FIITJEE

CBSE FULL TEST – I

ALL XTH STUDYING BATCHES

MATHS

Time: 3:00 Hours **Max Marks: 80**

General Instructions:

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part –A:

1. It consists three sections-I and II.
2. Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.
3. Section II has 4questions on case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part –B:

1. Question No 21to 26are Very short answer Type questions of 2mark each,
2. Question No 27 to 33 are Short Answer Type questions of3 marks each
3. Question No 34 to 36are Long Answer Type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks

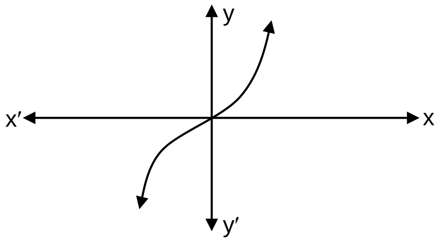
Name of the Candidate : ....

Enroll Number *:* ....

Date of Examination : ....

PART – A
SECTION – I
(16 question, 1 mark each)

1. The graph of $y = f(x)$ is given. How many zeroes are there of $f(x)$?



2. Find the product of the zeroes of $-2x^2 + kx + 6$.

3. If α, β are the zeroes of a polynomial, such that $\alpha + \beta = 10$ and $\alpha\beta = 6$, then write the polynomial.

4. For what value of k, are the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ real and equal?

OR

If the circumference and area of a circle are numerically equal, then find the diameter of the circle

5. For what value of p, are $2p + 1$, 13, $5p - 3$ three consecutive terms of an AP?

6. In an A.P., if $a = 3$, $n = 8$, $S_n = 192$, find d.

OR

Which term of the AP 21, 18, 15, ….., is zero?

7. The probability of getting a defective pen from a lot of 500 pens is $\frac{1}{25}$. Find the number of defective pens in the lot.

8. Find the roots/solution of the quadratic equation.
$x^2 - 9x + 20 = 0$

9 A box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears a prime number less than 23.

OR

A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant.

10. α, β are zeroes of the polynomial $x^2 - 6x + a$. Find the value of a, if $3\alpha + 2\beta = 20$.

OR

Find the sum of the zeroes of the given quadratic polynomial $-3x^2 + k$.

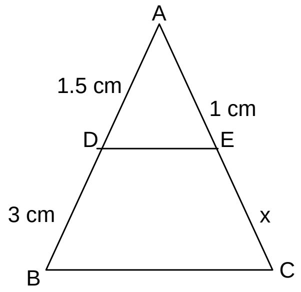
11. Find the angle of elevation of the sun when shadow of a pole x m high is $\sqrt{3}\,x\,m$ long.

12. Reciprocal of a number when subtracted from the number equals to $\frac{-24}{5}$. Find the number.

13. If (x, y) is equidistant from (7, –2) and (3, 1) express x in terms of y.

OR

In given figure if $DE \parallel BC$ then find value of x



14. Find the mean of the following data

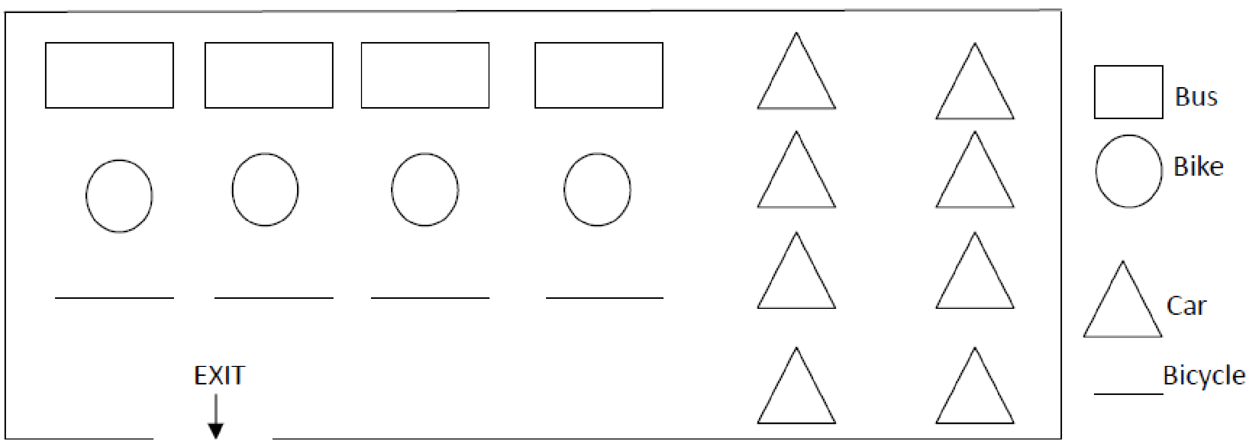
Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	3	5	9	5	3

15. Which term of the AP: 125, 121, 117,……is 1^{st} negative term?

16. Find the coordinates of a point P which lies on the line segment joining the points A (–2, 0) and B (0, 8) such that $AP = \frac{1}{4}AB$.

