FIITJEE

ALL INDIA TEST SERIES

PART TEST – II

JEE (Main)-2021

TEST DATE: 12-12-2020

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. D

Sol. $(U_e)_0 = \frac{\sigma R}{2\varepsilon_0}$; $(U_e)_p = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+H^2}-H\right)$

$(U_g)_0 = 0$; $(U_g)_p = mgH$

$(U_e)_0 + (U_g)_0 = (U_e)_p + (U_g)_p$

$\Rightarrow \ mgH = \frac{q\sigma}{2\varepsilon_0}\left(R+H-\sqrt{R^2+H^2}\right)$

$\Rightarrow \ gH = 2g\left(R+H-\sqrt{R^2+H^2}\right) \quad \left(\because \ \frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}\right)$

$\therefore \ H = \frac{4R}{3}$

2. A

Sol. W = 0

$U_i = 3\times\frac{5R}{2}\times 300 = 2250R$

$U_f = 2\times\frac{5R}{2}\times 400 + 2\times\frac{3R}{2}\times 400 = 3200R$

Q = ΔU = 950 R

3. B

Sol. Internal energy in liquid state is greater than that in the solid state.
Work done is negative as the volume of the system decreases.

4. B

Sol. Basic concepts of graph of thermodynamic processes.

5. D

Sol. Centre of mass is between C and X, which remains at rest and the whole body expands. So, C moves left and X moves right.

6. C

Sol. $T_1 = \frac{mv^2}{\ell} - mg$

As temperature increases, 'v' decreases and ℓ increases. So, T_1 decreases.

$(T_2)_i = \frac{mv_i^2}{\ell} + mg \quad \left(\because \frac{1}{2}mv_i^2 + mgx = \frac{1}{2}mv_f^2\right)$

$(T_2)_f = \frac{mv_i^2 + 2mgx}{\ell + x} + mg$

$(T_2)_f - (T_2)_i = \frac{m(2gl - v_i^2)x}{\ell(\ell + x)} < 0 \quad \therefore \ T_2$ decreases.

7. D

Sol. Basic concept of ferromagnetic substances.

8. C

Sol. Curie's Law

$M = C\frac{B_0}{T}$

$\Rightarrow \quad \chi = C\frac{\mu_0}{T}$

9. C

Sol. Both of then will have same values of power radiated and absorbed initially. But, rate of fall of temperature will be different as their masses are different.

10. B

Sol. Basic concept of electric field and graph.

11. C

Sol. ω = 1 rad/s

$\theta = \pi$ rad in $t = \pi$ s

$\therefore \quad z = +2 \times r = 2m$

$F_e = 1N \Rightarrow a_e = 1\ m/s^2$

$S_e = \frac{1}{2} \times 1 \times \pi^2 = 5\,m$

$\vec{S}_e = \frac{-5}{\sqrt{2}}\hat{i} + \frac{5}{\sqrt{2}}\hat{j}\ m$

$\therefore$ Coordinates are $\left(-\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}, 2\right)$m

12. A

Sol. $\frac{Q}{C} = B\ell v \Rightarrow Q = B\ell v\ C = 800\mu C$

Since, X is at higher potential than Y

$\therefore$ q_A is + 800 μC & q_B is – 800 μC.

13. D

Sol. Work done by magnetic field is always zero.

14. D

Sol. $V_1 = 0$

$$\Rightarrow \frac{K(x)}{r} + \frac{K(30)}{2r} + \frac{K(y)}{4r} = 0$$

$$\Rightarrow 4x + y + 60 = 0 \quad \ldots(1)$$

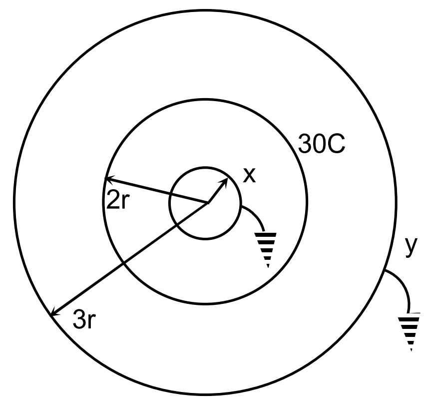
$V_3 = 0$

$$\Rightarrow \frac{K(x)}{4r} + \frac{K(30)}{4r} + \frac{K(y)}{4r} = 0$$

$$\Rightarrow x + y + 30 = 0 \quad \ldots(2)$$

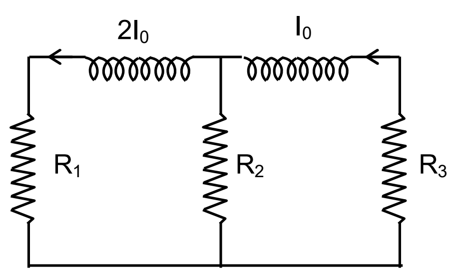
From (1) & (2), we get x = – 10 & y = – 20

Net charge = 0.



15. C

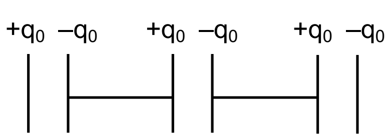
Sol. Just after key is open current in the inductors won't charge.



