FIITJEE

ALL INDIA TEST SERIES

PART TEST – I

JEE (Advanced)-2021

PAPER –1

TEST DATE: 06-12-2020

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. B

Sol. $K_e = \frac{1}{2}3Mv^2 + \frac{1}{2}Mv^2 = 2Mv^2$

$K_s = \frac{1}{2}(3M+M)\left(\frac{2Mv}{4M}\right)^2 = \frac{1}{2}Mv^2$

So, $\frac{K_e}{K_s} = 4$

2. A

Sol. $\tau_n = \frac{2v_n}{g}$, where $v_n = \alpha^n v_i$

So, $\tau_n = \tau_0\alpha^n$, where $\tau_0 = \frac{2v_i}{g}$

$t_n = \tau_0\Sigma\alpha^n = \tau_0\frac{1-\alpha^n}{1-\alpha} = \frac{\tau_0 - \tau_n}{1-\alpha}$

So, $\tau_n = \tau_0 - (1-\alpha)t_n$

3. C

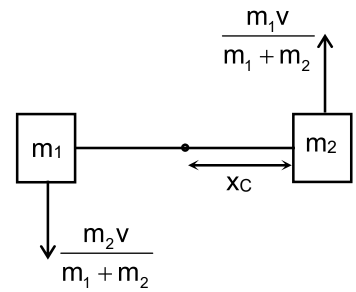
Sol. The side of the cube is $L \propto m^{1/3}$ (density is constant)
So, $A \propto m^{2/3}$
For constant speed,

$m \propto P(= Fv)$
So, $m \propto Av^3 \propto m^{2/3}v^3$
So, $v \propto m^{1/9}$

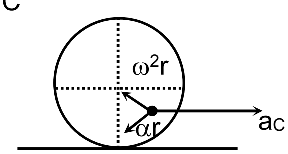
4. D
Sol. Relative to COM frame, the rod is undergoing pure rotatory motion with a constant angular velocity, $\omega = \frac{v}{L}$

$x_C = \frac{m_1 L}{m_1 + m_2}$

So, $t = \frac{2\pi L}{v}$



5. C
Sol.

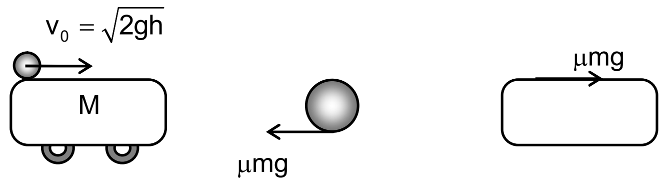


6. A
Sol. $(v_0 - \mu g t) - \alpha t R = \frac{\mu m g}{M} t$

So, $t = \frac{v_0}{\mu g\left(3 + \frac{m}{M}\right)}$

So, in the frame of the cart

$L = v_0 t - \frac{1}{2}\mu g\left(1 + \frac{m}{M}\right)t^2 = \frac{7}{8}m$



At t = 0 **F.B.D. of disc** **F.B.D. of cart**

7. A, C
Sol. $a_C = \frac{2FR^2}{3mR^2} = \frac{2F}{3m}$

So, $F - f_s = m\frac{2F}{3m}$

So, $f_S = \frac{F}{3}$

$f_S \le \mu mg$

So, $\frac{F}{3} \le 40$ N

So, $F \le 120$ N

So, the disc undergoes pure rolling motion upto t = 12 sec
Torque about point P = (10t) (0.1) N-m

So, L = $\frac{t^2}{2}$ kg-m^2/sec

8. A, D

Sol. For moving together, $a = \frac{mg}{M+m}$

So, $T = \frac{Mmg}{M+m}$

For toppling the cylinder $(T)\left(\frac{10}{2}\right) > (Mg)\left(\frac{5}{2}\right)$