SECTION – II

17. In a car park, there are 125 cars, 3p bikes, 10 bicycles and 20 buses. One of the vehicles leaves the park at random



1 Vehicle Coming out.

Based on the above situation, answer the following questions.

(I) The formula for calculating the probability of an event is
(A) Number of possible outcomes/ number of favourable outcomes.
(B) Number of favourable outcomes/ number of possible outcomes.
(C) Number of favourable outcomes x Total number of outcomes.
(D) Number of possible outcomes – Number of favourable out comes.

(II) The probability that the vehicle leaving the park is a bus is

(A) $\frac{20}{155+3p}$ (B) $\frac{125}{155+3p}$

(C) $\frac{20}{158p}$ (D) $\frac{20}{158}$

(III) If the probability that vehicle leaving the park is a bike is $\frac{9}{40}$, then the value of p is

(A) 20 (B) 25

(C) 15 (D) 30

(IV) The probability that the vehicle leaving the park is a car (using the value of p calculated in part (iii).

(A) $\frac{4}{8}$ (B) $\frac{6}{8}$

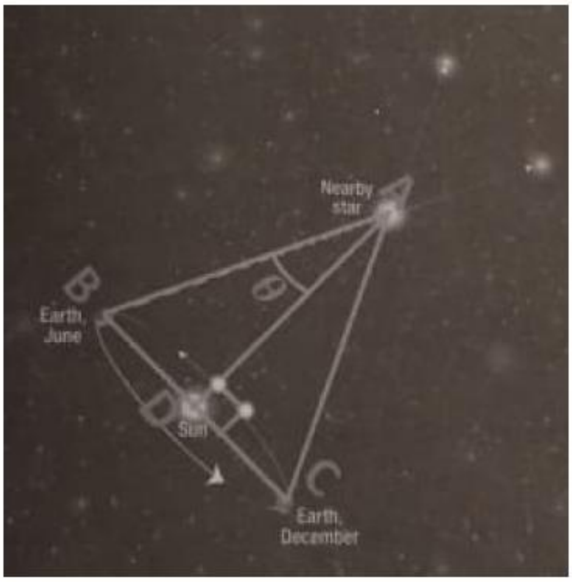
(C) $\frac{7}{8}$ (D) $\frac{5}{8}$

(V) Sum of the probabilities of vehicle leaving the park as car, bus, bike and bicycle is

(A) 1 (B) $\frac{155}{155+p}$

(C) 0 (D) $\frac{155+3}{155}$

18. The parallax angle is the angle between the Earth at one time of year and the Earth six months later as measured from near by star. Astronomers use this angle to find the distance from the Earth to that star.



Based on the above situation, answer the following question.

(I) In $\Delta ABD,$ the ratio represented by $\frac{AB}{BD}$ with respect to angle θ is

(A) $\sec\theta$ (B) $\cos\theta$

(C) $\operatorname{cosec}\theta$ (D) $\sin\theta$

(II) If AD = BD, then the angle θ is equal to

(A) 60^o (B) 45^o

(C) 30^o (D) 90^o

(III) If $\theta = 30^o$ and the horizontal distance between the Earth and Sun is 148 million km, then the distance between and the Earth (in June) and the star (in million km) is

(A) 74 (B) $\frac{296}{\sqrt{3}}$

(C) $\frac{74}{\sqrt{3}}$ (D) 296

(IV) For the data given in part (III), the vertical distance between the star and the sun (in million km) is

(A) $148\sqrt{3}$ (B) $\frac{148}{\sqrt{3}}$

(C) $\frac{296}{\sqrt{3}}$ (D) $296\sqrt{3}$

(V) $\angle DAC = \angle BAD = \theta,$ the type of triangle formed by the position of Earth in June and in December and the star is:

(A) Equilateral triangle (B) Isosceles triangle

(C) Right Angled triangle (D) Scalene triangle

19. A school planned for visit a Science Museum.

The Science Museum charges Rs. 14 for the adult admission and Rs. 11 for each child. The total bill for 68 people from the school field trip was Rs. 784.

Taking x as the number of adults and y as number of children answer the following questions:

(I) The pair of linear equations formed by the given situation is

(A) $11x + 14y = 784,\ x + y = 68$ (B) $11x + 14y = 68,\ x + y = 784$

(C) $14x + 11y = 784,\ x + y = 68$ (D) $14x - 11y = 784,\ x + y = 68$

(II) The lines formed by the pair of linear equations are

(A) Parallel (B) Intersecting

(C) Coincident (D) Collinear

(III) The condition for a pair of linear equation to have unique solution is

(A) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (B) $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (D) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(IV) The number of adults visiting the Museum are

