FIITJEE

ALL INDIA TEST SERIES

PART TEST – III

JEE (Advanced)-2021

PAPER –1

TEST DATE: 20-12-2020

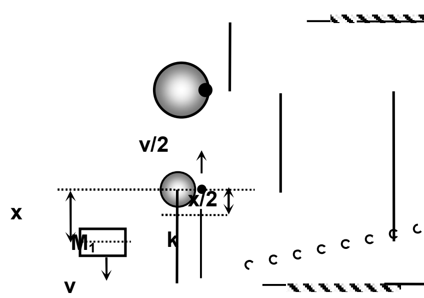
ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. C

Sol. If the mass is displaced by x and has speed v spring will extend by x/2 & M_2 will have speed v/2



Energy of this system can be written as

$$-M_1gx+\frac{1}{2}M_1v^2+M_2g\frac{x}{2}+\frac{1}{2}M_2\left(\frac{v}{2}\right)^2+\frac{1}{2}K\left(\frac{x}{2}\right)^2 = \text{constant}$$

$$-M_1gx+\frac{1}{2}M_1v^2+M_2g\frac{x}{2}+\frac{1}{8}M_2v^2+\frac{1}{8}Kx^2 = \text{constant}$$

Differentiate w.r.t. time

$$-M_1gv+M_1v\frac{dv}{dt}+\frac{M_2g}{2}v+\frac{1}{4}M_2v\frac{dv}{dt}+\frac{1}{4}Kx.v=0$$

$$\Rightarrow -M_1g+M_1\frac{dv}{dt}+\frac{M_2g}{2}+\frac{M_2}{4}\frac{dv}{dt}+\frac{1}{4}Kx=0$$

$$\left(M_1+\frac{M_2}{4}\right)\frac{dv}{dt}=-\frac{K}{4}x+M_1g-\frac{M_2g}{2}$$

$$a=\frac{dv}{dt}=-\left(\frac{K}{4M_1+M_2}\right)x+\frac{2(2M_1-M_2)g}{4M_1+M_2}$$

Here $\omega^2=\dfrac{K}{4M_1+M_2}$

$$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{4M_1+M_2}{K}}$$

2. B

Sol. use $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

3. A

Sol. For total internal reflection at face BC, $90^o - \theta > C$

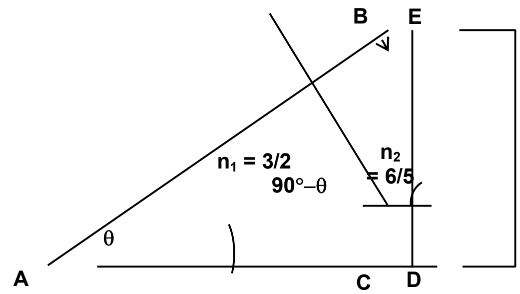
$90^o - C > \theta$ ..... (1)

For refraction at surface BC

$\frac{3}{2}\sin C = \frac{6}{5}\sin 90^\circ \Rightarrow \sin C = \frac{4}{5}$

C = 53° ..... (2)

From 1 & 2 $\Rightarrow 90^o - 53^o > \theta \Rightarrow 37^o > \theta$



4. C

Sol. If R_0 be the initial activity of the sample, then $R_1 = R_0 e^{-\lambda t_1}$ and $R_2 = R_0 e^{-\lambda t_2}$

Where $\lambda = \frac{1}{T}$ {∵ Mean life T = 1/λ}

$\Rightarrow \frac{R_2}{R_1} = \frac{e^{-\lambda t_2}}{e^{-\lambda t_1}} = e^{\lambda(t_1 - t_2)}$

$\Rightarrow R_2 = R_1 \exp\left(\frac{t_1 - t_2}{T}\right)$

5. C

Sol. ${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + p$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$

$3\,{}_1H^2 \rightarrow {}_2He^4 + p + n$

$\Rightarrow \quad \Delta m = m\left({}_2He^4\right) + m(p) + m(n) - 3m\left({}_1H^2\right)$

$\Rightarrow \quad \Delta m = \left[4.001 + 1.007 + 1.008 - 3(2.014)\right]u$

$\Delta m = -0.026u$

$\Rightarrow \quad |\Delta E| = c^2|\Delta m|$

$\Rightarrow \quad \Delta E = (9 \times 10^{16})(0.026 \times 1.67 \times 10^{-27})$

$\Rightarrow \quad \Delta E = (931.5)(0.026)$ MeV

$\Rightarrow \quad \Delta E = 3.87 \times 10^{-12}$J

As each reaction involves 3 deutrons, so total number of reactions involved in the process $= \frac{10^{40}}{3}$. If each reaction produces an energy ΔE, then

