FIITJEE

ALL INDIA TEST SERIES

FULL TEST – II

JEE (Advanced)-2021

PAPER –2

TEST DATE: 04-01-2021

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. B, C, D

Sol. If $\vec{F}$ is conservative, then

$$F_x = -\frac{\partial U}{\partial x}, F_y = -\frac{\partial U}{\partial y}$$

And so $\frac{\partial F_x}{\partial y} = -\frac{\partial^2 U}{\partial y \partial x} = -\frac{\partial^2 U}{\partial x \partial y} = \frac{\partial F_y}{\partial x}$

$$W = \int \vec{F}.\overrightarrow{ds} = (x^2y^2\hat{i} + x^2y^2\hat{j}).(dx\hat{i} + dy\hat{j})$$

$$= \int x^2y^2dx + \int x^2y^2dy$$

$$W_{BC} = \int_0^a a^2y^2dy = \frac{a^5}{3}$$

Thus, $W_{ABC} = W_{AB} + W_{BC} = \frac{a^5}{3}(J)$

$$W_{AC} = 2\int_0^a x^4dx = \frac{2a^5}{3}(J)$$

2. A, D

Sol. Kinetic energy of the sphere

$$K = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2} \times \frac{2}{5} MR^2 (2\pi n)^2$$

$$= \frac{4}{5} \pi^2 n^2 MR^2$$

Kinetic energy used to raise, the temperature

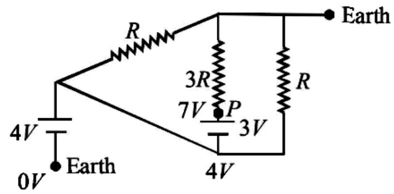
$$= 0.5\left(\frac{4}{5}\pi^2 n^2 MR^2\right)$$

$$= \frac{2}{5}\pi^2 n^2 mR^2$$

$\therefore$ $$\Delta T = \frac{2\pi^2 n^2 R^2}{5C}$$

3. A, C

Sol.



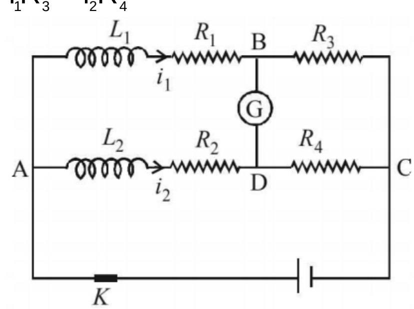
$\therefore$ $$V_P = 7V; i = \frac{7-0}{3R} = \frac{7}{3R}$$

4. A, B, C, D

Sol. $V_B = V_D$

$$L_1 \frac{di_1}{dt} + i_1 R_1 = L_2 \frac{di_2}{dt} + i_2 R_2 \quad \ldots(1)$$

$$i_1 R_3 = i_2 R_4 \quad \ldots(2)$$



From Equation (1) and (2)

$$\frac{di_1}{dt} R_3 = \frac{di_2}{dt} R_4$$

$$\text{or } \frac{di_1}{dt} = \frac{R_4}{R_3}\frac{di_2}{dt} \text{ and } i_1 = i_2 = \frac{R_4}{R_3}$$

$$\left(L_1 \frac{R_4}{R_3} - L_2\right)\frac{di_2}{dt} = i_2\left[R_2 - \frac{R_1 R_4}{R_3}\right] \quad \ldots.(3)$$

At t = 0, i_2 = 0

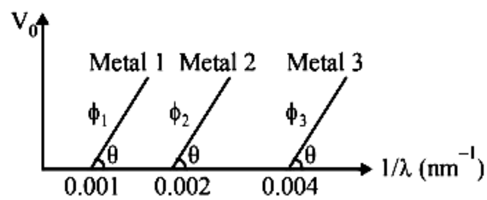
$\therefore$ $$\frac{L_1}{L_2} = \frac{R_3}{R_4} \quad \ldots(4)$$

$$R_2 - \frac{R_4R_1}{R_3} = 0 \therefore \frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad \ldots(5)$$

From Equation (4), (5), $\frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{R_3}{R_4}$

