FIITJEE

ALL INDIA TEST SERIES

PART TEST – II

JEE (Advanced)-2021

PAPER – 2

TEST DATE: 13-12-2020

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. B, D

Sol. Induced emf in the loops, $\varepsilon = \frac{R^2\theta}{2}\frac{dB}{dt} = \frac{a_0R^2\theta}{2}$

Induced current in the loop,

$$I = \frac{\varepsilon}{2R(1+\cos\theta)\lambda} = \frac{a_0R\theta}{4\lambda(1+\cos\theta)}$$

$$V_O - V_Q = I\lambda 2R\cos\theta = \frac{a_0R^2\theta\cos\theta}{2(1+\cos\theta)}$$

At θ = 45°,

$$V_O - V_Q = \frac{a_0R^2\pi}{8(\sqrt{2}+1)}$$

$$V_P - V_Q = \varepsilon - I\lambda R = \frac{a_0R^2\theta}{2} - \frac{a_0R^2\theta}{4(1+\cos\theta)} = \frac{a_0R^2\theta}{4}\left(\frac{1+2\cos\theta}{1+\cos\theta}\right)$$

For θ = 45°,

$$V_P - V_Q = \frac{a_0R^2\pi}{8\sqrt{2}}$$

2. A, B, D

Sol. $\Delta Q_{AB} = nC_P\Delta T = 2\times\frac{5R}{2}(1200-600) = 3000\text{ R}$

In the process BC, $T = \propto V^2$

PV^{-1} = constant

Molar heat capacity in the process BC,

$$C = C_V + \frac{R}{(1-x)} = \frac{3R}{2}+\frac{R}{2}$$

C = 2R

$\Delta Q_{BC} = nC\Delta T = 2 \times 2R\ (300 - 1200) = -3600\text{ R}$

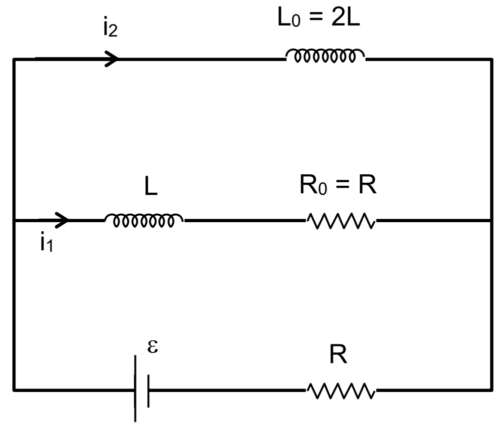
$\Delta Q_{CA} = nC_V\Delta T = 2 \times \frac{3R}{2}\ (600 - 300) = 900\text{ R}$

The total work done by the gas during the cyclic process.

$\Delta W_{cycle} = \Delta Q_{cycle} = 3000\text{ R} - 3600\text{ R} + 900\text{ R} = 300\text{ R}$

3. A, C

Sol. Current through the inductor 'L_0' in the steady state after closing the switch-2 will be $I_S = \frac{\varepsilon}{R}$



Now, $2L\frac{di_2}{dt} = L\frac{di_1}{dt} + i_1R_0$

$$2L\int_0^{\varepsilon/R} di_2 - L\int_{\varepsilon/2R}^{0} di_1 = R_0\int i_1 dt$$

$$2L\frac{\varepsilon}{R} - L\left(0-\frac{\varepsilon}{2R}\right) = R_0\Delta q$$

$$\frac{5L\varepsilon}{2R} = R_0\Delta q \Rightarrow \Delta q = \frac{5L\varepsilon}{2RR_0}$$

$$\Delta q = \frac{5L\varepsilon}{2R^2}$$

4. A, B, C, D

Sol. Cooling rate is inversely proportional to the radius of sphere.