$E_{total} = \frac{10^{40}}{3}\Delta E = 1.29 \times 10^{28}$ J

$E_{total} = Pt$

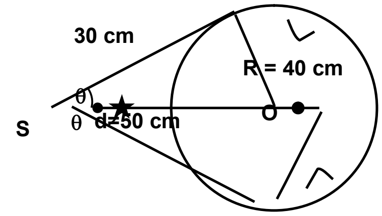
Time of exhaustion of the star

$t = \frac{1.29 \times 10^{28}}{10^{16}}$

$\Rightarrow t = 1.29 \times 10^{12}$ s

6. D

Sol. $P = \frac{P_0}{4\pi} \times 2\pi(1-\cos\theta)$



$P = \frac{P_0}{2}\left(1-\frac{3}{5}\right)$

$P = \frac{P_0}{5} = \frac{2\times10^{-3}}{5}$

P = 4×10^{-4} watt

No. of photoelectrons emitted per second

$n_e = \frac{4\times10^{-4}}{5\times1.6\times10^{-19}\times10^{6}} = 5\times10^{8}$ photoelectrons/sec

Let after time 't' the photoelectron emission will stop.

$\frac{q}{4\pi\varepsilon_0 R} = V_S \Rightarrow \frac{n_e e t}{4\pi\varepsilon_0 R} = V_S$

$\frac{9\times10^{9}\times5\times10^{8}\times1.6\times10^{-19}\times t}{0.4} = 3$

$t = \frac{30}{18} = \frac{5}{3}$

t = 1.67 sec

7. A, B, C

Sol. λ = 1m

$\lambda/4$ = x

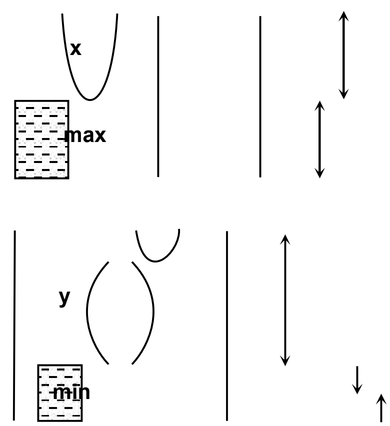
So, x = 25 cm

Maximum length of water column = 120 − 25 = 95 cm

y = $3\lambda/4$ = 75 cm

Minimum length of water column = 120 − 75 = 45 cm

Distance between the two successive nodes =$\lambda/2$ = 50 cm

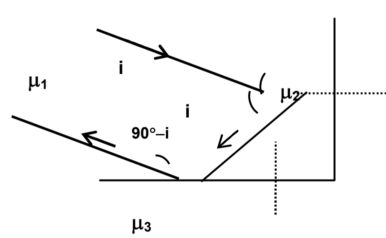


8. B, C, D

Sol. Initial extension is 5cm and equilibrium is at the extension of 2 cm. So Amplitude of SHM is 3cm

9. B, C, D

Sol. For T.I.R. at interface of μ_1&μ_2



i > $C_1 \Rightarrow$ sin i > sin C_1 .... (1)

sin i > sin C_1

Taking refraction at interface of μ_1&μ_2

$\mu_1 \sin C_1 = \mu_2 \sin 90^o \Rightarrow \sin C_1 = \frac{\mu_2}{\mu_1}$ .... (2)

Form (1) and (2)

sin i > $\frac{\mu_2}{\mu_1}$

For T.I.R. at interface of μ_2 andμ_3 $90^o - i > C_2$

$\sin(90^o - i) > \sin C_2 \Rightarrow \cos i > \sin C_2$ .... (3)

Taking refraction at interface of μ_2 & μ_3

$\mu_1 \sin C_2 = \mu_3 \sin 90^o \Rightarrow \sin C_2 = \dfrac{\mu_3}{\mu_1}$ .... (4)

From (3) & (4)

$\cos i > \dfrac{\mu_3}{\mu_1} \Rightarrow \sqrt{1-\sin^2 i} > \dfrac{\mu_3}{\mu_1}$