So, $\frac{Mmg}{M+m}\left(\frac{10}{2}\right) > Mg\left(\frac{5}{2}\right)$

So, M < m

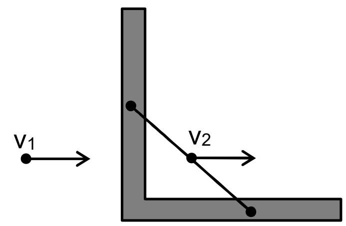
9. C, D

Sol. From conservation of linear momentum

$2mu = 2mv_1 + 2mv_2$

And for e, $\frac{1}{2}u = v_2 - v_1$

So, $v_1 = \frac{u}{4}$ and $v_2 = \frac{3u}{4}$

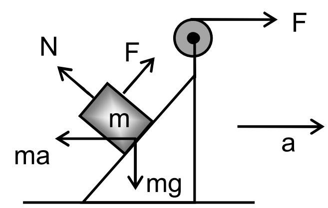


Impulse is passing through centre of mass of the body. So there is no rotation after the collision

Loss in kinetic energy = $\frac{1}{2}2mu^2 - \frac{1}{2}2m(v_1^2 + v_2^2) = \frac{3}{8}mu^2$

10. A, B, D

Sol. $a = \frac{F}{M+m}$



m is in equilibrium w.r.t to M

So, $F = \frac{(M+m)mg\sin\theta}{M+m-m\cos\theta}$

So, pseudo force on m w.r.t to M = $\frac{mF}{M+m}$

= $\frac{m^2 g\sin\theta}{M+m-m\cos\theta}$

Pseudo force on M w.r.t. to m = $\frac{MF}{M+m}$

Also, F cos θ = N sin θ + ma

$\Rightarrow$ N sin 45° = $\frac{F}{\sqrt{2}} - \frac{mF}{M+m}$

So, $N = \frac{mF}{M+m}\left[\frac{M+m-\sqrt{2}m}{m}\right]$

So, N > $\frac{mF}{M+m}$

11. A, C, D

Sol. $\frac{\mu mv^2}{R} = -m\frac{dv}{dt}$

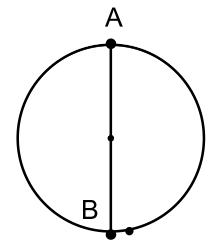
So, $\int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v^2} = -\frac{\mu}{R}\int_0^t dt \Rightarrow t = \frac{R}{\mu v_0}$

$|P_{avg}| = \frac{|\Delta K.E.|}{t} = \frac{3}{8}\frac{\mu m v_0^3}{R}$

At v = v_0/2, instantaneous power due to kinetic friction force = $-\frac{\mu m}{R}\frac{v_0^2}{4}\frac{v_0}{2} = -\frac{\mu m v_0^3}{8R}$

12. A, B

Sol. Extension in the spring at highest point A is L.



$F_{string} = \frac{4mg}{L}\times L = 4mg$

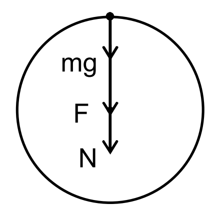
From conservation of energy at B and A

$\frac{1}{2}mu_0^2 = mg(2L) + \frac{1}{2}\left(\frac{4mg}{L}\right)L^2$

So, $u_0 = \sqrt{8gL}$

For u = 2u_0 at B.

$\frac{1}{2}m\left[2\sqrt{8gL}\right]^2 = \frac{1}{2}mv^2 + mg(2L) + \frac{1}{2}\left(\frac{4mg}{L}\right)L^2$



So, $v = \sqrt{24gL}$

So, F + mg + N = $\frac{m\left[\sqrt{24gL}\right]^2}{L}$

So, N = 19mg (radially inward)

At point D, string is stretched and gravity is also acting on the bead.