5. B, C, D

Sol. At t = 0, current through the inductor = 2A, charge on the capacitor = 4C

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{2}\sec^{-1}$, $T = \frac{2\pi}{\omega} = 4\pi$ sec

$$\frac{q_{max}^2}{4} = \frac{1}{2}\times 2(2)^2 + \frac{1}{2}\times 2(2)^2 \Rightarrow q_{max} = 4\sqrt{2}C$$

$$q = 4\sqrt{2}\sin\left(\omega t + \frac{3\pi}{4}\right)$$

$$i = 2\sqrt{2}\cos\left(\omega t + \frac{3\pi}{4}\right)$$

6. A, B, C, D

Sol. $X_C = -100\hat{j}$, $x_L = 200\hat{j}$

$$\frac{1}{z_{AB}} = \frac{1}{200\hat{j}} + \frac{1}{-100\hat{j}}$$

$Z_{AB} = -200\ \hat{j}$

For circuit, z = 100 – 200 $\hat{j}$

$$i_{0R} = \frac{V_0}{z} = \frac{100}{100(1-2j)} = \frac{1+2\hat{j}}{5}$$

$$i_{0R} = \frac{1}{\sqrt{5}}\sin(100t + \tan^{-1}2)$$

$$v_R = 20\sqrt{5}\sin(100t + \tan^{-1}2)$$

$$v_{AB} = \frac{1}{\sqrt{5}} \times (-200\hat{j}) = -40\sqrt{5}\hat{j}$$

$$V_L = V_C = 40\sqrt{5}\sin\left(100t + \tan^{-1}2 - \frac{\pi}{2}\right)$$

$$i_{0C} = \frac{V_C}{X_c} = \frac{40\sqrt{5}}{-100\hat{j}} = \frac{2}{\sqrt{5}}\hat{j}$$

$$i_C = \frac{2}{\sqrt{5}}\sin(100t + \tan^{-1}2)$$

$$i_{0L} = \frac{40\sqrt{5}}{200\hat{j}} = -\frac{1}{\sqrt{5}}\hat{j}$$

$$i_L = \frac{1}{\sqrt{5}}\sin(100t + \tan^{-1}2 - \pi)$$

At t = 0, $i_L = \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = -\frac{2}{5}A$, $i_C = +\frac{2}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$

$$V_C = V_L = -40\sqrt{5} \times \frac{1}{\sqrt{5}} = -40 \text{ Volt}$$

We gained by inductor = $\left(-\frac{2}{5}\right) \times (-40) = 16$ J/S

Power gained by capacitor = $\left(\frac{4}{5}\right) \times (-40)$

At t = 0, $i_R = \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{2}{5}A$

$$V_R = 20\sqrt{5} \times \frac{2}{\sqrt{5}} = 40V$$

Power consumed = $\frac{2}{5} \times 40 = 16$ J/s

SECTION – B

7. 8

Sol. $\frac{\sigma}{4\varepsilon_0} \times 10^{-3} + \frac{\sigma}{2\varepsilon_0} \times 10^{-3} = 30$

$$\Rightarrow \frac{\sigma}{\varepsilon_0} = 4 \times 10^4$$

$$\frac{q}{\varepsilon_0} = 4 \times 10^{+4} \times 2 \times 10^{-4} = 8\ \text{V} - \text{m}$$

8. 2

Sol. Resistance of coil is zero ⇒ emf

Developed will be zero so ϕ = constant

or $Li + B_0(1 - x)\pi R^2$ = constant

at t = 0, i = 0 ⇒ $\phi = B_0\pi R^2$

or $Li + B_0(1 - x)\pi R^2 = B_0\pi R^2$

⇒ $Li = B_0\pi R^2 x$

or $i = \frac{B_0\pi R^2}{L}x$

acceleration of coil

$$ma_2 = i \times 2\pi R \times 2B_0 = -\frac{4B_0^2\pi^2R^3}{L}x$$

$$a_x = -\frac{4B_0^2\pi^2R^3}{mL}x$$

As $a_x \alpha - x$ ⇒ motion in S.H.M. and $\omega^2 = \frac{4B_0^2\pi^2R^3}{mL}$