(A) 13 (B) 10

(C) 56 (D) 12

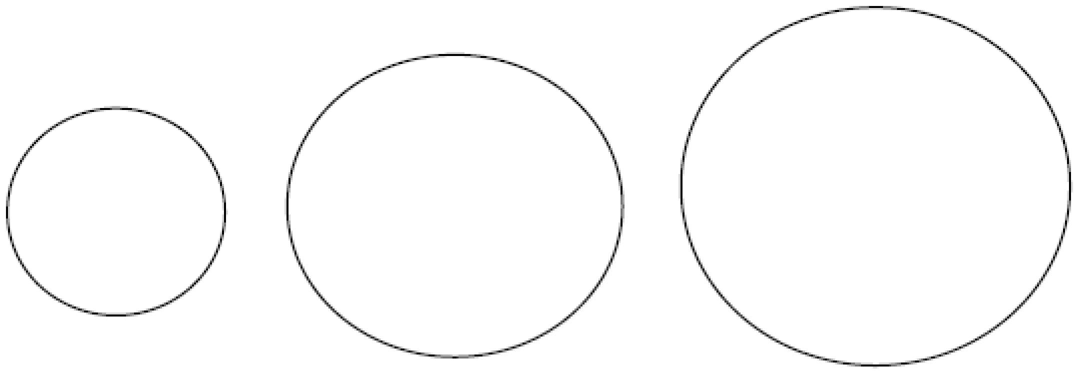
(V) The number of children visiting the Museum are

(A) 54 (B) 52

(C) 56 (D) 12

20. Coin Problem

The problem is to design a new set of coins. All coins will be circular and coloured silver, but of different diameter.



Researchers have found out that an ideal coin system meets the following requirements:

Diameters of coins should not be smaller than 14 mm and not be larger than 45 mm and thickness of each coin being 2 mm.

Given a coin, the diameter of the next coin must be at least 20% larger.

The machine can only produce coin with diameter of whole number of millimeters (for example, 17.3 mm is not allowed).

The diameter of first coin in the picture is 14 mm. Based on the given situation, answer the following questions.

(I) The diameter of second coin in the above diagram is

(A) 16 mm (B) 16.8 mm

(C) 17 mm (D) 16.5 mm

(II) The formula for calculating the surface area of a coin is

(A) $\pi r(r+h)$ (B) $2\pi r(r+h)$

(C) $2\pi r^2$ (D) $\pi r^2 h$

(III) The curved surface area of first coin is

(A) 44 mm^2 (B) 176 mm^2

(C) 352 mm^2 (D) 88 mm^2

(IV) The ratio of the volume of first coin to the volume of second coin is

(A) $\frac{289}{196}$ (B) $\frac{17}{14}$

(C) $\frac{196}{289}$ (D) $\frac{14}{17}$

(V) If 4 coins of first type are placed one above the other, then the volume of solid formed is

(A) 616 mm^3 (B) 2464 mm^3

(C) 1232 mm^3 (D) 308 mm^3

PART – B
(2 marks each)

21. If −4 is a root of the quadratic equation $x^2 + px - 4 = 0$ and the quadratic equation $x^2+px+k=0$ has equal roots, find the value of k.

22. In an A.P., the 24th term is twice the 10th term. Prove that the 36th term is twice the 16th term.

23. If product of the zeroes of the polynomial $kx^2 + 41x + 42$ is 7 then find the zeroes of the polynomial $(k - 4)x^2 + (k + 1)x + 5$.

OR

Find the zeroes of the quadratic polynomial $3x^2 - 2$ and verify the relationship between the zeroes and the coefficients.

24. Using quadratic formula, solve the following quadratic equation for x:
$p^2x^2 + (p^2 - q^2)x - q^2 = 0$.

OR

If a and b are roots of the equation $2x^2 + 7x + 5 = 0$ then write a quadratic equation whose roots are 2a + 3 and 2b + 3.

25. Seven years ago Varun's age was five times the square of Swati's age. Three years hence Swati's age will be two-fifth of Varun's age. Find their present ages.

26. Show that the sum of first n even natural numbers is equal to $\left(1+\frac{1}{n}\right)$ times the sum of the first n odd natural numbers.

(3 marks each)

27. One card is drawn from well – shuffled deck of 52 cards. Find the probability of getting
(i) an ace
(ii) '3' of spades
(iii) '9' of black suit

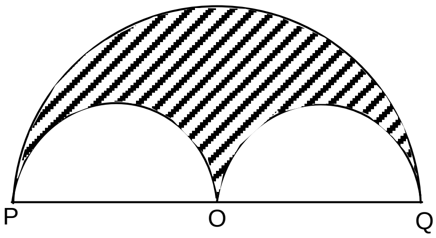
28. If l, b, h, s and v are length, breadth, height, total surface area and volume respective of a cuboid, prove that $\frac{1}{v} = \frac{2}{s}\left(\frac{1}{l} + \frac{1}{b} + \frac{1}{h}\right)$

29. D and E are respectively the points on the sides AB and AC of a triangle ABC such that AB = 5.6 cm , AD = 1.4 cm , AC = 7.2 cm and AE =1.8 cm , show that DE ||BC.

30. The area of an equilateral triangle is 17320.5 cm^2. With each vertex of the triangle as centre, circles are described with radius equal to half the length of the side of the triangle. Find the area of the triangle not included in the circles (use $\sqrt{3} = 1.73205$).

