FIITJEE

IOQM MOCK TEST - 2

Indian Olympiad Qualifier in Mathematics (IOQM) – 2020-21

Time: 3 hours **Maximum Marks: 30**

INSTRUCTIONS

Please read the instructions carefully.

A: General:

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball point pen.
2. Use ***Black Ball Point Pen only*** for writing particulars on Answer Sheet. ***Use of pencil is strictly prohibited.***
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed.
4. The answer sheet, a machine-gradable Objective Response Sheet (ORS) is provided separately.
5. Do not Tamper/mutilate the **ORS** or this booklet.
6. No additional sheets will be provided for rough work.
7. On completion of this test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***

B: Questions paper format & **Marking Schema:**

1. The question paper consists of **30 questions** of Mathematics.
2. Answers will be whole numbers from 0 to 99, where students must fill OMR corresponding to correct answer.
3. **If your answer is 9 then you will have to fill in as 09.**
4. For each question you will be awarded **1** marks if you darken the bubble corresponding to the correct answer ONLY and zero **(0)** marks if no bubbles are darkened. There is no negative Marking.

Mathematics

1. Find the value of $\sqrt[3]{26-15\sqrt{3}}+\sqrt[3]{26+15\sqrt{3}}$.

2. Find the number of positive integral solutions of the equation $xy=9$.

3. Find number of real solutions of the equation $x=\left(x-\frac{1}{x}\right)^{\frac{1}{2}}+\left(1-\frac{1}{x}\right)^{\frac{1}{2}}$.

4. If p, q, r are the roots of the cubic equation $x^3-3px^2+3q^2x-r^3=0$ then find $\frac{p+r}{q}$.

5. Find the number of polynomials of the form x^3+ax^2+bx+c which are divisible by x^2+1 and a, b, $c \in \{1, 2,....,10\}$.

6. If $n!$ has exactly 20 zeros at the end, then find how many such n are there?

7. Find the number of integers n such that $\sqrt{\frac{3n-5}{n+1}}$ is an integer

8. The number of positive integers ≤ 3600 that are coprime to 3600 is λ. Find $\left[\frac{\lambda}{100}\right]$. Where [.] is greatest integer function.

9. If $x^2y^3=6$, find the least value of $3x+4y$, x, $y \in R^+$.

10. In how many ways can 10 balls different colour be divided between two boys, one receiving two and other eight, balls?

11. Number of positive integral solution of the equation $2x+3y=7$.

12. 8 players compete in a tournament. Everyone plays everyone else just once. The winner of a game gets 1, the loser 0, or each gets $\frac{1}{2}$ if the game is drawn. The final result is that everyone gets a different score and the player placing second gets the same as the total of the four bottom players. What was the score of player placing?

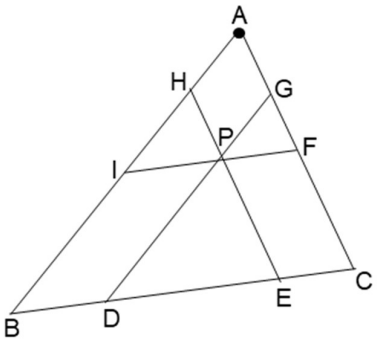
13. Find unit digit of $(2+1)(2^2+1)(2^4+1)...\left(2^{2^{10}}+1\right)+1$.

14. Calculate the unit digit of the expression $\left(\frac{-2a}{4+a}-\frac{\sqrt{|a|-3}+\sqrt{3-|a|}}{3-a}\right)^{2018}$.

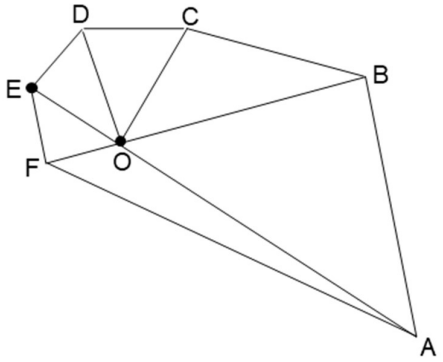
15. Let a, b, c, x, y be real numbers so that $a^3+ax+y=0$, $b^3+bx+y=0$ and $c^3+cx+y=0$. If a, b, c are distinct numbers different from 0, then find $a+b+c$.