$1 - \dfrac{\mu_2^2}{\mu_1^2} > \dfrac{\mu_3^2}{\mu_1^2}$

$\mu_1^2 - \mu_2^2 > \mu_3^2$ and $\mu_1^2 - \mu_3^2 > \mu_2^2$

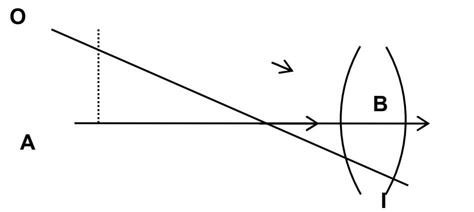
Obviously, $\mu_1^2 + \mu_2^2 > \mu_3^2$

10. A, C

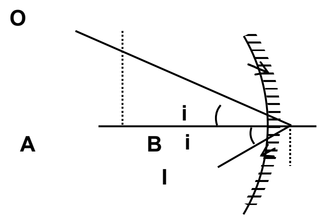
Sol. To find unit vector along reflected ray use the two laws of reflection.

11. A, C, D

Sol. (A)



(C)



(D) Image is inverted $\Rightarrow$ It should be real

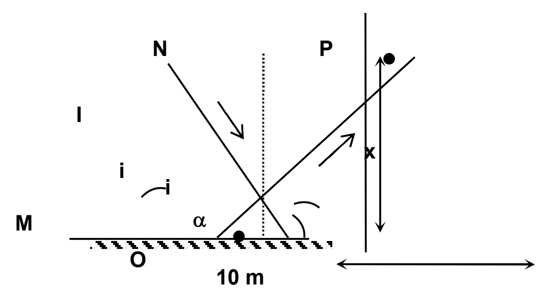
12. A, B, C

Sol. Due to a convex mirror of focal length 2.5 cm, due to a concave mirror having its pole at (2 cm, 0) real virtual pair.

SECTION – C

13. 01000.00

Sol. $x = 10 \tan \alpha$



$\dfrac{dx}{dt} = 10 \sec^2 \alpha \dfrac{d\alpha}{dt}$ …(1)

when the mirror is rotated by angle θ reflected ray will rotate by angle 2θ, so

$\dfrac{d\alpha}{dt} = 2\dfrac{d\theta}{dt}$ …(2)

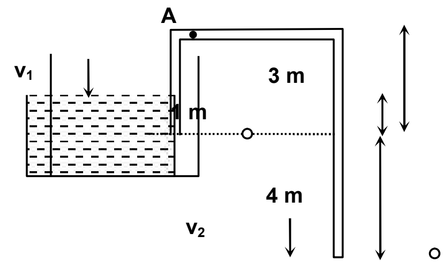
From (1) and (2)

$\dfrac{dx}{dt} = 10 \sec^2 \alpha \cdot 2\dfrac{d\theta}{dt} \Rightarrow \left|\dfrac{dx}{dt}\right| = 20 \sec^2 \alpha \times \omega$

$\left|\dfrac{dx}{dt}\right| = 20 \sec^2 \alpha \times 2\pi n \Rightarrow$ when $i = 37^o$, $\alpha = 53^o$

$\left|\dfrac{dx}{dt}\right| = 20 \sec^2 53^\circ \times 2\pi \times \dfrac{9}{\pi} \Rightarrow \left|\dfrac{dx}{dt}\right| = 20 \times \left(\dfrac{5}{3}\right)^2 \times 2 \times 9 = 1000$ m/s

14. 00006.50



Sol. $P_0 + \frac{\rho v_1^2}{2} + 0 = P_A + \rho g h + \frac{\rho v_2^2}{2}$

$(\because v_2 = \sqrt{2g \times 5} = 10 >> v_1)$

$\left(\frac{\rho v_1^2}{2} - \frac{\rho v_2^2}{2}\right) - \rho g h = (P_A - P_0)$

$(P_A - P_0) = -\rho g h - \frac{\rho v_2^2}{2}$

$P_A = 10^5 - \frac{10^3}{2} \times 10 \times 2 - \frac{10^3}{2} \times \frac{10 \times 10}{2}$

$P_A = 6.5 \times 10^4$ N/m^2

15. 00002.50

Sol. $(\mu - 1)t = 2\lambda$

$t = \frac{2\lambda}{(\mu - 1)}$

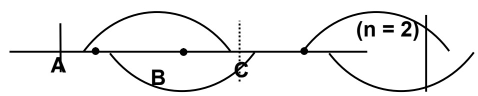
t = 25 × 10^{-7} m = 2.5 μm

16. 00001.50

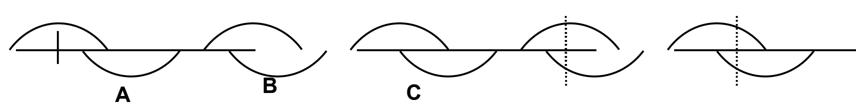
Sol. $\frac{P_0^2}{2\rho v} = I = \frac{\text{Power}}{4\pi r^2}$

Put the values to get the answer.

