FIITJEE

ALL INDIA TEST SERIES

PART TEST – II

JEE (Main)-2023

TEST DATE: 03-12-2022

ANSWERS, HINTS & SOLUTIONS

Physics PART – A

SECTION – A

1. C

Sol. $Pt = Q_{system} + Q_{lost}$

$750 \times 4 \times 60 = 0.5 \times 4200 \times 70 + Q_{lost}$

$\therefore\ P_{lost} = \frac{Q_{lost}}{time}$

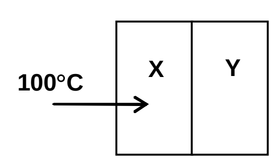
2. B

Sol. $(m_0 - \lambda t)\ CdT = Rdt$

3. A

Sol. At steady state

Let the temperature at the junction be T



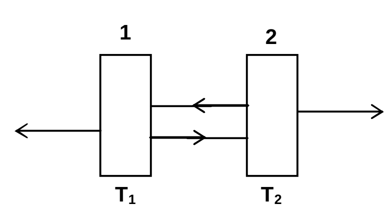
$\therefore\ \frac{kA}{\ell}(T - 100) = \frac{4nknA}{2n^2\ell}(0 - T)$

$\therefore$ T is independent of n.

4. A

Sol. $P_1 = 2\sigma AT_1^4 - \sigma AT_2^4$

$\sigma AT_1^4 = 2\sigma AT_2^4$



5. A

Sol. $\dfrac{300}{600}=\dfrac{2(2\pi R^2)\sigma e(500^4-300^4)}{2\pi R^2\sigma e\left[500^4-300^4\right]+2\pi R^2\sigma\left[500^4-300^4\right]}$

6. B

Sol. $\omega^2=\dfrac{1}{LC}$

7. D

Sol. $\cos\phi_1=0.6$ $\quad\cos\phi_2=0.5$ $\quad\cos\phi_3=1$

$\tan\phi_1=\dfrac{X_L}{R_1}$ $\quad\tan\phi_2=\dfrac{X_C}{R_2}$ $\quad\therefore X_L=X_C$

$R_1\tan\phi_1=R_2\tan\phi_2$

8. B

Sol. The force exerted by gas on the piston when spring has compression x is

$F=P_0A+kx$

$W=\int_0^{\ell}F\,dx$

9. D

Sol. Pitch (along x-axis) = V_0t

Magnetic force is towards negative z-axis

$\therefore\ \theta=\omega t=\pi$

$\therefore$ at this instant y = 0 and z = –2radius

10. C

Sol. $\vec{u}=u_x\hat{i}+u_y\hat{j}$

$\vec{E}=-E\hat{j}$

$\vec{B}=-B\hat{k}$

$\vec{F}=-e\left[\vec{E}+\vec{u}\times\vec{B}\right]=-e\left[-E\hat{j}+\left(u_x\hat{i}+u_y\hat{j}\right)\times\left(-B\hat{k}\right)\right]$

$F_x=eu_yB,\ F_y=e\left(E-u_xB\right)$

$\therefore\quad a_x=\dfrac{eB}{m}u_y$

$a_y=\dfrac{e}{m}\left[E-u_xB\right]$

$\therefore\quad u_y=A\sin\left(\omega t+\delta\right),\ a_y=A\omega\cos\left(\omega t+\delta\right)$

At t = 0, $u_y=0$ $\therefore\ \delta=0$

$a_y=\dfrac{F_y}{m}=\dfrac{eE}{m}$

$\therefore\quad A=\dfrac{eE}{m\omega}=\dfrac{E}{B}$

$\therefore\quad u_y=\dfrac{E}{B}\sin\omega t$

The velocity is along x-axis implies final velocity along y-axis is zero.

$$u_y = \frac{dy}{dt} \Rightarrow \int dy = \int_0^{\pi/\omega} u_y dt\ .$$

11. C

Sol. $\varepsilon = -\frac{d\phi}{dt}$

12. D

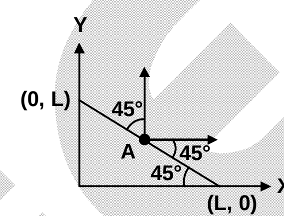
Sol. At point A, E along the rod = 0

x cos 45° = y sin 45°

$\Rightarrow$ x = y

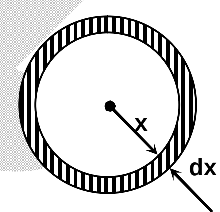
If the particle just reaches point A with velocity tending toward zero, then it will be able to reach the other end.

Apply work energy theorem



13. D

Sol. $\int_0^i di = \int_0^{R/2} J_0\left(\frac{x}{R}-1\right)2\pi x dx + \int_{R/2}^{R} J_0 \frac{x}{R}(2\pi x)dx$



14. D

Sol. Apply PV = nRT

$$\frac{P}{nT} = \frac{R}{V}$$

15. A

Sol. Magnetic field is along the tangent

16. B

Sol. $\vec{B} = \frac{\mu_0 I[\sin 60°](+\hat{k})}{4\pi(2R\sin 30°)} + \frac{\mu_0 I[\sin 30°](-\hat{k})}{4\pi(2R\sin 60°)}$

17. B

Sol. $\theta = \frac{1}{2}\beta t^2$

$$\varepsilon = -\frac{d\left(B\theta a^2/2\right)}{dt}$$

18. A

Sol. mg = kx

and $kx = \frac{q^2}{2\varepsilon_0 A}$

19. C

Sol. $T_0 \propto \frac{1}{\lambda_0}$

$P_0 \propto T_0^4$

$\therefore\ P_0 \propto \frac{1}{\lambda_0^4}$ and $\frac{256P_0}{81} \propto \frac{1}{\lambda^4}$

$\therefore\ \frac{P_0}{\frac{256P_0}{81}} = \frac{\lambda^4}{\lambda_0^4}$

20. D

Sol. PV = RT

and $P = \frac{A}{1+\frac{B^2}{V^2}}$;

$\frac{RT}{V} = \frac{A}{1+\frac{B^2}{V^2}}$

$T = \frac{VA}{R\left(1+\frac{B^2}{V^2}\right)}$

SECTION – B

21. 2

Sol. $V = I_1R_1$ and $V = I_2R_2$

NIAB = $k\theta$

$I_1 = \frac{k\theta_1}{N_1AB}$

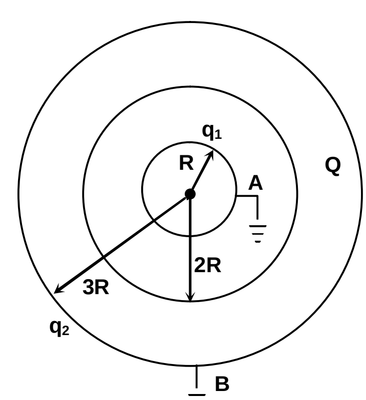
$\therefore\ I_2 = \frac{k\theta_2}{N_2AB}$