5. A, C

Sol. $\phi_1 : \phi_2 : \phi_3 = eV_{0_1} : eV_{0_2} : eV_{0_3}$



$= V = V_{0_1} : V_{02} : V_{03} = 0.001 : 0.002 : 0.004 = 1 : 2 : 4$

Therefore (a) is correct

$\Rightarrow \qquad V = \frac{hc}{e\lambda} - \frac{\phi}{e} \qquad \ldots(1)$

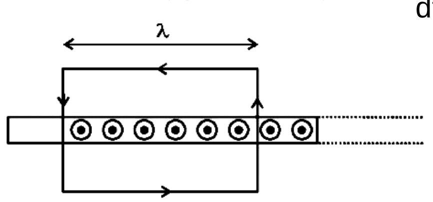
$m = \frac{hc}{e} = \tan\theta$

$\frac{1}{\lambda_{0_1}} = 0.001\text{nm}^{-1} \Rightarrow \lambda_{0_1} = \frac{1}{0.001} = 1000\text{mn}$

Also, $\frac{1}{\lambda_{0_2}} = 0.002\ \text{nm}^{-1} \Rightarrow \lambda_{02} = 500\text{mn}$ and $\lambda_{0_2} = 250\text{nm}$

6. B, D

Sol. Current flowing per unit length $= \sigma\frac{dx}{dt} = \sigma v$

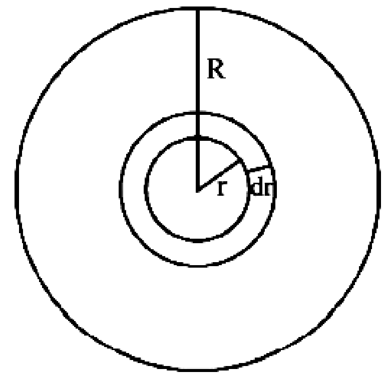


By Ampere's law

$B\ell + B\ell = \mu_0\sigma v\ell \Rightarrow B = \frac{\mu_0\sigma v}{2}$

SECTION – B

7. 6



Sol. $I = \int_0^R (dm)r^2$

$\therefore \quad I = \int_0^R \rho \times 4\pi r^2 dr \times r^2$

$\therefore \quad I = \int_0^R \rho \times 4\pi r^2 dr \times r^2$

$\therefore \quad I_A = 4\pi \int_0^R k\frac{r}{R} \times r^4 dr = \frac{4\pi K}{R}\int_0^R r^5 dr$

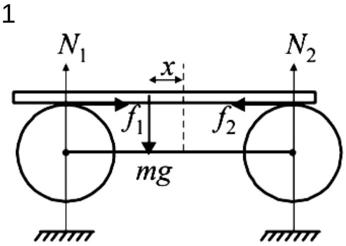
$= \frac{4\pi K}{R}\left(\frac{R^6}{6}\right) = 4\pi K\frac{R^5}{6}$

$I_B = 4\pi \int_0^R K\left(\frac{r}{R}\right)^5 r^4 dr = \frac{4\pi K}{R^5} \times \frac{R^{10}}{10} = 4\pi K\frac{R^5}{10}$

$\therefore \quad \frac{I_B}{I_A} = \frac{6}{10} \Rightarrow n = 6$

8. 1

Sol.



$F = -(f_1 - f_2)$

$= -(\mu N_1 - \mu N_2)$

$= -\mu(N_1 - N_2)$

$N_1 + N_2 = mg$ …(1)

$mg(\ell/2 - x) - N_2\ell = 0$ …(2)

$\therefore \quad N_2 = \frac{mg}{\ell}(\ell/2 - x)$

$= \frac{mg}{\ell}\left[\frac{\ell}{2} + x\right]$

$F = \mu\left[\frac{mg}{\ell}\left(\frac{\ell}{2} + x\right) - \frac{mg}{\ell}\left(\frac{\ell}{2} - x\right)\right]$

$= -\mu\frac{mg}{\ell}(2x)$

$= \pi\sqrt{\frac{2\ell}{\mu g}} = 10\,\text{sec}$

9. 7

Sol. $(T_A + F_e) - mg = \frac{mv_A^2}{\ell}$ …(i)

$(T_A + F_e) - mg = \frac{mv_A^2}{\ell}$

$mg - F_e = \frac{mv_B^2}{\ell}$ …(ii)

$F_e = \frac{1}{4\pi \epsilon_0} \frac{q^2}{\ell^2}$

$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mg(2\ell)$ …(iii)