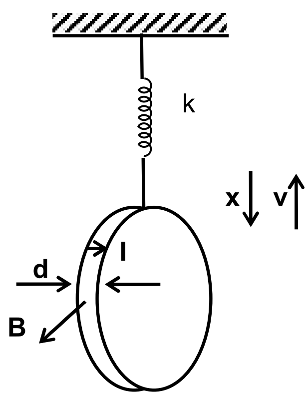
$x = A\sin(\omega t + \pi)$

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \pi)$$

at t = 0, $v = -v_0$

⇒ $v_0 = A\omega$ or $A = \frac{v_0}{\omega}$

9. 3



Sol. eE = evB

E = Bv …(i)

$q = \left(\frac{\varepsilon_0 A}{d}\right)V_0$ (V_0 = potential drop)

$q = \left(\frac{\varepsilon_0 A}{d}\right)Ed$

$q = \varepsilon_0 AE$

$q = \varepsilon_0 ABv$

$I = \frac{dq}{dt} = \varepsilon_0 AB\frac{dv}{dt}$ …(ii)

Now,

$$m\frac{dv}{dt} = kx - BId$$

$$m\frac{dv}{dt} = kx - Bd\left(\varepsilon_0 AB\frac{dv}{dt}\right)$$

$m\frac{dv}{dt} = kx - \varepsilon_0 VB^2\frac{dv}{dt}$ (where V = Ad = volume of the disc)

$$\left(m + \varepsilon_0 VB^2\right)\frac{dv}{dt} = kx$$

$$\frac{dv}{dt} = \frac{kx}{(m + \varepsilon_0 VB^2)}$$

$$\frac{d^2x}{dt^2} = -\frac{kx}{(m+\varepsilon_0 VB^2)}$$

Time period, $T = 2\pi\sqrt{\frac{m+\varepsilon_0 VB^2}{k}}$

10. 0

Sol. $2 - 2i - 10(i_1) = 0$

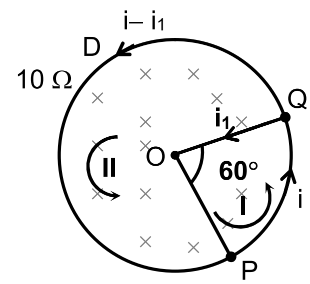
$i + 5i_1 = 1$ …(i)

$10 - 10(i - i_1) + 10i_1 = 0$

$i - 2i_1 = 1$ …(ii)

Solving (i) and (ii), $i_1 = 0$

⇒ Reading of voltmeter is zero.



11. 1

Sol. Work done by magnetic field zero.

$\Rightarrow mgy = \frac{1}{2}mv^2$

$v = \sqrt{2gy}$ …(i)

$ma_x = F_x$

$m\frac{dv_x}{dt} = qvB\cos\theta = qBv_y$

$\Rightarrow m\frac{dv_x}{dt} = qB\frac{dy}{dt}$

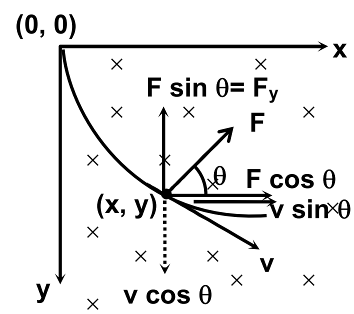
$mv_x = qBy$

$v_x = \frac{qBy}{m}$ …(ii)

Speed will be maximum at lowest point $v = v_x$ at lowest point.

