FIITJEE

ALL INDIA TEST SERIES

FULL TEST – II

JEE (Main)-2021

TEST DATE: 27-12-2020

Time Allotted: 3 Hours **Maximum Marks: 300**

General Instructions:

- The test consists of total 75 questions.
- Each subject (PCM) has 25 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each part has only three sections: **Section-A**, **Section-B and Section-C**.

Section-A (01 – 20, 26 – 45, 51 – 70) *contains 60 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

Section-B (21 – 22, 46 – 47, 71 – 72) *contains 6 Numerical based questions with answer as numerical value from* ***0 to 9*** *and each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

Section-C (23 – 25, 48 – 50, 73 – 75) *contains 9 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

Physics

PART – I

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A horizontal wind is blowing with a velocity *v* towards north east. A man starts running towards north with acceleration *a*. The time, after which man will feel the wind blowing towards east, is

(A) $\frac{v}{a}$

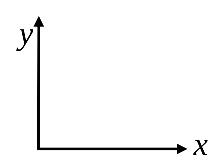
(B) $\frac{\sqrt{2}v}{a}$

(C) $\frac{v}{\sqrt{2}a}$

(D) $\frac{2v}{a}$

Ans. C

Sol. $\vec{V}_w = \frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}$



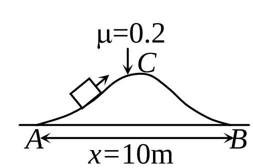
$\vec{V}_m = (at)\hat{j}$

$\vec{V}_{wm} = \frac{v}{\sqrt{2}}\hat{i} + \left(\frac{v}{\sqrt{2}} - at\right)\hat{j}$

It appears due east when, $\frac{v}{\sqrt{2}} - at = 0$

$\therefore t = \frac{v}{\sqrt{2}a}$

2. A block of mass 1 kg is pulled along the curve path *ACB* by a tangential force as shown in figure. The work done by the frictional force when the block moves from *A* to *B* is



(A) 5 J

(B) 10 J

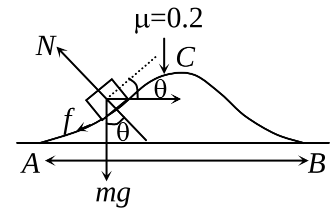
(C) 20 J

(D) none of these

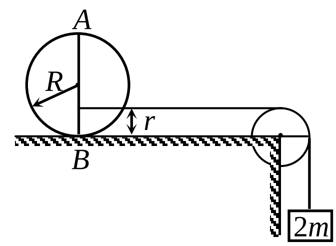
Ans. C

Sol. Work done by friction = $\int \vec{F}\cdot d\vec{s}$

$= \int_0^x \mu mg\cos\theta \frac{dx}{\cos\theta} = \mu mgx$ = 20 J



3. An uniform ring of radius *R*, is fitted with a massless rod *AB* along its diameter. An ideal horizontal string (whose one end is attached with the rod at a height *r*) passes over a smooth pulley and other end of the string is attached with a block of mass double the mass of ring as shown. The co-efficient of friction between the ring and the surface is μ. When the system is released from rest, the ring moves such that rod *AB* remains vertical. The value of *r* is



(A) $R\left(1-\frac{3\mu}{2(1+\mu)}\right)$

(B) $R\left(1-\frac{\mu}{2(1+\mu)}\right)$

(C) $R\left(2-\frac{3\mu}{2(1+\mu)}\right)$

(D) $R\left(1-\frac{3\mu}{(1+\mu)}\right)$

Ans. A

Sol. $T(R-r)=\mu mgR$, $2mg-T=2ma$, $T-\mu mg=ma$
On solving, we get
$\therefore r=R\left(1-\frac{3\mu}{2(1+\mu)}\right)$

4. A closed organ pipe of length 99.4 cm is vibrating in its first overtone and in always resonance with a tunning fork having frequency $f=(300-2t)$ Hz, where *t* is time in second. The rate by which radius of organ pipe changes when its radius is 1 cm, is (speed of sound in organ pipe = 320 m/s)

(A) $\frac{1}{72}$ m/s

(B) $\frac{1}{36}$ m/s

(C) $\frac{1}{18}$ m/s

(D) $\frac{1}{9}$ m/s

Ans. A

Sol. $f = \frac{3v}{4(L+0.6r)}$

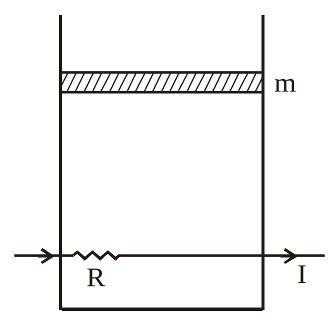
$$\frac{df}{dt} = \frac{3v}{4}\left(-\frac{1}{(L+0.6r)^2}\cdot(0.6)\frac{dr}{dt}\right)$$

$$-2 = -\frac{3v}{4}\left(0.6\frac{dr}{dt}\right)$$

$$\frac{8}{3v\times 0.6} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{72} \text{ m/s}$$

5. A coil of resistance R is connected to an external battery is placed inside an adiabatic cylinder fitted with a frictionless piston and containing an ideal gas. A current $I = a_0t$ flows through the coil (a_0 is a +ve constant). For time interval $t = 0$ to $t = t_0$, the piston goes upto a height of (Assume $\Delta U = 0$)



(A) $\frac{a_0^2R^2t_0^2}{2mg}$

(B) $\frac{a_0^2Rt_0^3}{2mg}$

(C) $\frac{a_0^2Rt_0^3}{3mg}$

(D) $\frac{a_0^2Rt_0^2}{3mg}$

Ans. C

Sol. $I^2 R\, dt = mg\, dx$, $\int_o^t (a_0 t)^2 R dt = \int_o^h mg\, dx$,

$a_o^2 R \int_o^{t_o} dt\, t^2 = mgh$, $\frac{a_o^2 R t_o^3}{3} = mgh$, $h = \frac{a_0^2 R t_0^3}{3mg}$

6. A rod of length *l* (laterally thermally insulated) of uniform cross-sectional area *A* consists of a material whose thermal conductivity varies with temperature as $K = \frac{K_o}{a + bT}$ where K_0, *a* & *b* are constants. T_1 and T_2 (< T_1) are the temperature of two ends of rod. Then rate of flow of heat across the rod is

(A) $\frac{AK_0}{b\,l}\left(\frac{a + bT_1}{a + bT_2}\right)$

(B) $\frac{AK_0}{b\,l}\left(\frac{a + bT_2}{a + bT_1}\right)$

(C) $\frac{AK_0}{b\,l}\ln\left[\frac{a + bT_1}{a + bT_2}\right]$

(D) $\frac{AK_0}{a\,l}\ln\left[\frac{a + bT_2}{a + bT_1}\right]$

Ans. C

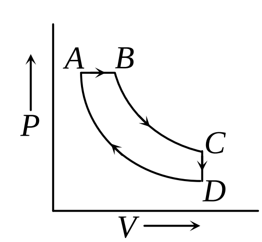
Sol. $\frac{dQ}{dt} = -KA\frac{dT}{dx}$

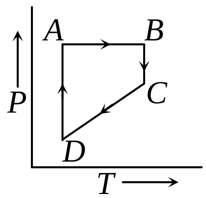
$\frac{dQ}{dt} = -\frac{K_0 A}{a + bT}\frac{dT}{dx}$

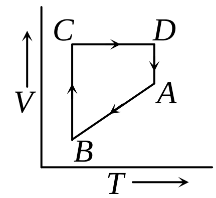
$\frac{dQ}{dt}\int_0^l dx = -K_0 A \int_{T_1}^{T_2} \frac{dT}{a + bT}$

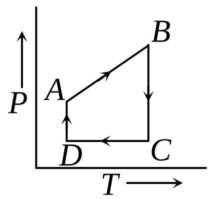
$\frac{dQ}{dt} = \frac{AK_0}{b\,l}\ln\left[\frac{a + bT_1}{a + bT_2}\right]$

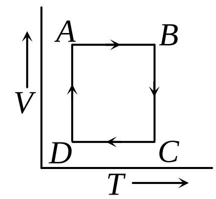
7. A cyclic process *ABCD* is shown in the *P–V* diagram.
Which of the following curves represent the same process?



(A) 

(B) 

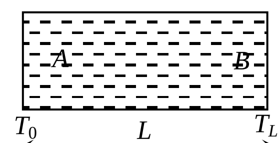
(C) 

(D) 

Ans. A

Sol. *AB* $\rightarrow$ constant *P. T* will be increasing with increasing *V*.
BC $\rightarrow$ constant *T. P* will be decreasing with increasing *V*.
CD $\rightarrow$ constant *V*, decreasing *P*; hence decreasing *T*.
DA $\rightarrow$ constant *T* , decreasing *V*, increasing *P*.
Also, *BC* is at a higher temperature than *AD*.