17. 00007.50



Sol. $f_1 = \frac{2v}{2L}$ …(i)



(n = 6)

$f_2 = \frac{6v}{2L}$ …(ii)

$\frac{f_2}{f_1} = \frac{6v/2L}{\frac{2v}{2L}} = 3$

$f_2 = 3f_1 = 3 \times 100 = 300$ Hz

18. 00022.40

Sol. $\left(\frac{D + x}{D - x}\right)^2 = 16$

$\Rightarrow \left(\frac{D + x}{D - x}\right) = 4$

$\frac{x}{D} = \frac{3}{5} \Rightarrow x = \frac{3D}{5} = \frac{3}{5} \times 140 = 84$ cm

Focal length of the lens

$f = \frac{D^2 - x^2}{4D}$

$f = \frac{(140)^2 - (84)^2}{4 \times 140} = 22.40$ cm

f = 22.40 cm

Chemistry

PART – II

SECTION – A

19. C
Sol. Oxidation number of Ni is zero and strong field ligands.

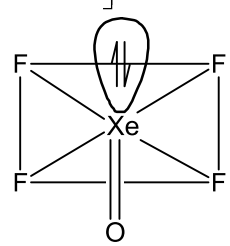
20. C
Sol. $FeO + SiO_2 \longrightarrow FeSiO_3$

21. D
Sol. Its autoreduction
$Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2$

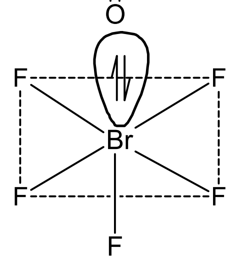
22. B
Sol. $BeO + NaOH \longrightarrow Na_2BeO_2$
$Cr(OH)_3 + NaOH + H_2O_2 \longrightarrow Na_2CrO_4 + H_2O$
$ZnO + NaOH \longrightarrow Na_2ZnO_2 + H_2O$
$Al(OH)_3 + NaOH \longrightarrow Na[Al(OH)_4]$

23. A
Sol. $XeOF_4 \longrightarrow 8 + 6 + (7 \times 4) = 42$
$BrF_5 = 7 \times 6 = 42$
Isoelectronic

$XeOF_4 \longrightarrow sp^3d^2$



$BrF_5 \longrightarrow sp^3d^2$



Both square pyramidal.

24. C
Sol. $SO_3^{2-} + \text{dil. } H_2SO_4 \longrightarrow SO_2$
$SO_2 + Ba(OH)_2 \longrightarrow \underset{\text{(White turbidity)}}{BaSO_3 \downarrow}$
$SO_2 + K_2Cr_2O_7 / H^+ \longrightarrow \underset{\text{(Green)}}{Cr^{3+}} + SO_4^{2-}$
$SO_2 + 2H_2S \longrightarrow 3S \downarrow + 2H_2O$ (Yellowish white turbidity)

25. B, C, D
Sol. Facts.

26. A, B, C, D
Sol. $CuSO_4 + 4NH_3 \longrightarrow [Cu(NH_3)_4]SO_4$ (blue solution)

$FeCl_3 + K_4[Fe(CN)_6] \longrightarrow KFe[Fe(CN)_6]\downarrow$ (blue)

$FeCl_2 + K_3[Fe(CN)_6] \longrightarrow Fe_3[Fe(CN_6)_2]\downarrow$ (blue)

$$Na + (x+y)NH_3 \longrightarrow \underbrace{\underset{\text{(Solvated cations)}}{[Na(NH_3)_x]^+} + \underset{\text{(Solvated electrons)}}{[e(NH_3)_y]^-}}_{\text{blue solution}}$$

27. A, C, D
Sol. $XeF_4(s) + Pt(s) \longrightarrow Xe(g) + PtF_4(s)$

$XeF_4(s) + [N(CH_3)_4]F \longrightarrow [N(CH_3)_4]^+ + [XeF_5]^-$

28. B, C, D
Sol. In negative deviation
$\Delta H = -ve$, $\Delta S = +ve$ and intermolecular forces are more than in pure liquids.

