FIITJEE

ALL INDIA TEST SERIES

OPEN TEST

JEE (Advanced)-2021

PAPER –2

TEST DATE: 24-01-2021

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. A, B, C

Sol. $\frac{F}{A} = \eta.\frac{dv}{dy}$

2. B, C

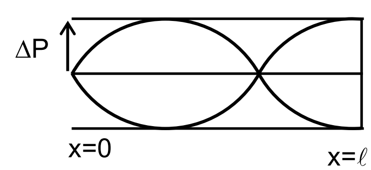
Sol. $mg - F_B = 6\pi\eta r v_\tau$

$\Rightarrow v_\tau = \frac{\rho g r^2}{9\eta}$

$mgH - F_B H + W_f = \frac{1}{2}mv_\tau^2 - 0$

3. A, C, D

Sol. $f = \frac{3v}{4\ell}$



$\ell = \frac{3v}{4f} = \frac{3\times 330}{4\times 550}$

$\ell = \frac{9}{20}m = 45cm$

The amplitude of pressure variation in the pipe is

$a = |\Delta P_0 \sin kx|$

$a = \left|\Delta P_0 \sin\left(\frac{3\pi}{2\ell}x\right)\right|$

at x = $\ell/2$

$a = \left|\Delta P_0 \sin\left(\frac{3\pi}{4}\right)\right| = \frac{\Delta P_0}{\sqrt{2}}$

The maximum pressure at the middle of the pipe

$= \left(P_0 + \frac{\Delta P_0}{\sqrt{2}}\right)$

The maximum pressure at the closed end of the pipe = $\left(P_0 + \Delta P_0\right)$

4. A, B

Sol. $\gamma = \frac{dV}{VdT} = \frac{2}{T}$

$\Rightarrow \int \frac{dv}{V} = 2\int \frac{dT}{T}$

$\Rightarrow V = kT^2$

i.e. $PV^{1/2} =$ constant

C = C_V + $\frac{R}{\left(1-\frac{1}{2}\right)} \Rightarrow C = C_V + 2R$

5. B, C, D

Sol. $T = 2\pi\sqrt{\frac{4m}{g}} = 4\pi\sqrt{\frac{m}{K}}$

$\omega^2 A = g$

$\Rightarrow A = \frac{g}{\omega^2} = \frac{gT^2}{4\pi^2} = \frac{4mg}{K}$

Equilibrium compression $X_0 = \frac{4mg}{K}$

6. B, C, D

Sol. $eV_0 = \frac{hc}{\lambda} - \phi$

$= \left(\frac{1240}{400} - 1.5\right)eV$

$= V_0 = 1.6\text{volt}$

For B, $V_0 = \frac{1240}{400} - 2 = 1.1$ volt

Energy is equally distributed through out spectrum but for B, $\lambda_{th} = 620$ nm for photo electric effect in metal B range of spectrum is 400 nm to 620 nm.

SECTION – B

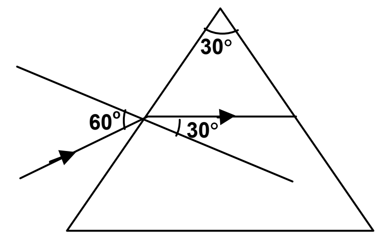
7. 3

Sol. $\delta = i + e - A$

$30^\circ = 60^\circ + e - 30^\circ$

$\Rightarrow e = 0 \Rightarrow$ ray is perpendicular on the second surface of prism, so that r=30°

$\mu = \frac{\sin 60^\circ}{\sin 30^\circ} = \sqrt{3}$



8. 6

Sol. Angular velocity about centre of mass is same.

So that

$v_A = \omega r_A$

$v_B = \omega r_B$

$\frac{k_{e(B)}}{k_{e(A)}} = \frac{\frac{1}{2}m_B v_B^2}{\frac{1}{2}m_A v_A^2}$

$= \frac{2\,M_S}{12\,M_S}\frac{\left(\omega\frac{6d}{7}\right)^2}{\left(\omega\frac{d}{7}\right)^2} = 6$