8. The temperature of a mono-atomic gas in an uniform container of length '*L*' varies linearly from T_0 to T_L as shown in the figure. If the molecular weight of the gas is *M*, then the time taken by a wave pulse in traveling from end *A* to end *B* is



(A) $\dfrac{2L}{\sqrt{T_L}+\sqrt{T_0}}\sqrt{\dfrac{3M}{5R}}$

(B) $\sqrt{\dfrac{3(T_L-T_0)}{5RML}}$

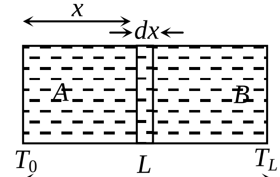
(C) $\dfrac{2L}{\sqrt{T_L}-\sqrt{T_0}}\sqrt{\dfrac{3M}{5R}}$

(D) $L\sqrt{\dfrac{M}{2R(T_L-T_0)}}$

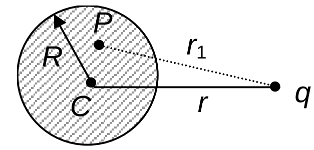
Ans. A

Sol. $v=\sqrt{\dfrac{\gamma RT}{M}}=\sqrt{\dfrac{5RT}{3M}}$

$dx = C\cdot dt=\sqrt{\dfrac{5R}{3M}\left[T_0+\left(\dfrac{T_L-T_0}{L}\right)x\right]}dt$, $\quad t=\dfrac{2L}{\sqrt{T_L}+\sqrt{T_0}}\sqrt{\dfrac{3M}{5R}}$



9. A point charge is placed at a distance r from center of a conducting neutral sphere of radius R ($r > R$). The potential at any point P inside the sphere at a distance r_1 from point charge due to induced charge of the sphere is given by [$k = \frac{1}{4\pi\varepsilon_0}$]



(A) kq/r_1

(B) kq/r

(C) $kq/r - kq/r_1$

(D) $-kq/R$

Ans. C

Sol. Potential of centre of sphere = $\frac{Kq}{r} + V_i = \frac{Kq}{r}$

where V_i = potential due to induced charge at centre = 0 [$\therefore$ $\Sigma q_i = 0$ and all induced charges are equidistance from centre]

$\therefore$ potential at point P = $\frac{Kq}{r} = \frac{Kq}{r_1} + V_i$ (For conductor all points are equipotential)

$\therefore$ $V_i = K\left(\frac{q}{r} - \frac{q}{r_1}\right)$

10. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of system, when it is displaced by a small distance x along the x-axis, is approximately proportional to

(A) x

(B) x^2

(C) x^3

(D) $1/x$

Ans. B

Sol. Potential energy of the system when charge Q is at O is $U_0 = \frac{qQ}{a} + \frac{qQ}{a} = \frac{2qQ}{a}$

When charge Q is shifted to position O', the potential energy will be

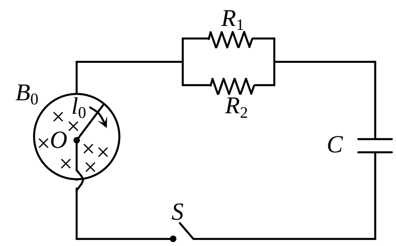
$$U = \frac{qQ}{(a+x)} + \frac{qQ}{(a-x)} = \frac{qQ(2a)}{(a^2 - x^2)} = \frac{2qQ}{a} \times \left(1 - \frac{x^2}{a^2}\right)^{-1}$$

$$= \frac{2qQ}{a} \times \left(1 + \frac{x^2}{a^2}\right) \qquad (\because x << a)$$

$\therefore$ $\Delta U = U - U_0 = \frac{2qQ}{a}\left(1+\frac{x^2}{a^2}\right) - \frac{2qQ}{a} = \frac{2qQ}{a^3}(x^2)$

Hence ΔU $\Delta U \propto x^2$.

11. There is a small metallic ring of radius l_0 and having negligible resistance placed perpendicular to a constant magnetic field B_0. One end of a rod is hinged at the centre of ring O and other end is placed on the ring. Now rod is rotated with constant angular velocity ω_0 by some external agent and circuit is connected as shown in the figure, initially switch is open and capacitor is uncharged. If switch S is closed at $t = 0$, then calculate heat loss from the resistor R_2 from $t = 0$ to the instant when voltage across the capacitor becomes V_0. (Assume plane of ring to be horizontal and friction to be an absent at all the contacts). (Assume, $R_2 = 2R_1$, $B_0 l_0^{\,2}\omega_0 = 4V_0$)



(A) $\frac{1}{2}CV_0^2$

(B) $\frac{1}{6}CV_0^2$

(C) $\frac{2}{3}CV_0^2$

(D) $\frac{1}{3}CV_0^2$

Ans. A

Sol. Voltage across rod = $\frac{1}{2}B_0 l_0^2 \omega_0$

Charge on capacitor = CV_0

$v \times q = \frac{1}{2}CV_0^2 + H_{R_1} + H_{R_2}$

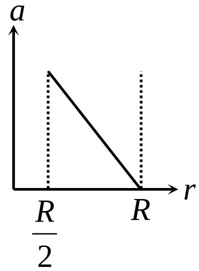
$CV_0 \times \frac{1}{2}B_0 l_0^2 \omega_0 = \frac{1}{2}CV_0^2 + \frac{R_2}{R_1}H_{R_2} + H_{R_2}$

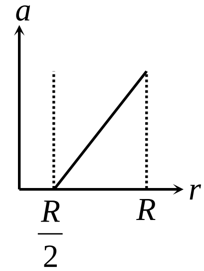
$\frac{1}{2}CV_0 B_0 l_0^2 \omega_0 - \frac{1}{2}CV_0^2 = \frac{H_{R_2}[R_2 + R_1]}{R_1}$

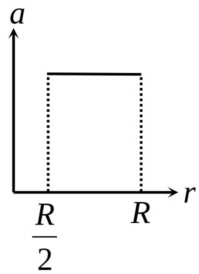
$H_{R_2} = \frac{R_1}{R_1 + R_2}\left[\frac{1}{2}CV_0 B_0 l_0^2 \omega_0 - \frac{1}{2}CV_0^2\right] = \frac{R_1}{R_1 + R_2} \times \frac{1}{2}CV_0[B_0 l_0^2 \omega_0 - V_0]$

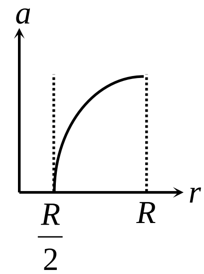
$= \frac{1}{2}CV_0^2$

12. A frictionless tunnel is dug along a chord of the earth at a perpendicular distance $\frac{R}{2}$ from the centre of earth (where R is radius of earth). An object is released from one end of the tunnel. The correct graph, showing the variation of acceleration of particle with its distance r from centre of earth is

(A) 

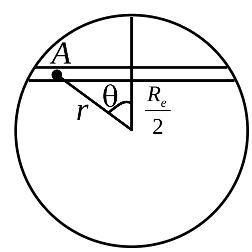
(B) 

(C) 

(D) 

Ans. D

Sol. Let mass of the earth is M_e and mass of object is m.

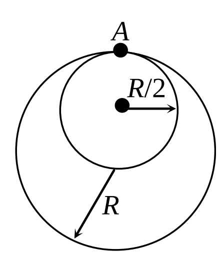


Force on object at $A = \left(\frac{GM_e m}{R^3} r\right)$ towards centre.

Force on object along the tunnel $= \left(\frac{GM_e m}{R^3} r\right) \frac{\sqrt{r^2 - \frac{R^2}{4}}}{r}$

Acceleration of object along the tunnel $= \left(\frac{GM_e}{R^3}\right) \sqrt{r^2 - \frac{R^2}{4}}$

13. Two rings made of same material and thickness, one of radius R and other of radius R/2 are welded together at point A. Now, it is hanged on a nail at wall, the nail touching both the rings at A. Now it is slightly displaced in the plane of rings and released. The period of small oscillations is



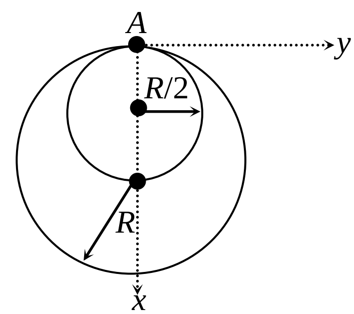
(A) $2\pi\sqrt{\frac{2R}{5g}}$

(B) $2\pi\sqrt{\dfrac{5R}{6g}}$

(C) $2\pi\sqrt{\dfrac{9R}{5g}}$

(D) $2\pi\sqrt{\dfrac{5R}{2g}}$

Ans. C



Sol. $x_{cm} = \dfrac{m\left(\dfrac{R}{2}\right) + 2mR}{m + 2m} = \dfrac{5}{6}R$

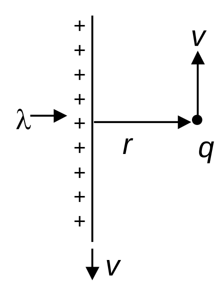
$I_A = I_1 + I_2$, $I_1 = m\left(\dfrac{R}{2}\right)^2 + m\left(\dfrac{R}{2}\right)^2 = \dfrac{mR^2}{2}$

$I_2 = 2mR^2 + 2mR^2 = 4mR^2 \Rightarrow I_A = \dfrac{9}{2}mR^2$

for compound pendulum $T = 2\pi\sqrt{\dfrac{I}{Mgd}}$

$\Rightarrow$ here M = 3m, $d = \dfrac{5}{6}R \Rightarrow T = 2\pi\sqrt{\dfrac{9R}{5g}}$

