chunk of concrete of specific gravity 2.5 are fastened together. The ratio of mass of concrete to the mass of wood which makes the combination to float with its entire volume submerged in water is

(A) $\frac{5}{3}$ (B) $\frac{5}{1}$ (C) $\frac{5}{2}$ (D) $\frac{4}{3}$

4. **A**

5. A sealed tank containing a liquid of density $\rho$ moves with a horizontal acceleration a, as shown in the figure. The difference in pressure between the points A and B is




(A) $h\rho g$ (B) $l\rho a$

(C) $h\rho g - \ell\rho a$ (D) $h\rho g + \ell\rho a$

5. **D**

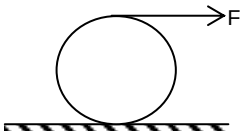
6. A uniform rod of density $\rho$ is placed in a wide tank containing a liquid of density $\rho_0 \left(\rho_0 > \rho\right)$. The depth of liquid in the tank is half the length of the rod. The rod is in equilibrium, with its lower end resting on the bottom of the tank. In this position the rod makes an angle $\theta$ with the horizontal.

(A) $\sin\theta = \frac{1}{2}\sqrt{\rho_0 / \rho}$ (B) $\sin\theta = \frac{1}{2}.\frac{\rho_0}{\rho}$

(C) $\sin\theta = \sqrt{\rho / \rho_0}$ (D) $\sin\theta = \rho_0 / \rho$

6. **A**

7. A force F is applied at the top of a ring of mass M and radius R placed on a rough horizontal surface as shown in figure. Friction is sufficient to prevent slipping. The frictional force acting on the ring is

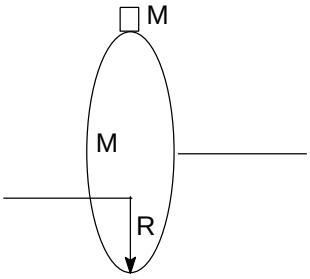



(A) $\frac{F}{2}$ towards right (B) $\frac{F}{2}$ towards left

(C) $\frac{2F}{3}$ towards right (D) zero

7. **D**

8. A uniform disc of mass M, object of mass M is attached to the rim and raised to the highest point above the centre. The unstable system is then released. The angular speed of the system when the attached object passes directly beneath the pivot is.




(A) $\sqrt{\frac{3R}{8g}}$ (B) $\sqrt{\frac{3g}{8R}}$ (C) $\sqrt{\frac{8R}{3g}}$ (D) $\sqrt{\frac{8g}{3R}}$

8. **D**

9. A boy is standing at the centre of a boat which is free to move on water. If the masses of the boy and the boat are $m_1$ & $m_2$ respectively and the boy move a distance of 1m forward then the movement of the boat is in meters

(A) $\frac{m_1}{m_1+m_2}$ (B) $\frac{m_2}{m_1+m_2}$ (C) $\frac{m_1}{m_2}$ (D) None of these

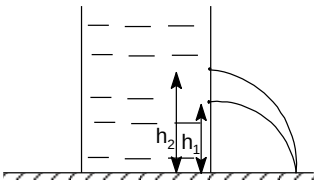
9. **A**

10. A bullet of mass 'm' strikes a pendulum bob of mass 2m with velocity V. It passes through and emerges out with a velocity $\left(\frac{v}{2}\right)$ from bob. If the length of pendulum is l. what should be minimum value of v if the pendulum bob will swing through a complete circle.

(A) $4\sqrt{5gl}$ (B) $\sqrt{5gl}$ (C) $8\sqrt{5gl}$ (D) None of these

10. **A**

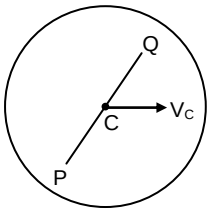
11. In a cylindrical vassal containing liquid as shown in figure. for these two holes at $h_1$ & $h_2$, Range of efflux is same at the bottom of vassal. The height of hole, for which range of efflux would be maximum will be




(A) $h_2 - h_1$ (B) $h_2 + h_1$ (C) $\frac{h_2-h_1}{2}$ (D) $\frac{h_2+h_1}{2}$

11. **D**

12. A disc is rolling (without slipping) on a horizontal surface. C is the centre and Q and P are two points equidistant from C. Let $V_P$, $V_Q$ and $V_C$ be the velocity of P, Q and C respectively, then