Where $r_A + r_B = d$

$m_A r_A = m_B r_B$

$\Rightarrow r_A = \frac{d}{7}$

$\Rightarrow r_B = \frac{6d}{7}$

9. 5

Sol. $e_{av} = \frac{\Delta\phi}{\Delta t} = \frac{2BA}{\Delta t}$

$\Rightarrow B = \frac{e_{av}\Delta t}{2A}$

$= \frac{20 \times 10^{-3} \times 0.2}{2 \times 4 \times 10^{-4}} = 5$

10. 9

Sol. $F - \mu N = ma$

$80 - \frac{1}{3} \times 6g = ma$

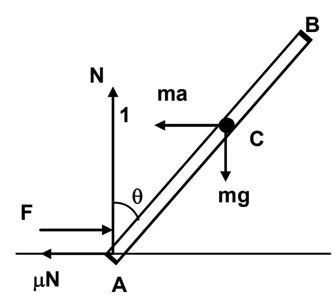
Ma =60

$\tau_A = 0$

$ma\frac{\ell}{2}\cos\theta - mg\frac{\ell}{2}\sin\theta = 0$

$\Rightarrow \tan\theta = 1$

$\theta = 45^\circ$



11. 5

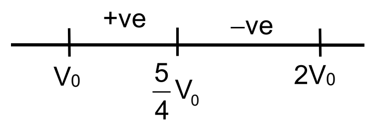
Sol. The equation of process A $\rightarrow$ B is

$$P = -\left(\frac{2P_0}{V_0}\right)V + 5P_0$$

$$T = -\left(\frac{2P_0}{nRV_0}\right)V^2 + \left(\frac{5P_0}{nR}\right)V$$

$$\frac{dT}{dV} = -\left(\frac{4P_0}{nRV_0}\right)V + \frac{5P_0}{nR}$$

Sign scheme for $\frac{dT}{dV}$

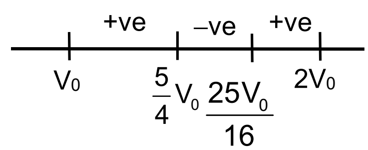


Now, $C = C_V + \frac{pdV}{ndT}$

$$C = \frac{3R}{2} + R\left(\frac{5P_0V_0 - 2P_0V}{5P_0V_0 - 4P_0V}\right)$$

$$C = \frac{R}{2}\left(\frac{25V_0 - 16V}{5V_0 - 4V}\right)$$

Sign scheme for C



Hence the heat is absorbed by the gas from V_0 to $\frac{25}{16}V_0$

Hence the process changes from endothermic to exothermic at the volume $\left(\frac{25V_0}{16}\right)$

12. 4

Sol. $Q = Q_0 \cos \omega t$

$i = Q_0\, \omega \sin \omega t$

$$\frac{Q_0}{n\sqrt{\pi\varepsilon_0 Lr}} = \frac{Q_0}{\sqrt{L.8\pi\varepsilon_0 r}}.\sin\left(\frac{1}{\sqrt{L.8\pi\varepsilon_0 r}} \times \frac{\pi}{4}\sqrt{8\pi L\varepsilon_0 r}\right)$$