16. Find the greatest integer x such that 23^{6+x} divides $2000!$

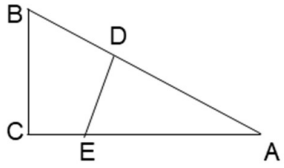
17. P is an inner point of ΔABC, $BC = a$, $CA = b$, $AB = c$, through P introduce $IF \| BC$, $DG \| AB$ & $EH \| CA$ respectively where I, H are on AB, D, E are on BC,F, G are on CA as shown in the give diagram below. Given $DE = a'$, $FG = b'$ & $HI = c'$ find. $\frac{a'}{a} + \frac{b'}{b} + \frac{c'}{c}$.



18. The diagram shows a hexagon ABCDEF made of five right angled isosceles triangles ABO, CDO, BCO, DEO, EPO and a triangle AOF, where O is the point of intersection of the lines BF & AE. Given that $OA = 8$ cm, find the area of ΔAOF in cm^2.



19. In the figure, $\angle C = 90^0$, $\angle A = 30^0$, D is the mid-point AB and $DE \perp AB$, $AE = 4$ cm. Find $(BC)^2$.



20. Let a_n equal $6^n + 8^n$. Determine the remainder upon dividing a_{83} by 49.

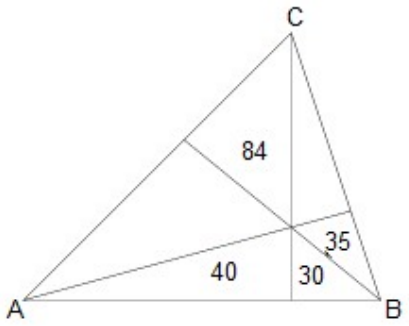
21. If $\frac{1}{x} - \frac{1}{y} = 4$, find (3). $\left(\frac{2x + 4xy - 2y}{x - y - 2xy}\right)$.

22. if $a_{n+1} = \frac{1}{1 + \frac{1}{a_n}} (n = 1, 2, ..., 2008)$ and $a_1 = 1$, find the value of $a_1a_2 + a_2a_3 + a_3a_4 + ... + a_{2008}a_{2009} = \frac{p}{q}$, where G.C.D. $(p, q) = 1$. Find $|p - q|$.

23. The number $\sqrt{104\sqrt{6} + 468\sqrt{10} + 144\sqrt{15} + 2006}$ can be written as $a\sqrt{2} + b\sqrt{3} + c\sqrt{5}$, where a, b, & c are positive integers. Find a.

24. Given the nine-sided regular polygon $A_1A_2....A_8A_9$ how many distinct equilateral triangles in the plane of the polygon have at least two vertices in the set $\{A_1, A_2, ..., A_9\}$?

25. As shown in the figure, triangle ABC is divided into six smaller triangles by lines drawn from the vertices through a common interior point. The areas of four of these triangles are as indicated. Find the area of triangle ABC.



26. Suppose that x & y are non-zero real numbers such that $\frac{3x+y}{x-3y}=-2$. What is the value of $\frac{x+3y}{3x-y}$?

27. Define $[a, b, c]$ to mean $\frac{a+b}{c}$, where $c \neq o$. What is the value of $[[60, 30, 90],[2, 1, 3],[10, 5, 15]]$?

28. The polynomial $1-x+x^2-x^3+...+x^{16}-x^{17}$ may be written in the form $a_0+a_1y+a_2y^2+....+a_{16}y^{16}+a_{17}y^{17}$ where $y=x+1$. If $a_2=\binom{p}{q}$. Find $p+q$.

29. Does there exist any polygon in which numbers of sides equal to the diagonals of the polygon. if exists then what is number of sides of the polygon?

30. Two circles with radii $\frac{9}{4}$ and 9 touch each other externally. Let r be the radius of another circle that touches these two circles as well as a common tangent to the two circles. Find r.

****** Best of Luck ******

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ANSWERS KEY

Q.No.	ANS
1.	04
2.	03
3.	01
4.	02
5.	10
6.	05
7.	02
8.	09
9.	10
10.	90
11.	01
12.	06
13.	06
14.	06
15.	00
16.	83
17.	01
18.	04
19.	12
20.	02
21.	02
22.	01
23.	13
24.	66
25.	01
26.	02
27.	02
28.	21
29.	05
30.	01