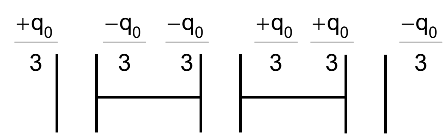
16. B

Sol.

Initial charge distribution



Final charge distribution



$$q_0 = \left(\frac{C\varepsilon}{3}\right)$$

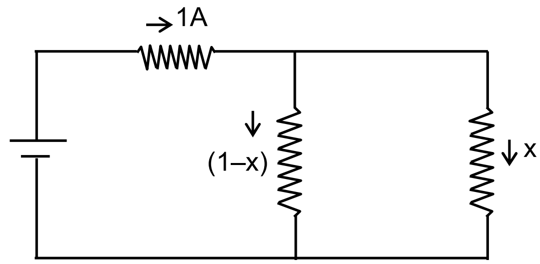
$$\therefore \text{ Heat dissipated} = 3 \times \left\{\frac{\left(\frac{C\varepsilon}{3}\right)^2}{2C} - \frac{\left(\frac{C\varepsilon}{9}\right)^2}{2C}\right\} = \frac{4}{27}C\varepsilon^2$$

17. B

Sol. Increase in potential across 2R + Increase in potential across R = 0

1(2R) + (1 – x) R = 0

$\Rightarrow$ x = 3



18. A

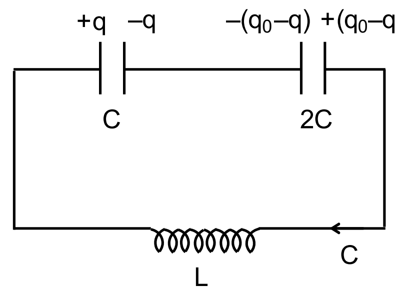
Sol. by KVL

$$\frac{q_0 - q}{2C} - \frac{Ldi}{dt} - \frac{q}{c} = 0$$

$$\Rightarrow \left(q - \frac{q_0}{3}\right) = A\sin\left(\frac{\sqrt{3}t}{\sqrt{2LC}} + \phi\right)$$

At t = 0: q = 0 & I = 0 $\Rightarrow$ $A = \frac{-q_0}{3}$ & $\phi = -\frac{\pi}{2}$

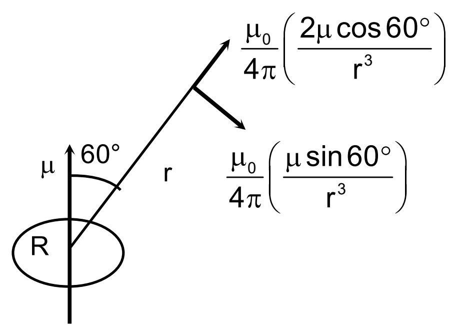
$\therefore$ $I_{max} = \frac{q_0}{\sqrt{6LC}}$



19. D

Sol. Magnetic moment, $\mu = I(\pi R^2)$

$$\therefore \quad B_{net} = \left(\frac{\mu_0}{4\pi}\right)\frac{\mu}{r^3}\sqrt{(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$



20. D

Sol. Initial P.E. of the system = $\frac{KQ^2}{4r} + \frac{KQ^2}{2r} + \frac{KQ^2}{2r}$

Final P.E. of the system = $\frac{2KQ^2}{4r} + \frac{KQ^2}{2r} + \frac{4KQ^2}{2r}$

$$W = \Delta U = \frac{7}{4}\left(\frac{KQ^2}{r}\right)$$

SECTION – B

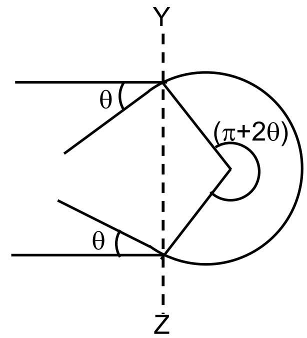
21. 5

Sol. $\omega = \frac{qB}{m} = \alpha B$

Clearly,

$t = \frac{\pi + 2 \times \pi / 3}{\omega}$

$= \frac{5\pi}{3\alpha B}$



22. 5

Sol. $P \propto \frac{1}{T} \Rightarrow PT = \text{constant}$ …(1)

PV = nRT …(2)

From (1) & (2),

P^2V = constant

Or $PV^{1/2}$ = constant

For this polytrophic process, α = ½.

$\therefore \quad C = \frac{R}{\gamma - 1} + \frac{R}{1 - \alpha}$

$\Rightarrow \quad 4.5R = \frac{R}{\gamma - 1} + 2R$

$\Rightarrow \quad \gamma = 7/5$

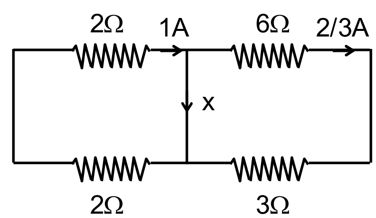
$\therefore$ Degree of freedom = 5

SECTION – C

23. 00000.33

Sol. Ideal ammeter can be treated as a live wire.