$$= \frac{Q_0}{\sqrt{L.8\pi\varepsilon_0 r}} \times \frac{1}{\sqrt{2}}$$

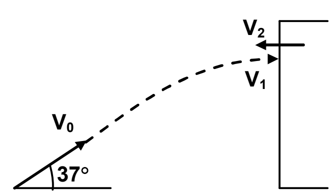
$n = 4$

SECTION – C

13. 00006.25

Sol. $V_{sep} = e\, V_{app}$

$V_1 - V_2 = e(V_1 + V_2)$



$$V_0 \times \frac{4}{5}(1-e) = V_2(1+e)$$

$$\frac{V_0}{V_2} = \frac{(1+e)}{1-e} \times \frac{5}{4} = 6.25$$

14. 00014.14

Sol. $\therefore$ $a_x = 0$

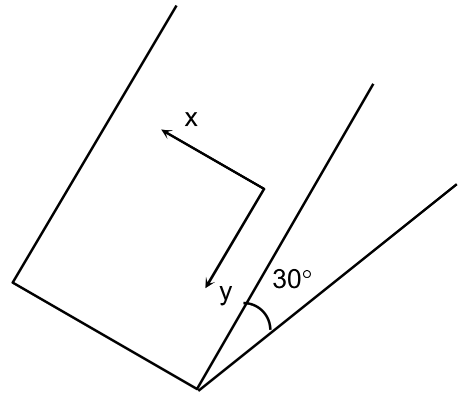
$a_y = g \sin 30°$

$\vec{v} = \vec{u} + \vec{a}t$

$\vec{v} = 5\hat{i} + (5\hat{j})1$

$(5\hat{i} + 5\hat{j})$m / s

$R = \dfrac{v^2}{a_n} = \dfrac{5^2 + 5^2}{(5/\sqrt{2})} = 14.14\text{m}$



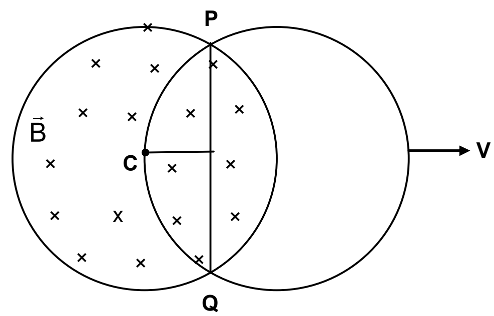
15. 00001.73

Sol. induced emf

$e = Bv\ell$

$\ell = PQ = 2\sqrt{(10^2 - 5^2)} = 10\sqrt{3}$

e = 1.73 volt



16. 00003.25

Sol. When it is at 45° from horizontal

$mg\dfrac{\ell}{4}\cos 45° = \left(\dfrac{m\ell^2}{12} + m\dfrac{\ell^2}{16}\right)\alpha$

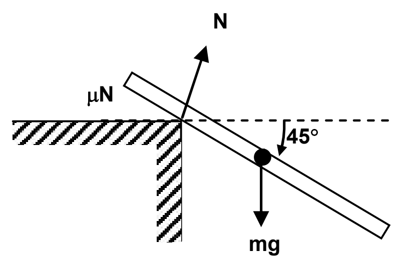
$\Rightarrow \alpha = \dfrac{12g}{7\sqrt{2}\ell}$

Conservation of energy gives : $\omega^2 = \dfrac{24g}{7\sqrt{2}\ell}$

$\mu N - mg\sin 45° = m\left(\omega^2 \dfrac{\ell}{4}\right)$

mg cos 45° – N = $m\left(\dfrac{\ell}{4}\alpha\right) \Rightarrow N = \dfrac{4mg}{7\sqrt{2}}$

By solving $\mu = 3.25$



17. 00002.50

Sol. COM

$4 \times 2.5 = 4 V_C + 4 V_1$

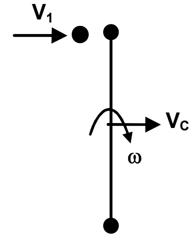
$\Rightarrow V_c + V_1 = 2.5$

COAM

$4 \times 2.5 \times \dfrac{1}{2} = 4 \times V_1 \times \dfrac{1}{2} + [2 \times 2 (\dfrac{1}{2})^2]\omega$

$\Rightarrow 2.5 = V_1 + \dfrac{\omega}{2}$

$V_{sep} = e\, V_{app}$



$V_C + \omega\frac{1}{2} - V_1 = \frac{1}{2}\times 2.5$

$\Rightarrow V_C + \frac{\omega}{2} = V_1 + \frac{2.5}{2}$

By solving equation, $\omega = 2.5$ rad / s, $V_C = \frac{2.5}{2}$ m / s, $V_1 = \frac{2.5}{2}$m / s

Loss of energy = $K_{e(i)} - K_{e(f)} = 3.12$ J

18. 00038.46

Sol. V = 2.5 × 60 = 150 volt

$I = \sqrt{I_R^2 + I_L^2} = \sqrt{15.25}$

$Z = \frac{V}{I} = \frac{150}{\sqrt{15.25}} = 38.46\Omega$

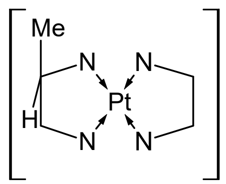
Chemistry

PART – II

SECTION – A

19. A, B, C

Sol. (A) $K_2[Zn(Cl)(Br)(CN)(NO_2)]$ has four constitutional isomers and each one is chiral.

(B)  has one chiral centre and hence it exists as a pair of enantiomer.