$V = \frac{k\theta_1R_1}{N_1AB} = \frac{k\theta_2R_2}{N_2AB}$

22. 4

Sol. $V_A = 0 = \frac{kq_1}{R} + \frac{kQ}{2R} + \frac{k(q_2)}{3R}$

$V_B = 0 = \frac{k(q_1+Q+q_2)}{3R}$



23. 3

Sol. $S = \frac{I_g G}{I - I_g} = \frac{\rho \ell}{A}$

24. 75

Sol. $\Delta H = \frac{(30)^2}{2}\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{6}\right) - \left[\frac{(15)^2}{2}\left(\frac{1}{3}+\frac{1}{6}\right)+\frac{(45)^2}{2\times 6}\right] = 75\ \mu J$

25. 3

Sol. $\Delta P = \frac{n_A RT}{2V} - \frac{n_A RT}{V}$ …(i)

$1.5\Delta P = \frac{n_B RT}{2V} - \frac{n_B RT}{V}$ …(ii)

Solve equation (i) and (ii)

26. 5

Sol. PT = constant

$PV^{1/2}$ = constant

$x = \frac{1}{2}$

$C = C_v + \frac{R}{1-x}$

$\frac{9R}{2} = \frac{fR}{2} + 2R$

$\therefore$ f = 5

27. 7

Sol. $\Delta Q = nC_P\Delta T$

$\Delta Q = \frac{7}{2} nR\Delta T$

$\Delta W = P\Delta V = nR\Delta T$

$\frac{\Delta Q}{\Delta W} = \frac{7}{2}$

28. 3

Sol. $P \propto V$

PV^{-1} = constant

$\Rightarrow$ x = –1

$C = C_v + \frac{R}{1-x}$

$= \frac{5R}{2} + \frac{R}{2} = \frac{6R}{2} = 3R$

29. 1

Sol. Resistance, $R = \frac{\rho 2\pi R_1}{\pi r^2}$

$m = d(\pi r^2) 2\pi R_1$

$R = \rho (2\pi R_1)^2 \frac{d}{m}$

Induced current in the loop, $I = \frac{dB}{dt}\frac{\pi R_1^2}{R}$

30. 3

Sol. $I = \frac{\varepsilon}{R} = 3\,A$

$H = \frac{1}{2}LI^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (3)^2 = 9\,\mu J$

$H_1 = \frac{H}{3} = \frac{9}{3} = 3\,\mu J$

Chemistry

PART – B

SECTION – A

31. B

Sol. $P(C_2H_4O)$ and $Q(C_3H_6O)$ must be carbonyl compounds as they both show +ve $NaHSO_3$ test. P must be ethanal as it gives haloform test.

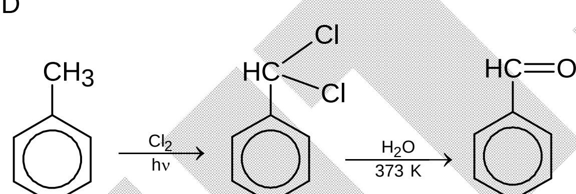
$P = CH_3-\underset{\underset{O}{\|}}{C}-H, Q = CH_3-CH_2-CHO$

$$CH_3-CHO + CH_3-CH_2-CHO \xrightarrow[2.\ \Delta]{1.\ NaOH} CH_3-CH=CH-CHO$$
$$+$$
$$CH_3-CH_2-CH=\overset{\overset{CH_3}{|}}{C}-CHO$$
$$+$$
$$CH_3-CH=\overset{\overset{CH_3}{|}}{C}-CHO$$
$$+$$
$$CH_3-CH_2-CH=CH-CHO$$

Total 4 products obtained if we exclude stereoisomerism.

32. D

Sol.



(P) (Q) (R)

Free-radical substitution is involved in conversion of P to Q.

33. C

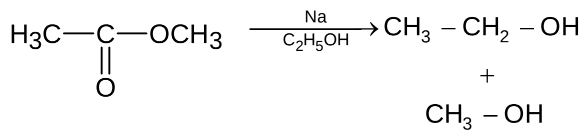
Sol. $H_3C-\underset{\underset{O}{\|}}{C}-OC_2H_5 \xrightarrow[(ii)\ H_2O]{(i)\ LiAlH_4} 2CH_3-CH_2OH$

$H_3C-\underset{\underset{O}{\|}}{C}-O-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow[(ii)\ H_2O]{(i)\ LiAlH_4} 2CH_3-CH_2OH$

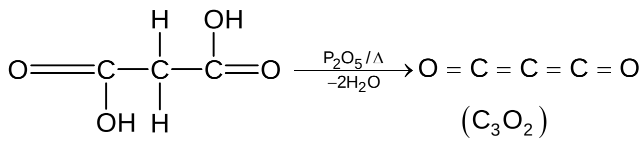
$H_3C-\underset{\underset{O}{\|}}{C}-OCH_2CH_3 \xrightarrow[(ii)\ H_2O]{(i)\ DIBAL-H} H_3C-\underset{\underset{O}{\|}}{C}-H$

$+$

CH_3-CH_2-OH



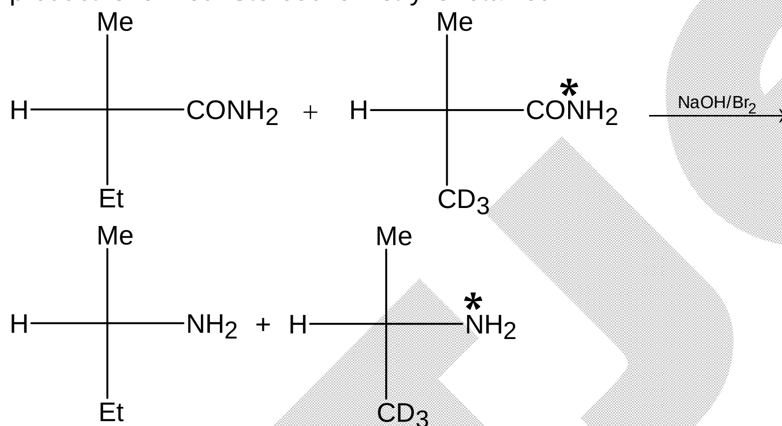
34. C

Sol. 

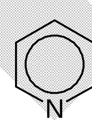
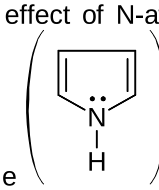
Malonic acid

35. D

Sol. This is Hoffmann bromamide reaction which involved intramolecular migration. No cross-over product is formed. Stereochemistry is retained.