OR

In the given figure, O is the centre of the semi circle. Two small semi circles of diameters PO = OQ = 14 cm are taken out of the semi circular shape. Find the area of the shaded region. (Use $\pi = \frac{22}{7}$)

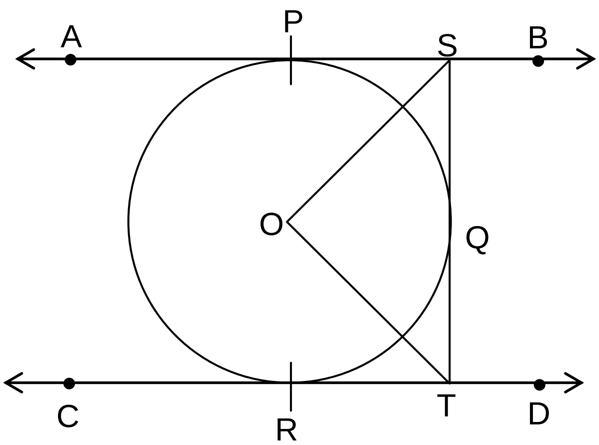


31. Draw two concentric circles of radius 3 cm and 6 cm. Taking a point on the outer circle, construct the pair of tangents to the other circle.

OR

Draw a circle of radius 3 cm. Take two points P and Q on either side of the extended diameter at a distance of 8 cm and 5 cm respectively from the centre. Draw tangents to the circle from the points P and Q. Write steps of construction.

32. In figure, AB and CD are two parallel tangents to a circle with centre O. ST is tangent segment between the two parallel tangents touching the circle at Q. Show that $\angle SOT = 90^o$.



33. If the angles of depression and elevation of the bottom and top of a tower as observed from a building h meters high are x and y respectively. then what is the height of tower?

(5 marks each)

34. If mean of following data is 15.45 then find missing frequencies f_1 and f_2

C I	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30	Total
F	6	f_1	10	9	f_2	40

35. Solve for $x : 2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5;$ given that $x \neq -3, x \neq \frac{1}{2}$.

36. An A.P., consists of 21 terms. The sum of the three terms in the middle is 129 and of the last three is 237. Find the A.P.

OR

The first and the last terms of an AP are 8 and 350 respectively. If its common difference is 9, how many terms are there and what is their sum?

HINTS AND SOLUTIONS

1. 1

Sol. Graph of y = f(x) intersect x-axis in one point only.
Therefore number of zeroes of f(x) is one.

2. - 3

Sol. Here a = –2, b = k, c = 6

$$\text{Product of zeroes } = \frac{c}{a}$$

i.e., $\alpha \times \beta = \frac{6}{-2} = -3$

3. $k(x^2 - 10x + 6)$

Sol. α, β are the zeroes of a polynomial.
Sum of zeroes, S = $\alpha + \beta = 10$, product of zeroes, P = $\alpha\beta = 6$
The required polynomial g(x) is given by

$$g(x) = k(x^2 - Sx + P)$$
$$g(x) = k(x^2 - 10x + 6)$$

where k is any non zero real number.

4. $k = \pm 9$

Sol. $D = b^2 - 4ac \Rightarrow D = (2k)^2 - 4 \times 3 \times 27 = 4k^2 - 324$
For real and equal roots, $D = 0 \Rightarrow 4k^2 - 324 = 0 \Rightarrow 4k^2 = 324$

$\Rightarrow \quad k^2 = \frac{324}{4} \quad \Rightarrow k^2 = 81 \quad \Rightarrow k = \pm 9$

OR

$2\pi r = \pi r^2 \Rightarrow r = 2$
So diameter = 4

5. 4

Sol. If terms are in A.P., then 13 – (2p + 1) = (5p – 3) – 13
$\Rightarrow 13 - 2p - 1 = 5p - 3 - 13 \Rightarrow 28 = 7p \Rightarrow p = 4$

6. 6

Sol. $S_n = \frac{n}{2}\left[2a + (n-1)d\right] \Rightarrow 192 = \frac{8}{2}\left[2 \times 3 + (8-1) \times d\right]$

$\frac{192}{4} = 6 + 7d \Rightarrow 7d = 48 - 6 = 42 \Rightarrow d = 6$

OR

Here, a = 21, d = 18 – 21 = –3
Let $a_n = 0$

$\Rightarrow \quad a + (n-1)d = 0 \quad \Rightarrow \quad 21 + (n-1)(-3) = 0$

$\Rightarrow \quad (n-1)(-3) = -21 \quad \Rightarrow \quad n - 1 = \frac{-21}{-3} = 7 \Rightarrow n = 8$

7. **20**

Sol. Let defective pens = x

$\frac{x}{50} = \frac{1}{25} \Rightarrow x = 20$ **(2 marks)**

8. 4, 5

Sol. Given equation is $x^2 - 9x + 20 = 0 \Rightarrow x^2 - 5x - 4x + 20 = 0$

$\Rightarrow x(x-5) - 4(x-5) = 0 \Rightarrow (x-5)(x-4) = 0$

$\Rightarrow$ either $x - 5 = 0$ or $x - 4 = 0 \Rightarrow x = 5$ or $x = 4$

$\therefore$ x = 4 or 5 are the roots/solution of the given quadratic equation.