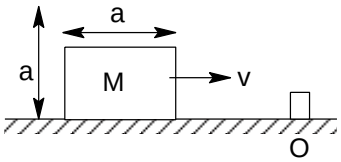



(A) $V_Q > V_C > V_P$ (B) $V_Q < V_C < V_P$

(C) $V_Q = V_P, V_C = \frac{V_P}{2}$ (D) $V_Q < V_C > V_P$

12. **A**

13. A cubical block of side a moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits o is

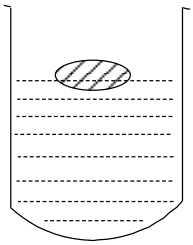



(A) 3V / 4a (B) 3V/2a

(C) $\sqrt{3}$ / $\sqrt{2}$a (D) Zero

13. **A**

14. A body floats in a liquid contained in a beaher. The whole system as shown in figure falls freely under gravity. The upthrust on the body is.



(A) Zero
(B) equal to the weight of liquid displead
(C) equal to the weight of body in air
(D) equal to the weight of the immersed portion of the body.

14. **A**

15. A spherical drop of liquid having surface tension T contains 9 $cm^3$ of liquid. The drop is broken into 1000 identical drops. The work done by the breaking agent is.

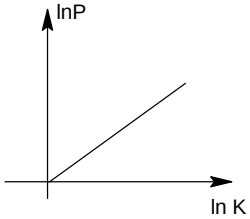
(A) $9(4\pi)^{1/3}$ T erg (B) $81(4\pi)^{1/3}$ T erg

(C) $27(4\pi)^{1/3}$ T erg (D) None of these

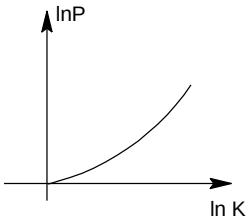
15. **B**

16. Which of the following graph represents the graphical relation between momentum (p) and kinetic energy (k) for a body in motion.

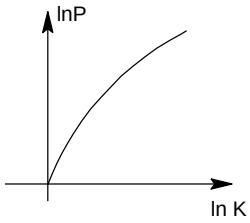
(A)




(B)



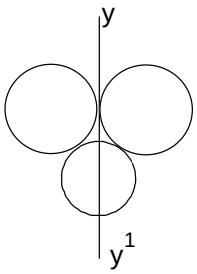
(C)




(D) None of these

16. **D**

17. Three rings of mass P and radius R are arranged of as shown in figure. The moment of inertia of the arrangement about yy' axis will be.




(A) $\frac{7}{2}PQ^2$ (B) $\frac{3}{7}PQ^2$

(C) $\frac{2}{5}PQ^2$ (D) $\frac{5}{2}PQ^2$

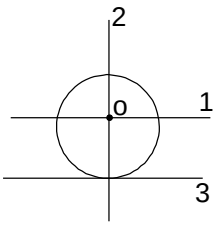
17. **A**

18. The angular velocity of a body is $\vec{w} = 2\hat{i}+3\hat{j}+4\hat{k}$ and a torque $\vec{\tau} = \hat{i}+2\hat{j}+3\hat{k}$ acts on it. The rational power will be.

(A) 20 watt (B) 15 watt (C) $\sqrt{17}$ watt (D) $\sqrt{14}$ watt

18. **A**

19. Figure shows a uniform sphere with centre O. moment of inertia about axes 0,1,2,3 are I, $I_1$, $I_2$, $I_3$ respectively which one is the correct option




(A) $I = I_1 + I_2$

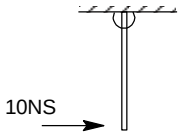
(B) $I = I_1 = I_2 < I_3$

(C) Both (A) and (B) are correct

(D) Both (A) and (B) are wrong

19. **B**

20. A thin uniform straight rod of mass 2 kg and length 1m is free to rotate about its upper end at rest. It receives an impulsive blow of 10NS at its lowest point normal to its length as shown in figure. the kinetic energy of rod just after impact is.




(A) 75 J (B) 100J

(C) 200J (D) None of these

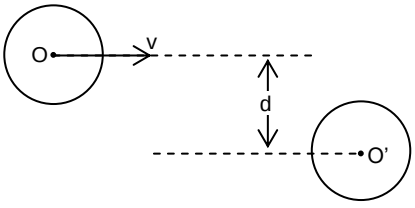
20. **A**

## (PART – B)

**(Integer Type)**

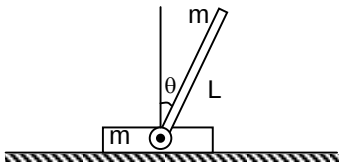
**Part-C (01-05)** contains five (05) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+4 marks** for correct answer and **there will be no negative marking.**