29. A, C, D
Sol. $NaCl + K_2Cr_2O_7 \xrightarrow{H_2SO_4} \underset{\text{(Red vapour)}}{CrO_2Cl_2} + Na_2SO_4 + \underset{\text{(Small amount)}}{Cl_2}$

$CrO_2Cl_2 + NaOH \longrightarrow \underset{\text{(Yellow sol.)}}{Na_2CrO_4} + NaCl$

$Na_2CrO_4 + AgNO_3 \longrightarrow \underset{\text{(Brick red)}}{Ag_2CrO_4}\downarrow$

$Na_2CrO_4 + Pb(CH_3COO)_2 \longrightarrow \underset{\text{(Yellow)}}{PbCrO_4}\downarrow$

30. B, C
Sol. Osmotic pressure and boiling points of a solution decreases on increasing dilution of a solution while vapour pressure and freezing point of solution increases on increasing dilution.

SECTION – C

31. 00071.68
Sol. $d = \dfrac{n \times M}{N_{av} \times a^3}$

$\therefore M = \dfrac{(1.95) \times (6.25 \times 10^{-8})^3 \times 6.023 \times 10^{23}}{4}$

$= 71.6848$

≈ 71.68

32. 00000.69
Sol. $m_1 = 0.1\,kg, m_2 = 0.012\,kg$

T_b of sol. $= 100.57°C$ T_b of water $= 100°C$.

Molar mass of $Ba(NO_3)_2 = 261.3$ g mol^{-1}

$\Delta T_b = K_b m \times i$

$0.57 = 0.52 \times \frac{(0.012/0.2613)}{0.1} \times i$

$i = 2.3868 \simeq 2.38$

$i = 1 + (n-1)\alpha$

$\frac{2.38-1}{(3-1)} = \alpha$

$\alpha = 0.69$

33. 00065.00

Sol. $E_{cell} = \left(E^o_{Cl^-|Hg_2Cl_2|Hg} - E^o_{Q,QH_2,H^+|Pt}\right) + \frac{0.0591}{1}\log\frac{1}{[H^+]}$

$E_{cell} = (0.280 - 0.6996) + \frac{0.0591}{1} \times 6.0$

$= -0.065$ V $= -65$ m.volt

$|E_{cell}| = 65$ m volt

34. 00067.84

Sol. Partial pressure of oxygen $(P_{O_2}) = 0.18$ atm

$= 0.18 \times 1.01 \times 10^5$ Pa

= 18180 Pa

Partial pressure of $N_2 (P_{N_2}) = 0.82$ atm

= 82820 Pa

Fraction of oxygen dissolved in $H_2O = \frac{18180}{2.53 \times 10^9}$

$= 7185.77 \times 10^{-9}$

$\simeq 7.186 \times 10^{-6}$

Fraction of nitrogen dissolved in $H_2O = \frac{82820}{5.47 \times 10^9}$

$= 15.140 \times 10^{-6}$

$\frac{\text{Amount of dissolved } N_2}{\text{Amount of dissolved } O_2} = \frac{15.140 \times 10^{-6}}{7.186 \times 10^{-6}} = 2.108$

$\simeq 2.11$

$\therefore \% N_2 = \frac{2.11}{3.11} \times 100 = 67.845$

$\simeq 67.84$

35. 00005.76

Sol. $\Delta S_A = 0.5 R \ell n \frac{2}{1}$

= 2.88 Joule

Same for B, $\Delta S_B = 2.88$ Joule

$\therefore \Delta S_{Total} = 2.88 + 2.88 = 5.76$ Joule

36. 00002.29

Sol. $AgCl(s) + \frac{1}{2}H_2(g) \rightleftharpoons H^+(aq) + Cl^-(aq) + Ag(s)$

$E = E^o - \frac{0.0591}{1}\log\frac{[H^+][Cl^-]}{p_{H_2}^{1/2}}$

$p_{H_2} = 1\,atm$ & $[H^+] = [Cl^-]$

$\therefore E = E^o - \frac{0.059}{1}\log[H^+]^2$

0.493 = 0.222 + 2((0.0592) pH

$\therefore pH = 2.29$

Mathematics PART – III

SECTION – A

37. C

Sol. Taking dot product with $\bar{a}, \bar{b}, \bar{c}$, we get $5\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right] = \bar{a}\cdot\bar{a}$, $3\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right] = \bar{a}\cdot\bar{c}$ and $\bar{a}\cdot\bar{b} = 0$

So, $\dfrac{\left(2\bar{a}+\bar{c}+3\bar{d}\right)\cdot\bar{a}}{\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right]} = 13$, as $\bar{d}\cdot\bar{a} = \left(\lambda_1\bar{b}+\lambda_2\bar{a}\times\bar{b}\right)\cdot\bar{a} = 0$

38. B

Sol. Let p be (α, β, γ) so points A, B and C are $(\alpha, \beta, 0)$, $(\alpha, 0, \gamma)$ and $(0, \beta, \gamma)$ respectively.