9. $\frac{4}{45}$

Sol. Favourable cases = $\{2, 3, 5, 7, 11, 13, 17, 19\}$

$\therefore$ Required probability = $\frac{8}{90} = \frac{4}{45}$

OR

There are 21 consonants in English alphabets.

$\therefore$ Required probability = $\frac{21}{26}$

10. - 16

Sol. $\alpha + \beta = 6$, $\alpha\beta = a$

Now $3\alpha + 2\beta = 20$

$\Rightarrow \alpha + 2\alpha + 2\beta = 20 \Rightarrow \alpha + 2(\alpha + \beta) = 20$

$\Rightarrow \alpha + 2 \times 6 = 20 \Rightarrow \alpha = 20 - 12 = 8$

$\therefore \beta = -2$

Now $\alpha\beta = a$

$\therefore 8 \times (-2) = a \Rightarrow -16$

OR

Since polynomial is is $-3x^2 + 0x + k$ $\therefore$ a = –3, b = 0, c = k

and sum of zeroes $= \frac{-b}{a}$

i.e., $\alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{0}{-3} = 0$

11. $\frac{\pi}{6}$

Sol. $\frac{x}{\sqrt{3}x} = \tan\theta$

$\theta = \frac{\pi}{6}$ **(1 marks)**

12. $\frac{1}{5}, -5$

Sol. $a - \frac{1}{a} = \frac{-24}{5}$

$$\frac{a^2-1}{a}+\frac{24}{5}=0$$

$5a^2-5+24a=0$ **(1/2 marks)**

$5a^2+24a-5=0$

$5a^2+25a-a-5=0$

$5a(a+5)-1(a+5)=0$

$a=\frac{1}{5}, -5$ **(1 marks)**

13. $x=\frac{6y+43}{8}$

Sol. $(x-7)^2+(y+2)^2=(x-3)^2+(y-1)^2$

$-14x+49+4y+4=-6x+9+1-2y$ **(1/2 marks)**

$6y+43=8x$

$x=\frac{1}{8}(6y+43)$ **(1 marks)**

OR

13. 2

Sol. $\frac{AD}{BD}=\frac{AE}{x}\Rightarrow\frac{1.5}{3}=\frac{1}{x}\Rightarrow x=2\text{ cm}$

14. **25**

Sol. $\bar{x}=\frac{\sum x_i f_i}{\sum f_i}=\frac{3\times5+5\times15+9\times25+5\times35+3\times45}{25}$ **(1 marks)**

= 25

15. **33**

Sol. $a_n=125+(n-1)(-4)$ **(1/2 marks)**

$a_n<0$

$125-4n+4<0$

$4n>129$ **(1/2 marks)**

$n>\frac{129}{4}$

$n = 33$

16. $\left(\frac{-3}{2}, 2\right)$

Sol. $P(x_1 y)$

$x = \frac{1x0 + 3(-2)}{1+3} = \frac{-6}{4} = \frac{-3}{2}$ **(1/2 marks)**

$y = \frac{3(0) + 1(8)}{1+3} = 2$ **(1/2 marks)**

17.

(I) B

Sol. Probability of an event

$= \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$

(II) A

Sol. P (bus) $= \frac{20}{155 + 3p}$

(III) C

Sol. $\frac{3p}{155 + 3p} = \frac{9}{40} \Rightarrow p = 15$

(IV) D

Sol. $P(\text{car}) = \frac{125}{155 + 3 \times 15} = \frac{125}{200} = \frac{5}{8}$

(V) A

Sol. P (car) + P (bike) + P (bus) + P (bicycle)

$= \frac{200}{200} = 1$

18.

(I) C

Sol. $\frac{AB}{BD} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \operatorname{cosec}\theta$

(II) B

Sol. $AD = BD \Rightarrow \theta = 45^o$

(III) D

Sol. $\frac{AB}{BD} = \operatorname{cosec}\theta \Rightarrow \frac{AB}{148} = \operatorname{cosec} 30^o = 2$

$\Rightarrow AB = 296$ million km

(IV) A

Sol. $\frac{BD}{DA} = \tan\theta \Rightarrow \frac{148}{DA} = \tan 30^o = \frac{1}{\sqrt{3}}$

$\Rightarrow DA = 148\sqrt{3}$ million km

(V) B

Sol. When $\angle DAC = \angle BAD = \theta$ then $\Delta ADB \cong \Delta ADC \Rightarrow AB = AC$

So, ABC will be isosceles.

19.

(I) C

Sol. Total people = x + y = 68

Total bill = 14x + 11y = 784

(II) B

Sol. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ Intersecting lines

(III) A

Sol. For unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(IV) C

Sol. On solving both equations we get x = 56

(V) D

Sol. On solving both equation we get y = 12

20.