$v = \frac{qB}{m} \times \frac{v^2}{2g}$

$\Rightarrow v = \frac{2mg}{qB} = \frac{2\times 0.1\times 10}{1\times 2} = 1\,m/s$



12. 4

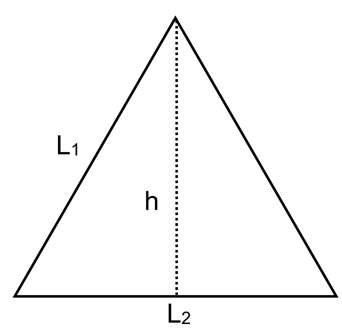
Sol. $h^2 = L_1^2 - \frac{L_2^2}{4}$

Differentiating and using $dL = L\,\alpha\,\Delta t$

$0 = 2L_1\alpha_1 L_1\Delta t - \frac{1}{4}2L_2\alpha_2 L_2\Delta t$

$L_1L_1\alpha_1\Delta t = \frac{1}{4}L_2L_2\alpha_2\Delta t$

$\Rightarrow \frac{L_2}{L_1} = 2\sqrt{\frac{\alpha_1}{\alpha_2}} = 2\sqrt{\frac{4\times 10^{-6}}{1\times 10^{-6}}} = 4$



SECTION – C

13. 00006.97
Range 6.90 to 7.00

Sol. $\vec{\tau} = \vec{M} \times \vec{B}$

$= (2 \times 20 \times 10^{-4})\left\{\cos 30\hat{i} + \sin 30\hat{j}\right\} \times (4\hat{i} - \sqrt{3}\hat{j})$

$= 20 \times 10^{-4}\left\{-3\hat{k} + 4(-\hat{k})\right\} = -140 \times 10^{-4}\hat{k}$ N-m

$I_0 = \frac{mR^2}{2} + md^2$

$= \frac{1}{4} \times \frac{20 \times 10^{-4}}{\pi} + \frac{1}{2} \times 37 \times 10^{-4}$

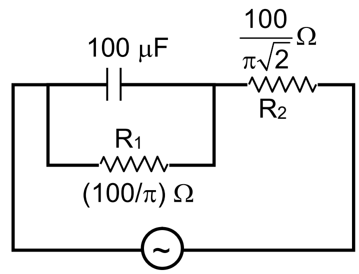
$= \frac{(10 + 37\pi)}{2\pi} \times 10^{-4}$

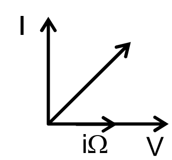
$\alpha = \frac{140 \times 10^{-4} \times 2\pi}{(10 + 37\pi) \times 10^{-4}} = \frac{140 \times 2\pi}{10 + 37\pi}$

$\frac{879.646}{126.239} = 6.968$ rad/s^2

14. 00391.96
Range 390.96 to 392.96

Sol. $\frac{1}{Z_1} = \sqrt{\left(\frac{\pi}{100}\right)^2 + \left(\frac{\pi}{100}\right)^2} = \frac{\pi\sqrt{2}}{100}$

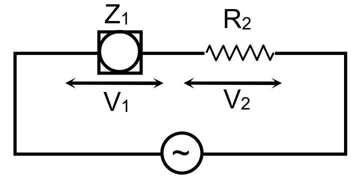




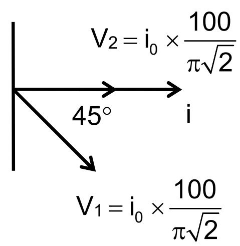
$Z_1 = \frac{100}{\pi\sqrt{2}}$

$i_0 \times \frac{100}{\pi\sqrt{2}} \le 150\sqrt{2}$

$i_0 \le \frac{300\pi}{100} = 3\pi$



$v_0 = i_0\sqrt{\left(\frac{100}{\pi\sqrt{2}}\right)^2 + \left(\frac{100}{\pi\sqrt{2}}\right)^2 + 2\left(\frac{100}{\pi\sqrt{2}}\right)\left(\frac{100}{\pi\sqrt{2}}\right) \times \frac{1}{\sqrt{2}}}$