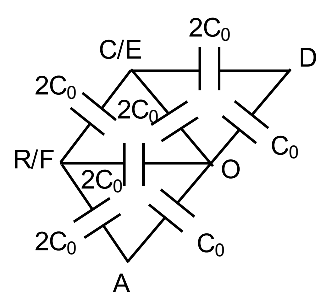
$x = \left(1 - \frac{2}{3}\right)A = \frac{1}{3}A$



24. 00002.22

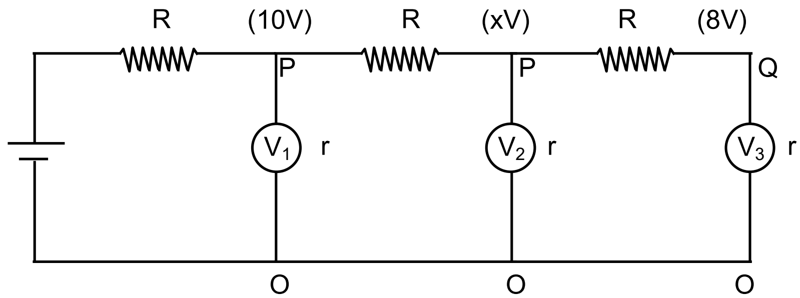
Sol. Using the parallel axis theorem, it can be reduced to the following circuit.

Solving this circuit will give us equivalent capacitance between A and O as $\left(\frac{20}{9}\right)C_0$



25. 00008.64

Sol.



If the resistance of each voltmeter is r, then,

At P:

$$\frac{x-10}{R}+\frac{x-8}{R}+\frac{x-0}{r}=0$$

$$\Rightarrow \qquad \frac{R}{r}=\frac{18-2x}{x} \qquad \ldots(1)$$

At Q:

$$\frac{8-x}{R}+\frac{8-0}{r}=0$$

$$\Rightarrow \qquad \frac{R}{r}=\frac{x-8}{8} \qquad \ldots(2)$$

From (1) & (2); we get ;

x = 8.64

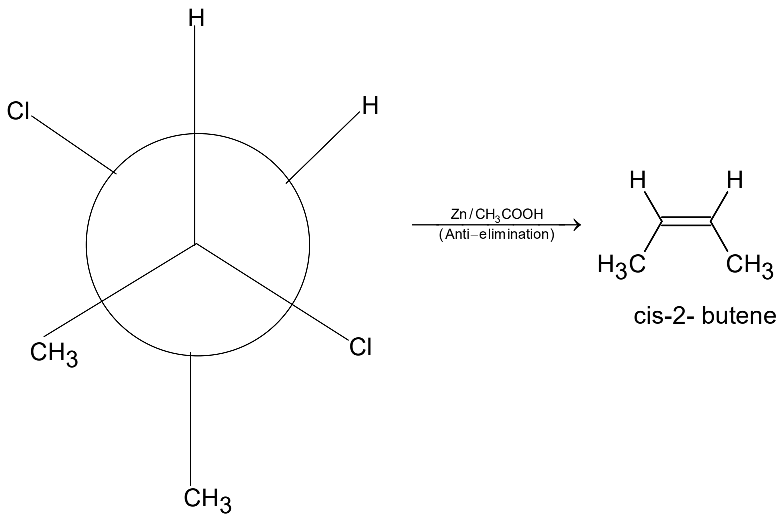
Chemistry

PART – II

SECTION – A

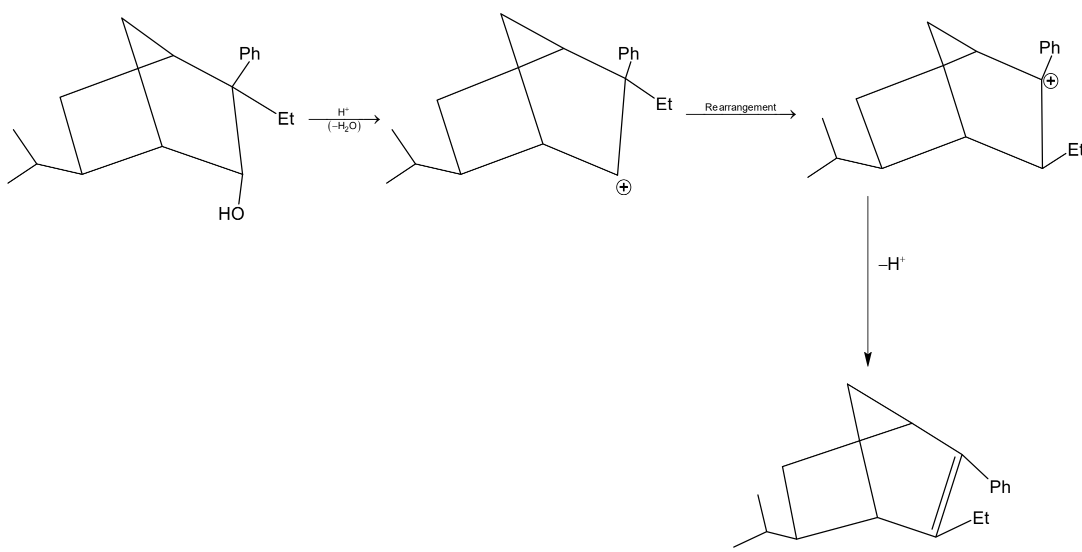
26. A

Sol.