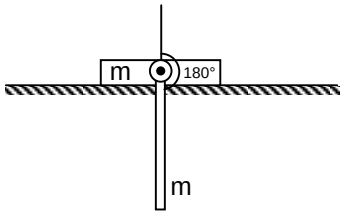
21. A disc of radius r and mass m moving on perfectly smooth surface at a speed v= $\frac{15}{\sqrt{17}}$ m/s undergoes an elastic collision with an identical stationary disc of mass 2m. the magnitude of velocity (in m/s) of the first disc after the collision will be( given d= $\frac{8r}{5}$ )




21. **3**

22. A rod of mass *m* and length *L* (= 5/3m) is hinged in plank of same mass *m*. The plank is kept on a smooth horizontal surface and rod makes $\theta$ with the vertical. The system is released from $\theta = 0°$. Find the velocity(in m/s) of plank when the rod makes $\theta = 180°$. ($g$ = 10 m/s$^2$)




22. **1**

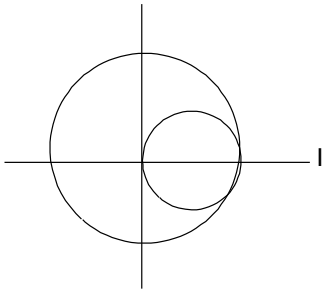
23. A body of mass 4 kg floats in a liquid. What is the value of m if buoyant force acting on the body is $\frac{49}{10}$m newton?

23. **8**

24. A force $\vec{f} = \left(\hat{i} - \hat{j} + \hat{k}\right)$N acts at P (2, 3, –1). The magnitude of torque of force $\vec{f}$ about Q (1, 0, 1) is nearly equal to

24. **5.10**
Range: 5.00 to 5.20

25. A uniform disc of surface mass density ($\sigma$) exists in space. Its radius is R. A small disc of radius $\left(\frac{R}{2}\right)$ is cut from it as shown in the figure. The moment of inertia (I) about axis perpendicular to plane of figure and passing through the centre of disc of radius 'R' is $\frac{3}{k}\sigma\pi R^4$. Find the value of k.




25. **7.38**
Range: 7.31 to 7.40

# SECTION – II : CHEMISTRY

## (PART – A)

### Single Correct Questions

This section contains 20 Single Correct Questions out of which only one option is correct

1. At constant temperature, the equilibrium constant ($K_p$) for the decomposition reaction $N_2O_4 \rightleftharpoons 2NO_2$ is expressed by
$K_p = \left(4x^2p\right)/\left(1-x^2\right)$, where p = pressure, which of the following statement is true?
(A) $K_p$ increases with increase of p
(B) $K_p$ increases with increases of x
(C) $K_p$ increases with decreases of x
(D) $K_p$ remains constant with change in p and x.

1. **D**

2. Four solution of $NH_4Cl$ are taken with concentrations 1 M, 0.1 M, 0.01 M, & 0.001 M. Their degree of hydrolysis are $h_1$, $h_2$, $h_3$ & $h_4$ respectively. What is the gradation of degree of hydrolysis?
(A) $h_1 > h_2 > h_3 > h_4$
(B) $h_1 = h_2 = h_3 = h_4$
(C) $h_4 > h_3 > h_2 > h_1$
(D) none of these

2. **C**

3. When equal volume of the following solutions are mixed precipitation of AgCl ($K_{sp}$ = 1.8 $\times$ $10^{-10}$) will occur only with
(A) $10^{-4}M\,Ag^+ + 10^{-4}M\,Cl^-$
(B) $10^{-5}M\,Ag^+ + 10^{-5}M\,Cl^-$
(C) $10^{-6}M\,Ag^+ + 10^{-6}M\,Cl^-$
(D) $10^{-10}M\,Ag^+ + 10^{-10}M\,Cl^-$

3. **A**

4. Which of the following change takes place in increasing the temperature of a chemical reaction?
(A) Half – life increases
(B) Activation energy decreases
(C) More number of reactant molecules cross the energy barrier
(D) Energy of transition state increases

4. **C**

5. At a temp, T, a compound $AB_4(g)$ dissociates as $2AB_4(g) \rightleftharpoons A_2(g) + 4B_2(g)$ with a degree of dissociation x, which is small compared with unity. The expression of $K_P$ in terms of x and total pressure P is:
(A) $8P^3x^5$
(B) $256P^3x^5$
(C) $4Px^2$
(D) None of these