SECTION – C

13. 00010.25

Sol. $\frac{dv}{dt} = \omega\frac{dr}{dt} = \omega\frac{\omega}{2\pi}d$

Where $\frac{dv}{dt}$ = acceleration of the mass and r = radius of the reel at any instant

So, T – Mg = $M\frac{\omega^2 d}{2\pi}$

So, T = $M\left(g + \frac{16\pi^2\times10^{-2}}{2\pi}\right) = 10.25\,N$

14. 00013.89

Sol. Centripetal force F_C= $Pd\ell.d$

Where $d\ell$ is a small section of pipe

So, $Pd\ell.d$ = $\frac{\pi d^2}{4}\rho(d\ell)\frac{v^2}{R}$

and $v = \frac{4m}{\pi d^2 \rho t}$

So, $P = \frac{4m^2}{\pi\rho R d^3 t^2} = 13.89\,N/m^2$

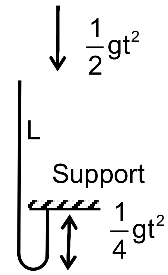
15. 00017.25

Sol. $\rho Lg - T = \frac{d}{dt}\left(\rho Lgt - \frac{\rho g^2 t^3}{4}\right)$

So, $T = \frac{3}{4}\rho g^2 t^2$

So, at t = 1 sec

$T = \frac{3}{4}\left(\frac{2.3}{10}\right)(10)^2(1)^2 = 17.25\text{ N}$



16. 00014.50

Sol. $d_{air} = \frac{v^2 \sin 2\theta}{g}$

Impulse of normal reaction from ground = mV sin θ

So, after collision, $v_x = v\cos\theta - \mu v\sin\theta$

So, $d_{ground} = \frac{v^2(\cos\theta - \mu\sin\theta)^2}{2\mu g}$

So, $d_{total} = \frac{v^2}{2\mu g}\left[2\mu\sin 2\theta + (\cos\theta - \mu\sin\theta)^2\right]$

So, $\frac{d}{dt}(d_{total}) = 0$ So tan θ = μ

So, $d_{total}^{maximum} = \frac{v^2}{2\mu g}(1+\mu^2) = 14.50\text{ m}$

17. 00006.25

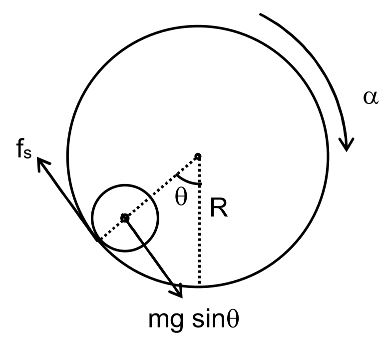
Sol. $f_s = mg\sin\theta$

$\alpha_{sphere} = \frac{R\alpha}{r}$

So, $\tau = I\alpha_{sphere}$

So, mgr sin θ = $\frac{2}{5}mr^2\frac{R}{r}\alpha$

So, $\alpha = \frac{5g\sin\theta}{2R} = \frac{(5)(10)\left(\frac{1}{2}\right)}{(2)(2)} = 6.25\text{ rad/s}^2$



18. 00001.50

Sol. $v_3 = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} = \sqrt{v_1 v_2}$

After solving $\frac{s_1}{s_2} = \sqrt{\frac{v_1}{v_2}} = 1.50$

Chemistry PART – II

SECTION – A

19. B

20. A

Sol. $-\frac{1}{m}\frac{dA}{dt} = k[A]^m$

$-\frac{dA}{dt} = mk[A]^m$

$\log_{10}\left[-\frac{dA}{dt}\right] = \log_{10} mk + m\log[A]$

m = 1, $\log_{10} mk = 0.6 = \log 3.98$

k = 3.98, first order reaction.

21. C

Sol. $NH_3 + H_2O + CO_2 \longrightarrow NH_4HCO_3$

$NH_4HCO_3 + NaCl \longrightarrow NaHCO_3 + NH_4Cl$

22. C

Sol. $T\ell I_3 \longrightarrow T\ell^{+1} + I_3^-$

$B(OC_2H_5)_3$ Burns with green edge flame.