14. A long wire having linear charge density λ moving with constant velocity v along its length. A point charge moving with same speed in opposite direction and at that instant it is r distance from the wire. The net force acting on the charge is given by



(A) $\dfrac{\lambda q}{2\pi r}\left[\dfrac{1}{\varepsilon_0} + v^2\mu_0\right]$

(B) $\dfrac{\lambda q}{2\pi r}\left[\dfrac{1}{\varepsilon_0} - \mu_0 v^2\right]$

(C) $\dfrac{\lambda q}{2\pi r}\sqrt{\left(\dfrac{1}{\varepsilon_0}\right)^2 + v^4\mu_0^2}$

(D) zero

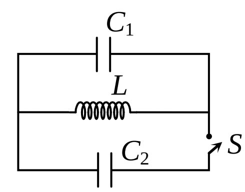
Ans. A

Sol. Electrostatics force on $q = \frac{\lambda q}{2\pi\varepsilon_0 r}$ away from line charge

Magnetic force = $\frac{\mu_0 \lambda v}{2\pi r} \times q \times v$ away from line charge

$\therefore$ total force = $\frac{\lambda q}{2\pi r}\left[\frac{1}{\varepsilon_0} + \mu_0 v^2\right]$

15. At a moment (t = 0), when the charge on capacitor C_1 is zero, the switch is closed. If I_0 be the current through inductor at t = 0 , for t > 0



(A) maximum current through inductor equals $I_0/2$.

(B) maximum current through inductor equals $\frac{C_1 I_0}{C_1 + C_2}$.

(C) maximum charge on $C_1 = \frac{C_1 I_0 \sqrt{LC_2}}{C_1 + C_2}$.

(D) maximum charge on $C_1 = C_1 I_0 \sqrt{\frac{L}{C_1 + C_2}}$.

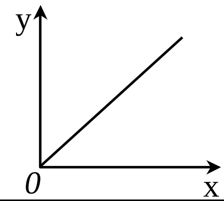
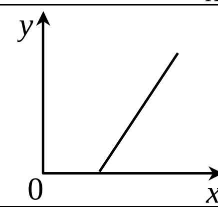
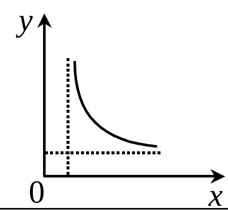
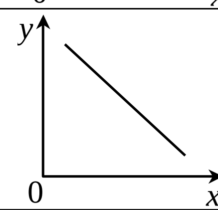
Ans. D

Sol. $\frac{1}{2} L I_0{}^2 = \frac{1}{2}(C_1 + C_2)V^2$

$$V = \left[\frac{L I_0^2}{(C_1 + C_2)}\right]^{1/2}$$

$$Q_1 = C_1 V = C_1 I_0 \sqrt{\frac{L}{C_1 + C_2}}$$

16. The graphs given apply to a convex lens of focal length *f*, producing a real image at a distance *v* from the optical centre when self, luminous object is at distance *u* from the optical centre. The magnitude of magnification is *m*. Identify the following graphs with the first named quantity being plotted along *y*-axis.

Column–I	Column–II
I. *v* against *u*	A. 
II. $\frac{1}{v}$ against $\frac{1}{u}$	B. 
III. *m* against *v*	C. 
IV. (*m* + 1) against $\frac{v}{f}$	D. 

(A) (I - C), (II - D), (III - A), (IV - B)

(B) (I - D), (II - C), (III - B), (IV - A)

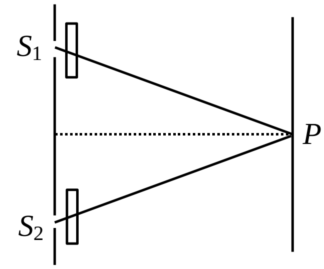
(C) (I - C), (II - D), (III - B), (IV - A)

(D) (I - A), (II - B), (III - C), (IV - D)

Ans. C

Sol. $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}, m=\frac{v}{u}$

17. One slit in a *YDSE* set up is covered by a glass plate (Refractive index = μ_1) and other by another glass plate (Refractive index = μ_2) of same thickness. If I_0 is the intensity of light through each slit and λ is the wavelength of light, then intensity at point *P* is (P is symmetrical w.r.t. slits S_1 and S_2)



(A) $4I_0 \cos^2 \frac{\pi}{\lambda}(\mu_1 - \mu_2)t$

(B) $4I_0 \cos^2 \frac{\pi}{\lambda}(\mu_1 + \mu_2)t$

(C) $4I_0 \sin \frac{\pi}{\lambda}(\mu_1 - \mu_2)t$

(D) $4I_0 \sin \frac{\pi}{\lambda}(\mu_1 + \mu_2)t$

Ans. A

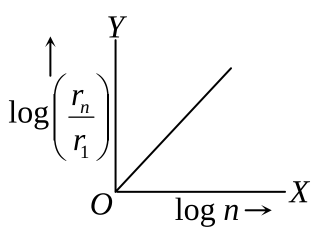
Sol. $\Delta x = (\mu_1 - \mu_2)\, t$

$$I_P = 2I_0\left[1 + \cos\left(\frac{2\pi}{\lambda}\right)(\mu_1 - \mu_2)\, t\right]$$

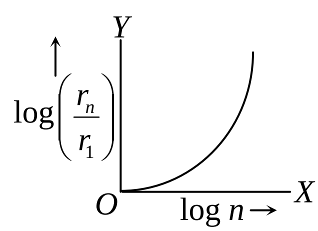
$$= 4I_0 \cos^2 \frac{\pi}{\lambda}(\mu_1 - \mu_2)\, t$$

18. In hydrogen atom, the radius of n^{th} Bohr's orbit is r_n. The graph between log $\left(\frac{r_n}{r_1}\right)$ and log *n* will be

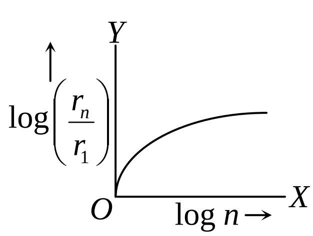
(A)



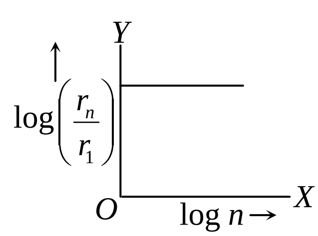
(B)



(C)



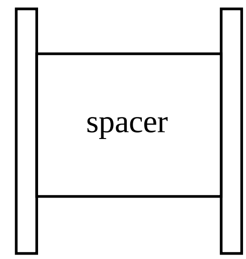
(D)



Ans. A

Sol. $r_n \propto n^2$; $\log \frac{r_n}{r_1} = 2\log n$

19. A capacitor is to be designed to operate, with constant capacitance, in an environment of fluctuating temperature. As shown in the figure, the capacitor is a parallel plate capacitor with 'spacer' to change the distance for compensation of temperature effect. If α_1 be the co-efficient of linear expansion of plates and α_2 that of spacer, the condition for no change in capacitance with change of temperature is (The capacitance of the capacitor is equal to C and spacer have insulated ends)



(A) $\alpha_1 = \alpha_2$

(B) $\alpha_1 = 2\alpha_2$

(C) $2\alpha_1 = \alpha_2$

(D) $2\alpha_1 = 3\alpha_2$

Ans. C

Sol. $C = \frac{\varepsilon_0 A}{L}$

$\therefore \quad \log C = \log \varepsilon_0 + \log A - \log L$

$\frac{dC}{C} = \frac{dA}{A} - \frac{dL}{L}$

$\frac{dC}{C} = 2\alpha_1 dT - \alpha_2 dT = 0$

$\therefore \quad 2\alpha_1 = \alpha_2$

20. A 15 gm ball is shot from a spring gun whose spring has a force constant of 600 N/m. The spring is compressed by 5 cm. The greatest possible horizontal range of the ball for this compression is (g = 10 m/s^2)

(A) 6.0 m

(B) 12.0 m

(C) 10.0 m

(D) 8.0 m

Ans. C

Sol. $R_{\max} = \frac{u^2}{g} \text{ at } \theta = 45^0 = \left(\frac{mu^2}{2}\right)\left(\frac{2}{mg}\right) = \left(\frac{1}{2}kx^2\right)\left(\frac{2}{mg}\right)$

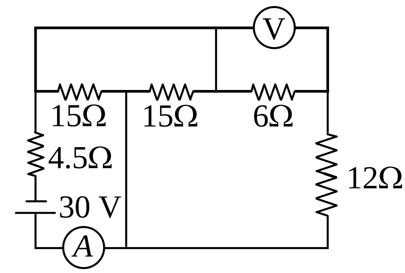
$\because \quad \left(\frac{1}{2}mu^2 = \frac{1}{2}kx^2\right)$

Substituting the values $R_{\max} = \frac{(600)(5\times10^{-2})^2}{(15\times10^{-3})(10)} = 10.0$ m

SECTION – B
(Single digit integer type)

This section contains **02** questions. The answer to each question is a **single Digit integer** ranging from **0 to 9, both inclusive**.