(I) C

Sol. Diameter of second coin

$= 14 + 14 \times \frac{20}{100} = 16.8 \text{ mm} = 17 \text{mm}$ (approx)

(II) B

Sol. Surface Area of coin $= 2\pi r\left(h + r\right)$

(III) D

Sol. $CSA = 2\pi rh = 2\pi(7)(2) = 88 \text{mm}^2$

(IV) A

Sol. $\frac{V_1}{V_2} = \frac{\pi\left(\frac{17}{2}\right)^2 \times 2}{\pi(7)^2 \times 2} = \frac{289}{196}$

(V) C

Sol. $V = \pi(7)^2 \times 8 = 1232 \text{ mm}^3$

21. $\Rightarrow k = \frac{9}{4}$

Sol. -4 is a root of quadratic equation $x^2 + px - 4 = 0$

$\therefore$ $(-4)^2 + p(-4) - 4 = 0 \Rightarrow 16 - 4p - 4 = 0 \Rightarrow p = 3$

Putting p = 3 in equation $x^2 + px + k = 0$, we get $x^2 + 3x + k = 0$

Equation has equal roots D = 0 $\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (3)^2 - 4 \times 1 \times k = 0 \Rightarrow -4k = -9 \Rightarrow k = \frac{9}{4}$

22. Let 1st term = a, common difference = d

$a_{10} = a + 9d$, $a_{24} = a + 23d$

According to the question, $a_{24} = 2 \times a_{10}$

$\Rightarrow a + 23d = 2(a + 9d) \Rightarrow a + 23d = 2a + 18d \Rightarrow a = 5d$

Now, $a_{16} = a + 15d = 5d + 15d = 20d$

$a_{36} = a + 35d = 5d + 35d = 40d$

From (i) and (ii), we get

$a_{36} = 2 \times a_{16}$

23. $x = -1, x = \frac{-5}{2}$

Sol. Here $f(x) = kx^2 + 41x + 42$

a = k, b = 41, c = 42

A.T.Q. product of zeroes = 7

$\Rightarrow \frac{c}{a} = 7$

$\Rightarrow \frac{42}{k} = 7 \Rightarrow 42 = 7k \Rightarrow k = 6$

Putting k = 6 in polynomial

$p(x) = (k - 4)x^2 + (k + 1)x + 5$

we get $p(x) = (6 - 4)x^2 + (6 + 1)x + 5$

$\Rightarrow p(x) = 2x^2 + 7x + 5$

For zeroes of p(x), $2x^2 + 7x + 5 = 0$

$2x^2 + 5x + 2x + 5 = 0$

$\Rightarrow x(2x + 5) + 1(2x + 5) = 0$

$\Rightarrow (x + 1)(2x + 5) = 0$

$\Rightarrow x = -1, x = \frac{-5}{2}$

$\therefore$ zeroes are $-1, \frac{-5}{2}$

OR

Here $p(x) = 3x^2 - 2$

For zeroes of p(x), $p(x) = 0$

$\Rightarrow 3x^2 - 2 = 0 \Rightarrow 3x^2 = 2$

$\Rightarrow x^2 = \frac{2}{3} \Rightarrow x = \pm\sqrt{\frac{2}{3}}$

$\therefore$ zeroes are $\sqrt{\frac{2}{3}}$ and $-\sqrt{\frac{2}{3}}$

Also a = 3, b = 0 and c = –2

Now sum of zeroes $= \sqrt{\frac{2}{3}} + \left(-\sqrt{\frac{2}{3}}\right) = 0$

Also $\frac{-b}{a} = \frac{-0}{3} = 0 \Rightarrow \text{Sum of zeroes} = \frac{-b}{a}$

and product of zeroes $= \sqrt{\frac{2}{3}} \times -\sqrt{\frac{2}{3}} = \frac{-2}{3}$

Also $\frac{c}{a} = \frac{-2}{3} \Rightarrow \text{Product of zeroes} = \frac{c}{a}$

24. $x = \frac{q^2}{p^2}, -1$

Sol. $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Here $a = p^2, b = \left(p^2 - q^2\right), c = -q^2,$

$D = b^2 - 4ac = \left(p^2 - q^2\right)^2 - 4 \times p^2 \times \left(-q^2\right) = \left(p^2 + q^2\right)^2$

Now $x = \frac{-b + \sqrt{D}}{2a} \cdot \frac{-b - \sqrt{D}}{2a}$

$\Rightarrow$ $x = \frac{-\left(p^2 - q^2\right) + \sqrt{\left(p^2 + q^2\right)^2}}{2 \times p^2}$; $x = \frac{-\left(p^2 - q^2\right) - \sqrt{\left(p^2 + q^2\right)^2}}{2 \times p^2}$

$\Rightarrow$ $x = \frac{q^2}{p^2}, -1$

OR

Here given quadratic equation is $2x^2 + 7x + 5 = 0$

$\because$ a and b are roots $\therefore$ $a + b = \frac{-7}{2}$ …(i)

and $a \cdot b = \frac{5}{2}$ …(ii)