36. B

Sol. The lone pair on N-atom in pyridine  is localised but – I effect of N-atom deactivates benzene ring. On the other hand, the lone pair on N-atom in pyrrole  is de-localized and hence activates the ring towards EAS.

$\therefore$ Order of reactivity is : III > II > I.

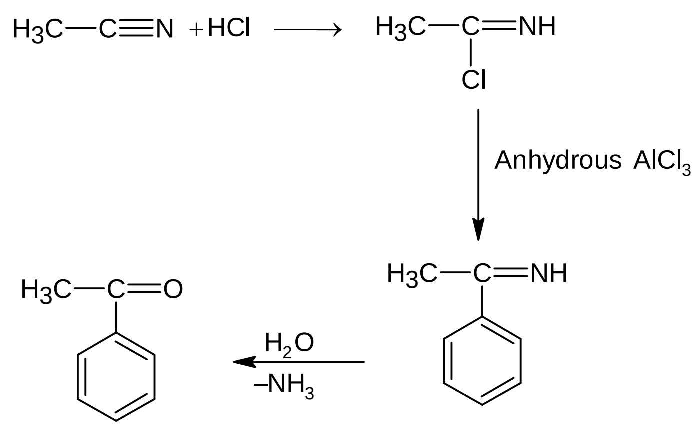
37. A

Sol. In the first reaction, the lone pair on nitrogen atom takes part in resonance with benzene ring and hence activates benzene ring towards EAS. But in second reaction, the lone pair on nitrogen atom does not take part in resonance with benzene ring due to SIR-effect and hence does not activate benzene ring.

$\therefore r_1 > r_2$.

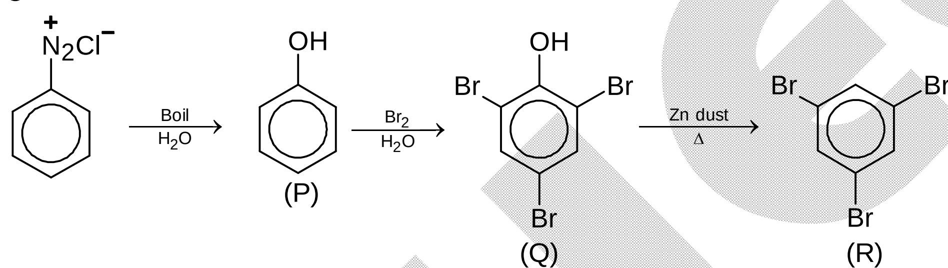
38. D

Sol.



39. C

Sol.

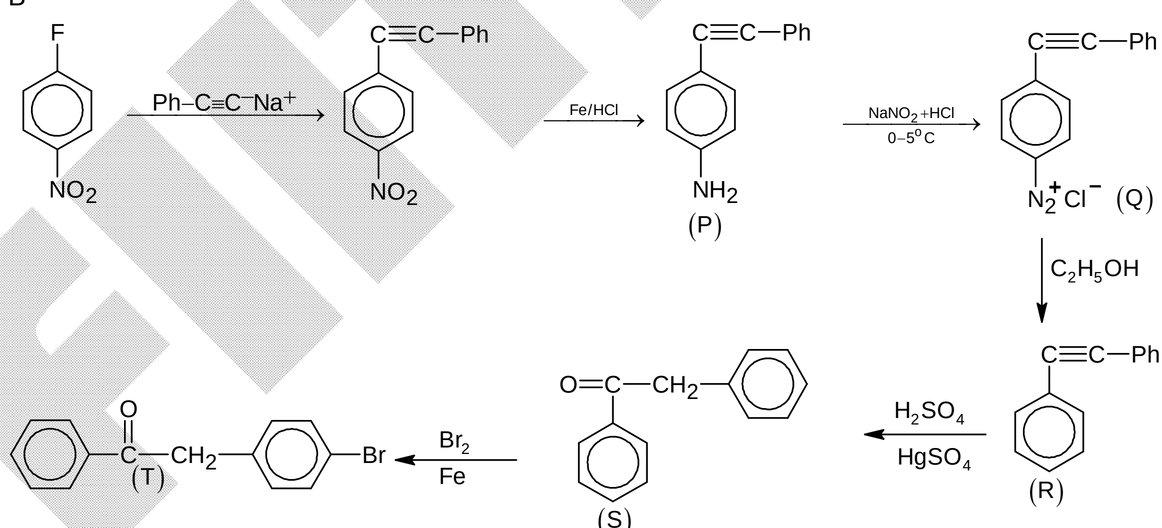


Q is polar while R is non-polar.

$\therefore \mu_Q > \mu_R$.

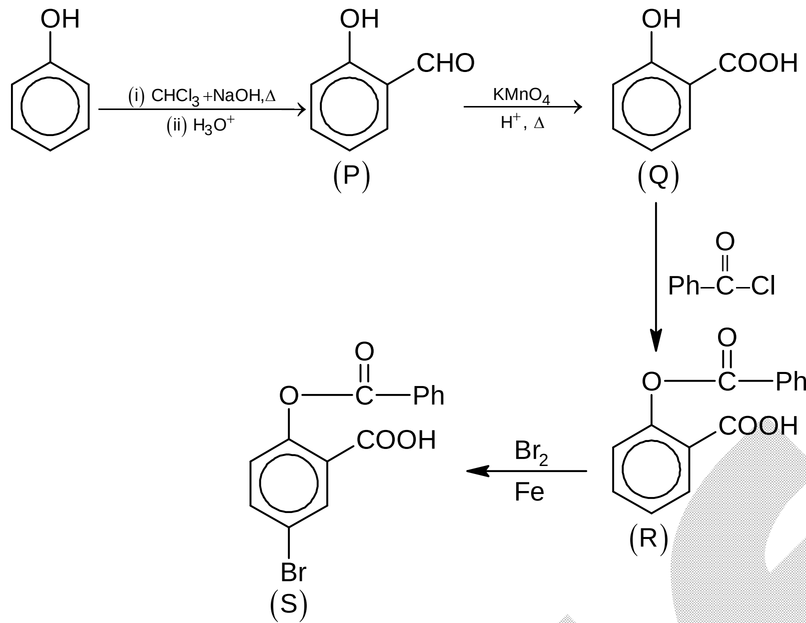
40. B

Sol.



41. C

Sol.