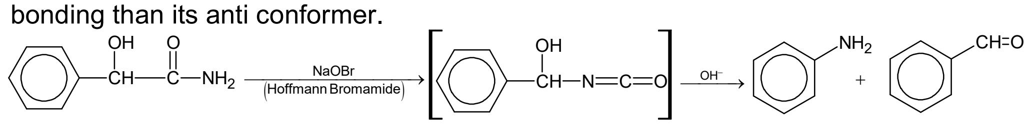
(C) $[Co(ox)(F)_2(Cl_2)]^{3-}$ it has three geometrical isomers and one of them is chiral. So, total stereoisomers are 4.

(D) $K_3[FeF_6]$ is a colourless complex

20. B, D

Sol. (A) Increasing order of stability of conformers of cyclohexane is
chair > twist boat > boat > half-chair

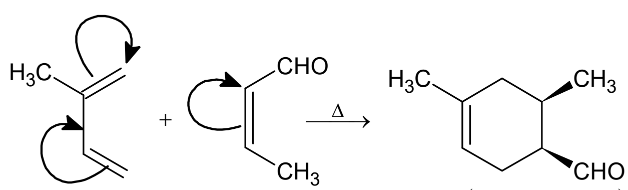
(B) Gauche conformer of 2-fluoroethanol is more stable due to intramolecular hydrogen bonding than its anti conformer.

(C) 

(D) During ozonolysis, cross-ozonides are also formed.

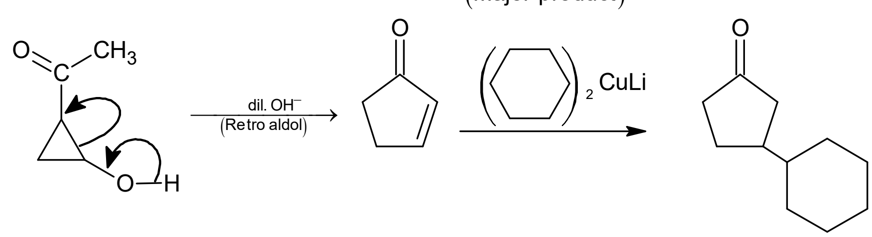
21. B, C, D

Sol. (A)



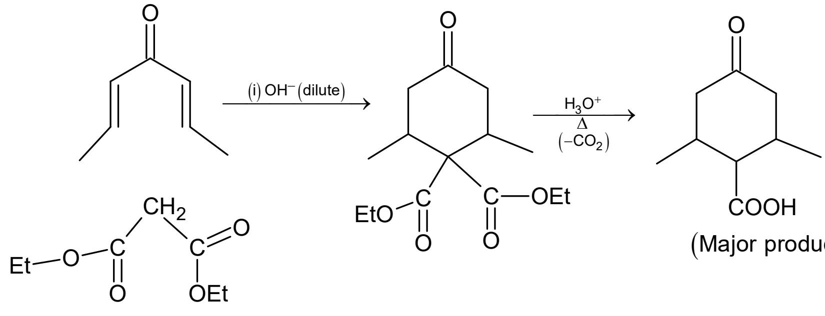
(Major product)

(B)



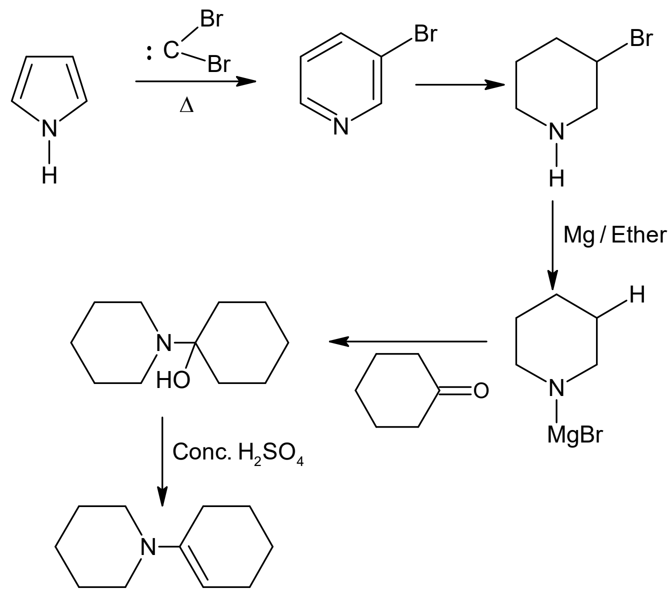
(Major product)

(C)