23. C

24. C

25. A, B, D

26. A, B, C

Sol. $\ell = 3$

n = 4

orbital angular momentum $= 4 \times \frac{h}{2\pi} = \frac{2h}{\pi}$

27. A, B

Sol.

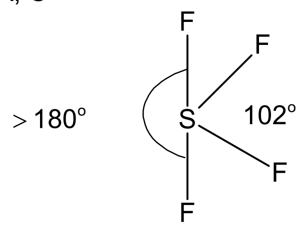
$$A + B \rightleftharpoons 2C$$

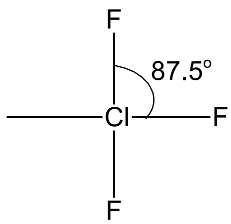
At equilibrium $1+x \quad 1+x \quad 2-2x$

$1 = \frac{(2-2x)^2}{(1+x)(1+x)}$

28. A, C

Sol.





Bond angle of $OBr_2 = 112^\circ$

$OCl_2 = 110^\circ$

29. A, D

30. A, D

SECTION – C

31. 00150.00

Sol. $pH = pK_{a_2} + \log\frac{[HPO_4^{2-}]}{[H_2PO_4^-]}$

$8.3 = 8 + \log\frac{[HPO_4^{2-}]}{[H_2PO_4^-]}$

$\frac{[HPO_4^{2-}]}{[H_2PO_4^-]} = 2$

$H_3PO_4 + NaOH \longrightarrow NaH_2PO_4$

9	x	0
0	$x-9$	9

$NaH_2PO_4 + NaOH \longrightarrow Na_2HPO_4$

9	$x-9$	0
$18-x$	0	$x-9$

$\frac{x-9}{18-x} = 2$

$x = 15$

$15 = V \times 0.1,\ V = 150$ ml

32. 00009.48

Sol. $NH_4OH + HCl \longrightarrow NH_4Cl$

40	10	0
30	0	10

$pOH = 5 + \log\frac{1}{3};\ pH = 9.48$

33. 00004.20

Sol. $C_6H_5NH_3^+ + NaOH \longrightarrow C_6H_5NH_2 + NaCl$

5	4	0	0
1	0	4	

$pOH = pK_b + \log\frac{1}{4}$

= 10.4 – log4 = 10.4 – 0.6 = 9.8

pH = 4.2

34. 00014.06

Sol. $a\ eV = E_1\left[\frac{1}{3^2} - \frac{1}{5^2}\right]$

$E_1 = \frac{225}{16} a\ eV$

x = 14.06

35. 00014.50

Sol. $15eV = \frac{13.6}{4^2} eV + KE$

$KE = 14.15eV$

36. 00000.71

Sol. $K_1 + K_2 = \frac{2.303}{0.5} \log\frac{1}{1-x}$

Hence, $(1-x) = 0.71$ molar

Mathematics PART – III

SECTION – A

37. A

Sol. Stretch the figure by a factor 3 along the y-axis. So that the point (x, y) goes to (x, 3y), then new line is 2y – 3x

So, for R = R′, $m = \frac{2}{9}$

38. B

Sol. $\frac{1-f(x)}{x}, \frac{1+f(x)}{x} \geq 0 \ \forall x \in [1, 3]$

$$\int_1^2 \frac{1-f(x)}{x}dx \geq \int_1^2 \frac{1-f(x)}{2}dx \quad \text{..... (1)}$$

$$\int_2^3 \frac{1+f(x)}{x}dx \leq \int_2^3 \frac{1+f(x)}{2}dx$$

$$\Rightarrow -\int_2^3 \frac{1+f(x)}{x}dx \geq -\int_2^3 \frac{1+f(x)}{2}dx \quad \text{..... (2)}$$