Now, quadratic equation whose roots are 2a + 3 and 2b + 3 is

$x^2 - \left[2a + 3 + 2b + 3\right]x + \left(2a + 3\right)\left(2b + 3\right) = 0$

$\Rightarrow$ $x^2 - \left[2\left(a + b\right) + 6\right]x + (4ab + 6\left(a + b\right) + 9] = 0$

$\Rightarrow$ $x^2 - \left[2\left(\frac{-7}{2}\right) + 6\right]x + \left[4 \times \frac{5}{2} + 6 \times \left(\frac{-7}{2}\right) + 9\right] = 0$ [using eq. (i) and (ii)]

$\Rightarrow$ $x^2 + x - 2 = 0$

25. Present age of Swati = 9 years and Varun = 27 years

Sol. Let Varun's present age be x years and Swati's present age be y years.

Case-I: 7 years ago

Varun's age was (x – 7) years and Swati's age was (y – 7) years

ATQ $(x - 7) = 5(y - 7)^2 \Rightarrow x = 5\,(y - 7)^2 + 7$ …(i)

Case – II: 3 years hence

Varun's age will be (x + 3) years and Swati's age will be (y + 3) years

ATQ $y + 3 = \frac{2}{5}\left(x + 3\right)$

$\Rightarrow$ $y + 3 = \frac{2}{5}\left[\left(5\left(y - 7\right)^2 + 7\right) + 3\right]$ [Using eq. (i)]

$\Rightarrow \quad y+3=\frac{2}{5}\times 5(y-7)^2+\frac{2}{5}\times 10 \quad \Rightarrow y+3=2\left(y^2-14y+49\right)+4$

$\Rightarrow \quad y+3=2y^2-28y+98+4 \quad \Rightarrow 2y^2-29y+99=0$

$\Rightarrow \quad 2y^2-18y-11y+99=0 \quad \Rightarrow 2y(y-9)-11(y-9)=0$

$\Rightarrow \quad (y-9)(2y-11)=0 \quad \Rightarrow y=9, y=\frac{11}{2}(\text{rejecting})$

$\therefore \quad y=9 \quad \therefore x=5(9-7)^2+7$ [From (i)]

$\therefore$ Present age of Swati = 9 years and Varun = 27 years.

26. Let S_1 be the sum of first n even natural numbers.

Then $S_1=2+4+6+.....+2n$

$\Rightarrow \quad S_1=\frac{n}{2}\left[2\times 2+(n-1)2\right]$

$\Rightarrow \quad S_1=\frac{n}{2}\left[4+2n-2\right]=n(n+1)$ …(i)

Let S_2 be the sum of first n odd natural numbers.

Then $S_2=\frac{n}{2}\left[2\times 1+(n-1)2\right]=n^2$

$S_1=\left(1+\frac{1}{n}\right)S_2$ [From (i)]

27. **(i)** $\frac{1}{13}$ **(ii)** $\frac{1}{52}$ **(iii)** $\frac{1}{26}$

Sol. (i) $P(E)=\frac{4}{52}=\frac{1}{13}$ **(1 marks)**

(ii) $P(E)=\frac{1}{52}$ **(1 marks)**

(iii) $P(E)=\frac{2}{52}=\frac{1}{26}$ **(2 marks)**

28.

Sol. $S=2(\ell b+bh+h\ell)$ **(1 marks)**

$v=\ell bh$

$S=2\left(\frac{V}{h}+\frac{V}{\ell}+\frac{V}{6}\right)$ **(1 marks)**

$\frac{1}{V}=\frac{2}{S}\left(\frac{1}{h}+\frac{1}{\ell}+\frac{1}{b}\right)$ **(2 marks)**

29.

Sol. $\frac{AD}{AB}=\frac{AE}{AC}$ **(2 marks)**

$\frac{1.4}{5.6} = \frac{1.8}{7.2}$

$\frac{1}{4} = \frac{1}{4}$ ($\therefore$ by converse of Basic Proportionality Theorem)

$DE \parallel BC$ **(2 marks)**

30. 1620.5 cm^2

Sol. $\frac{\sqrt{3}}{4} \times a^2 = 17320.5$ **(1 marks)**

$a = 200$

$A = \frac{\theta_1 + \theta_2 + \theta_3}{360} \times \pi \times \left(\frac{a}{2}\right)^2$ **(1 marks)**

$= \frac{180}{360} \times \frac{22}{7} \times (100)^2$

$= 15714.2$

$17320.5 - 15714.2 = 1620.5\ cm^2$ **(2 marks)**

OR

31. 154 cm^2

Sol. Area $= \frac{\pi r_1^2}{2} - \pi r_2^2$ **(2 marks)**

$= \pi\left(\frac{14^2}{2} - (7)^2\right)$

$= \pi(98 - 49)$

$= 49\pi = 154\ cm^2$ **(2 marks)**

32.