(Major product)

(D)



(Major product)

22. A, B, C, D

Sol. (A)

	H_3PO_4	$+PO_4^{3-}$	$\longrightarrow$	$H_2PO_4^-$	$+HPO_4^{2-}$
Initial m. mol	10	10		0	0
m.mol. after neutralization	0	0		10	10

So, $pH = pK_{a_2} = 8$ (Buffer)

(B) (H_3PO_4) $(H_2PO_4^-)$ (PO_4^{3-})

(10) (10) (10)

	H_3PO_4	$+PO_4^{3-}$	$\longrightarrow$	$H_2PO_4^-$	$+HPO_4^{2-}$
Initial m. mol	10	10		10	0
After neutralization	0	0		20	10

Again it is a buffer

$$pH = pK_{a_2} + \log\frac{[HPO_4^{2-}]}{[H_2PO_2^-]} = 8 + \log\frac{10}{20} = 7.70$$

(C) (H_3PO_4) (PO_4^{3-}) $(H_2PO_4^{2-})$ (HCl)

(10) (10) (10) (10)

$\Rightarrow$ $PO_4^{3-} + HCl \longrightarrow HPO_4^{2-} + Cl^-$

10	10	0	0
0	0	10	10

$\Rightarrow$ $H_3PO_4 + HPO_4^{2-} \longrightarrow 2H_2PO_4^-$

10	10	10
0	0	30

$$\text{So, } pH = \frac{pK_{a_1} + pK_{a_2}}{2} = \frac{3+8}{2} = 5.5$$

(D) pH will remain same as in option (B).

23. A, C, D
Sol. Formula of ideal solid 'x' = AB_3

$\Delta T_f = i \times k_f \times m$

$0.2976 = 1.6 \times 1.86 \times \frac{0.60 \times 1000}{m_x \times 100}$

$\therefore m_x = 60 \text{ g/mole}$

$r_{A^{3+}} + r_{B^-} = \frac{a}{\sqrt{2}} \quad \therefore a = 2\overset{\circ}{A}$

- Density (d) $= \frac{60 \times 1}{6 \times 10^{23} \times 8 \times 10^{-24}} = 12.5 \text{ g/cm}^3$

  Total number of unit cells = Total number of molecules of $AB_3 = \frac{0.60}{60} \times 6 \times 10^{23} = 6 \times 10^{21}$

- If there are 12% cation vacancies it means formula of defected solid would be $A_{0.88}B_{3.00}$

  So, $\%A^{4+} \text{ ions} = \frac{36}{88} \times 100 = 40.9\%$

24. A, C
Sol. Gas is undergoing free expansion, so, process is irreversible adiabatic as well as isothermal.

$\Delta S_{syst} = nR\ell n\frac{V_2}{V_1}$

$= nR\ell n\frac{2.73V_i}{V_i}$

= nR

$\Delta S_{syst} = \frac{P_iV_i}{T_i}$

SECTION – B

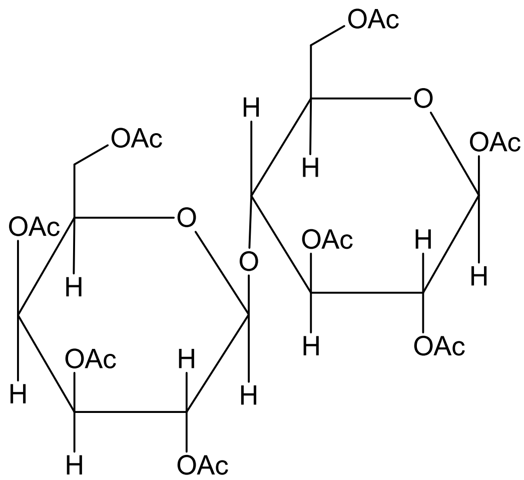
25. 0
Sol.

	Species	**Geometry**	**Hybridization**	**d-orbitals involved**
I.	$\left[VO(acac)_2\right]$	Square pyramidal	dsp^3	$d_{x^2-y^2}$
II.	$\left[Fe(CO)_5\right]$	Trigonal bi-pyramidal	dsp^3	d_{z^2}
III.	$\left[PtCl_4\right]^{2-}$	Square planar	dsp^2	$d_{x^2-y^2}$
IV.	$K_4\left[Fe(CN)_6\right]$	Octahedral	d^2sp^3	$d_{x^2-y^2}$, d_{z^2}
V.	$\left[IF_7\right]$	Pentagonal bi-pyramidal	sp^3d^3	$d_{x^2-y^2}$, d_{z^2} and d_{xy}

So, x = 3, y = 4 and z = 1

26. 8

Sol.