21. A galvanometer of coil resistance 1Ω is converted into voltmeter by using a resistance of 5Ω in series and same galvanometer is converted into ammeter by using a shunt of 1Ω. Now ammeter and voltmeter connected in circuit as shown, find the reading of voltmeter and ammeter.



Ans. 3

Sol. $R_{\text{Voltmeter}} = 6\Omega$, $R_{\text{ammeter}} = 0.5\Omega$
$R_{\text{eq}} = 10\Omega$
$$I = \frac{30}{10} = 3A$$
Reading of voltmeter = 1 × 3 = 3 volt.

22. Suppose potential energy between electron and proton at separation *r* is given by *U* = *k* ln *r*, where *k* is constant. For such hypothetical hydrogen atom, the ratio of energy difference between energy levels (*n* = 1 and *n* = 2) and (*n* = 2 and *n* = 4) is

Ans. 1

Sol. $F = -\frac{dU}{dr} = -\frac{k}{r}$, $\frac{k}{r} = \frac{mv^2}{r}$ ..... (i)
$$E_n = \frac{1}{2}mv^2 + k\ln r \quad \text{..... (ii)}$$
$$mvr = \frac{nh}{2\pi} \quad \text{..... (iii)}$$
Solving these $E_n = \frac{k}{2}\left(1 + \ln\left(\frac{n^2h^2}{4\pi^2 mk}\right)\right)$
required ratio = $\frac{E_2 - E_1}{E_4 - E_2} = 1$.

SECTION – C
(Numerical Answer Type)

This section contains **03** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

23. The frequency of a sonometer wire is 100 Hz. When the weights producing the tensions are completely immersed in water the frequency becomes 80 Hz and on immersing the weights in a certain liquid the frequency becomes 60 Hz. The specific gravity of the liquid is

Ans. 00001.77 (1.75 – 1.80)

Sol. $f \propto \sqrt{g}$

In water $f_w = 0.8 f_{air}$ $\therefore$ $\frac{g'}{g} = (0.8)^2 = 0.64$

or $\frac{\rho_w}{\rho_m} = 0.36$ …(i)

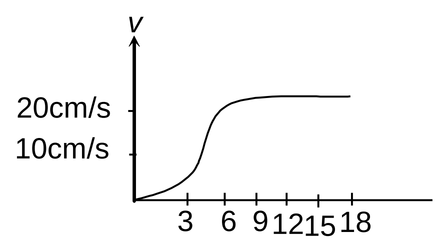
In liquid , $\frac{g'}{g} = (0.6)^2 = 0.36$

or $\frac{\rho_L}{\rho_m} = 0.64$

From equations (i) and (ii) $\frac{\rho_L}{\rho_w} = \frac{0.64}{0.36}$

$\therefore S_L = \rho_L/\rho_w = 1.77$

24. At $t = 0$, a spherical body of radius 2 cm begins to fall under gravity through a long column of a liquid of coefficient of viscosity $\frac{2}{\pi}$ poise. Its speed time variation is as shown in the figure. If density of the body is 3 times the density of liquid, buoyancy force acting on the body at t = 12 sec is

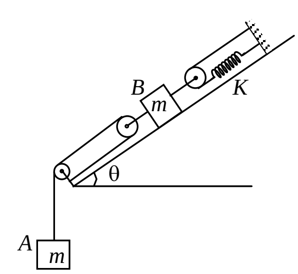


Ans. 00240.00

Sol. $F_v + F_B = mg$; $F_v + F_B = 3F_B$ $(F_B = \frac{mg}{3})$; $F_B = \frac{F_v}{2}$ = 240 dyne

$F_v = 6\pi\eta rv = 480$ dyne

25. Two blocks *A* and *B*, each of mass *m* are connected by means of a pulley-spring system on a smooth inclined plane of inclination θ as shown in the figure. All the pulleys and spring are ideal. Now, *B* is slightly displaced from its equilibrium position. It starts to oscillate. Time period of oscillation of *B* will be (Take *m* = 4 kg, *K* = 5 N/m, π = 3.14)



Ans. 00006.28

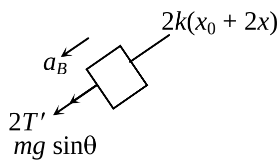
Sol. Let elongation of spring be x_0 in equilibrium. Then,

$$2T + mg\sin\theta = 2kx_0 \quad \ldots(i)$$

And $T = mg$ …(ii)

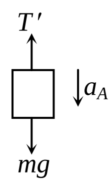
Let Block *B* is displaced by *x* down the inclination

F.B.D. of *B*



$$-ma_B = 2k(x_0 + 2x) - 2T' - mg\sin\theta \quad \ldots(iii)$$

F.B.D. of *A*



$mg - T' = ma_A$

Also, $a_A = 2a_B$

$$T' = mg - 2ma_B$$

$$-ma_B = 2kx_0 + 4kx - 2mg + 4ma_B - mg\sin\theta$$

$$-ma_B = 4kx + 4ma_B$$

$$a_B = -\frac{4k}{5m}x$$

$$\therefore T = 2\pi\sqrt{\frac{5m}{4k}}$$

T = 6.28 s.

Chemistry PART – II

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

26. If a balloon fill with CO gas is opened in a chamber filled with N_2 gas when the temperature and pressure of both the gases are the same, what will happen to the balloon?

(A) It will shrink

(B) It will swell

(C) It will burst

(D) It will remain unchanged

Ans. D

Sol. CO and N_2 both have equal molecular masses and so equal densities.

27. The third nearest neighbours of Na^+ in NaCl are:

(A) $12Na^+$

(B) $8Cl^-$

(C) $8Na^+$

(D) $6Cl^-$

Ans. B

Sol. At 8 corners of cube.

28. When the letters shown have their usual meanings, the velocity of photoelectrons is:

(A) $\left[\frac{2hc}{m}\left\{\frac{\lambda_0-\lambda}{\lambda\lambda_0}\right\}\right]^{1/2}$

(B) $\left[\frac{2hc}{m}\left\{\frac{\lambda_0-\lambda}{\lambda\lambda_0}\right\}\right]^{2}$

(C) $\frac{2hc}{m}$

(D) $\left(\frac{m}{2hc}\right)^2$

Ans. A

Sol. $KE = h(v - v_0)$

$$\frac{1}{2}mv^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$v^2 = \frac{2hc}{m}\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)$$

29. Which of the following properties is not shown by NO?

(A) It is a neutral oxide

(B) It combines with oxygen to form nitrogen dioxide

(C) Its bond order is 2.5

(D) It is diamagnetic in gaseous state

Ans. D

Sol. NO has total 15 electrons and one electron is unpaired.

30. Heat of the following reaction in bomb calorimeter is -1415kJ.
$C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(l)$
What is the heat released if 1.4g C_2H_4 is combusted in open atmosphere at 27°C?
$(R = 8.3\,JK^{-1}mol^{-1})$

(A) -71.0 kJ

(B) -1415 kJ

(C) -710 kJ

(D) -1420 kJ

Ans. A

Sol. $\Delta n_g = 2 - 4 = -2$

$\Delta n_g RT = -2 \times 8.314 \times 10^{-3} \times 300 = -5.0$ kJ

$\Delta H = -1415 - 5 = 1420\,kJ$

Heat released for 1.4 g $C_2H_4 = \dfrac{-1.4 \times 1420}{28}$

$= -71\,kJ$

31. The dissociation equilibrium of a gas AB_2 can be represented as:
$2AB_2(g) \rightleftharpoons 2AB(g) + B_2(g)$
The degree of dissociation is 'x' and is small compared to 1. The expression relating the degree of dissociation (x) with equilibrium constant K_p and total pressure P is:

(A) $(2K_p / P)^{1/3}$

(B) $(2K_p / P)^{1/2}$

(C) (K_p / P)

(D) $(2K_p / P)$

Ans. A

Sol.

	$2AB_2(g)$	$\rightleftharpoons$	$2AB(g) + B_2(g)$	
at t = 0	2moles(let)	-	-	(x is degree of dissociation)
at eqm.	(2-2x)moles		2xmoles x mole	

Total = 2- 2x = 2x + x = (2 + x) moles;
Total pressure = P

$$K_p = \frac{P_{AB}^{o}.P_{B_2}}{P_{AB_2}^{2}}$$

$$\Rightarrow x = \left[\frac{2K_p}{P}\right]^{1/3}$$

32. $AlCl_3$ and $FeCl_3$ can be separated from their mixture by using:

(A) NH_4OH

(B) NaOH

(C) H_2O

(D) magnetic method

Ans. B

Sol. $AlCl_3$ dissolves in aqueous NaOH while $FeCl_3$ gives brown ppt. of $Fe(OH)_3$ which on separation can again be converted to respective chlorides.

$$AlCl_3 + 3NaOH \xrightarrow[-3NaCl]{} Al(OH)_3 \xrightarrow{NaOH} \underset{\text{solution}}{Na[Al(OH)_4]}$$

$$FeCl_3 + 3NaOH \xrightarrow[-3NaCl]{} \underset{\text{(brown ppt.)}}{Fe(OH)_3}$$

33. In PO_4^{3-}, the bond order of P-O bond and formal charge on O-atom are, respectively:

(A) 0.25, -0.25

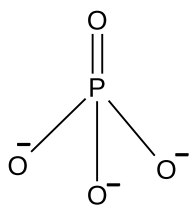
(B) 0.50, -0.50

(C) 1.25, -0.75

(D) 0.75, -1.25

Ans. C

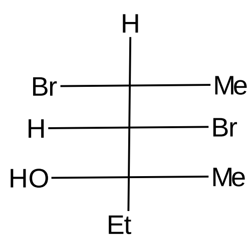
Sol.