$= 3\pi \times \frac{100}{\pi\sqrt{2}}\sqrt{2 + \sqrt{2}}$

$= 300\sqrt{1 + \frac{1}{\sqrt{2}}} = 391.96$

390.96 to 392.96 volts

15. 00001.66
Range 1.60 to 1.70

Sol. $ms\left(\frac{dT}{dt}\right) = a\sigma AT_0^4 - e\sigma AT_b^4 + 2$

$= 2.177 - 0.6889 + 2$

= 3.4831 watt

$2 \times 0.25 \times 4.2\left(\frac{dT}{dt}\right) = 3.4831$

$\frac{dT}{dt} = 1.66\text{ K/sec}$

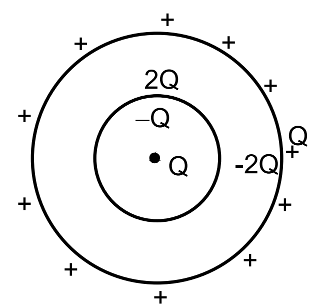
1.60 to 1.70 K/sec

16. 00040.00

Sol. When switch S is open

Energy = $U_0 + \frac{kQ^2}{2R} + \frac{kQ^2}{4R} + \frac{kQ^2}{R} - \frac{kQ^2}{2R} - \frac{kQ^2}{2R}$

$= U_0 + \frac{3kQ^2}{4R}$



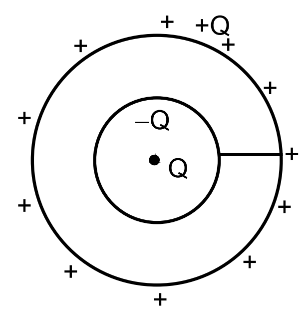
When switch S is closed

Energy $U_2 = U_0 + \frac{kQ^2}{2R} + \frac{kQ^2}{4R} - \frac{kQ^2}{R} + \frac{kQ^2}{2R} - \frac{kQ^2}{2R}$

$= U_0 - \frac{kQ^2}{4R}$

Loss of energy = $U_1 - U_2 = \frac{kQ^2}{R}$

$= \frac{9 \times 10^9 \times 400 \times 10^{-12}}{9 \times 10^{-2}} = 40J$



17. 00000.82
Range 0.80 to 0.83

Sol. Force acting on hemisphere

$F \propto \sigma SE$

Where σ is charge ensily

$S \rightarrow$ surface area

$E \rightarrow$ electric field present

as$\sigma \propto E$

$\Rightarrow F \propto SE^2$

$\propto R^2E^2$

For spherical shell of thickness 2t and radius 3R will require 6 timesforceto tear apart compared to first shell.

$6F \propto (3R)^2.E^2$

$F \propto R^2E_0^2$

$\Rightarrow 6 = 9\left(\frac{E}{E_0}\right)^2 \Rightarrow \frac{E}{E_0} = \sqrt{\frac{2}{3}}$

$E = \sqrt{\frac{2}{3}}E_0$

$\frac{E}{Eo} = 0.816$

0.80 to 0.83

18. 00068.48

Range 67.50 to 69.50

Sol. $2 \times 10^{-6} \times 4200 \times 12 = 4 \times \frac{1}{2} \times 20 \times 10^{-4} \times 5.67 \times 10^{-8} \times (300)^3 (\Delta T)$

$\Delta T = \frac{2 \times 4.2 \times 12 \times 10^{-3}}{40 \times 5.65 \times 27 \times 10^{-6}} = 16.5°C$