27. 5

Sol. $RH_2 + Ca^{2+} \longrightarrow RCa + 2H^+$

3×10^{-4} M

So, $\left[Ca^{2+}\right] = \frac{3}{2}\times10^{-4}$ M

So, mass of $CaCO_3$ present in 1 L hard water $= \frac{3}{2}\times10^{-4}\times100 = 1.5\times10^{-2}$ g

So, degree of hardness of water $= \frac{1.5\times10^{-2}}{1500}\times10^{6} = 10$ g

So, degree of hardness = 10 ppm.

So, 2x = 10 and x = 5.

28. 5

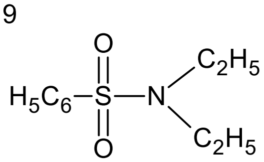
Sol. Only, reaction II, IV, V, VI, VII will give tert-butyl benzene as the major product.

29. 6

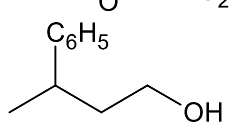
Sol. Only KNO_3, Ag_2O, $KClO_3$, HgO, $NaNO_3$ and H_2O_2 will decompose on heating to give O_2 as the only gaseous product.

30. 9

Sol.



; $C_6H_5NH_2$; $C_6H_5-CH_3$; $C_6H_5-\underset{}{\overset{CH_3}{\overset{|}{C}}}H-C\equiv C-H$;



; $C_6H_5-NO_2$ are insoluble in aq. NaOH and rest are soluble.

SECTION – C

31. 00001.48

Sol. The 4th nearest distance in CsCl structure $\left(d_1\right) = \frac{\sqrt{11}a}{2}$

and the 5th nearest distance in NaCl structure $\left(d_2\right) = \frac{\sqrt{5}a}{2}$

$$\frac{d_1}{d_2} = \sqrt{\frac{11}{5}} = \sqrt{2.2} = 1.48$$

32. 00004.60

Sol. $Mg\left(NH_4\right)\left(PO_4\right) \rightleftharpoons Mg^{2+} + NH_4^+ + PO_4^{3-}$

$K_{sp} = \left[Mg^{2+}\right]\left[NH_4^{2+}\right]\left[PO_4^{3-}\right]$ … (1)

Now,

$H_3PO_4 + H_2O \rightleftharpoons H_2PO_4^- + H_3O^+$; $pK_{a_1} = 2.2$

$H_2PO_4^- + H_2O \rightleftharpoons HPO_4^{2-} + H_3O^+$; $pK_{a_2} = 7.2$

and $HPO_4^{2-} + H_2O \rightleftharpoons PO_4^{3-} + H_3O^+$; $pK_{a_3} = 12.4$

from the above data, it is very much clear that at pH = 10, the main component is HPO_4^{2-}.

Let, the solubility of $Mg\left(NH_4\right)PO_4$ be 'S' mol dm^{-3}, then

$S = \left[Mg^{2+}\right] = \left[HPO_4^{2-}\right]$ … (2)

$K_{a_3} = \frac{\left[PO_4^{3-}\right]\left[H_3O^+\right]}{\left[HPO_4^{2-}\right]}$ or $\left[PO_4^{3-}\right] = \frac{K_{a_3} \times \left[HPO_4^{2-}\right]}{\left[H_3O^+\right]}$ … (3)

So, from Eq. (1), (2) and (3), we get

$$K_{sp} = \left[NH_4^+\right] \times S \times K_{a_3} \times \frac{S}{\left[H_3O^+\right]}$$

$$K_{sp} = \frac{\left[NH_4^+\right] \times K_{a_3} \times S^2}{\left[H_3O^+\right]} \qquad \therefore S = \left(\frac{K_{sp} \times \left[H_3O^+\right]}{\left[NH_4^+\right]K_{a_3}}\right)^{1/2}$$

So, $p\left(S\right) = \frac{1}{2}\left[pK_{sp} + pH + \log_{10}\left[NH_4^+\right] - pK_{a_3}\right]$

$$p\left(S\right) = \frac{1}{2}\left[12.6 + 10 - 1 - 12.4\right]$$

p(S) = 4.60

33. 00001.33

Sol. In solid truncated cube, each vertex becomes a triangular face and each square face becomes an octagonal face.