27. B

Sol.

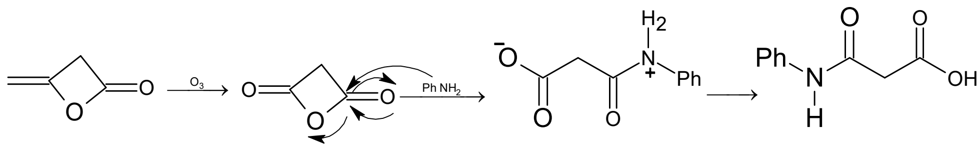


28. C

Sol. Due to H – bonding, the most stable conformation is gauche at 60°.
Stability order w.r.t θ = 60° > 180° > 120° > 0°.

29. D

Sol. $NaBH_4$ reduces ketonic group to alcoholic group without affecting Br and benzene ring.

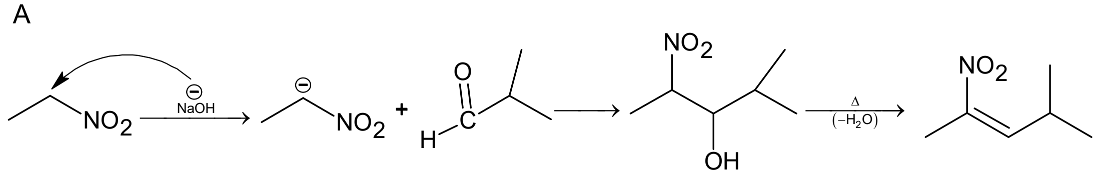
30. B
Sol.


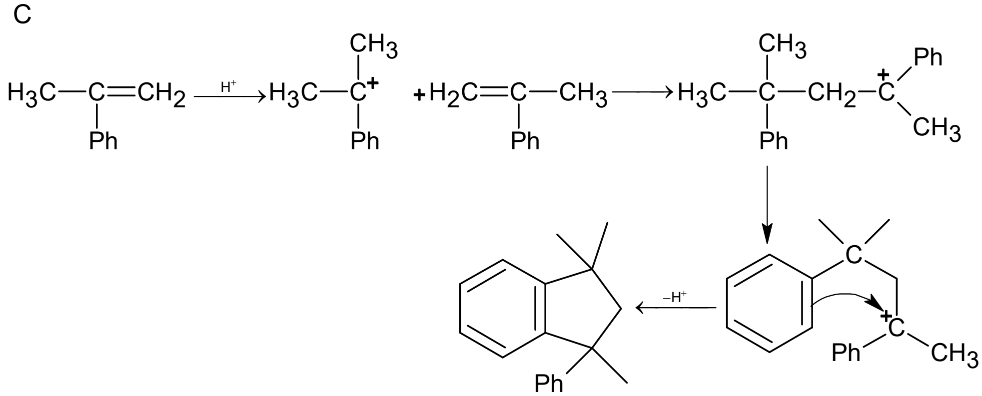
31. C
Sol. Molich and Barfoed are test for carbohydrate.
Xanthoprotic is for for aromatic amino acids.
Biuret is for proteins.

32. A
Sol. Natural rubber is bound by weak vander Waal's interactions.

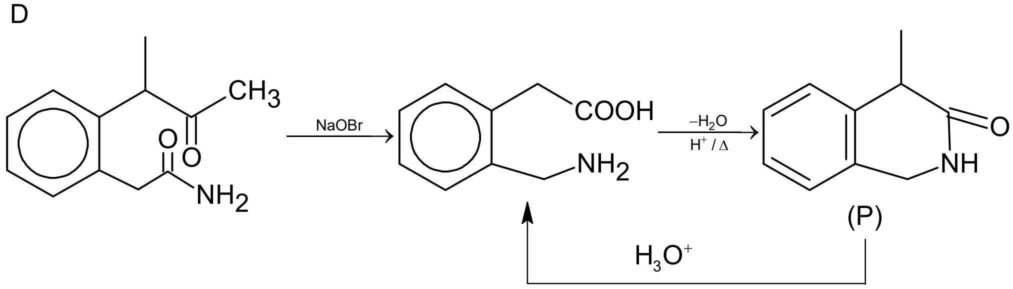
33. D
Sol. Strongest acid is HI.