5. **A**

6. Which of the following is not known?
(A) $KHF_2$
(B) $PbCl_2$
(C) $PbI_4$
(D) $BiF_5$

6. **C**

7. The volume of 0.2 M NaOH needed to prepare a buffer of pH 4.74 with 50 mL of 0.2 M acetic acid ($pK_b$ of $CH_3COO^-$ = 9.26) is :
(A) 50 mL (B) 25 mL
(C) 20 mL (D) 10 mL

7. **B**

8. In the reaction: $2X + B_2H_6 \rightarrow \left[BH_2(X)_2\right]^+ [BH_4]^-$. Which of the following amine does not give reaction?
(A) $NH_3$ (B) $CH_3NH_2$
(C) $(CH_3)_2NH$ (D) $(CH_3)_3N$

8. **D**

9. What is the conjugate base of $NH_4^+$ ion?
(A) $NH_3$ (B) $NH_5^{2+}$
(C) $NH_2^-$ (D) $NH^{2-}$

9. **A**

10. Which of the following reaction produces a most thermally stable product?
(A) $2X + Y \longrightarrow 3Z + 100kJ$ (B) $P + Q \longrightarrow 3R + 200kJ$
(C) $R + S + 100kJ \longrightarrow 2T$ (D) $X + W \longrightarrow 2Y + 400kJ$

10. **D**

11. Which of the following substance on combustion produces pure $CO_2$ gas?
(A) Coal (B) Petroleum
(C) Coke (D) Fuel oil

11. **C**

12. For which of the following reaction , $K_P = K_C$?
(A) $2H_2(g) + O_2(g) \rightleftharpoons 2H_2O(\ell)$ (B) $H_2(g) + Cl_2(g) \rightleftharpoons 2HCl(g)$
(C) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$ (D) $H_2(g) \rightleftharpoons H(g) + H(g)$

12. **B**

13. Which of the following compound illustrates the strongest $p\pi$ - $d\pi$ bond?
(A) $CCl_4$ (B) $SiCl_4$
(C) $GeCl_4$ (D) $SnCl_4$

13. **B**

14. Which of the following solution will display common ion effect if HCl is added to it?
(A) KH (B) HCN
(C) $NH_4OH$ (D) $HNO_3$

14. **B**

15. For a certain reaction, the product of rate constant and half-life period is a constant which changes only with temperature. Therefore, the order of reaction is:
(A) zero (B) first
(C) second (D) third

15. **B**

16. 0.5 mole of each of $NH_4Cl(s)$ and $NH_3(g)$ were taken in a vessel and the mixture was heated. If the equilibrium gaseous mole in the container is 0.8. How many moles of HCl was produced.

$$NH_4Cl(s) \rightleftharpoons NH_3(g) + HCl(g)$$

(A) 0.4 (B) 0.3
(C) 0.15 (D) 0.24

16. **C**

17. Which group-13 oxide is most basic?
(A) $B_2O_3$ (B) $Al_2O_3$
(C) $Ga_2O_3$ (D) $Tl_2O$

17. **D**

18. Which of the following thermodynamic quantity is zero for a reaction at equilibrium?
(A) Enthalpy (B) Entropy (system)
(C) Free energy (D) Internal energy

18. **C**

19. The variation of rate constant of a chemical reaction with temperature is given by the relation:
(A) $k = \frac{A \times E_a}{RT}$ (B) $k = A\, e^{-E_a/RT}$
(C) $k = E_a \times e^{-A/RT}$ (D) $k = \frac{RT}{A} e^{-E_a}$

19. **B**

20. $NH_4OH$ is a weak base due to presence of hydrogen bonding between
(A) $NH_4^+$ and $H_2O$ (B) $NH_3$ and $H^+$
(C) $NH_4^+$ and $OH^-$ (D) $OH^-$ and $H_2O$

20. **C**

## (PART – B)

**(Integer Type)**

**Part-C (01-05)** contains five (05) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+4 marks** for correct answer and **there will be no negative marking.**

1. What is the pH of 0.2 M $CH_3COONH_4$ solution?
[Given $K_a$ of $CH_3COOH$ = $K_b$ of $NH_4OH$ = $10^{-5}$]

1. **7**

2. The half-life of a zero order reaction become 2 min, when it starts with an initial concentration of 8 M of the reactant. What is the rate constant, k of the reaction in mol $L^{-1}s^{-1}$ unit?