Av. charge= $\frac{-3}{4} = -0.75$

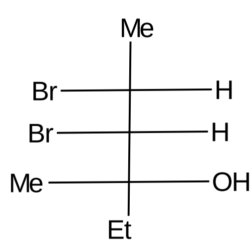
Av.B.O = $\frac{2+1+1+1}{4} = 1.25$

34. Enantiomer of

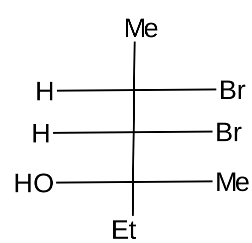


is

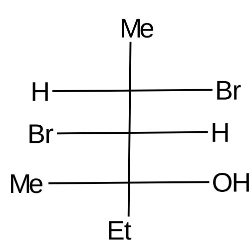
(A)



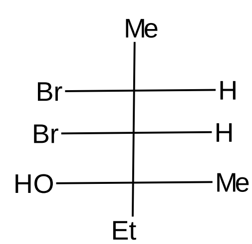
(B)



(C)



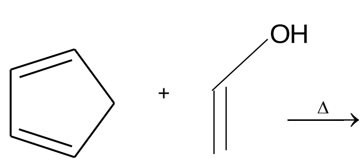
(D)



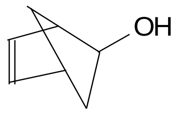
Ans. A

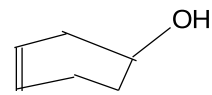
Sol. Factual

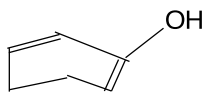
35.

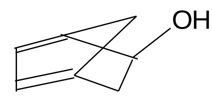


; Major product is ....

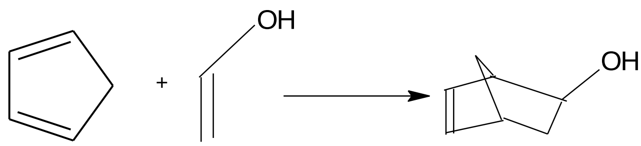
(A) 

(B) 

(C) 

(D) 

Ans. A

Sol. 

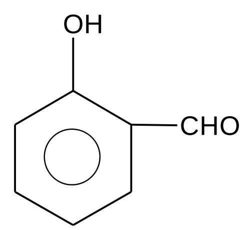
(for 1,4-addition)

36. Which of the following cannot be prepared from $CHCl_3$?

(A) $CH \equiv CH$

(B) C_6H_5CN

(C) $O_2N - CCl_3$

(D) 

Ans. B

Sol. Factual

37. Gold numbers of starch, gelatin and albumin are respectively 25, 0.01 and 0.15. Which of the following is the correct decreasing order of protective power?

(A) Starch > gelatin > albumin

(B) Starch > albumin > gelatin

(C) Gelatin > albumin > starch

(D) Data is insufficient to decide.

Ans. C

Sol. Lower the gold number, higher is the protective power. Hence, the choice (c).

38. In silicon dioxide:

(A) each silicon atom is surrounded by four oxygen atoms and each oxygen atom is bonded to two silicon atoms

(B) each silicon atom is surrounded by two oxygen atoms and each oxygen atom is bonded to two silicon atoms

(C) silicon atom is bonded to two oxygen atoms

(D) there are double bonds between silicon and oxygen atoms

Ans. A

Sol. Factual

39. The rate determining step in the reaction of CH_4 with Cl_2 is:

(A) $CH_4 + \dot{Cl} \rightarrow CH_3Cl + \dot{H}$

(B) $CH_4 + \dot{Cl} \rightarrow \dot{C}H_3 + HCl$

(C) $\dot{C}H_3 + \dot{Cl} \rightarrow CH_3Cl$

(D) $Cl_2 \rightarrow 2\dot{Cl}$

Ans. B

Sol. $\text{Rate} \propto [CH_4][Cl]; 2^{nd}$ order reaction.

40. The total number of alkenes including geometrical isomers possible by dehydrobromination of 3-bromo-3-cyclopentylhexane using alcoholic KOH is:

(A) 2

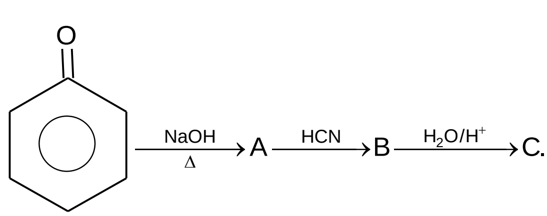
(B) 3

(C) 4

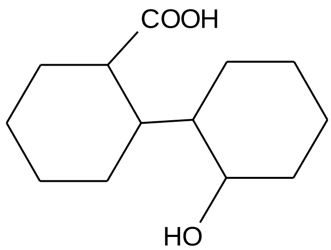
(D) 5

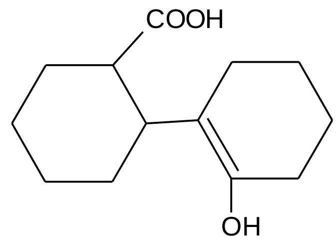
Ans. D

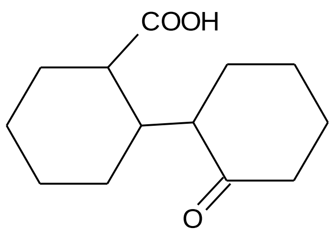
Sol. Factual

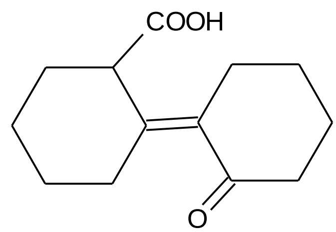
41. 

What is C?

(A) 

(B) 

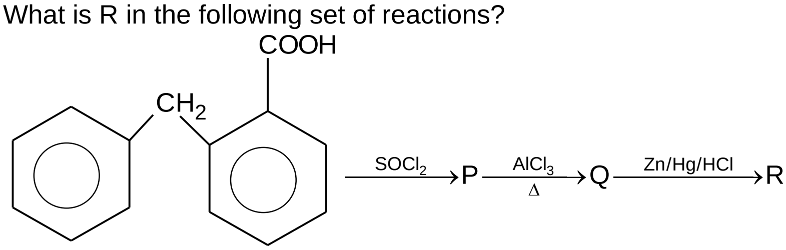
(C) 

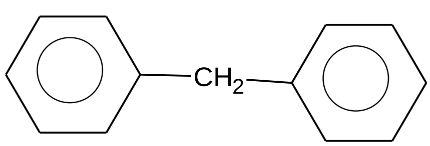
(D) 

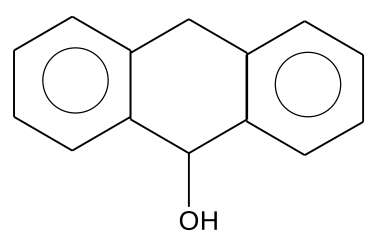
Ans. C

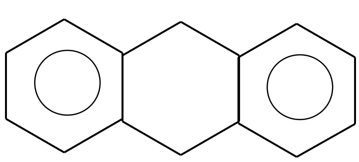
Sol. Factual

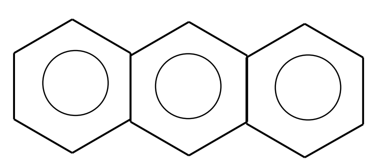
42. What is R in the following set of reactions?



(A) 

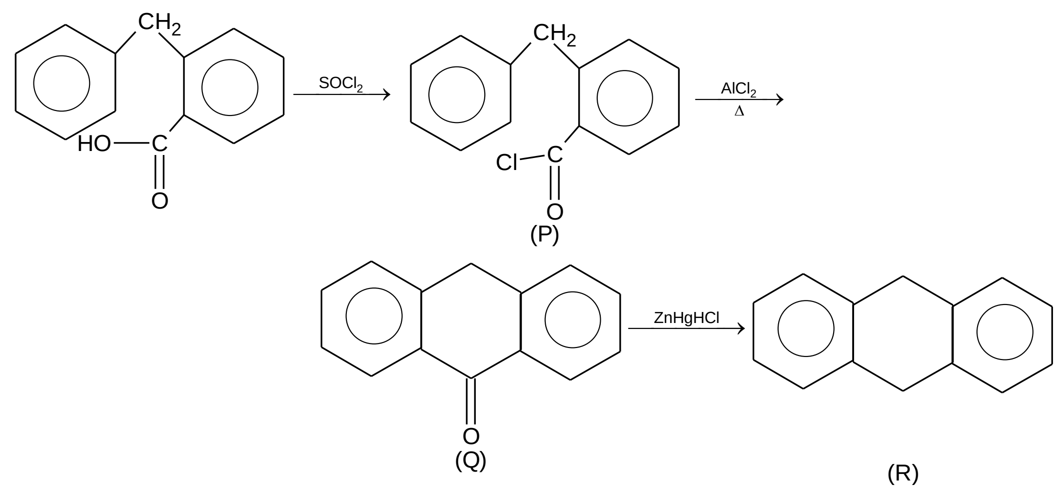
(B) 

(C) 

(D) 

Ans. C

Sol.