So, plane passing through these points is $\dfrac{x}{\alpha}+\dfrac{y}{\beta}+\dfrac{z}{\gamma} = 2$ also $\dfrac{\alpha}{a}+\dfrac{\beta}{b}+\dfrac{\gamma}{c} = p$

If (x_1, y_1, z_1) is foot of perpendicular from origin, then equation of plane

$x_1(x - x_1) + y_1(y - y_1) + z_1(z - z_1) = 0$

Comparing with $\dfrac{x}{\alpha}+\dfrac{y}{\beta}+\dfrac{z}{\gamma} = 2$, we get $2\alpha x_1 = 2\beta y_1 = 2\gamma z_1 = x_1^2 + y_1^2 + z_1^2$

So locus is $\left(x_1^2+y_1^2+z_1^2\right)\left(\dfrac{1}{ax_1}+\dfrac{1}{by_1}+\dfrac{1}{cz_1}\right) = 2p$

39. A

Sol. For non trivial (infinitely many solution)

$\dfrac{b+c-a}{ab} = \dfrac{c}{(c+a-b)(a+b-c)}$

$(a + b - c)(b + c - a)(c + a - b) - abc = 0$

But for distinct the values of a, b, c L.H.S is always negative. c

Let $a > b > c$ so $(a + b - c)$ and $(c + a - b) > 0$ and for it $b + c - a \le 0$ result is obvious for $b + c > a$ consider a, b, c as side of triangle

L.H.S = $8(s - a)(s - b)(s - c) - abc = \dfrac{8\Delta^2}{s} - 4\Delta R = 4\Delta\left(\dfrac{2\Delta}{s} - R\right) = 4\Delta(2r - R) < 0$

40. C

Sol. Probability he gets atleast one head = $\left(1-\dfrac{1}{2^n}\right)$ probability he gets head 3k times

$= \left(\dfrac{1+\left(\dfrac{1+\omega}{2}\right)^n+\left(\dfrac{1+\omega^2}{2}\right)^n}{3} - \dfrac{1}{2^n}\right) = \dfrac{2^n-3+(-1)^{n+1}}{3\left(2^n\right)}$

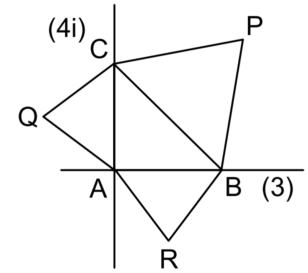
So, required probability is $\dfrac{2^n-3+(-1)^{n+1}}{3\left(2^n-1\right)}$

41. A

Sol. Using rotation, we get complex number corresponding to P, Q, R as

$3 + (3 - 4i)\omega$, $4i + 4i\omega$ and -3ω respectively

So, centroid of $\Delta PQR = \frac{3+4i}{3}$



42. A

Sol. Let $T_{k+1} = \frac{1}{a+kd}$ so $\frac{1}{a+5d}, \frac{1}{a+9d}, \frac{1}{a+17d}$ are in Arithmetic progression

$\Rightarrow \frac{1}{a+9d} - \frac{1}{a+5d} = \frac{1}{a+17d} - \frac{1}{a+9d} \Rightarrow a = 7d$

$\Rightarrow T_p = \frac{1}{(p+6)d}, \ T_{12} = \frac{1}{18d}, \ T_q = \frac{1}{(q+6)d}$

$\Rightarrow 18^2 = (p + 6)(q + 6)$ so maximum value of q is 30 (as $p + 6 \ge 7$)

So, minimum value of p + 6 = 9

$\Rightarrow q + 6 = 36 \Rightarrow q = 30$

43. A, C, D

Sol. Let $xy + yz + zx = -p$, $xyz = q$

So, x, y, z are roots of equation $t^3 - pt - q = 0$

$\Rightarrow x^3 = px + q$, $y^3 = py + q$, $z^3 = pz + q$

So, $x^3y + y^3z + z^2x = p(xy + yz + zx) + q(x + y + z) = -p^2 = -9$

$\Rightarrow p = \pm 3$

$xy + yz + zx = xy - (x + y)^2 = -(x^2 + y^2 + yx) < 0 \ \ \forall x, y \in R$

if x = y, we get $z = -2x$, $-3x^2 = \pm 3$, $x = \pm 1$ or $x = \pm i$

44. A, B, C, D

Sol. 6k + 4 is such number which leaves remainder 1 when divided by 3. So as a whole number of the form 3k – 1 (i.e. 2, 11) should be used even number of times and of form (3k + 1) (i.e., 7) can be used any number of time. So total cases = $3 \times (2 \times 3 + 1 \times 3) = 27$