$v_A = 7$ m/s

10. 7

Sol. $B = \frac{\mu_0}{4\pi} \frac{2\pi i R^2}{(R^2 + 3R^2)^{3/2}} = \frac{\mu_0 i}{16R}$

$M = \frac{N\phi}{i} = \frac{2}{i}\left[\frac{\mu_0 i}{16R} \times a^2 \cos 45°\right]$

$\therefore \quad M = \frac{\mu_0 a^2}{\frac{7}{2^2 R}}$

11. 9

Sol. The intensity is given by

$I = \frac{1}{2} \epsilon_0 E^2 C$

or $2.5 \times 10^{14} = \frac{1}{2} \times (8.86 \times 10^{-12}) \times E_0^2 \times (3 \times 10^8)$

$\therefore$ $E_0 = 4.3 \times 10^8$ V/m

and $B_0 = \frac{E_0}{C} = 1.44\text{T}$

12. 2

Sol. Applying first law of thermodynamics to path iaf

$Q_{iaf} = \Delta U_{iaf} + W_{iaf}$

$500 = \Delta U_{iaf} + 200$

$\therefore$ $\Delta U_{iaf} = 300$ J

Now,

$Q_{iaf} = \Delta U_{ibf} + W_{ib} + W_{bf}$

$= 300 + 50 + 100$

$Q_{ib} + Q_{bf} = 450$ J …(1)

Also, $Q_{ib} = \Delta_{ib} + W_{ib}$

$\therefore$ $Q_{ib} = 100 + 50 = 150$ J …(2)

From (1) & (2) $\frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$

SECTION – C

13. 01732.50

Sol. Probability that a particular nucleus will decay in next 2 hr is

$\frac{N_0 - N}{N_0} = 8 \times 10^{-4}$ [N = surviving population after 2 hr]

$N_0[1 - 8 \times 10^{-4}] = N$

$\Rightarrow \quad 1 - 8 \times 10^{-4} = e^{-\lambda t} \qquad \left[\because \frac{N}{N_0} = e^{-\lambda t}\right]$

$\Rightarrow \quad e^{-\lambda t} = 0.9992 \Rightarrow -\lambda t = \ell n(0.9992)$

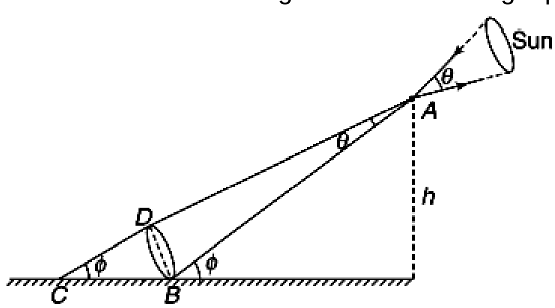
$-\frac{\ell n2}{t_{1/2}} \times 2 = \ell n(0.9992) \quad [\because t = 2hr]$

$t_{1/2} = \frac{2\ell n2}{\ell n(0.9992)} hr$

$= \frac{2 \times 0.693}{8 \times 10^{-4}} = 1732.5\, hr$

14. 00003.44

Sol. The cone of rays passing through the hole at a produce an elliptical spot on the floor. The circular base having diameter BD will get projected on the floor as an ellipse.



CB = major axis = 12 cm

DB = Minor axis = 6 cm

$DB \simeq AB(\theta) = \frac{h\theta}{\sin\phi} \Rightarrow h = (6cm)\frac{\sin\phi}{\phi}$

But $\quad \sin\phi = \frac{DB}{CB} = \frac{6}{12} = \frac{1}{2}$

$\therefore \quad h = \frac{(6\,cm)}{2 \times \theta}$

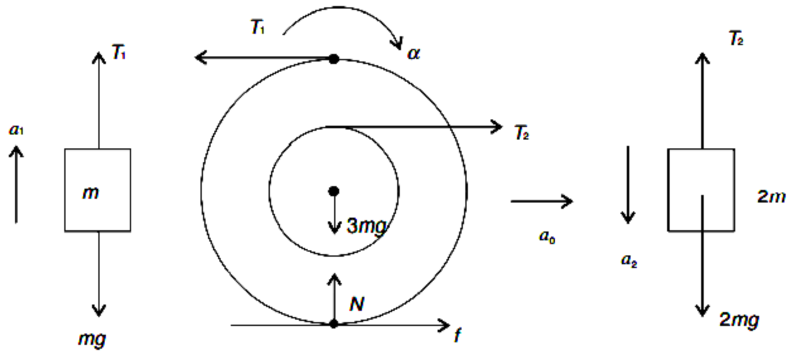
$= \frac{3cm \times 180}{0.5 \times \pi} \qquad \left[0.5^\circ = \frac{0.5 \times \pi}{180} radian\right]$