43. Aromatic nitriles (ArCN) are not prepared by reacton:

(A) $ArX + KCN$

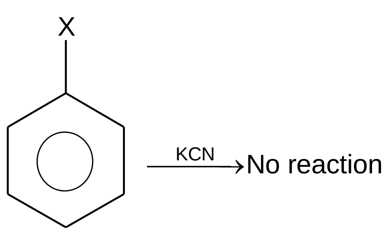
(B) $ArN_2^+ + CuCN$

(C) $ArCONH_2 + P_2O_5$

(D) $ArCONH_2 + SOCl_2$

Ans. A

Sol.



44. Which of the following is not a polyamide?

(A) Natural silk

(B) Nylon-6

(C) Bakelite

(D) Nylon -6, 6

Ans. C

Sol. Bakelite is not a polyamide., it is phenol formaldehyde resin.

45. Peptide bond between two α-amino acids is actually:

(A) ether bond

(B) ester bond

(C) amide bond

(D) glycoside bond

Ans. C

Sol. $-\overset{\overset{\displaystyle O}{\|}}{C}-NH-$ is amide bond which, in peptides, is called peptide bond.

SECTION – B
(Single digit integer type)

This section contains **02** questions. The answer to each question is a **single Digit integer** ranging from **0 to 9, both inclusive**.

46. 5 mole of A and 12 mol of B are taken for a reaction shown by the equation, $A + 3B \rightleftharpoons 2C$.
How many moles of C will be produced?

Ans. 8

Sol. $A + 3B \rightleftharpoons 2C$
1 mol of A reacts with 3 mol of B.
5 mol of A will react with 15 mol of B.
Given mol of B are 12 which is limiting.
Now, 3 mol of B will give 2 mol of C.
So, 12 mol of B will give $\frac{2 \times 12}{3}$, i.e., 8 mol of C.

47. Ammonium carbamate is in equilibrium with NH_3 and CO_2 as:
$NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$
If total pressure at equilibrium is 3 atm, K_p is:

Ans. 4

Sol. $NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$

At t = 0	solid	–	–
At eqm.	–	2 moles	1 mole ; n_{total}=3 moles

$K_{kp} = (P_{NH_3})^2 (P_{CO_2})$
For $P = 3$ atm
$K_p = \left(\frac{2}{3} \times 3\right)^2 \times \left(\frac{1}{3} \times 3\right) = 4$

SECTION – C
(Numerical Answer Type)

This section contains **03** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

48. Two what temperature should an open vessel be heated to expel two-fifth of air, if initial temperature is 27^oC?

Ans. 00500.00

Sol. Let the initial number of moles in vessel be 5 at 27^oC, i.e., 300 K. Moles of air left at new temperature

$$= = 5 = \frac{2}{5} \times 5 = 3$$

Pressure and volume remaining the same,

$$n_1RT_1 = n_2RT_2$$

49. On the basis of the information available from the reaction;

$$\frac{4}{3}Al + O_2 \rightarrow \frac{2}{3}Al_2O_3; \Delta G = -826.040 \text{ kJ mol}^{-1} \text{ of } O_2$$

The minimum EMF required to carry out electrolysis of Al_2O_3 is:
(1 F = 96500) C mol^{-1} of electrons)

Ans. 00002.14

Sol. 1 mol Al, n = 3 (electron exchanged)

$$\frac{4}{3} \text{mol Al}, n = \frac{4}{3} \times 3 = 4$$

$$\Delta G^o = -nFE^0 \Rightarrow E^0 = 2.14V$$

50. 0.63 g of a dibasic acid is available in 100 mL solution. 20 mL of this solution required 20 mL of $\frac{M}{10}$ NaOH solution for complete neutralization. The molar mass of the acid is:

Ans. 00126.00

Sol. 20 mL solution of acid contains = $\frac{0.63}{100} \times 20 = 0.126g$

$$E = \frac{W \times 1000}{N'V'(\text{alkali})}$$

Molar mass = 126 g

Mathematics

PART – III

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

51. If $f(x)=\dfrac{\sin^{-1}\left(\dfrac{x}{^{10}C_x}\right)+\cos^{-1}\left(\dfrac{x}{^{10}C_x}\right)}{\sqrt{5[x]^2+6\{x\}^2-60[x]+77\{x\}+160}}$, (where [.] denotes the GIF and {.} denotes the fractional part) then domain of f(x) is :

(A) $\{x \mid x \in \mathbf{I}, 0 \le x \le 10\}$

(B) {0, 1, 2, 3, 4, 9}

(C) {0, 1, 2, 3, 4, 9, 10}

(D) {0, 1, 2, 3, 9}

Ans. D

Sol. For $^{10}C_x$ to be defined $x \in \mathbf{I}, 0 \le x \le 10$.

$\dfrac{x}{^{10}C_x} \le 1 \Rightarrow x \ne 10$ $\quad [x]=x, \{x\}=0$ as $x \in \mathbf{I}$

$\Rightarrow 5x^2-60x+160>0$

$\Rightarrow x^2-12x+32>0$

$\Rightarrow x \notin [4,8]$

52. Maximum distance of any point on the curve $5x^2 - 2xy + 5y^2 = 1$ form origin is

(A) 5

(B) 2

(C) 1/2

(D) 1/5

Ans. C

Sol. Put $x = r\cos\theta,\ y = r\sin\theta$

$\Rightarrow 5r^2 - r^2\sin 2\theta = 1 \Rightarrow r^2 = \dfrac{1}{5-\sin 2\theta}$

r is maximum, when $5-\sin 2\theta$ is minimum

$5-\sin 2\theta\,|_{min} = 4 \Rightarrow r^2\,|_{max} = \dfrac{1}{4} \Rightarrow r\,|_{max} = \dfrac{1}{2}$

53. If f(x) is monotonic and differentiable function, a, b $\in$ R and $I_1 = \int_a^b \left((f(x))^2 - (f(a))^2\right)dx$ and $I_2 = \int_{f^{-1}(a)}^{f^{-1}(b)} x\left(b - f^{-1}(x)\right)dx$ then $\frac{I_1}{I_2}$ is equal to

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) None of these

Ans. B

Sol. Let $f^{-1}(x) = y \quad \Rightarrow x = f(y) \qquad \Rightarrow dx = f^{-1}(y)dx$

$$I_2 = \int_a^b f(y)(b-y)f'(y)dy = \int_a^b (b-y)f(y)f'(y)dx$$

$$= \frac{1}{2}I_1 \qquad \Rightarrow \frac{I_1}{I_2} = 2$$

$\therefore$ option 'B' is correct

54. $\int_0^{5n}\left[3 + \frac{[x]}{5}\right]dx$ is equal to $(n \in N)$ (where [.] denotes greatest integer function)

(A) 3n

(B) $\frac{5n^2 + 25n}{2}$

(C) 15n

(D) $\frac{20n^2 - 5n}{2}$

Ans. B

Sol. $\int_0^{5n}\left[3 + \frac{[x]}{5}\right]dx = \int_0^{5n}\left(3 + \left[\frac{[x]}{5}\right]\right)dx = 15n + \int_0^{5n}\left[\frac{x}{5}\right]dx$

$\left(\because \left[\frac{[x]}{5}\right] = \left[\frac{x}{5}\right]\right)$

55. Area bounded by the curve $y=\sqrt{\sin[x]+[\sin x]}$, (where [.] greatest integer function) and lines $x=1\,\&\,x=\frac{\pi}{2}$ and x-axis is equal to –

(A) $\frac{\pi}{2}-1$

(B) $\sqrt{\frac{\pi}{2}}\left(\frac{\pi}{2}-1\right)$

(C) $\sqrt{\sin 1}\left(\frac{\pi}{2}-1\right)$

(D) $\sqrt{\cos 1}\left(\frac{\pi}{2}-1\right)$

Ans. C

Sol. For $x\in\left[1,\frac{\pi}{2}\right], f(x)=\sqrt{\sin 1}$

$\therefore$ Area $=\int_0^{\pi/2}\sqrt{\sin 1}\,dx=\sqrt{\sin 1}\left(\frac{\pi}{2}-1\right)$

56. If $A=\{1,3,5,7,11,13,15,17,19,21\}$ and $B=\{2,4,6,....,22\}$
U is universal set, then $A'\cup((A\cup B)\cap B')$ is-

(A) U

(B) A'

(C) $A\cap B$

(D) A

Ans. A

Sol. $(A\cup B)\cap B'=A$
$\Rightarrow A'\cup((A\cup B)\cap B')=A'\cup A=U$

57. The set of all values of λ for which the lines $\lambda x+2y+2=0$, $2x+\lambda y+3=0$ and $3x+3y+\lambda=0$ are concurrent is

(A) $\{2, 3, -5\}$

(B) $\{3, -5\}$

(C) $\{2, 3, 5\}$

(D) $\{-3, 5\}$

Ans. B

Sol. Three non parallel lines are
Concurrent

$$\begin{vmatrix} \lambda & 2 & 2 \\ 2 & \lambda & 3 \\ 3 & 3 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda = 2, 3, -5$

But λ = 2, Ist two lines are parallel.