34. B
Sol. $(CH_3)_2CuLi$ causes 1, 4 - addition and the stereo is given by the existing neighbouring group.

35. A
Sol.


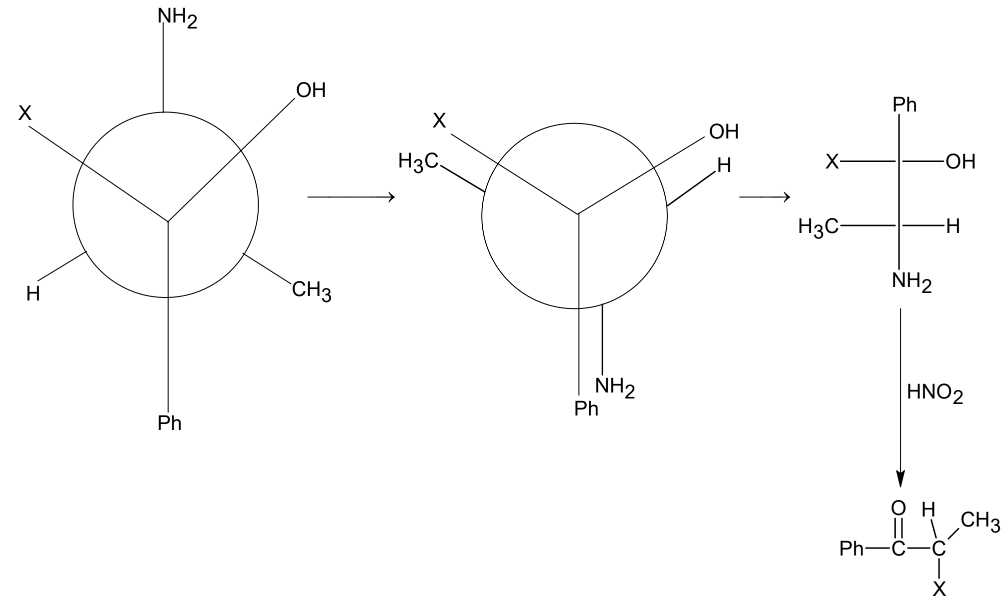
36. C
Sol.


37. A
Sol. Factual

38. D
Sol.


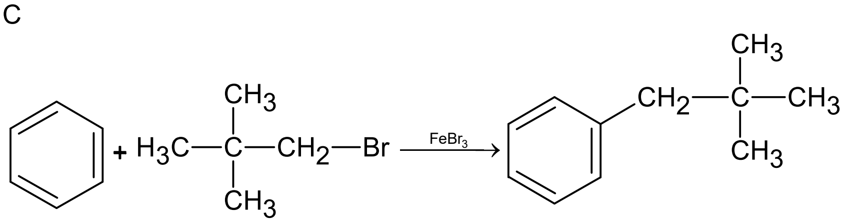
39. B

Sol.



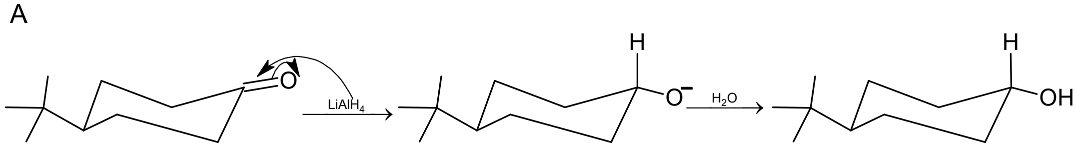
40. C

Sol.



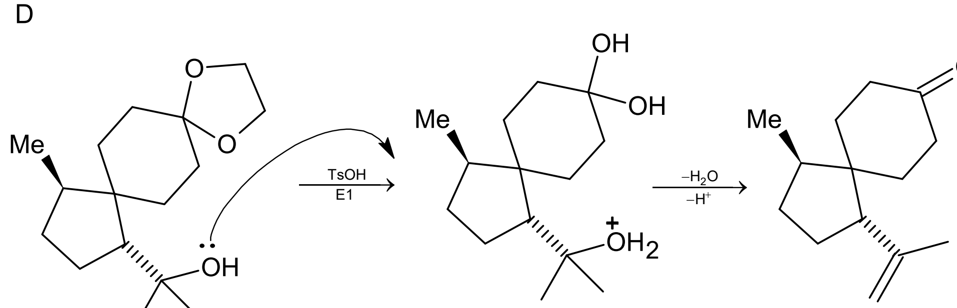
41. A

Sol.



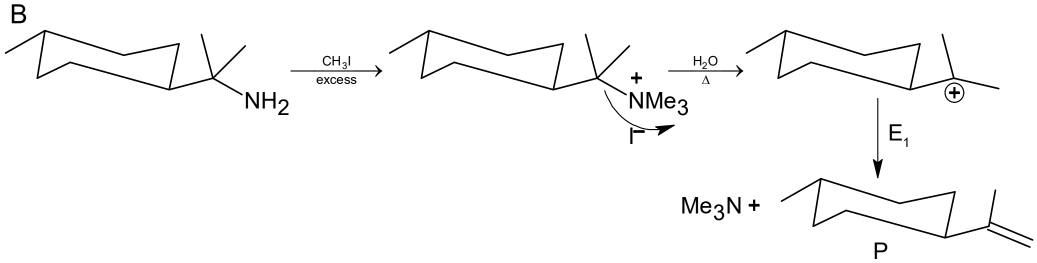
42. D

Sol.