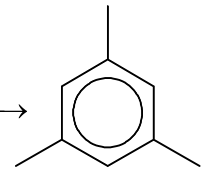
34. 00004.50

Sol. y = 0, x = 9

35. 00007.00

Sol.

$$CH_2 = C = CH_2 \xrightarrow{H_3O^+} \underset{(X)}{CH - \overset{\overset{\Large O}{||}}{C} - CH_3} \xrightarrow[\Delta]{\text{conc. } H_2SO_4}$$



(Y)

36. 00006.75

Sol. Wt. of pay load $= 80 \times 10^3$ g

$$80 \times 10^3 + 0 + 100 \times x = \frac{nRT}{P} \times 1.25 \times x$$

$\therefore x = 26.8 \approx 27$

So, minimum number of balloon required is 27.

Mathematics PART – III

SECTION – A

37. A, B, C, D

Sol. $\beta_1, \beta_2, \beta_3, \ldots\ldots \beta_n$ are in H.P.

So, $\frac{2}{\beta_k} = \frac{1}{\beta_{k-1}} + \frac{1}{\beta_{k+1}}$ for k = 2, 3, ..., n – 1

38. B, C, D

Sol. $b_{n+1} = a_n + b_n + c_n$

$c_{n+1} = b_{n+1}$

$a_{n+1} = b_n + c_n$

$\Rightarrow b_{n+1} = a_{n+1} + a_n$

$a_{n+1} = 2(a_n + a_{n-1})$

$r^2 - 2r - 2 = 0$

$r = \frac{2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3}$

$a_n = A\left(1+\sqrt{3}\right)^n + B\left(1-\sqrt{3}\right)^n$ put $a_1 = 1$, $a_2 = 2$

$A = \frac{1}{2\sqrt{3}}$ and $B = -\frac{1}{2\sqrt{3}}$

39. A, C

Sol. $(x^2 - 3x)^2 + x^2 - 3x + a = 0$

40. A, B, C

Sol. $\left(A^2\right)_{ij} = \sum_{r=1}^{n} f(i)f(r)\cdot f(r)f(j) = \left(\sum_{r=1}^{n} f(r)^2\right)A_{ij} \Rightarrow A^2 = (\text{tr}(A))A$

41. A, B

Sol. $\ell_1^2 = OI_1^2 - R^2 = 2r_1R$

42. A, B, C, D

Sol. (A) $\frac{{}^7C_1 \times 6! \times 4}{3! \times 3! \times 2!} + {}^7C_4 \times 2 + {}^7C_7$

SECTION – B

43. 5

Sol. The equation of circle will be $(x – 1)(x – 8) + y^2 + \lambda y = 0$ will touch y = x

$\lambda = 17, 1$ as ordinate of P is positive so $\lambda = 1$

$x^2 + y^2 – 9x + y + 8 = 0$

$r = \sqrt{\frac{81}{4} + \frac{1}{4} - 8} = \frac{\sqrt{50}}{2} = \frac{5}{\sqrt{2}}$

44. 0

Sol. Let $h(x)=\int_0^x \frac{t^{10}+1}{t^{10}+t^2+1}dt - x^2 - x$

$h'(x)=\frac{x^{10}+1}{x^{10}+x^2+1}-2x-1 = \frac{-x^2}{x^{10}+x^2+1}-2x \le 0$

$\Rightarrow$ h(x) decreases in (0, ∞), h(0) = 0 so no positive real root of the equation

45. 2

Sol. Put x = sin θ, then $\int_0^1 f(x)dx = \int_0^{\pi/2} f(\sin\theta)\cos\theta d\theta$ ..... (1)

Put x = cos θ, $\int_0^1 f(x)dx = \int_0^{\pi/2} f(\cos\theta)\sin\theta d\theta$ ..... (2)

Adding equation (1) and (2), we get $2\int_0^1 f(x)dx \le \int_0^{\pi/2} 1\,dx$, $\int_0^1 f(x)dx \le \frac{\pi}{4}$