58. If α and β are the eccentric angles of the extremities of a focal chord of a standard ellipse, then the eccentricity of the ellipse is

(A) $\dfrac{\cos\alpha + \cos\beta}{\cos(\alpha+\beta)}$

(B) $\dfrac{\sin\alpha + \sin\beta}{\sin(\alpha+\beta)}$

(C) $\dfrac{\cos\alpha - \cos\beta}{\cos(\alpha-\beta)}$

(D) $\dfrac{\sin\alpha + \sin\beta}{\sin(\alpha+\beta)}$

Ans. D

Sol. $$\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2} \Rightarrow e = \frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}.\frac{2\sin\frac{\alpha+\beta}{2}}{2\sin\frac{\alpha+\beta}{2}} = \frac{\sin\alpha + \sin\beta}{\sin(\alpha+\beta)}$$

59. A tangent to the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ meets the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ at A and B. The angle between tangents at A and B of the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is

(A) $\dfrac{\pi}{2}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{\pi}{4}$

(D) $\dfrac{\pi}{6}$

Ans. A

Sol. Equation of tangent to $\frac{x^2}{4}+y^2=1$ at $(2\cos\theta, \sin\theta)$ is $x\cos\theta+2y\sin\theta=2$ ....(1)

Equation of chord of contact of $P(h, k)$ to $\frac{x^2}{6}+\frac{y^2}{3}=1$ is hx + 2ky = 6 ....(2)

Equation (1) & (2) are identical

$\frac{h}{\cos\theta}=\frac{k}{\sin\theta}=3$

$\Rightarrow h=3\cos\theta$ and $k=3\sin\theta$

$\Rightarrow h^2+k^2=9$

$\Rightarrow h^2+k^2=6+3$

$\therefore$ it lies on director circle = angle = $\frac{\pi}{2}$.

60. If a horizontal tangent of xy (x – 6y) =$9a^3$(a $\neq$ 0) is an asymptote of the hyperbola xy+ x + 2y – 8 = 0 then the value of a equals

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) 3

Ans. A

Sol. $\frac{dy}{dx}=0\Rightarrow P(-3a, -a)\Rightarrow$ Tangent is $y+a=0$

Which is asymptote $\Rightarrow$ a = 1

61. The area bounded by the curves $|y+x|\le 1, |y-x|\le 1$ and $2x^2+2y^2\ge 1$ is

(A) $\left(2+\frac{\pi}{2}\right)$

(B) $\left(2-\frac{\pi}{2}\right)$

(C) $\left(4-\frac{\pi}{2}\right)$

(D) $\left(4+\frac{\pi}{2}\right)$

Ans. B

Sol. Area = area inside square formed $x+y=\pm 1, x-y=\pm 1$ and outside circle $x^2+y^2=\dfrac{1}{2}$

62. If the line $y=\sqrt{3}x$ cuts the curve $x^4 + ax^2y + bxy + cx + dy + 6 = 0$ at A, B, C and D, then OA.OB.OC.OD (where O is the origin) is

(A) a – 2b +c

(B) $2c^2d$

(C) 96

(D) 6

Ans. C

Sol. The line $y=\sqrt{3}x$ can be written as $x=\dfrac{r}{2}$, $y=\dfrac{r\sqrt{3}}{2}$. If this line cuts the given curve, then

$$\frac{r^4}{16}+\frac{ar^3\sqrt{3}}{8}+\frac{br^2\sqrt{3}}{4}+\frac{cr}{2}+\frac{dr\sqrt{3}}{2}+6=0.$$

Therefore OA. OB.OC.OD = $|r_1|\,|r_2|\,|r_3|\,|r_4| = |r_1 r_2 r_3 r_4| = 96$

63. The equation formed by decreasing each root of $ax^2+bx+c=0$ by 1 is $2x^2 + 8x + 2 = 0$ then

(A) a = – b

(B) b = – c

(C) c = – a

(D) b = a + c

Ans. B

Sol. Let roots of $2x^2 + 8x + 2 = 0$ are α, β then roots of $ax^2 + bx + c = 0$ are $\alpha + 1$, $\beta + 1$ sum of roots = $\alpha + 1 + \beta + 1 = -b/a$

$\Rightarrow \quad -4+2=-\dfrac{b}{a} \quad \Rightarrow \dfrac{b}{a}=2$ ....(1)

And product of roots $=(\alpha+1)(\beta+1)=\dfrac{c}{a}$

$\Rightarrow \dfrac{c}{a}=-2$ ...(2)

$\{\because\ \alpha+\beta=-4 \text{ and } \alpha\beta=1\}$

Adding (1) and (2) gives b + c = 0

64. Let P be any point on $x^2 + y^2 = 4$ and PA and PB are two tangents drawn to parabola $y^2=4(x-4)$. If Q is image of point P w.r.t. line AB, then the maximum area of quadrilateral PAQB is :

(A) $48\sqrt{6}$

(B) $16\sqrt{6}$

(C) $40\sqrt{6}$

(D) $54\sqrt{3}$

Ans. A

Sol. Let point P is $(2\cos\theta, 2\sin\theta)$

$\Rightarrow$ area = 2 $\times$ area of PAB

$= 2\left[\frac{(S_1)^{3/2}}{2a}\right] = \frac{\left[4\sin^2 0 - 4(2\cos\theta - 4)\right]^{3/2}}{1}$

$= 8\left[1 - \cos^2\theta - 2\cos\theta + 4\right]^{3/2} = 8\left[6 - (\cos\theta + 1)^2\right]^{3/2}$

$\Rightarrow$ Area $\leq 8(6)^{3/2}$

Area $\leq 48\sqrt{6}$

65. If the standard deviation of $x_1, x_2, ...x_n$ is 3.5, then the standard deviation of $-2x_1 - 3, -2x_2 - 3, ..., -2x_n - 3$ is

(A) -7

(B) -4

(C) 7

(D) 1.75

Ans. C

Sol. We know that if $d_i = \frac{x_i - A}{h}$ then $\sigma_x = |h|\sigma_d$.

In this case $-2x_i - 3 = \frac{x_i - 3/2}{-1/2}$

So $h = -\frac{1}{2}$.

Thus $\sigma_d = \frac{1}{|h|}\sigma_x = 2 \times 3.5 = 7$

66. Using all of 0, 0, 0, 1, 1, 1, -1, -1, -1 if set of all 3×3 matrices formed is S, and if number of symmetric matrices in S is K and if number of matrices having trace zero in S is P then

(A) $9P = 140K$

(B) $P = 8K$

(C) P = 12K

(D) $P = 140K$

Ans. D

Sol. $K = 3! \times 3! \times 1 = 36$
$P = 6! \times 3! + 6! = 5040$

67. Let
$\vec{a} = (2+\sin\theta)\hat{i} + \cos\theta\hat{j} + \sin 2\theta\hat{k},\ \vec{b} = \sin\left(\theta + \frac{2\pi}{3}\right)\hat{i} + \cos\left(\theta + \frac{2\pi}{3}\right)\hat{j} + \sin\left(2\theta + \frac{4\pi}{3}\right)\hat{k},$
$\vec{c} = \sin\left(\theta - \frac{2\pi}{3}\right)\hat{i} + \cos\left(\theta - \frac{2\pi}{3}\right)\hat{j} + \sin\left(2\theta - \frac{4\pi}{3}\right)\hat{k}$, be three vectors where $\theta \in \left(0, \frac{\pi}{2}\right)$.
Then the maximum volume of tetrahedron whose coterminous edges are represented be the vectors $2\vec{b} \times \vec{c}, 3\vec{c} \times \vec{a}\ \vec{a} \times 4\vec{b}$ is

(A) $4\sqrt{3}$

(B) 16

(C) 12

(D) 8

Ans. C

Sol. Conceptual

68. There are two possible values of A in the solution of the matrix equation
$\begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}^{-1} \begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 14 & D \\ E & F \end{bmatrix}$ where A, B, C, D, E, F are real numbers.
The absolute value of the difference of these two solutions of A is

(A) $\frac{8}{3}$

(B) $\frac{11}{3}$

(C) $\frac{1}{3}$

(D) $\frac{19}{3}$

Ans. D

Sol. If $P = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \text{adj. } P = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
Also $A^{-1} = \frac{\text{adj.} A}{\det . A}$

$$\therefore \begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}^{-1} = \frac{\begin{bmatrix} A & 5 \\ 4 & 2A+1 \end{bmatrix}}{2A^2+A-20}$$

$$\therefore \frac{1}{2A^2+A-20}\begin{bmatrix} A & 5 \\ 4 & 2A+1 \end{bmatrix}\begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 14 & D \\ E & F \end{bmatrix}$$