Adding equation (1) and (2), we get $\ln 2 - \ln\left(\frac{3}{2}\right) \geq \int_2^3 \frac{f(x)}{x}dx$

Equality holds for $f(x) = \begin{cases} 1 & ; \ 1 \leq x < 2 \\ -1 & ; \ 2 \leq x \leq 3 \end{cases}$

39. B

Sol. $f(x) = \left(x^3 + \frac{1}{x^3}\right) - 6\left(x^2 + \frac{1}{x^2}\right) + 12\left(x + \frac{1}{x}\right) + a \geq 0 \ \forall x > 0$ and

$f(x) = \left(x^3 + \frac{1}{x^3}\right) - 6\left(x^2 + \frac{1}{x^2}\right) + 12\left(x + \frac{1}{x}\right) + a \leq 0 \ \forall x > 0$

Let $x + \frac{1}{x} = t$

So, $g(t) = t^3 - 6t^2 + 9t + 12t + a \geq 0; t \geq 2$

$g'(t) = 3(t - 1)(t - 3) \Rightarrow g(3) \geq 0$ and $g(t) \leq 0 \ \forall t \leq -2 \Rightarrow -12 \leq a \leq 38$

40. A

Sol. $2|f'(x)| \leq |f(1)| + |f(-1)| + \left|(1-x)^2 \frac{f^2(h)}{2}\right| + \left|\frac{(1+x^2)f''(k)}{2}\right| \Rightarrow |f'(x)| \leq \frac{3+x^2}{2} \leq 2$

41. A

Sol. Let $I(a) = \int_0^\infty \frac{\tan^{-1} ax - \tan^{-1} x}{x}$

$$I'(a) = \frac{1}{a^2}\int_0^\infty \frac{dx}{x^2 + \frac{1}{a^2}} = \frac{\pi}{2a}$$

$\therefore \ I = \frac{\pi}{2}\ln a + c$

So, $I(a) = \frac{\pi}{2}\ln a$

42. D

Sol. **Case-I:** Let $x \le 0$ then $2.3^x \le 2$ and $\frac{4x^2+x+2}{x^2+x+1} \ge 2 \Rightarrow x \le 0$ or $x \ge \frac{1}{2}$. So, $x \le 0$

Case-II: Let x > 0, we prove that $\frac{4x^2+x+2}{x^2+x+1} < 2.3^x$

Assume the opposite i.e. $\frac{4x^2+x+2}{x^2+x+1} \ge 2.3^x$

$\because \frac{4x^2+x+2}{x^2+x+1} > 2.3^0 = 2 \Rightarrow x < 0$ or $x > \frac{1}{2}$

Since, x > 0. So, $x > \frac{1}{2}$. Hence, $\frac{4x^2+x+2}{x^2+x+1} \ge 2.3^x > 2\sqrt{3} > 3$

$\Rightarrow x^2 - 2x - 1 > 0 \Rightarrow x \in \left(1+\sqrt{2}, \infty\right)$

Thus, $\frac{4x^2+x+2}{x^2+x+1} \ge 2.3^x > 23^{1+\sqrt{2}} > 2\cdot 3^2 = 18$

But $\frac{4x^2+x+2}{x^2+x+1} < 4$ for any x > 0, we get a contradiction

So, domain is R

43. B, C

Sol. (A) f(x) = {x}$\sin^2 \pi x$ differentiable $\forall$ x $\in$ R

(B) not differentiable at x = (2n + 1)π

(C) not differentiable at x = 0, 2, 6, 4

(D) not differentiable at all integers

44. A, B, D

Sol. Let $k(x) = f(x)^2 + f'(x)^2$ using MVT, $|f'(a)| = \left|\frac{f(2)-f(0)}{2}\right| \le 1$: a $\in$ (0, 2)

So, $k(a) \le 2$

Similarly; $k(a) \le 2$, b $\in$ (–2, 0)

$\because$ k (0) = 4. So k(x) has a maximum at some interior point of (–2, 2).