Sol. $\angle POR = 180^o$ (linear pair) **(1 marks)**

$\angle POS = x, \angle ROT = y$

$2x + 2y = 180^o$ **(1 marks)**

$x + y = 90^o$

$\angle SOT = 90^o$ **(2 marks)**

33. $h(1 + \cot x \tan y)$

Sol. $\frac{H-h}{AB}=\text{Tan}\,y$ **(1 marks)**

$AB=\frac{H-h}{\text{Tan}\,y}$

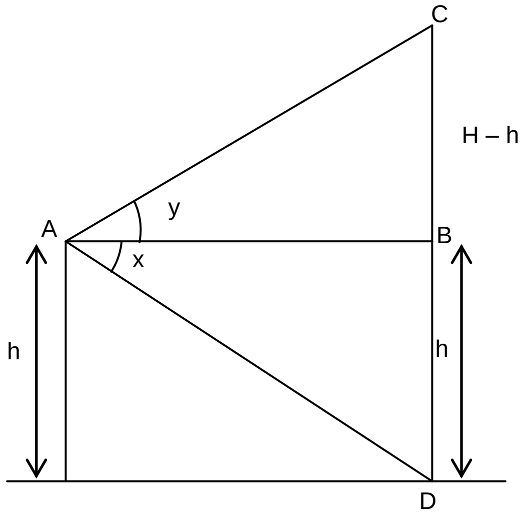
$\frac{h}{AB}=\text{Tan}\,x$

$\frac{h}{\text{Tan}\,x}=\frac{H-h}{\text{Tan}\,y}$ **(1 marks)**

$h\,\text{Tan}\,y=H\,\text{Tan}\,x-h\,\text{Tan}\,x$

$H\,\text{Tan}\,x=h(\text{Tan}\,x+\text{Tan}\,y)$ **(1 marks)**

$H=h(1+\cot x\,\text{Tan}\,y)$ **(1 marks)**



34. $f_1=8, f_2=7$

Sol. $\bar{x}=\frac{\sum x_i f_i}{\sum f_i}\Rightarrow 15.45=\frac{3\times 6+9f_1+15\times 10+21\times 8+27f_2}{40}$ **(1 marks)**

$f_1+3f_2=29$

$25+f_1+f_2=40$ **(1 marks)**

$F_1=8, F_2=7$ **(2 marks)**

35. $x=-10,\frac{-1}{5}$

35. $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5$

Let $\frac{2x-1}{x+3}=y$

$\therefore$ Given equation becomes $2y-3\times\frac{1}{y}=5 \Rightarrow 2y^2-3=5y \Rightarrow 2y^2-5y-3=0$

$\Rightarrow$ $2y^2-6y+y-3=0 \quad \Rightarrow 2y(y-3)+1(y-3)=0$

$\Rightarrow$ $(y-3)(2y+1)=0 \quad \Rightarrow y=3, y=-\frac{1}{2}$

$\Rightarrow$ $\frac{2x-1}{x+3}=3$ or $\frac{2x-1}{x+3}=-\frac{1}{2} \quad \Rightarrow 2x-1=3x+9$ or $4x-2=-x-3$

$\Rightarrow$ $-x=10$ or $5x=-1 \quad \Rightarrow x=-10$ or $x=\frac{-1}{5}$

$\Rightarrow$ $x=-10,\frac{-1}{5}$

36. 3, 7, 11, 15, ……

Sol. Let 1st term of A.P. be a and common difference be d.

Now, three middle terms of this A.P. are a_{10}, a_{11} and a_{12}

A.T.Q., $a_{10} + a_{11} + a_{12} = 129$

$\Rightarrow (a + 9d) + (a + 10d) + (a + 11d) = 129$

$\Rightarrow 3a + 30d = 129$

$\Rightarrow a = 10d = 43 \Rightarrow a = 43 - 10d$ …(i)

Also, last three terms are a_{19}, a_{20} and a_{21}

$\therefore a_{19} + a_{20} + a_{21} = 237$

$\Rightarrow (a + 18d) + (a + 19d) + (a + 20d) = 237$

$\Rightarrow 3a + 57d = 237 \Rightarrow a + 19d = 79$

$\Rightarrow 43 - 10d + 19d = 79$ [Using (i)]

$\Rightarrow 9d = 36 \Rightarrow d = 4$

When d = 4, equation (i) becomes

$a = 43 - 10 \times 4 = 3$

$\therefore$ A.P. is 3, 7, 11, 15, ……

OR

Here, a = 8, $\ell = 350$, d = 9

Using formula, $\ell = a + (n - 1)d$, we get

$a + (n - 1)d = 350$

$\Rightarrow 8 + (n - 1)9 = 350 \Rightarrow (n - 1)9 = 350 - 8$

$\Rightarrow (n-1)9 = 342 \Rightarrow n - 1 = \frac{342}{9}$

$\Rightarrow n - 1 = 38 \Rightarrow n = 38 + 1 = 39$

From formula, $S_n = \frac{n}{2}(a + \ell)$, we get

$$S_{39} = \frac{39}{2}(8 + 350) = \frac{39}{2} \times 358 = 6981$$

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