The gives, $\frac{A^2-5A-10}{2A^2+A-20} = 14 \qquad \Rightarrow A = 5 \text{ or } -\frac{10}{3}$

$$\therefore 3 + \frac{10}{3} = \frac{19}{3}$$

69. If two parallel chords of a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the centre respectively, then the distance between these chords, is:

(A) $\frac{4}{\sqrt{7}}$

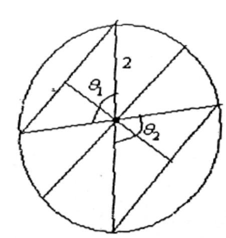
(B) $\frac{8}{\sqrt{7}}$

(C) $\frac{8}{7}$

(D) $\frac{16}{7}$

Ans. B

Sol. $\theta_1 = \cos^{-1}\left(\frac{1}{7}\right)$



$\theta_2 = \cos^{-1} 7$

$d_1 + d_2 = 2\cos\frac{\theta_1}{2} + 2\cos\frac{\theta_2}{2}$

$= 4\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{7}\right)$

Let $\cos^{-1}\frac{1}{7} = \theta \Leftrightarrow \frac{1}{7} = \cos\theta$

Then $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{7}\right) = \cos\frac{\theta}{2}$

$\cos\theta/2 = \frac{2}{\sqrt{7}}$

$\therefore d_1 + d_2 = \frac{8}{\sqrt{7}}$

70. A pair of unbiased dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is

(A) 2/5

(B) 3/5

(C) 4/5

(D) none of these

Ans. A

Sol. Let A denote the event that a sum of 5 occurs, B the event that a sum of 7 occurs and C the event that neither a sum of 5 nor a sum of 7 occurs. We have

$P(A) = \frac{4}{36} = \frac{1}{9}$, $P(B) = \frac{6}{36} = \frac{1}{6}$ and $P(C) = \frac{26}{36} = \frac{13}{18}$.

Thus,

P (A occurs before B) $= P[A \text{ or } (C \cap A) \text{ or } (C \cap C \cap A) \text{ or } ...]$

$= P(A) + P(C \cap A) + P(C \cap C \cap A) + ...$

$= P(A) + P(C)P(A) + P(C)^2 P(A) + ...$

$= \frac{1}{9} + \left(\frac{13}{18}\right)\frac{1}{9} + \left(\frac{13}{18}\right)^2 \frac{1}{9} + ...$

$= \frac{1/9}{1 - 13/18} = \frac{2}{5}$ [sum of an infinite G.P.]

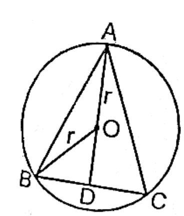
SECTION – B
(Single digit integer type)

This section contains **02** questions. The answer to each question is a **single Digit integer** ranging from **0 to 9, both inclusive**.

71. ABC is an isosceles triangle inscribed in a circle of radius r. If AB = AC and h is the altitude from A to BC. If the triangle ABC has perimeter P and area Δ then $\lim_{h \to 0} 512r\frac{\Delta}{P^3}$ is equal to

Ans. 4

Sol. We have BC = 2BD, AD = h and OD = h – r.



$\therefore BC = 2\sqrt{r^2 - (h - r)^2} = 2\sqrt{2hr - h^2}$

$\Rightarrow AB = \sqrt{2hr - h^2 + h^2} = \sqrt{2hr}$

So that $P = 2AB + BC = 2\left[\sqrt{2hr - h^2} + \sqrt{2hr}\right]$

Also, the area of Δ ABC is

$\Delta = BD \times AD = h\sqrt{2hr - h^2}$

$\therefore \frac{\Delta}{P^3} = \frac{h\sqrt{2hr - h^2}}{8\left(\sqrt{2hr - h^2} + \sqrt{2hr}\right)^3}$; $\frac{\sqrt{2r - h}}{8\left(\sqrt{2r - h} + \sqrt{2r}\right)^3}$

$\Rightarrow \lim_{h \to 0} 512r\frac{\Delta}{P^3} = 512r\frac{\sqrt{2r}}{8(2\sqrt{2r})^3} = 4$

72. Find the smallest integer satisfying the equation $\left|\log_{1/6} x - 1\right| + 2 = \left|\log_{1/6} x - 3\right|$

Ans. 1

Sol. Clearly $x > 0$. Put $\log_{1/6} x = t$ so that
$|1-t| + 2 = |3-t| \Rightarrow t \le 1$
$\Rightarrow x \ge 1/6$

SECTION – C
(Numerical Answer Type)

This section contains **03** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

73. If $x = \log 2,\ y = \log 3$ and
$a + bx + cy = \left[\log(1) + \log(1+3) + ... + \log(1+3+5+....+19)\right] - 2\left[\log 1 + \log 2 + \log 3 + ...\log 7\right]$,
where a, b, and c are positive integers, then find the value of 2a +3b + 5c. [where log x = $\log_{10}$ x]

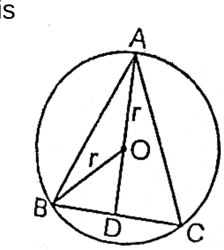
Ans. 00042.00

Sol. $a + bx + cy = \log 1^2 + \log 2^2 + ... + \log 10^2 - 2\log \underline{|7}$
$\Rightarrow a + bx + cy = 2\left[\log \underline{|10} - \log \underline{|7}\right] = 2\log(10 \times 9 \times 8)$
$\Rightarrow a + bx + cy = 2\left[1 + 3\log 2 + 2\log 3\right] = 2 + 6x + 4y$
$\Rightarrow a = 2, b = 6, c = 4 \Rightarrow 2a + 3b + 5c = 4 + 18 + 20 = 42.$
Hence option 'B' is correct.

74. Let θ (x) denotes an angle measured in radians which is subtended by a fixed closed interval [1, 3] on the y-axis at a point x on the positive x-axis. If θ_0 is the maximum value of θ(x). Then $\frac{102}{50}\left(\frac{\pi}{\theta_0}\right)$ is equal to:

Ans. 00012.24

Sol. Since the slope of AP is $\tan^{-1}(-1/x)$ and the slope of BP is $\tan^{-1}(-3/x)$ so
$\theta(x) = \tan^{-1}(-1/x) - \tan^{-1}(-3/x)$
$= \tan^{-1}(3/x) - \tan^{-1}(1/x)$
$= \tan^{-1}(3/x) - \tan^{-1}\frac{2x}{x^2+3}$
Since tan x is an increasing function so tan θ(x) is maximum when θ(x) is maximum. But $A = \tan\theta(x) = \frac{2x}{x^2+3}$
$\frac{dA}{dx} = 2\frac{x^2+3-2x^2}{(x^2+3)^2} = 2\frac{-x^2+3}{(x^2+3)^2}$
$\frac{dA}{dx} = 0 \Rightarrow x = \pm\sqrt{3}.$



Also, $\frac{dA}{dx} > 0$ for $x^2 < 3$ and $\frac{dA}{dx} < 0$ for $x^2 > 3$. Thus A is maximum when $x = \sqrt{3}$.

Hence max $\theta(x) = \tan^{-1}\frac{2\sqrt{3}}{3+3} = \frac{\sqrt{3}}{3} = \tan^{-1}1/\sqrt{3} = \pi/6$

75. Let vectors $\vec{v}_1 = (\cos\theta)\hat{i} - (3\sin\theta)\hat{j} - 2\hat{k}$

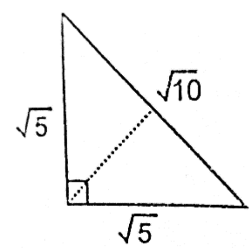
$\vec{v}_2 = (2\cos\theta)\hat{i} - (\sin\theta)\hat{j} + \hat{k}$

And $\vec{v}_3 = (\cos\theta)\hat{i} - (2\sin\theta)\hat{j} + 3\hat{k}$

Represent sides of a triangle, where $\theta \in \left[0, \frac{\pi}{2}\right)$. If area of the triangle is minimum and length of the median which bisects the side represented by $\vec{v}_3$ is ℓ, then find the value of $\frac{1024}{500}(2\ell^2)$.

Ans. 00010.24

Sol. $\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & -3\sin\theta & -2 \\ 2\cos\theta & -\sin\theta & 1 \end{vmatrix}$



$= -5\sin\theta\hat{i} - 5\cos\theta\hat{j} + 5\sin\theta\cos\theta\hat{k}$

$\therefore$ Area of the $\Delta = \frac{1}{2}|\vec{v}_1 \times \vec{v}_2|$

$\frac{1}{2}.5\sqrt{1+\sin^2\theta\cos^2\theta} = \frac{5}{2}\sqrt{1+\frac{1}{4}\sin^2 2\theta}$

For area of Δ to be minimum $\sin 2\theta = 0 \Rightarrow \theta = 0$

Now, $\vec{v}_1 = \hat{i} - 2\hat{k}, \vec{v}_2 = 2\hat{i} + \hat{k}, \vec{v}_3 = \hat{i} + 3\hat{k}$

$|\vec{v}_1| = \sqrt{5}, |\vec{v}_2| = \sqrt{5}, |\vec{v}_3| = \sqrt{10}$

$l = \frac{\sqrt{10}}{2} \Rightarrow 2l^2 = 5$