Temperature of rheostat wire = 27 + 16.5 = 43.5°C

Resistance of Rh at 43.5°

$R = R_0(1 + \alpha\Delta t)$

Potential drop across Rh = $\sqrt{\text{Power} \times \text{Resistance}}$

$= \sqrt{2 \times 10^{-3} \times 1 \times 12 \times 4.2 \times R}$

$= \sqrt{2 \times 12 \times 4.2 \times 10^{-3} \times 100}(1 + \Delta t)^{1/2}$

$= 3.18(1 + 2 \times 10^{-3} \times 4.35) = 3.44$ Volt

PD across AB = 12 – 3.44 = 8.56 volt

Balance length = 40 cm

$\Rightarrow \frac{V_{AB}}{AB} \times 40 = 5$

AB = 68.48 cm

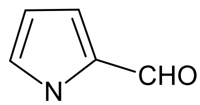
Length of wire AB = 68.48 cm

Range 67.5 to 69.5 cm

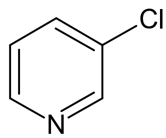
Chemistry PART – II

SECTION – A

19. A, C
Sol. Reimer-Tiemann reaction.



Normal RTR product

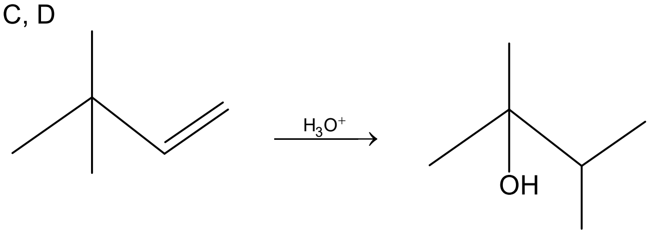


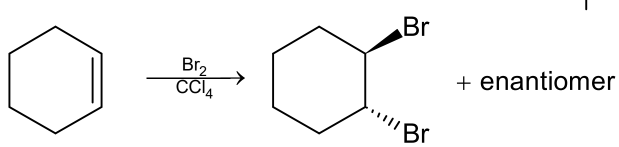
Abnormal RTR product

20. A, B, C
Sol. Ketones having $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-$ give haloform reaction. CH_3CH_2OH and secondary alcohol containing $CH_3-\overset{\overset{\displaystyle OH}{|}}{CH}-$ also give haloform reaction.

21. A, C, D
Sol. This cumulene is optically active because it is chiral.

22. C, D
Sol.



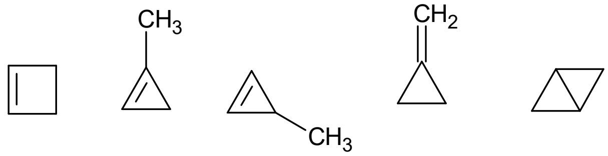


23. A, D
Sol. HIO_4 does not cleave compounds in which an intervening – CH_2 – group separate – OH groups and compounds in which – OH group is adjacent to – OR group.

24. A, B, C, D
Sol. All the given compounds are more acidic than H_2CO_3.

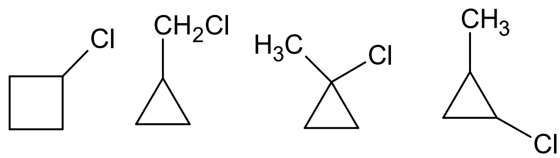
SECTION – B

25. 9
Sol. $CH_2=CH-CH=CH_2$, $CH_2=C=CH-CH_3$
$CH\equiv C-CH_2-CH_3$, $CH_3-C\equiv C-CH_3$



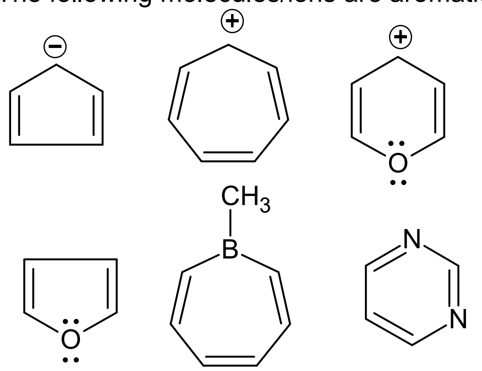
26. 4

Sol.



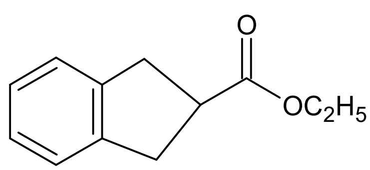
27. 6

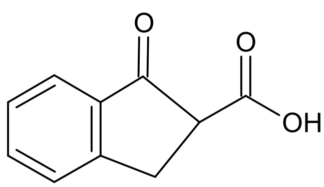
Sol. The following molecules/ions are aromatic



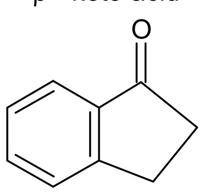
28. 6

Sol.