Number of divisors of $n^2 = 7 \times 9 \times 5 \times 11$

Number of divisors of $n = 4 \times 5 \times 3 \times 6$

So, number of divisors of n^2, which are not divisors of n is $7 \times 9 \times 5 \times 11 - 4 \times 5 \times 3 \times 6 = 3105$

Number of divisor of n^2 which are less than equal to n = $\frac{7\times9\times5\times11+1}{2}$

So, divisors of n^2 which are less than n but does not divide n

$\Rightarrow \frac{7\times9\times5\times11+1}{2} - 360 = 1373$

45. A, D

Sol. $(MN)^2 = 3MN$

$\Rightarrow NMNMN\ M = 3NMNM$

$\Rightarrow (NM)^3 = 3(NM)^2 \Rightarrow (NM) = 3I$

$P = \frac{1}{3}I$ so, $P + P^2 + ... = \left(\frac{1}{3} + \frac{1}{3^2} + ...\right)I = \frac{1}{2}I$

46. A, B, D

Sol. for 1 and 4 option we use P.I.E

Option 1 : $\frac{3^6 - 3C_1 2^6 + 3}{3^6} = \frac{20}{27}$

Option 2 : $\frac{3C_2\left(2^6 - 2\right)}{3^6} = \frac{62}{243}$

for favourable cases in 3 and 4 we use exponential generating function

Option 3 : favourable cases = coefficient of x^6 in $\frac{6!\left(e^x + e^{-x}\right)\left(e^x - e^{-x}\right)e^x}{4}$

Option 4 : favourable cases = coefficient of x^6 in $6!\left(\frac{e^x + e^{-x}}{2} - 1\right)\left(\frac{e^x - e^{-x}}{2}\right)\left(e^x - 1\right) = \frac{3^6 - 2^8 - 1}{4}$

47. A, B, C, D

Sol. f(n) = coefficient of x^n in ${}^nC_0(1+x)^{3n} - {}^nC_1(1+x)^{3n-3} + {}^nC_2(1+x)^{3n-6} + ...$

$\Rightarrow$ coefficient of x^n in $((1 + x)^3 - 1)^n$

$\Rightarrow$ f(n) = 3^n, so f(ff(n)) = $3^{3^{3^n}} = (27)^{3^{\left(3^n - 1\right)}}$

$(28-1)^{3^{\left(3^n-1\right)}}$, so remainder is –1 or 6

$f\,f(n) = 3^{\left(3^n\right)} = 3^{\left(1 + {}^nC_1 2 + {}^nC_2 2^2 + ...\right)} = 3\ 9^{\left({}^nC_1 + {}^nC_2 2^1 + ... {}^nC_n 2^{n-1}\right)}$

$3(10-1)^{\left({}^nC_1 + {}^nC_2 2^1 + ... {}^nC_n 2^{n-1}\right)}$, so remainder is –3 if n is odd + 3 if n is even

f(n) $3^n = (1+2)^n$ so remainder is 1

48. A, B, C, D

Sol. a + 2b = 9c, c + 2d = 9a, 2ab = –4d, 2cd = –4b

$\Rightarrow$ ac = 4, also $a^2 - 9ac - 4d = 0$, $c^2 - 9ac - 4b = 0$

adding two, we get $(a + c)^2 - 16(a + c) - 80 = 0$

$\Rightarrow$ a + c = 20 or –4

SECTION – C

49. 00012.00

Sol. Let $x = \frac{\alpha}{a}, y = \frac{\beta}{b}, z = \frac{\gamma}{c}$

L.H.S. reduced to $\left(a\alpha^2 + b\beta^2 + c\gamma^2\right)\left(\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma}\right)^2$

Using wt. A.M. $\geq$ wt. G.M.

$$\frac{a\alpha^2 + b\beta^2 + c\gamma^2}{a+b+c} \geq \left(\alpha^{2a}\beta^{2b}\ \gamma^{2c}\right)^{\frac{1}{a+b+c}}$$

$$\frac{\left(\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma}\right)}{a+b+c} \geq \left(\frac{1}{\alpha^a}\ \frac{1}{\beta^b}\ \frac{1}{\gamma^c}\right)^{\frac{1}{a+b+c}}$$

$$\Rightarrow \left(a\alpha^2 + b\beta^2 + c\gamma^2\right)\left(\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma}\right)^2 \geq (a+b+c)^3$$