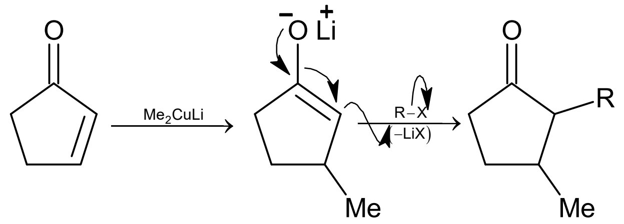
43. B

Sol.



44. C

Sol.



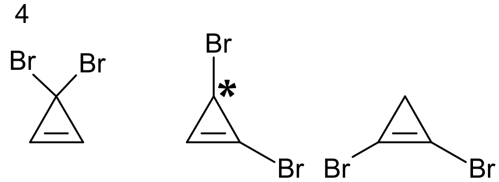
45. C

Sol. $Na + S \longrightarrow Na_2S \xrightarrow{Na_2[Fe(CN)_5NO]}$ Violet colour confirm sulphur

SECTION – B

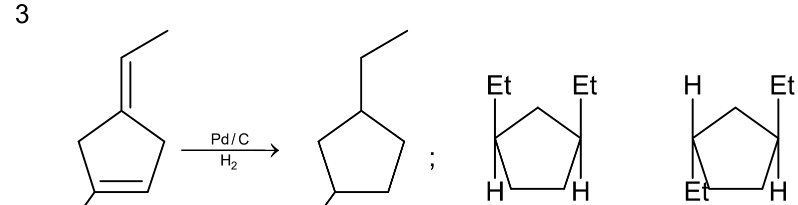
46. 4

Sol.



47. 3

Sol.



1 isomer 2 isomers

SECTION – C

48. 00009.59

Sol. $pI = \frac{8.90 + 10.28}{2}$

49. 00000.33

Sol. Product → 2HCHO + 3 HCOOH + CO_2

$\therefore \frac{1}{3} = 0.33$

50. 00000.67

Sol. X = 2

Teflon, Polyacrylonitrile

Y = 3

Nylon-6, 6, Nylon-6, Dacron

$\frac{X}{Y} = \frac{2}{3} = 0.67$

Mathematics

PART – III

SECTION – A

51. C

Sol. Since xy = k $\Rightarrow$ 2xy = 2k
Equation of circle is $x^2 + y^2 = k^2$
$(x + y)^2 - 2xy = k^2$
$(x + y)^2 = k^2 + 2k$ so, k = –1 does not satisfy this equation
Similarly, $(x - y)^2 = k^2 - 2k$, for k = 1 does not satisfy this equation
This only two values of k = 1, –1 does not satisfy

52. B

Sol. Since $S_1F_1 \cdot S_2F_2 = 25$
Now $\frac{S_1F_1 + S_2F_2}{2} \geq (S_1F_1 \cdot S_2F_2)^{\frac{1}{2}}$, $\frac{S_1F_1 + S_2F_2}{2} \geq 5 \Rightarrow S_1F_1 + S_2F_2 \geq 10$

53. C

Sol. Since, $\alpha + \beta + \gamma + \delta = (2n+1)\pi \Rightarrow$ (C) is correct

54. B

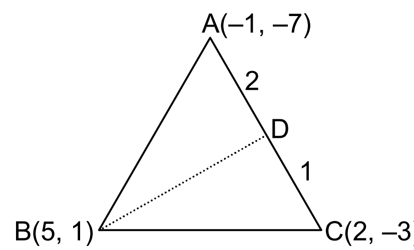
Sol. $AB = \sqrt{6^2 + 8^2}$, $BC = \sqrt{3^2 + 4^2} = 5$

$\frac{AD}{DC} = \frac{AB}{BC} = \frac{10}{5} = \frac{2}{1} \Rightarrow$ D divides AC in 2 : 1

$D\left(\frac{2 \times 2 + 1 \times -1}{2+1}, \frac{2 \times (-3) + 1(-7)}{2+1}\right)$

$\left(\frac{3}{3}, \frac{-6-7}{3}\right)$, $\left(1, \frac{-13}{3}\right)$

$BD = \left(\sqrt{(5-1)^2 + \left(1 + \frac{13}{3}\right)^2}\right) = \sqrt{16 + \frac{256}{9}} = \frac{20}{3}$

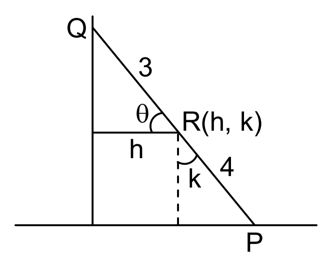


55. A

Sol. $\cos\theta = \frac{h}{3}$, $\cos(90 - \theta) = \frac{k}{4}$

$\sin\theta = \frac{k}{4}$

$\frac{x^2}{3^2} + \frac{y^2}{4^2} = 1$



56. C

Sol. Let the origin is shifted to (x_1, y_1) then $x = X + x_1$, $y = Y + y_1$
Now, $(X + x_1)^2 + 7(X + x_1)(Y + y_1) - 2(Y + y_1)^2 + 17(X + x_1) - 26(Y + y_1) - 60 = 0$
$x_1 = 2$ and $y_1 = -3$