Let that point be x_0 then certainly $k(x_0) \ge k(0) = 4 \Rightarrow k'(x_0) = 0$

$\Rightarrow f'(x_0)(f'' + f(x_0)) = 0$: clearly, $f'(x_0) \ne 0$

45. A, C

Sol. h(f(x)) is continuous $\forall$ x $\in$ R and not differentiable at x = –1, 1, 2

46. B, C

Sol. f(x) has maximum value 2 when x = –5 and g(x) has maximum value 3 when x = –5

47. A, B, C

Sol. (A) $\because \frac{1}{1-\alpha}\int_{\alpha}^{1} g(x)dx \le \frac{1}{\alpha}\int_{0}^{\alpha} g(x)dx$

$\alpha\int_{\alpha}^{1} g(x)dx \le (1-\alpha)\int_{0}^{\alpha} g(x)dx$

Add $\int_0^{\alpha} f(x)dx$ both sides

$\alpha\int_0^1 f(x)dx \le \int_0^{\alpha} f(x)dx$

(B) Cauchy-schwarz inequality to the functions f(t) and g(t) = 1 $\forall$ t $\in$ [0, 1]

48. A, D

Sol. $\lim\limits_{x\to 0}\frac{1-\cos nx}{1-\cos x} = n^2$

So, absolute value of the integrand is bounded as $x \to 0$ and hence the integral converges

$\frac{I_{n+1}+I_{n-1}}{2} = \int_0^{\pi}\frac{1-\cos nx\cdot\cos x}{1-\cos x}$ = $\int_0^{\pi}\frac{(1-\cos nx)+\cos nx(1-\cos x)}{1-\cos x} = I_n$

So, $I_n = \frac{1}{2}(I_{n+1}+I_{n-1})$; $n \ge 1$

$\because$ $I_0 = 0, I_1 = \pi, \ldots\ldots, I_n = n\pi$

SECTION – C

49. 00000.00

Sol. $\frac{f\left(\left(3+\sqrt{7}\right)^n T_1\right)}{g\left(\left(2+\sqrt{2}\right)^n T_2\right)} = \frac{f\left(\left(3+\sqrt{7}\right)^2 T_1+\left(3-\sqrt{7}\right)^n T_2-\left(3-\sqrt{7}\right)^n T_1\right)}{g\left(\left(2+\sqrt{2}\right)^n T_2+\left(2-\sqrt{2}\right)^n T_2-\left(2-\sqrt{2}\right)^n T_2\right)}$ = $\frac{f\left(-\left(3-\sqrt{7}\right)^n T_1\right)}{g\left(-\left(2-\sqrt{2}\right)^n T_2\right)}$ $\forall$ n $\ge$ 1

So, $\lim\limits_{n\to\infty}\frac{f\left(T_1\left(3+\sqrt{7}\right)^n\right)}{f\left(T_2\left(2+\sqrt{2}\right)^n\right)} = \frac{T_1}{T_2}\lim\limits_{n\to\infty}\left\{\frac{f\left(-\left(3-\sqrt{7}\right)^n T_1\right)}{-\left(3-\sqrt{7}\right)^n T_1}\cdot\frac{-\left(2-\sqrt{2}\right)^n T_2}{g\left(-\left(2-\sqrt{2}\right)^n T_2\right)}\cdot\frac{\left(3-\sqrt{7}\right)^n}{\left(2-\sqrt{2}\right)^n}\right\} = 0$

50. 00012.50

Sol. $f'\left(\frac{a}{x}\right) = \frac{x}{f(x)}$ ..... (1)

$x \to \frac{a}{x}$

$f'(x) = \frac{a}{xf\left(\frac{a}{x}\right)}$ ..... (2)

$f''(x) = -\frac{a}{x^2 f\left(\frac{a}{x}\right)} + \frac{a^2 f'\left(\frac{a}{x}\right)}{x^3\left(f\left(\frac{a}{x}\right)\right)^2}$ ..... (3)