2. **2**

3. If the oxidation number of silicon in silicon carbide is +x, the value of 'x' is:

3. **4**

4. The bond energy of $X_2$, $Y_2$ and XY gases are respectively 4, 7 and 3 kJ $mol^{-1}$. What will be the enthalpy change of the following reaction in kJ unit?

$$X_2(g) + Y_2(g) \longrightarrow 2XY(g)$$

4. **5**

5. What is the pH of $10^{-2}$ M NaOH solution?

5. **12**

# SECTION – I : MATHEMATICS

## (PART – A)

### Single Correct Questions

This section contains 20 Single Correct Questions out of which only one option is correct

1. The solution set of $\log_{1/2}(2^{x+2} - 4^x) \geq -2$ is

(A) $(-\infty, 2 - \sqrt{13})$ (B) $(-\infty, 2 + \sqrt{13})$

(C) $(-\infty, 2)$ (D) none of these

**1. C**

Sol: $\log_{1/2}(2^{x+2} - 4^x) \geq -2 \Rightarrow 2^{x+2} - 4^x \leq \left(\frac{1}{2}\right)^{-2} \Rightarrow 2^{2x} - 4\,2^x + 4 \geq 0$

$\Rightarrow (2^x - 2)^2 \geq 0$ which is always true

also $2^{x+2} - 4^x > 0 \Rightarrow 2^x(4 - 2^x) > 0 \Rightarrow 2^x(2^x - 4) < 0$

$\Rightarrow 2^x < 4 \Rightarrow x < 2$ so required interval = $(-\infty, 2)$

Hence (C) is the correct answer

2. The equations $ax^2 + bx + a = 0$, $x^3 - 2x^2 + 2x - 1 = 0$ have two roots in common. Then $a + b$ must be equal to

(A) 1 (B) –1

(C) 0 (D) none of these

2. **C**

Sol: $x^3 - 2x^2 + 2x - 1 = 0 \Rightarrow (x - 1)(x^2 - x + 1) = 0$ … (1)

$ax^2 + bx + a = 0$ … (2)

since equation (1) having one real and two imaginary roots. So equation (2) will also be having two same imaginary roots

So sum of the roots will be same for equation (2) and for equation $x^2 - x + 1 = 0$

or $-\frac{b}{a} = 1 \Rightarrow a + b = 0$

Hence (C) is the correct answer

3. A parabola having directrix $x + y + 2 = 0$ touches a line $2x + y - 5 = 0$ at (2, 1). Then the semi latus rectum of the parabola is

(A) $\frac{8}{\sqrt{2}}$ (B) $\frac{9}{\sqrt{2}}$ (C) $\frac{10}{\sqrt{2}}$ (D) None of these

**3. B**

**Sol.** Find foot of perpendicular from (2, 1) on the directrix say (α, β). Now, image of (α, β) about tangent is focus

On solving (α,β)=$\left(-\frac{1}{2}, \frac{3}{2}\right)$; focus $\left(\frac{11}{2}, \frac{3}{2}\right)$

4. PQ and PR are tangents drawn from a point P to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$. If equation of QR is $4x - 3y - 6 = 0$, then the coordinates of P are

(A) (2, 6) (B) (6, 2)

(C) (3, 4) (D) none of these

**4. B**

Sol: Let the coordinates of P be (h, k). Then equation of QR is

$\frac{hx}{9} - \frac{ky}{4} = 1$ …(1)

But equation of QR is

$4x - 3y = 6$ …(2)

$\Rightarrow \frac{h}{36} = \frac{k}{12} = \frac{1}{6} \Rightarrow h = 6, k = 2.$

Hence (B) is the correct answer.

5. Locus of mid-point of chords of the parabola $y^2 = 4ax$ that pass through the point (3a, a) is

(A) $y^2 + 2ax + ay - 6a^2 = 0$ (B) $y^2 + 2ax - ay + 6a^2 = 0$

(C) $y^2 - 2ax + ay + 6a^2 = 0$ (D) $y^2 - 2ax - ay + 6a^2 = 0$

**5. D**

**Sol.** Let the mid-point of chord be P(h,k) then its equation is

$T = S_1$

i.e. $yk + 2a(x + h) = k^2 - 4ah$

It must pass through (3a, a) hence $ak-2a(3a+h) = k^2 - 4ah$

Thus, locus of P is $y^2 - 2ax - ay + 6a^2 = 0$

$y^2 - 2ax - ay + 6a^2 = 0$

6. The curve $y = \frac{-x^2}{2} + x + \sin^2 x + \sin^2\left(\frac{5\pi}{2} - x\right)$ is symmetric with respect to line having equation:

(A) $x + y + 1 = 0$ (B) $y + 2 = 0$ (C) $x + 1 = 0$ (D) $x - 1 = 0$

**6. D**

**Sol:** Parabola is symmetric about its axis

7. If both roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie on the interval [1,5] then m lies in the interval

(A) (4, 5) (B) (– 5, – 4) (C) (-4, 5) (D) (3, 4)

**7. A**

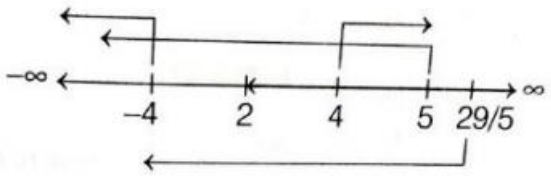
**Sol.** (i) $D > 0 \Rightarrow b^2 - 4ac > 0$

$\Rightarrow m \in (-\infty, -4) \cup (4, \infty)$

(ii) x = 1 and x = 5

$\therefore -\frac{b}{2a} \in (1,5)$

$\Rightarrow m \in (2,10)$




(iii) $f(1) > 0 \Rightarrow 1 - m + 4 > 0$

(iv) $f(5) > 0 \Rightarrow 25 - 5m + 4 > 0 \Rightarrow 5m < 29$

**From the values of m obtained in (i), (ii), (iii) and (iv), we get** $m \in (4,5)$

8. A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the X-axis. Then the eccentricity of the hyperbola is

(A) 2 (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{3}{2}$ (D) $\sqrt{3}$

**8. B**

**Sol.** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore \frac{16}{4} - \frac{4}{b^2} = 1 \Rightarrow 4 - \frac{4}{b^2} = 1 \Rightarrow b^2 = \frac{4}{3} \Rightarrow b = \frac{2}{\sqrt{3}}$

Now, eccentricity

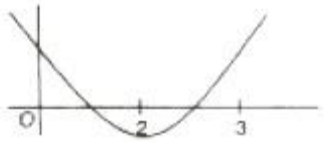
$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{\frac{4}{3}}{4}} = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}}$

9. Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0, c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2, 3). Then the number of elements in S is

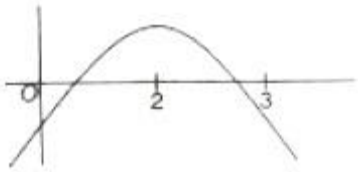
(A) 11 (B) 10 (C) 12 (D) 18

**9.** **A**

**Sol.** let $f(x) = (c-5)x^2 - 2cx + (c-4) = 0$. Then, according to problem, the graph of $y = f(x)$ will be either of the two ways, shown below.




Or




In both cases. f(0). f(2) < 0 and f(2) f(3) < 0
Now, consider

f(0) f(2)<0

$\Rightarrow (c-4)\left[4(c-5) - 4c + (c-4)\right] < 0$

$\Rightarrow \quad (c-4)(c-24) < 0$

$\Rightarrow \quad c \in (4, 24)$

$$\underset{4}{+} \quad \underset{}{-} \quad \underset{24}{+}$$

Similarly y, f(2). f(3)<0

$\Rightarrow \left[4(c-5) - 4c + (c-4)\right]$

$\Rightarrow c \in \left[\frac{49}{4}, 24\right]$

From (i) and (ii) we get $c \in \left[\frac{49}{4}, 24\right]$

$\therefore$ Integral values of c are 13, 14,……,23
Thus, 11 integral value of c are possible.

10. Let $S = \left\{(x,y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where $r \neq \pm 1$. Then S represents

(A) a hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$ when $0 < r < 1$.

(B) a hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$ when $0 < r < 1$.

(C) an ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$ when r > 1.

(D) an ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$ when r > 1.

**10.** **C**

**Sol.** Given, $S=\left\{(x,y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$

$= \left\{(x,y) \in R^2 : \frac{y^2}{1+r} + \frac{x^2}{1-r} = 1\right\}$

$\left[\because \text{For } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b, e = \sqrt{1 - \frac{a^2}{b^2}}\right]$

$= \sqrt{\frac{(r+1)-(r-1)}{r+1}} = \sqrt{\frac{2}{r+1}}$

11. If $a^2 - 2a\cos x + 1 = 674$ and $\tan\left(\frac{x}{2}\right) = 7$, then the integral value of a is

(A) 25 (B) 49 (C) 67 (D) 74

**11.** A

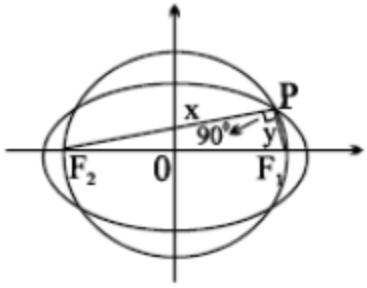
Sol. $(a - 25)(25a + 673) = 0$

12. A circle has the same centre as an ellipse & passes through the foci $F_1$ & $F_2$ of the ellipse and the two curves intersect in 4 points. Let 'P' be any one of their point of intersection. If the major axis of the ellipse is 17 & the area of the triangle $PF_1F_2$ is 30, then the distance between the foci is