A = 

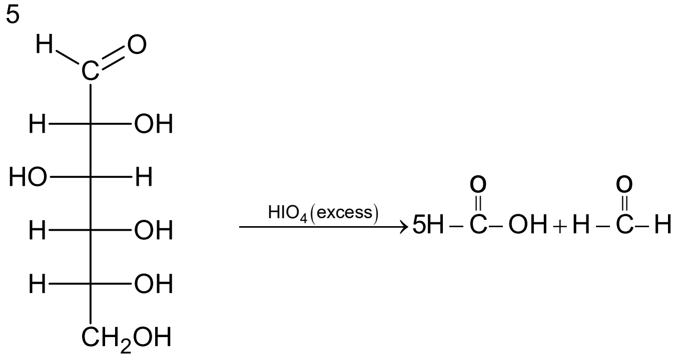
B = 

β – keto acid

C = 

29. 5

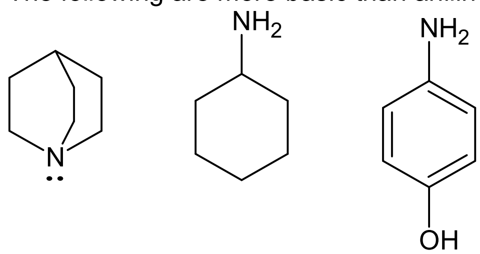
Sol.



D – Glucose

30. 3

Sol. The following are more basic than aniline



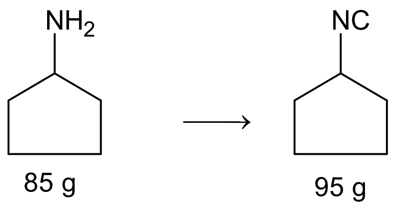
SECTION – C

31. 00009.75

Sol. $P^{I} = \frac{9+10.5}{2} = 9.75$

32. 00023.75

Sol.



Weight of product (P) formed $= \frac{95}{85} \times 21.25 = 23.75$ g

33. 00038.80

Sol. Optically purity $= \frac{-2.8}{12.5} \times 100$

$= 22.4(-)$

Amount of racemic mixture = 100 – 22.4

= 77.6%

% of $(-) = \frac{77.6}{2} + 22.4 = 61.20\%$

% of $(+) = 100 - 61.2 = 38.8\%$

34. 00007.50

Sol. $C_2H_6 + Br_2 \longrightarrow C_2H_5Br + HBr$

30 g 109 g

$2C_2H_5Br + 2Na \longrightarrow C_4H_{10} + 2NaBr$

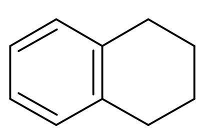
2×109 58

Weight of C_2H_6 required to produce 7.25 g $C_4H_{10} = \frac{60}{58} \times 7.25$

= 7.50 g

35. 00016.50

Sol.

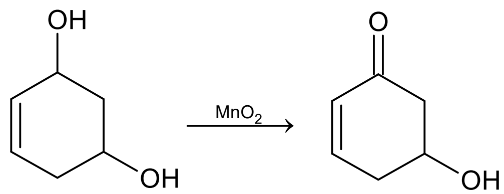
Product$(Q) =$ 

MW of Q = 132

$$\therefore \frac{x}{8} = \frac{132}{8} = 16.50$$

36. 00022.40

Sol.