42. B

Sol. $CH_3CH_2OH \xrightarrow[H_2SO_4]{\text{Cold conc.}} \left[CH_3 - CH_2 - \overset{+}{O}H_2\right]HSO_4^-$

Oxonium salt

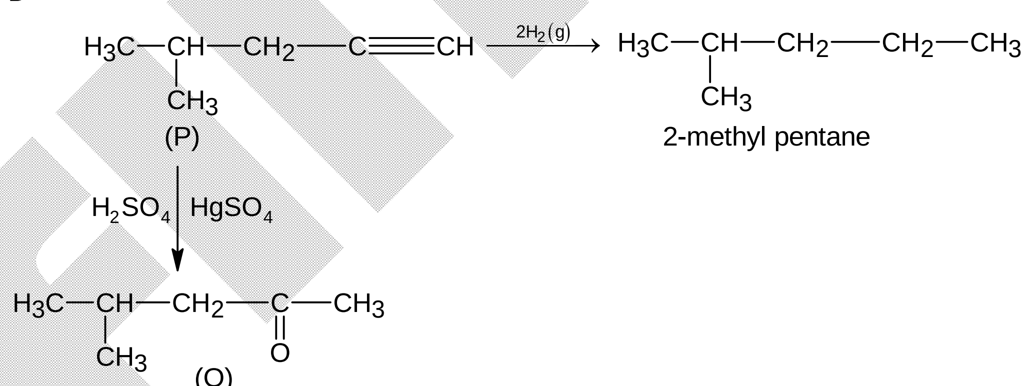
$(i \simeq 2)$

$Ph_3C - OH \xrightarrow[H_2SO_4]{\text{Cold conc.}} Ph_3C^+ + H_3O^+ + 2HSO_4^-$

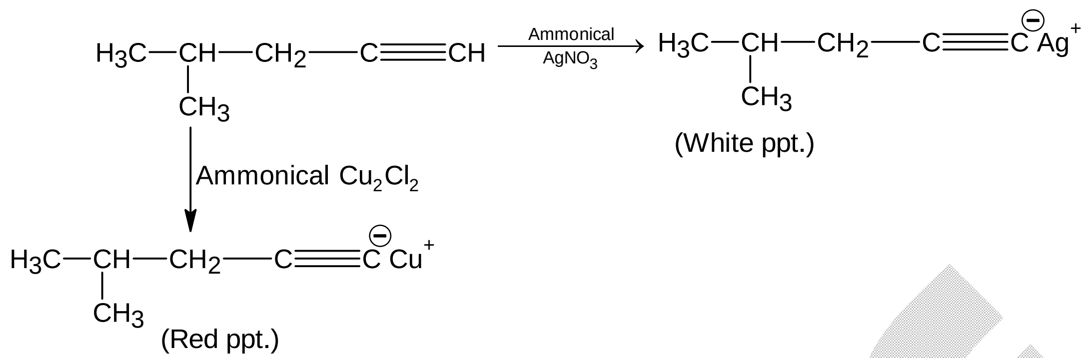
$(i \simeq 4)$

43. D

Sol.



The ketone (Q) gives positive Iodoform test and shows 2,4-DNP test. It also forms white crystalline product with $NaHSO_3$.



44. A

Sol. $\text{Phenol} + \text{H}-\text{CHO} \xrightarrow{\Delta} \text{Novolac}(\text{P}) \xrightarrow[\text{H}-\text{CHO}]{\Delta} \text{Bakelite}(\text{Q})$

45. A

Sol. It's a Reimer-Teimann reaction. Dichloro carbene (Singlet) is formed as an intermediate in this reaction.

46. B

Sol. Order of basic strength in aqueous medium is

$Me_2NH > MeNH_2 > Me_3N$

(II) (I) (III)

47. A

Sol.

Picric acid	$pK_a = 0.4$
Squaric acid	$pK_{a_1} = 1.5$
Phenol (Carbolic acid)	$pK_a = 9.98$
Carbonic acid	$pK_{a_1} = 6.37$

48. A

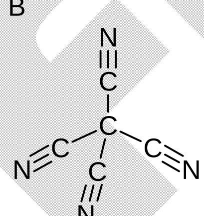
Sol. There is only geometrical isomerism in the given molecule around C = C. There are two geometrical isomers possible for the given molecule.

49. C

Sol. Assymetric compounds are P, Q and R as these do not contain POS (Plane of Symmetry).

50. B

Sol.



No. of σ bonds = 8

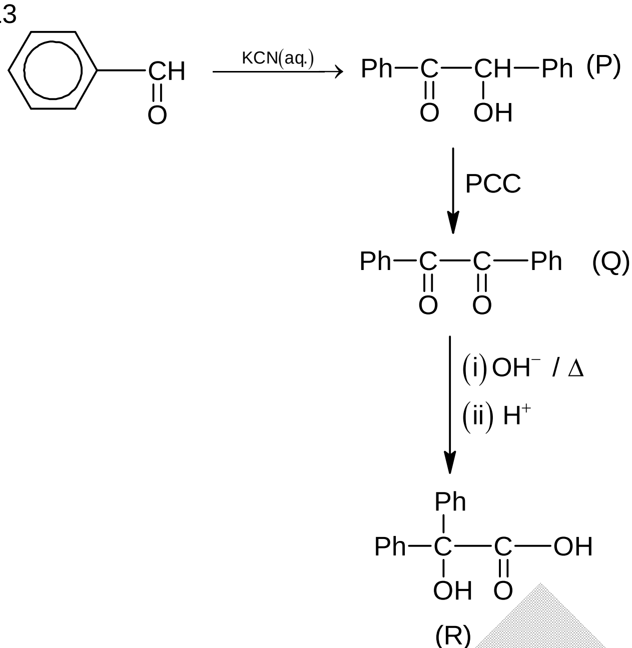
No. of π bonds = 8

$\therefore \frac{\sigma}{\pi} = 1:1.$

SECTION – B

51. 13

Sol.



Number of sp^2 hybridised carbons in R = 13.

52. 3

Sol. Soda lime is a mixture of 3 moles of NaOH and 1 mole of CaO.

53. 4

Sol. Pressure = 802 – 42 = 760 mm Hg
= 1 atmosphere

Volume = 110 mL = 0.11 L

$$n = \frac{PV}{RT} = \frac{1 \times 0.11}{0.0821 \times 300} = 4.47 \times 10^{-3} \text{ moles}$$

Weight of $N_2 = n \times 28$

$= 4.47 \times 10^{-3} \times 28 = 0.125 \text{ g}$

$$\% \text{ of nitrogen } = \frac{\text{Wt. of nitrogen}}{\text{Wt. of organic compound}} \times 100$$

$$= \frac{0.125}{0.5} \times 100 = 25\%$$

Molecular formula $= C_{12}H_{24}N_x$

Molecular weight = 168 + 14x

$$\% \text{ of nitrogen } = \frac{14x}{168 + 14x} \times 100 = 25$$

x = 4

$\therefore$ Molecular formula = $C_{12}H_{24}N_4$

54. 0

Sol. $$S^{-2} + Na_2\left[Fe(CN)_5(NO)\right] \longrightarrow Na_4\left[Fe(CN)_5(NOS)\right]$$
('X') Violet colour

Compound 'X' is sodium nitroprusside and its magnetic moment is (0.0 B.M.)