(A) 11 (B) 12 (C) 13 (D) None of these

**12. C**

**Sol.** $x + y = 17$ ; $xy = 60$, To find $\sqrt{x^2 + y^2}$




Now , $x^2 + y^2 = (x + y)^2 - 2xy$

$= 289 - 120 = 169$

$\Rightarrow \sqrt{x^2 + y^2} = 13$

13 A hyperbola having the transverse axis of length $2\sin\theta$ is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

(A) $x^2\,\text{cosec}^2\theta - y^2\sec^2\theta = 1$ (B) $x^2\sec^2\theta - y^2\,\text{cosec}^2\theta = 1$

(C) $x^2\sin^2\theta - y^2\cos^2\theta = 1$ (D) $x^2\cos^2\theta - y^2\sin^2\theta = 1$

**13. A**

**Sol.** The length of transverse axis is

$2\sin\theta = 2a$

or $a=\sin\theta$

Also, for the ellipse

$3x^2 + 4y^2 = 12$

or $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$a^2 = 4, b^2 = 3$

$\therefore e = \sqrt{1 - \frac{b^2}{a^2}}$

$= \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

Hence the focus of ellipse is (2x1/2,0) = (1, 0)

As the hyperbola is confocal with the ellipse the focus of the hyperbola is (1,0) now

ae=1

or $\sin\theta$ x $e = 1$

$e = \text{cosec}\theta$

$\therefore b^2 = a^2(e^2 - 1) = \sin^2\theta(\text{cosec}^2\theta - 1) = \cos^2\theta$

Therefore, the equation of hyperbola is

$\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$

or $x^2\,\text{cosec}^2\theta - y^2\sec^2\theta = 1$

14. If $\log_4 5 = a \,\&\, \log_5 6 = b$, then $\log_3 2 =$

(A) $\frac{1}{2a+1}$ (B) $\frac{1}{2b+1}$ (C) 2ab + 1 (D) $\frac{1}{2ab-1}$

14. D

Sol. $2ab = \log_2 3 + 1$

15. A chord AB whose equation is $ax + by + c = 0$ (where a, b, c are in A.P.) cuts the parabola $y^2 + 4y - 8x - 4 = 0$ at points A and B. The angle between the tangents drawn to the parabola at points A and B is

(A) $60^o$ (B) $30^o$ (C) $90^o$ (D) None of these

**15. C**

**Sol.** focal chord

16. If $ax + by = 1, cx^2 + dy^2 = 1$ have only one solution, then

(A) $\frac{a^2}{c} + \frac{b^2}{d} = 2$ (B) $x = \frac{a}{c}$

(C) $y = \frac{b}{a}$ (D) None of these

**16. B**

Sol. We have $ax + by = 1 \Rightarrow \frac{1-ax}{b}$

Putting this value in the second equation, we get $cx^2 + \frac{d}{b^2}(1-ax)^2 = 1$

$\Rightarrow \left(b^2c + a^2d\right)x^2 - 2adx + d - b^2 = 0$ (1)

This quadratic equation will have equal roots if $D = 4a^2d^2 - 4\left(b^2c + a^2d\right)\left(d - b^2\right) = 0$

$\Rightarrow \quad a^2d^2 + \left(b^2c + a^2d\right)b^2 - b^2cd - a^2d^2 = 0$

$\Rightarrow \quad b^2\left[b^2c + a^2d - cd\right] = 0$

$\Rightarrow \quad b^2c + a^2d = cd \Rightarrow \frac{b^2}{d} + \frac{a^2}{c} = 1$

Also, in this case

$x = \frac{2ad}{2\left(a^2c + a^2d\right)} = \frac{ad}{cd} = \frac{a}{c}$

$y = \frac{1-ax}{b} = \frac{1}{b}\left(1 - \frac{a^2}{c}\right) = \frac{1}{b}.\frac{b^2}{d} = \frac{b}{d}$

17. If two roots of $x^3 - ax^2 + bx - c = 0$ are equal in magnitude but opposite in sign, then

(A) $a + bc = 0$ (B) $a^2 = bc$ (C) $ab = c$ (D) None of these

**17. C**

**Sol.** Let the roots be $X_1, -X_1, X_2$.

Then $X_1 - X_1 + X_2 = a$

Hence $x = a$ is a root of the given equation.