57. D

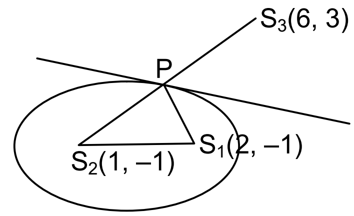
Sol. $\because PS_1 + PS_2 = 2a$

Let S_3 be the image of S_1 w.r.t tangent at P

So, $PS_1 = PS_3$

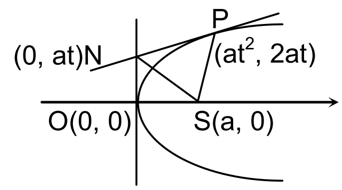
$\Rightarrow$ P, S_2, S_3 are collinear $S_3(6, 3)$

$\Rightarrow P\left(\frac{34}{9}, \frac{11}{9}\right)$



58. B

Sol. OS, SN, SP are in GP

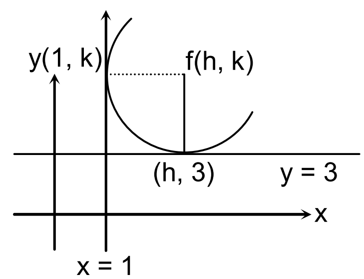


59. B

Sol. Let (h, k) be the focus and foot of perpendicular from (h, k) to y = 3 is (h, 3) and foot of perpendicular from (h, k) to x = 1 is (1, k)

As perpendicular from focus on any tangent meet it on the tangent at vertex so, slope of vertex is $\frac{3-k}{h-1} = 2$

$\Rightarrow 2h - 2 = 3 - k,\ k + 2h - 5 = 0,\ 2x + y - 5 = 0$



60. C

Sol. We know two line which cut the coordinate axis in concyclic points if $m_1m_2 = 1$

Equation of tangent is $y = m_1x + \sqrt{4m_1^2 - 9}$, $y = m_2x + \sqrt{4m_2^2 - 9}$

Let point of intersection is (h, k). So, $(k - mh)^2 = 4m^2 - 9$, $k^2 + m^2h^2 - 2mkh = 4m^2 - 9$

$m^2(h^2 - 4) - 2khm + k^2 + 9 = 0$, $m_1m_2 = 1$

$\frac{k^2+9}{h^2-4} = 1 \Rightarrow k^2 + 9 = h^2 - 4 \Rightarrow h^2 - k^2 = 13,\ x^2 - y^2 = 13$

61. B

Sol. Since equation of chord is $\frac{x}{a}\cos\frac{\alpha-\beta}{2} - \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha+\beta}{2}$

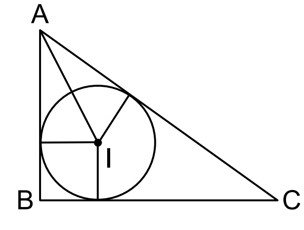
If $\alpha + \beta = (2n + 1)\pi$ then $\frac{x}{a}\cos\frac{\alpha-\beta}{2} = \pm\frac{y}{b}$ which always passes through centre

62. A

Sol. $IA = \frac{r}{\sin\frac{A}{2}}$, $IB = \frac{r}{\sin\frac{B}{2}}$, $IC = \frac{r}{\sin\frac{C}{2}}$

$$IA \cdot IB \cdot IC = \frac{r^3}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}} = \frac{r^3 \cdot 4R}{4R\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}}$$

$= \frac{4Rr^3}{r} = 4Rr^2$



63. B

Sol. $\cos(A-B)=\frac{4}{5}$, $2\cos^2\left(\frac{A-B}{2}\right)-1=\frac{4}{5}$, $\tan\frac{A-B}{2}=\frac{1}{3}=\frac{6-3}{6+3}\cot\frac{C}{2} \Rightarrow C=\frac{\pi}{2}$

64. B

Sol. Let $t=\frac{1-x}{1+x}$, if $0\le x\le 1 \Rightarrow 0\le t\le 1$

$\tan^{-1}t\in\left[0,\frac{\pi}{4}\right]$

65. C

Sol. $\pi-3\sin^{-1}x$, $\frac{1}{2}<x\le 1$

66. B

Sol. Solution is possible if sin x = 1 and cos ax = 1

$x=(4n+1)\frac{\pi}{2}$ and ax = 2mπ ⇒ $x=\frac{2m\pi}{a} \Rightarrow (4n+1)\frac{\pi}{2}=\frac{2m\pi}{a}$

$a=\frac{4m\pi}{4n+1}$

67. B

Sol. Expand by Binomial theorem

68. D

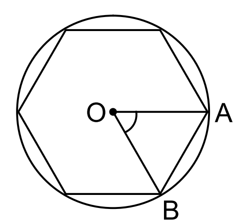
Sol. $\angle AOB=\angle OAB=\angle OBA=\frac{\pi}{3}$