So, $(a + b + c)^3 = 216$, a + b + c = 6

Using m^{th} power inequality, we get $\frac{a^2+b^2+c^2}{3} \geq \left(\frac{a+b+c}{3}\right)^2 \Rightarrow a^2 + b^2 + c^2 \geq 12$

50. 00002.50

Sol. $z^3 + (-\alpha z_1)^3 + (-(1-\alpha)z_2)^3 = 3z(-\alpha z_1)(-(1-\alpha)z_2)$
$\Rightarrow z - \alpha z_1 - (-1-\alpha)z_2 = 0,\ \alpha \in R$
So, $|z|$ is perpendicular distance from (0, 0) to line joining the points (3, 4) and (1, –2)

51. 00000.00

Sol. As three planes have a common line of intersection so $\bar{x}_1, \bar{x}_2, \bar{x}_3$ are coplanar
also $8\bar{x}_2 \times \bar{x}_3 + 5\bar{x}_3 \times \bar{x}_1 + 3\bar{x}_1 \times \bar{x}_2 = 0$

52. 04585.00

Sol. Case-1 > $1 \leq i, j, k \leq 10$, sum is ${}^{11}C_3$
Case-2 > $1 \leq i \leq 10$ and $11 \leq j, k \leq 20$, sum is $({}^{10}C_1 \cdot {}^{11}C_2) \times 4$
Case-3 > $1 \leq i, j \leq 10$, $11 \leq k \leq 20$ sum is $({}^{10}C_2 \cdot {}^{10}C_1) \times 2$
Case-4 > $11 \leq i, 5, k \leq 20$, ${}^{11}C_3 \times 2^3$
So, total sum is ${}^{11}C_3 + {}^{10}C_1\ {}^{11}C_2 \times 4 + {}^{10}C_2 \times {}^{10}C_1 \times 2 + {}^{11}C_3 \times 8 = 4585$

53. 00000.50

Sol. $3x_{n+1}x_n - x_n x_{n-1} = 1$. Let $a_n = 3^n x_{n-1}x_n$

$a_{n+1} - a_n = 3^n \Rightarrow a_n = \frac{3^n + 2a_1 - 3}{2}$ (Telescopic sum)

Hence, $3^n x_n x_{n-1} = \frac{3^n + 2a_1 - 3}{2}$, $3^{n+1}x_{n+1}x_n = \frac{3^{n+1} + 2a_1 - 3}{2}$

$\Rightarrow \frac{x_{n+1}}{x_{n-1}} = \frac{3^{n+1} + (2a_1 - 3)}{3^{n+1} + 3(2a_1 - 3)}$ for sequence to be periodic $2a_1 - 3 = 0$

$\Rightarrow 3x_0x_1 = \frac{3}{2}$

54. 00000.50

Sol. A = UV

$$U = \begin{bmatrix} (-1)^1(2\cdot 1^2+1) & 0 & 0 & 0 & . & . \\ (-1)^2(2\cdot 2^2+1) & 0 & 0 & . & . & . \\ \vdots & & & & & \\ (-1)^n(2\cdot n^2+1) & 0 & . & . & . & 0 \end{bmatrix} \quad V = \begin{bmatrix} \frac{1}{4+1} & + & \frac{1}{4\cdot 2^4+1} & \cdots & \frac{1}{4\cdot n^4+1} \\ 0 & & 0 & & 0 \\ 0 & & 0 & . & 0 \end{bmatrix}$$

$$\text{So, } VU = \begin{bmatrix} \text{trace A} & 0 & 0 & . & 0 \\ 0 & & & & 0 \\ 0 & & & . & 0 \end{bmatrix}$$

Hence, $A^2 = UVUV = (\text{trace A})UV = \text{trace}(A)\cdot A$
$\Rightarrow A^n = (\text{trace A})^{n-1} A \Rightarrow \text{trace}(A^n) = (\text{trace A})^n$

$$\text{trace A} = \frac{1}{2}\sum_{i=1}^{n}(-1)^i\left(\frac{2i^2+1}{4i^4+1}\right) = \sum_{i=1}^{n}\frac{1}{2}(-1)^i\left(\frac{1}{2i^2-2i+1} + \frac{1}{(2i^2+2i+1)}\right)$$

$$= \lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}\left(\frac{(-1)^i}{2i^2-2i+1} - \frac{(-1)^{i+1}}{2i^2+2i+1}\right) = -\frac{1}{2}$$