114 g　　　112 g

Weight of product (P) $= \frac{112}{114} \times 22.8 = 22.40$ g

Mathematics PART – III

SECTION – A

37. A, D

Sol. $\sum_{P=0}^{2020} \sin(3^P\theta)\sec(3^{P+1}\theta) = \frac{1}{2}[\tan 3^{2021}\theta - \tan\theta]$

38. A, C, D

Sol. Circles intersect at (1, 1), $\left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right)$

So, centroid = $\left(2, \frac{11}{18}\right)$

Orthocentre = $\left(\frac{-1}{6}, -6\right)$

Circumcentre = $\left(\frac{37}{12}, \frac{81}{16}\right)$

39. A, B

Sol. $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\frac{de}{dt} < 0$ $\forall$ t > 0 $\Rightarrow$ e(t) is a decreasing function $\forall$ t > 0

Rod will break off contact when 4 + 2t = 10 i.e. at t = 3s

40. A, B

Sol. Reflection of directrix in given tangents passes through focus
Reflection of directrix in given tangents will be 3x – 4y + 6 = 0 and 3x + 4y + 6 = 0
So, focus will be (–2, 0)

41. A, D

Sol. Draw graphs and check n(A) = 0, n(C) = n(B) + 1

42. A, D

Sol. $\sin^{-1}x + \cos^{-1} = \frac{5\pi}{2}$ and $\frac{\sin^{-1}x}{3\pi} = \frac{\cos^{-1}x}{4\pi}$

$\sin^{-1}x = \frac{15\pi}{14}$, $\cos^{-1}x = \frac{10\pi}{7}$

SECTION – B

43. 1

Sol. A : x + y = 4
B : y = 2x + 2
n(A $\cap$ B) = 1

44. 6

Sol. cos A + cos B + cos C = $1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{3}{2}$

45. 3

Sol. $\sin^{-1}\left(\frac{2\left(\frac{x}{2}\right)}{1+\left(\frac{x}{2}\right)^2}\right) = \cos^{-1}\left(\frac{1-\left(\frac{x}{2}\right)^2}{1+\left(\frac{x}{2}\right)^2}\right)$

Now, make cases based on x

Equation satisfies for x = 0, 1, 2

46. 5

Sol. Mid-point of segment of a chord between asymptotes coincides with mid-point of the chord

47. 5

Sol. Area of Δ = $\frac{1}{2}(\text{shortest side}) \times (\text{longest altitude})$

48. 0

Sol. Either $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x = 0$ or $\sin^{-1} x = \cos^{-1} x = \tan^{-1} x$

Both of which are not possible $\forall\ x \in R$

SECTION – C

49. 00001.83

Sol. f(a, b) represents distance between circle $x^2 + y^2 = 1$ and line $x + y = 4$

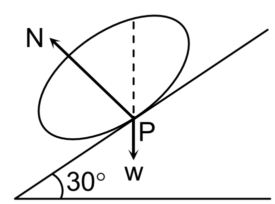
So, minimum value of f(a, b) is $2\sqrt{2} - 1$

50. 00012.00

Sol. Reflection is $x^2 + 2x + 4y + 9 = 0$

51. 00000.67

Sol. For minimum eccentricity angle between normal at point of contact and vertical will be 30°



If ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then point of contact can be taken as

(a cos θ, b cos θ) then $\frac{\frac{a}{b}\tan\theta - \frac{b}{a}\tan\theta}{1+\tan^2\theta} = \tan 30°$

$\Rightarrow \frac{e^2 \sin 2\theta}{2\sqrt{1-e^2}} = \frac{1}{\sqrt{3}}$. For minimum eccentricity $e = \sqrt{\frac{2}{3}}$

52. 00000.25

Sol. P lies on directrix of the parabola

53. 00000.16

Sol. $PR^2 - 2PR\cdot PQ\cos(\angle QPR) + PQ^2 - QR^2 = 0$

$\Delta_1 = 2\Delta_2 \Rightarrow$ one root of the equation is twice the other

$\Rightarrow \sin(\angle QPR) = \frac{\sqrt{5}}{4\sqrt{2}}$

54. 00002.00

Sol. $f(x) = x^x$ is decreasing in $\left(0, \frac{1}{e}\right)$ and increasing in $\left(\frac{1}{e}, \infty\right)$

So, if $x^x = c$ where $c \in \left(\left(\frac{1}{e}\right)^{\frac{1}{e}}, 1\right)$ then equation has two solutions