So, ΔOAB is equilateral so AB = radius of circle

∵ Area of circle is 8π ⇒ π(side)^2 = 8π, (side)^2 = 8

Area of hexagon = 6 × ΔOAB

= $6\times\frac{\sqrt{3}}{4}\cdot 8=12\sqrt{3}\ m^2$



69. A

Sol. $\angle I_1AI_2=\angle I_1AX+\angle XAI_2=\frac{\angle BAX}{2}+\frac{\angle CAX}{2}=\frac{\angle A}{2}$

Area of $\Delta AI_1A_2=\frac{1}{2}(AI_1)\times(AI_2)(\sin\angle I_1AI_2)$

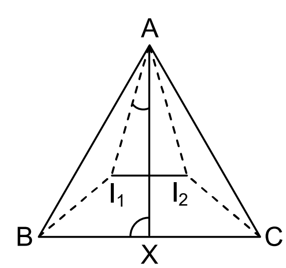
Let BC = a, AB = c, AC = b, ∠AXB = α

$\angle AI_1B$ = π – ($\angle I_1AB$ + ∠IBA) = $\pi-\frac{1}{2}(\pi-\alpha)=\frac{\pi}{2}+\frac{\alpha}{2}$

Applying since Rule in ΔABI_1

$\frac{AI_1}{AB}=\frac{\sin\angle ABI_1}{\sin(AI_1B)} \Rightarrow AF_1=\frac{C\sin\frac{B}{2}}{\sin\left(\frac{\pi}{2}+\frac{\alpha}{2}\right)}=\frac{C\sin\frac{B}{2}}{\cos\frac{\alpha}{2}}$

similarly $AF_2=\frac{b\sin\frac{C}{2}}{\sin\frac{\alpha}{2}}$



$$4AI_1F_2 = \frac{1}{2}\frac{c\sin\frac{B}{2}}{\cos\frac{\alpha}{2}}\cdot\frac{b\sin\frac{C}{2}}{\sin\frac{\alpha}{2}}\cdot\sin\frac{A}{2} = \frac{1}{2}\frac{bc\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{\sin\alpha}$$

So, minimum are is $bc\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{bc(a-b+c)(b-c+a)(c-a+b)}{9abc}$

$\because$ c = 10, a = 12, b = 14 = $\frac{14\times10(12-14+10)(14-10+12)(10-12+14)}{8\times10\times12\times14}$

= $\frac{8\times16\times12}{8\times12} = 16$

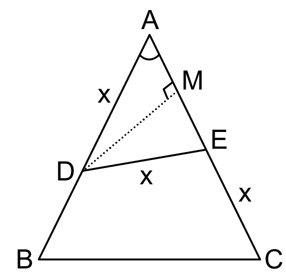
70. B

Sol. Since, $\cos A = \frac{7}{25}$

If M is mid-point of AE then AM = $\frac{10-x}{2}$

$\cos A = \frac{10-x}{2x} = \frac{7}{25}$

$x = \frac{250}{39}$



SECTION – B

71. 5

Sol. Since hyperbola is rectangular and P will be the mid-point
So, CP will be the radius.

72. 6

Sol. $x\sin x + \frac{9}{x\sin x} \ge 2\left(x\sin x \times \frac{9}{x\sin x}\right)^{\frac{1}{2}} \ge 6$

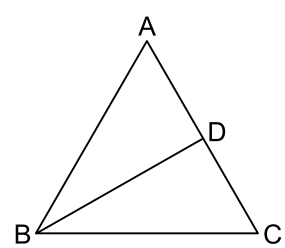
SECTION – C

73. 00033.00

Sol. $\because \frac{CD}{AD} = \frac{BC}{BA}$

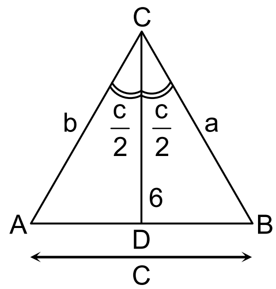
$\frac{BC}{CD} = \frac{AB}{AD} = \frac{BC}{8} = \frac{AB}{3}$

So, AB + BC = 11.K for lowest K = 1
AB + BC = 11 = AC which is not possible
If AB = BC = 22 (i.e., AB = 6, BC = 16)
So, perimeter must be 22 + 11 = 33



74. 00018.00

Sol. Area of ACD + Area of BCD = ΔABC



$$\left(\frac{1}{2}6b\sin\frac{C}{2}+\frac{1}{2}6a\sin\frac{C}{2}\right)=\frac{1}{2}ab\sin C$$

$$6b\sin\frac{C}{2}+6a\sin\frac{C}{2}=ab\,2\sin\frac{C}{2}\cos\frac{C}{2}$$

$$6b+6a=2ab\frac{1}{3}$$

$$6(a+b)=ab\frac{2}{3}\ \therefore\ \frac{2ab}{a+b}=18$$

75. 00001.50

Sol. Equation of chord in mid-point form is T = S_1

This passes through (2, 1) so $4\alpha^2 - 12\alpha + 9 = 0$

$$\Rightarrow\ \alpha=\frac{3}{2}$$