55. 8

Sol. Only primary alcohols give aldehydes on oxidation with PCC. 8 structural isomers are possible for such alcohol.

1. $CH_3-CH_2-CH_2-CH_2-CH_2-CH_2-OH$
2. $H_3C-CH_2-CH_2-CH(CH_3)-CH_2-OH$
3. $H_2C-CH_2-CH(CH_3)-CH_2-CH_2-OH$
4. $H_3C-CH(CH_3)-CH_2-CH_2-CH_2-OH$
5. $H_2C-CH_2-C(CH_3)_2-CH_2-OH$
6. $H_2C-CH(CH_3)-CH(CH_3)-CH_2OH$
7. $H_3C-C(CH_3)_2-CH_2-CH_2-OH$
8. $H_2C-CH_2-CH(CH_2CH_3)-CH_2OH$

56. 6

Sol. With $C\equiv CH \rightarrow$ 1 mole G.R. reacts

With $-OH \rightarrow$ 1 mole G.R. reacts

With $CN \rightarrow$ 1 mole G.R. reacts

With $CHO \rightarrow$ 1 mole G.R. reacts

With $COOCH_3 \rightarrow$ 2 mole of G.R. reacts

Thus, total 6 moles of G.R. are consumed

57. 4

Sol. On acetylation, –OH group gets converted into $-OCOCH_3$ per –OH group. It means that molecular weight increases by 42.

Molecular weight of 'A' = 166

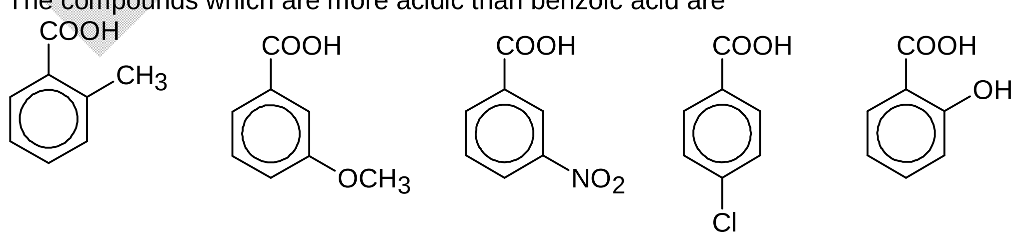
Molecular weight after acetylation = 334

Increase in molecular weight = 168

$\therefore$ Number of OH group $= \dfrac{168}{42} = 4$

58. 5

Sol. The compounds which are more acidic than benzoic acid are



59. 4

Sol.
1. $CH_3-CH=CH-CH_2-CH_2-CH_3$
2. $H_3C-CH=CH-CH-CH_3$
   $\quad\quad\quad\quad\quad\quad\quad\quad\ |$
   $\quad\quad\quad\quad\quad\quad\quad\quad CH_3$
3. $H_2C-CH=C-CH_2-CH_3$
   $\quad\quad\quad\quad\quad\quad\ |$
   $\quad\quad\quad\quad\quad\quad CH_3$
4. $H_2C-CH_2-CH=CH-CH_2-CH_3$

60. 4

Sol. Number of meso-isomers for a symmetrical molecule having odd number of chiral carbon

$=2^{\left(\frac{n-1}{2}\right)}$, where n = number of chiral carbon

$\therefore$ Number of meso isomers $=2^{\left(\frac{5-1}{2}\right)}=2^2=4$.

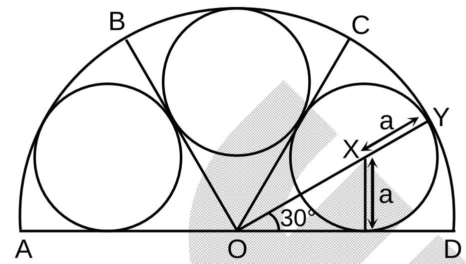
Mathematics

PART – C

SECTION – A

61. C

Sol. From the diagram, $\angle AOB = \angle BOC = \angle COD = 60°$



$\Rightarrow \quad \angle YOD = \frac{\pi}{6}$

$\Rightarrow$ Let X be the centre of right hand circle, OXsin30° = a

$\Rightarrow \quad OX = 2a$

Now

$r = OY = 2a + a$

$a = \frac{r}{3}$

62. A

Sol. Let AB = BO = R and radius of smallest circle is r, OA = 2R, ON = 2R – r and BN = R + r

In ΔOBN

$BN^2 + OB^2 = ON^2$

$(R + r)^2 + R^2 = (2R - r)^2$

$2R^2 - 6Rr = 0$

$R(R - 3r) = 0$

Considering ΔOBN, perimeter

$OB + BN + NO = R + (R + r) + (2R - r) = 8$

$\Rightarrow \quad R = 2$

$r = \frac{1}{3}R = \frac{2}{3}$

63. A

Sol. Square and add

$24(\sin A \cos B + \cos A \sin B) = 12$

$\Rightarrow \quad \sin(A + B) = \sin(\pi - C) = \frac{1}{2}$

$\Rightarrow \quad \sin C = \frac{1}{2}$

$\Rightarrow \quad C = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

but if $C = \frac{5\pi}{6}$, then $A < \frac{\pi}{6}$ so $3\sin A + 4\cos B = \frac{3}{2} + 4 < 6$

$\Rightarrow \quad C = \frac{\pi}{6}$ only.

64. B

Sol. $\frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$

$\Rightarrow \quad \frac{1}{12}[3(\sin^4 x + \cos^4 x) - 2(\sin^6 x + \cos^6 x)]$

$\Rightarrow \quad \frac{1}{12}[3((\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x) - 2\{(\sin^2 x + \cos^2 x)(1 - 3\sin^2 x\cos^2 x)\}]$

$\Rightarrow \quad \frac{1}{12}\{3 - 6\sin^2x\cos^2x - 2(1 - 3\sin^2x\cos^2x)\}$

$\Rightarrow \quad \frac{1}{12} = \frac{2}{24}$

65. B

Sol. $\frac{\frac{a}{b}-1}{\frac{a}{b}+1} = \tan\left(\frac{A-B}{2}\right)\tan\frac{C}{2}$

$\Rightarrow \quad \frac{1+\sqrt{3}}{\sqrt{3}+3} = \tan\left(\frac{A-B}{2}\right)\tan\frac{C}{2} = \tan\left(\frac{A-B}{2}\right)\frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\left(\frac{A-B}{2}\right) = 1$

$\Rightarrow \quad A - B = \frac{\pi}{2}$ and $A + B = \frac{2\pi}{3}$

$\Rightarrow \quad A = 105°, B = 15°$

66. D

Sol. $AO^2 = OB^2 = OC^2$

$\alpha^2 + 4 = \beta^2 + 4 \Rightarrow \alpha = \pm\beta$

$\Rightarrow \alpha = -\beta$ (if $\alpha = \beta$ then no triangle is formed)

ABC is right angled triangle at A

$\therefore$ $(\alpha, 2)$ is orthocentre $\therefore$ $\alpha = 3, \gamma = 2$

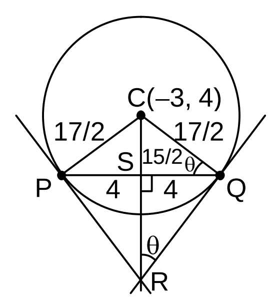
$\therefore$ vertices are (3, 2), (3, –2) and (–3, 2).