$\Rightarrow a^3 - a^3 + ab - c = 0 \quad \Rightarrow ab = c$

18. The set of real values of x such that $||(x - 1)| - 1| \le 1$ is

(A) [–1, 3] (B) [0, 2]

(C) [–1, 1] (D) none of these

18. A

Sol. Case –1. $x \le 1$ $|-x + 1 - 1| \le 1$

$|-x| \le 1$

$x \in [-1, 1]$ .....(1)

Case – 2. $x > 1$ $|x - 1 - 1| \le 1$
$|x - 2| \le 1$
$-1 \le x - 2 \le 1$
$1 \le x \le 3$ .....(2)
Taking union of (1) and (2)
$x \in [-1, 3]$

19. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed is another ellipse that passes through the point (4, 0) then the equation of the ellipse is
(A) $x^2 + 12y^2 = 16$ (B) $4x^2 + 48y^2 = 48$ (C) $4x^2 + 64y^2 = 48$ (D) $x^2 + 16y^2 = 16$

19. A

Sol. Let $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ be the required ellipse. It passes through (2, 1)
$\therefore \frac{1}{b^2} = \frac{3}{4}$

20. Consider two points $A\left(at_1^2, 2at_1\right)$ and $B\left(at_2^2, 2at_2\right)$ lying on the parabola $y^2 = 4ax$. If the line joining the points A and B passes through the point P (b, 0), then $t_1t_2$ is equal to
(A) $\frac{a}{b}$ (B) $-\frac{a}{b}$ (C) $\frac{b}{a}$ (D) $-\frac{b}{a}$

**20. D**

**Sol.** Equation of line joining A and B is $y(t_1 + t_2) = 2(x + at, t_2)$
It passes through $(b,0) \Rightarrow 0 = 2(b - at_1t_2) \Rightarrow t_1t_2 = -\frac{b}{a}$

## (PART – B)

**(Integer Type)**

**Part-C (01-05)** contains five (05) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+4 marks** for correct answer and **there will be no negative marking.**

21. The number of normals drawn from a point (3, 0) to the parabola $y^2 = 4x$ are k ,then (125.8)k equals to

**21. 377.40**

Sol: $y = mx - 2m - m^3$, this passes through (3, 0) hence
$0 = 3m - 2m - m^3 \Rightarrow m^3 - m = 0 \Rightarrow m = 0, m = \pm 1$
Hence three normals can be drawn. $9^{\log_3(\log_2 x)} = \log_2 x - (\log_2 x)^2 + 1$

22. If the radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having its centre at (0, 3) is r , then (11)r equals to

**22. 44**

Sol. We have $a^2 = 16$ and $9 = b^2 = a^2\left(1 - e^2\right) = 16\left(1 - e^2\right)$

So, $e = \frac{\sqrt{7}}{4}$, foci are $\left(\pm\sqrt{7}, 0\right)$; $r = \sqrt{\left(\sqrt{7} - 0\right)^2 + 3^2} = 4$

23. Number of values of x which satisfies the equation $9^{\log_3(\log_2 x)} = \log_2 x - (\log_2 x)^2 + 1$ equals to 'p' then (21.54)p equals to

**Ans. 43.08**

**Sol :**

Put $\log_2 x = t$, we get $2t^2 - t - 1 = 0$

$T = 1, -\frac{1}{2}$

24. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is $ax + by + 4 = 0$. Find $a + b$ .

**24. 3**

Sol. we know that, $y = mx + \frac{a}{m}$ is the equation of tangent of the parabola $y^2 = 4ax$

$y = mx + \frac{1}{m}$ in $xy = 2$ we get $x\left(mx + \frac{1}{m}\right) = 2$

$\Rightarrow m = \frac{-1}{2}$

Required equation of tangent is

$\Rightarrow x + 2y - 4 = 0$

25. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then find value of 'a' where the length of its latus rectum lies in the interval $(a, \infty)$

**25. 3**

**Sol.** For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$\Rightarrow 1 + \tan^2\theta > 4$

$\Rightarrow \tan\theta \in \left(-\infty, -\sqrt{3}\right) \cup \left(\sqrt{3}, \infty\right)$

$\Rightarrow \theta \in \left\{\frac{\pi}{3}, \frac{\pi}{2}\right\}$

Hence latus rectum length $\in (3, \infty)$