46. 3

Sol. $\int_0^1 f'(x)dx = f(1)-f(0)=1 = \int_0^1 (1+x^2)^{-1/4}(1+x^2)^{1/4} f'(x)dx$ (C.S. inequality)

$\le \left(\int_0^1 (1+x^2)^{-1/2}dx\right)^{\frac{1}{2}}\left(\int_0^1 (1+x^2)^{1/2}\left(f'(x)^2 dx\right)\right)^{\frac{1}{2}}$

$\Rightarrow \int_0^1 (1+x^2)^{1/2}(f'(x))^2 dx \ge \frac{1}{\ln(1+\sqrt{2})}$

47. 0

Sol. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \le \tan^{-1}\left(\frac{x+\frac{1}{x}}{2\sqrt{3}}\right) \le \tan^{-1}(\sqrt{3})$

$\frac{6}{\pi}\cdot\frac{\pi}{6} \le \frac{6}{\pi}\tan^{-1}\left(\frac{x+\frac{1}{x}}{2\sqrt{3}}\right) \le \frac{6}{\pi}\cdot\frac{\pi}{3}$

$1 \le \frac{6}{\pi}\tan^{-1}\left(\frac{x+\frac{1}{x}}{2\sqrt{3}}\right) \le 2$

$\Rightarrow \int_1^{3+2\sqrt{2}} \frac{1}{x^2+2}dx \le \int_1^{3+2\sqrt{2}} f'(x)dx \le \int_1^{3+2\sqrt{2}} \frac{1}{x^2+1}dx$

$\left|\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}\right|_1^{3+2\sqrt{2}} \le f(3+2\sqrt{3}) \le \tan^{-1}(3+2\sqrt{2})-\frac{\pi}{4}$

48. 1

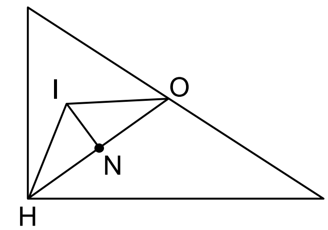
Sol. $\sin(15\times 4^\circ) = 2^{14}\prod_{r=0}^{14}\sin\left(4^\circ+\frac{r\pi}{15}\right)$

SECTION – C

49. 00000.25

Sol. N is mid-point of OH

$IH^2 + IO^2 = 2ON^2 + 2IN^2$



50. 00125.00

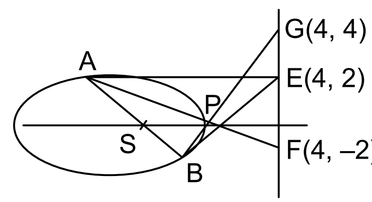
Sol. $a^3 \cos 3B + 3a^2b \cos(A - 2B) + 3ab^2 \cos(2A - B) + b^3 \cos(2A)$ = Real part of $(ae^{-iB} + be^{iA})^3$

$= ((a \cos B + b \cos A) + i(a \sin B - b \sin A))^3 = c^3 . \left\{\frac{a}{\sin A} = \frac{b}{\sin B}\right\}$

51. 00007.20

Sol. G(4, 3.2)

Tangents of A and B will intersect at corresponding directrix. So E and F lies on directrix x = 4. ES is perpendicular to AB where S is focal chord, also $\angle FSG$ is also $\frac{\pi}{2}$



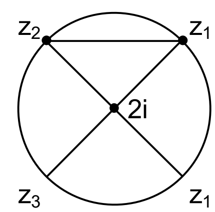
52. 00017.00

Sol. $(z - 2i)^4 = 1 + i$

$z - 2i = (\sqrt{2})^{1/4} e^{i\left(\frac{2k\pi + \pi/4}{4}\right)}$

Put k = 0, 1, 2, 3

Minimum value of $\sum_{i=1}^{4} |z - z_i| = 4 \cdot 2^{\frac{1}{8}} = 2^{2+\frac{1}{8}} = 2^{\frac{17}{8}}$



53. 00000.50

Sol. $P_A = \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^6 + 8\left(\frac{1}{2}\right)^8 + ... = \frac{1}{2}$

54. 00005.00

Sol. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{\frac{k}{n^2} + \frac{k^2}{n^3}}{\sqrt{1 + \frac{k}{n^2} + \frac{k^2}{n^3}} + 1} \right) = \frac{1}{2}\int_0^1 (x + x^2)\, dx = \frac{5}{12}$