67. B

Sol. $\tan\theta = \frac{15/2}{4} = \frac{15}{8}$

In ΔRSQ $\tan\theta = \frac{4}{RS}$

$\Rightarrow \quad \frac{15}{8} = \frac{4}{RS} \Rightarrow RS = \frac{32}{15}$



68. D

Sol. Image of vertex of P_1 in $3x - y + 10 = 0$ will be vertex of P_2

$\frac{x_2 - 1}{3} = \frac{y_2 - 3}{-1} = \frac{-2(3 - 8 + 10)}{10}$

$\therefore$ vertex of $P_2 = (-2, 9)$

Similarly focus of P_2 is (–9, 3) $\therefore$ $\alpha = -9$ and $\beta = 3$

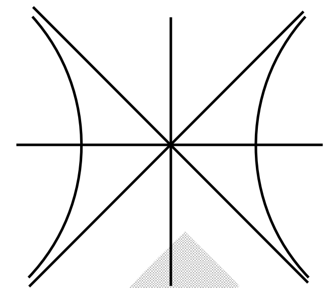
Slope of directrix = $-\frac{7}{6}$

Equation of directrix $y - 15 = -\frac{7}{6}(x - 5)$

$7x + 6y = 125 \therefore \gamma = 125$.

69. C

Sol. Equation of asymptotes $x - 3y = 0$ and $x + 3y = 0$
If tangents to different branches are drawn then
$(\alpha^2 - 9\beta^2 - 9) < 0$ and $(\alpha - 3\beta)(\alpha + 3\beta) < 0$
i.e. $\alpha^2 < 9\beta^2 + 9$ and $\alpha^2 < 9\beta^2$.



70. B

Sol. $a^2 = \frac{1}{4}$, $b^2 = \frac{1}{9}$ let P be (x_1, y_1)

Equation of normal $\frac{1x}{4x_1} - \frac{1y}{9y_1} = \frac{5}{36}$

It cuts y-axis at $\left(0, -\frac{5y_1}{4}\right) \Rightarrow \frac{-5y_1}{4} = \frac{-5}{8\sqrt{3}} \Rightarrow y_1 = \frac{1}{2\sqrt{3}}$ $\therefore x_1 = \frac{1}{4}$

Thus $\alpha = \frac{5}{36}$

Area of $\Delta OAB = \frac{1}{2} \times \frac{5}{36} \times \frac{5}{8\sqrt{3}} = \frac{25}{576\sqrt{3}}$

71. B

Sol. $x^2 + 24x = 25 \Rightarrow x = -25, 1$

$\therefore$ $P\left(1, 2\sqrt{6}\right)$ and $Q\left(1, -2\sqrt{6}\right)$

Equation of tangent at P and Q to the circle

$x + 2\sqrt{6}y = 25$ and $x - 2\sqrt{6}y = 25$

$\therefore$ R is (25, 0)

Equation of tangent at P and Q to the parabola

$2\sqrt{6}y = 12(x+1)$ and $-2\sqrt{6}y = 12(x+1)$

$\therefore$ S is (–1, 0)

Area of $\Delta PRS = \frac{1}{2} \times 2\sqrt{6} \times 26 = 26\sqrt{6}$

$$R = \frac{abc}{4\Delta} = \frac{26 \times 2\sqrt{7} \times 10\sqrt{6}}{4 \times 26\sqrt{6}} = 5\sqrt{7}$$

72. D

Sol. $a^2 + b^2 = 18$ and $b^2 = 6$

$\Rightarrow a^2 = 12 \Rightarrow 6 = 12(1 - e^2) \Rightarrow e = \frac{1}{\sqrt{2}}$

73. B

Sol. $x(y - 1) = 8$

Equation of directrix $x + (y - 1) = 2\sqrt{2} \times \sqrt{2}$

$\Rightarrow x + y = 5 \Rightarrow y = -x + 5$ touches

$\frac{x^2}{30} - \frac{y^2}{30/k} = 1$ $\therefore 25 = 30 \times 1 - \frac{30}{k}$ $\Rightarrow k = 6.$

74. A

Sol. $P \equiv (|OP| \cos\theta, |OP| \sin\theta)$

$Q \equiv \left(|OQ| \left(\cos\left(\theta \pm \frac{\pi}{2}\right), \sin\left(\theta \pm \frac{\pi}{2}\right)\right)\right)$

$\Rightarrow \quad \frac{1}{|OP|^2} = \frac{\sin^2\theta}{b^2} + \frac{\cos^2\theta}{a^2}$

$\Rightarrow \quad \frac{1}{|OQ|^2} = \frac{\sin^2\theta}{a^2} + \frac{\cos^2\theta}{b^2}$

$\Rightarrow \quad \frac{1}{|OP|^2} + \frac{1}{|OQ|^2} = \frac{1}{a^2} + \frac{1}{b^2}$

$\Rightarrow \quad |OP \times |OQ|$ reaches $\frac{2a^2b^2}{a^2+b^2}$

75. B

Sol. Suppose there is a point P(4cosθ, 3sinθ) on the ellipse, then distance of P to AB is

$= \left|\frac{3(4\cos\theta) + 4(3\sin\theta) - 12}{5}\right|$

$= \frac{12}{5}(\cos\theta + \sin\theta - 1)$

$\le \frac{12}{5}\left(\sqrt{2} - 1\right) \le \frac{6}{5}$

But AB = 5 so $\Delta APB \le \frac{1}{2} \times 5 \times \frac{6}{5} = 3$

So exactly two points.