Using equation (1), (2) and (3), we get $f''(x) = -\frac{f'(x)}{x} + \frac{(f'(x))^2}{f(x)}$

$\Rightarrow$ xf(x) f″(x) + f(x) f′(x) = x(f′(x))^2 $\Rightarrow$ $\frac{f'(x)}{f(x)} + \frac{xf''(x)}{f(x)} - \frac{x(f(x))^2}{(f(x))^2} = 0$

Observe $\left(\frac{xf'(x)}{f(x)}\right)' = 0$. So, $\frac{xf'(x)}{f(x)} = c \Rightarrow \frac{f'(x)}{f(x)} = \frac{c}{x}$

51. 00002.60

Sol. Put n = 1 ; f′(x) = f(x + 1) – f(x), n = 2 ; f′(x) = $\frac{f(x+2)-f(x)}{2}$

So, $f'(x)=\frac{f(x+2)-f(x+1)+f(x+1)-f(x)}{2}$, $f'(x)=\frac{1}{2}f'(x+1)+\frac{1}{2}f'(x)$

$\Rightarrow f'(x) = f'(x + 1)\ \forall\ x \in R \Rightarrow (f(x + 1) - f(x))' = 0\ \forall\ x \in R$

$\Rightarrow f(x + 1) - f(x) = c$ for a constant $c \in R \Rightarrow f'(x) = c \Rightarrow f(x) = cx + d$

52. 00000.50

Sol. $\because \lim\limits_{x\to 0} x^x = 1$

Fix $\in > 0$ and choose $\delta > 0$ such that for $0 < x < \delta$, $|x^x - 1| < \in$ then for $n \ge \frac{1}{\delta}$, we have

$$\left|n^2\int_0^{\frac{1}{n}}\left(x^{x+1}-x\right)dx\right| \le n^2\int_0^{\frac{1}{n}}\left|x^{x+1}-x\right|dx = n^2\int_0^{\frac{1}{n}}x\left|x^x-1\right|dx < \in n^2\int_0^{\frac{1}{n}}xdx = \frac{\in}{2}$$

So, $n^2\int_0^{\frac{1}{n}}x^{x+1}dx = \frac{\in}{2}+n^2\int_0^{\frac{1}{n}}xdx = \frac{\in}{2}+n^2\cdot\frac{1}{2n^2}=\frac{1}{2}$

53. 00002.00

Sol. $S_n = \lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{2^{\frac{r}{n}}}{1+\frac{1}{m}}\ \because\ \frac{2^{\frac{r}{n}}}{1+\frac{1}{m}} = \frac{2^{\left(\frac{r-1}{n}\right)}2^{\frac{1}{n}}}{1+\frac{1}{m}} = \frac{2^{\left(\frac{r-1}{n}\right)}e^{\frac{\ln 2}{n}}}{1+\frac{1}{rn}} = \alpha$ and

for; $r \ge 2$; $2^{\frac{r}{n}} > \alpha > 2^{\frac{r-1}{n}}\left(\frac{1+\frac{\ln 2}{n}}{1+\frac{1}{rn}}\right) > 2^{\frac{r-1}{n}}$ ($\because$ $e^x > 1 + x$)

So, $\frac{2^{\frac{1}{n}}+2^{\frac{2}{n}}+.....+2^{\frac{n-1}{n}}}{n} < S_n < \frac{2^{\frac{2}{n}}+2^{\frac{3}{n}}+.....+2^{\frac{n}{n}}}{n}$. Hence, $S_n = \int_0^1 2^x dx = \frac{1}{\ln 2}$

54. 00003.00

Sol. Let f(x) = sin(sin(sin(sin(sin x))))

$f'(0) = 1 > \frac{1}{3}$. Therefore, $f(x) > \frac{x}{3}$ in some neighbourhood of 0

So, there are 3 solutions