76. C

Sol. $PF_1 + PF_2 = 2a = 6$

$\frac{PF_1}{PF_2} = \frac{2}{1}$; $PF_1 = 4$ and $PF_2 = 2$

$F_1F_2 = 2\sqrt{5}$

there $PF_1^2 + PF_2^2 = (F_1F_2)^2$

$16 + 4 = \left(2\sqrt{5}\right)^2$

So ΔPF_1F_2 is right angle triangle

area = $\frac{1}{2} \times PF_1 \times PF_2 = \frac{1}{2} \times 4 \times 2 = 4$

77. B

Sol. Since $\sqrt{2} + \sqrt{3} > \pi, \frac{\pi}{2} - \sqrt{2} < \sqrt{3} - \frac{\pi}{2} < \frac{\pi}{2}$

$\Rightarrow \quad \cos\left(\frac{\pi}{2} - \sqrt{2}\right) > \cos\left(\sqrt{3} - \frac{\pi}{2}\right)$

$\Rightarrow \quad \sin\left(\sqrt{2}\right) > \sin\left(\sqrt{3}\right)$

again $\cos\left(\sqrt{2}\right) > 0$ and $\cos\left(\sqrt{3}\right) < 0$

$\cos\sqrt{2} - \cos\sqrt{3} > 0$

$\Rightarrow$ given curve will be ellipse

Now $\left(\sin\sqrt{2}-\sin\sqrt{3}\right)-\left(\cos\sqrt{2}-\cos\sqrt{3}\right)$

$= 2\sqrt{2}\sin\left(\frac{\sqrt{2}-\sqrt{3}}{2}\right)\sin\left(\frac{\sqrt{2}+\sqrt{3}}{2}+\frac{\pi}{4}\right)$

and $-\frac{\pi}{2}<\frac{\sqrt{2}-\sqrt{3}}{2}<0$

$\Rightarrow$ $\sin\left(\frac{\sqrt{2}-\sqrt{3}}{2}\right)<0$ and $\frac{\pi}{2}<\frac{\sqrt{2}+\sqrt{3}}{2}<\frac{3\pi}{4}$

$\Rightarrow$ $\sin\left(\frac{\sqrt{2}+\sqrt{3}}{2}+\frac{\pi}{4}\right)>0$

$\Rightarrow$ $2\sqrt{2}\sin\left(\frac{\sqrt{2}-\sqrt{3}}{2}\right)\sin\left(\frac{\sqrt{2}+\sqrt{3}}{2}+\frac{\pi}{4}\right)<0$

$\Rightarrow$ ellipse is vertical ellipse.

78. C

Sol. Assume AB is on the line y = 2x − 17 and coordinate of other two are on parabola are $C(x_1, y_1)$ and $D(x_2, y_2)$, then CD is on the line L whose equation is y – 2x = b

by solving with curve we get

$x^2 - 2x - b = 0 \Rightarrow x_1, x_2 = 1\pm\sqrt{b+1}$

assume length of one side of square is a

$\Rightarrow$ $a^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$= 5(x_1 - x_2)^2 = 20(b + 1)$ …(1)

Now $a=\left|\frac{17+b}{\sqrt{5}}\right|$ …(2)

$\Rightarrow$ b = 3 or 63 $\Rightarrow$ a^2 = 80 or 1280.

79. A

Sol. Equation of AB $\Rightarrow y=\sqrt{3}x$ by solving with parabola $y^2 = 8(x + 2)$

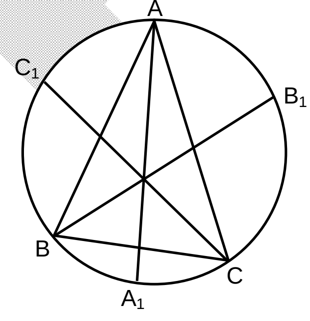
$3x^2 - 8x - 16 = 0$

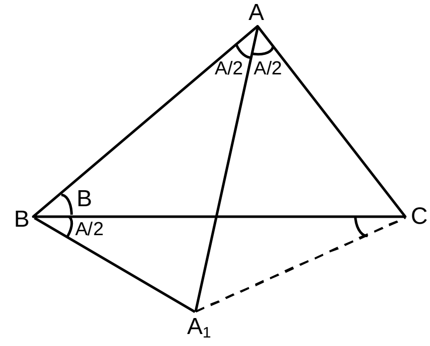
E is mid point of AB then x coordinate of E is $\frac{4}{3}$, then

$|FE| = \frac{1}{\cos 60^\circ}\times\frac{4}{3}=\frac{8}{3}$, $|PF| = 2\times|FE| = \frac{16}{3}$ where F is focus

80. A

Sol. In ΔABC





$\angle B = \angle ABC + \frac{A}{2}$

$\frac{AA_1}{\sin\left(B+\frac{A}{2}\right)} = 2R = 2$

$AA_1 = 2\sin\left(B+\frac{A}{2}\right) = 2\cos\left(\frac{B}{2}-\frac{C}{2}\right)$

$\Rightarrow \quad AA_1\cos\frac{A}{2} = 2\cos\left(\frac{B-C}{2}\right)\cos\left(\frac{A}{2}\right)$

$= \cos\left(\frac{\pi}{2}-C\right)+\cos\left(\frac{\pi}{2}-B\right) = \sin B + \sin C$

$\Rightarrow$ similarly $BB_1\cos\frac{B}{2} = \sin A + \sin C$

$CC_1\cos\frac{C}{2} = \sin A + \sin B$

SECTION – B

81. 2022

Sol. Let $x_n = \tan a_n$

$\Rightarrow \quad \tan a_{n+1} = \frac{1+\tan a_n}{1-\tan a_n} = \tan\left(\frac{\pi}{4}+a_n\right)$

$\Rightarrow \quad \tan a_{n+1} = \tan\left(\frac{\pi}{4}+a_n\right)$

$\Rightarrow \quad a_{n+1} = a_n + \frac{\pi}{4} + n\pi$

$\Rightarrow \quad a_{n+4} = a_n + k.\pi \; ; \; k \in I$

$\Rightarrow \quad x_{n+4} = \tan a_{n+4} = \tan a_n = x_n$

$\Rightarrow \quad x_n$ is periodic for period 4

$\Rightarrow \quad x_{2024} = x_0 = 2022$

82. 36

Sol. Let assume $x = \cos^2\alpha\cos^2\beta$, $y = \cos^2\alpha\sin^2\beta$, $z = \sin^2\alpha$

Now $x + y + z = 1$

$\Rightarrow \quad P = \sec^2\alpha\sec^2\beta + 4\sec^2\alpha\,\text{cosec}^2\beta + 9\,\text{cosec}^2\alpha$

$\Rightarrow \quad P = (\tan^2\alpha + 1)(\tan^2\beta + 1) + 4(\tan^2\alpha + 1)(\cot^2\beta + 1) + 9(\cot^2\alpha + 1)$

$= 14 + 5\tan^2\alpha + 9\cot^2\alpha + (\tan^2\beta + 4\cot^2\beta)(1 + \tan^2\alpha)$

$\geq 14 + 5\tan^2\alpha + 9\cot^2\alpha + 2\tan\beta.2\cot\beta(1 + \tan^2\alpha)$

$= 18 + 9(\tan^2\alpha + \cot^2\alpha) \geq 18 + 9.2\tan\alpha\cot\alpha = 36$

Equality hold when $\cos^2\alpha = \frac{1}{2}$, $\cos^2\beta = \frac{1}{3}$

$\Rightarrow \quad x = \frac{1}{6}, y = \frac{1}{3}, z = \frac{1}{2}$

83. 2

Sol. $(1 - \cot 22°)(1 - \cot 23°)$

$\Rightarrow \quad \left(1-\frac{\cos 22°}{\sin 22°}\right)\left(1-\frac{\cos 23°}{\sin 23°}\right)$

$\Rightarrow \left(\frac{\sin 22^\circ - \cos 22^\circ}{\sin 22^\circ}\right)\left(\frac{\sin 23^\circ - \cos 23^\circ}{\sin 23^\circ}\right)$

$\Rightarrow \frac{\sqrt{2}\left(\sin\left(22^\circ - 45^\circ\right)\right)\sqrt{2}\sin\left(23^\circ - 45^\circ\right)}{\left(\sin 22^\circ \sin 23^\circ\right)}$

$\Rightarrow \frac{2\sin\left(-22^\circ\right)\sin\left(-23^\circ\right)}{\left(\sin 22^\circ\right)\left(\sin 23^\circ\right)} = 2$

84. 2

Sol. $\frac{\frac{\sqrt{3}-1}{4}}{\sin x} + \frac{\frac{\sqrt{3}+1}{4}}{\cos x} = \sqrt{2}$

$\Rightarrow \frac{\sin\left(\frac{\pi}{12}\right)}{\sin x} + \frac{\cos\left(\frac{\pi}{12}\right)}{\cos x} = 2$

$\Rightarrow \sin\left(\frac{\pi}{12}\right)\cos x + \cos\left(\frac{\pi}{12}\right)\sin x = 2\sin x\cos x$

$\Rightarrow \sin\left(\frac{\pi}{12} + x\right) = \sin 2x$

$\Rightarrow \frac{\pi}{12} + x = 2x$ or $\frac{\pi}{12} + x = \pi - 2x$

$\Rightarrow x = \frac{\pi}{12}$ or $x = \frac{11\pi}{36}$

85. 1011

Sol. Let P denote the desired product and Q = $\sin\alpha.\sin2\alpha.\sin3\alpha$ …… $\sin(1011\alpha)$

then $2^{1011}PQ = \sin2\alpha.\sin4\alpha.\sin6\alpha$ ….. $\sin(2022)\alpha$

= $(\sin2\alpha.\sin4\alpha$ ….. $\sin(1010\alpha)$ $(\sin(\pi - 1012\alpha))$ …. $(\sin(\pi - 2020\alpha)$ $(\sin(\pi - 2022)\alpha)$

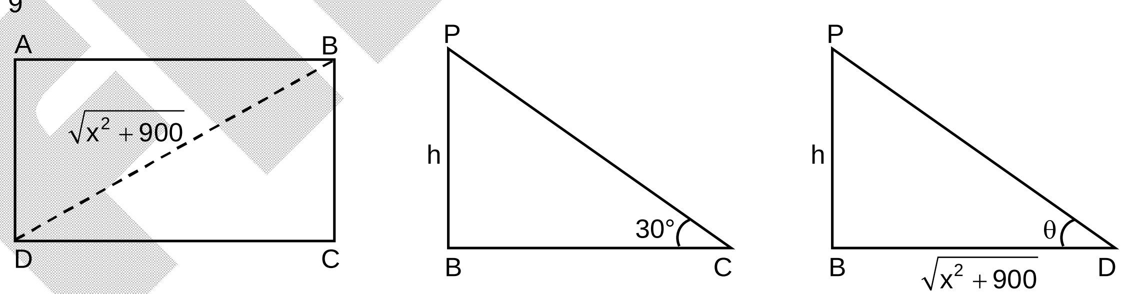
$\Rightarrow 2^{1011}PQ = \sin\alpha.\sin2\alpha.\sin3\alpha$ ….. $\sin(1011\alpha) = Q$

$\Rightarrow 2^{1011}PQ = Q$

$P = \frac{1}{2^{1011}}$

86. 9

Sol.



$\tan 30^\circ = \frac{h}{x} \Rightarrow x = h\sqrt{3}$

$\tan\theta = \frac{h}{\sqrt{x^2+900}} = \frac{8}{17\sqrt{3}}$

$\frac{h}{\sqrt{3h^2+900}} = \frac{8}{17\sqrt{3}}$

$289 \times 3 \times h^2 = 192h^2 + 64 \times 900$
$675h^2 = 64 \times 900$
$\Rightarrow h = \frac{16}{\sqrt{3}}$

87. 21
Sol. Asymptotes $3(x - 1) - 2(y + 3) = 0$ and $3(x - 1) + 2(y + 3) = 0$
$3x - 2y - 9 = 0$ and $3x + 2y + 3 = 0$
It touches $x^2 + y^2 - 2x + k/13 = 0$
$\therefore \quad A\left(-\frac{5}{13}, -\frac{12}{13}\right)$ and $B\left(\frac{31}{13}, -\frac{12}{13}\right)$
$\therefore \quad k = -23$

88. 1
Sol. $(\sin^2\alpha)^3 + (\cos^2\beta)^3 + (-1)^3 - 3\sin^2\alpha\cos^2\beta(-1) = 0$
$\Rightarrow \quad \sin^2\alpha + \cos^2\beta - 1 = 0$ or $\underbrace{\sin^2\alpha = \cos^2\beta = -1}_{\text{not possible}}$
$\Rightarrow \quad \sin^2\alpha = 1 - \cos^2\beta$
$\Rightarrow \quad \sin^2\alpha = \sin^2\beta$
$\alpha = \beta$

89. 315
Sol. Let M is mid point of AB, $M = \left(\frac{a+b}{2}, 24\right)$
$OM \perp MA$, $|OM| = \sqrt{3}\, MA$
$\Rightarrow \quad \frac{a+b}{2} = 13\sqrt{3}, \frac{a-b}{2} = 8\sqrt{3}$
$\Rightarrow \quad a = 21\sqrt{3}$ and $b = 5\sqrt{3}$

90. 1
Sol. $P(x, y) = (14\cos\theta - 5, 14\sin\theta + 12)$
minimum value of $g(\theta)$ where
$g(\theta) = (14\cos\theta - 5)^2 + (14\sin\theta + 12)^2$
$= 365 + 28(12\sin\theta - 5\cos\theta)$
$g(\theta) \geq 1$