= 344 cm

= 3.44 m

15. 00000.11

Sol. Let acceleration of COM of the spool be a_0 towards right, and acceleration of m_2 be a_2 downward and that of m_1 be a_1 upward



f = friction, α = acceleration of the spool

$2R\alpha = a_0$ ...(1)

$R\alpha + a_0 = a_2$ ...(2)

$2R\alpha + a_0 = a_1$ ...(3)

$T_1 - mg = ma_1$

$T_1 - mg = m(a_0 + 2R\alpha) = 2ma_0$ ....(4) $\left[\because R\alpha = \frac{a_0}{2}\right]$

$2mg - T_2 = 2ma_2$

$2mg - T_2 = 2m[a_2 + R\alpha] = 3ma_a$ ...(5)

$T_2 + f - T_1 = 3ma_0$ ...(6)

$T_2R - T_12R - f2R = I\alpha$

$T_2 - 2T_1 - 2f = 2mR\alpha \quad [\because I = 2mR^2]$

$T_2 - 2T_1 - 2f = ma_0$ ...(7)

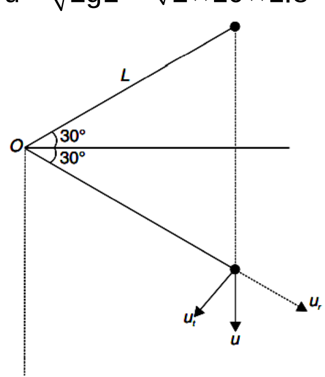
Solving, (4), (5), (6) & (7) we get $f = -\frac{mg}{3}$

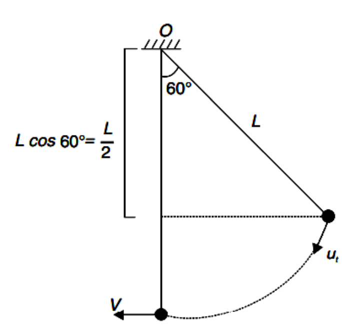
$\because f \le \mu N \quad \therefore \frac{mg}{3} \le \mu 3mg \quad \therefore \frac{1}{9} \le \mu$

16. 00035.00

Sol. The bob will experience free fall for a distance of L = 1.8 m.
Speed of the bob just before the string gets taut is

$u = \sqrt{2gL} = \sqrt{2 \times 10 \times 1.8} = 6$ m/s





$u_t = u\cos 30^\circ = \frac{\sqrt{3}u}{2} = 3\sqrt{3}$ m/s

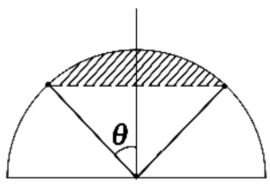
$\frac{1}{2}mV^2 = \frac{1}{2}mu_t^2 + mg\frac{L}{2}$

$V = 3\sqrt{5}$ m/s

$T = 10 + \frac{45}{1.8} = 35N$

17. 00002.45

Sol.

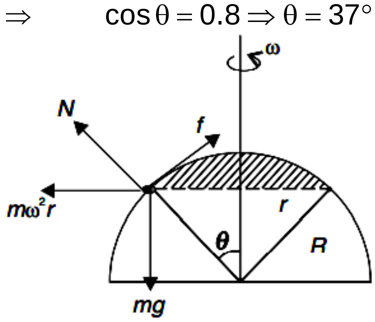


$s = \Omega R^2 = 2\pi R^2(1-\cos\theta)$

If s = (0.2) ($2\pi R^2$) then

$0.2 = 1 - \cos\theta$

$\Rightarrow \quad \cos\theta = 0.8 \Rightarrow \theta = 37^\circ$



$N = mg\cos\theta - m\omega^2 r\sin\theta$

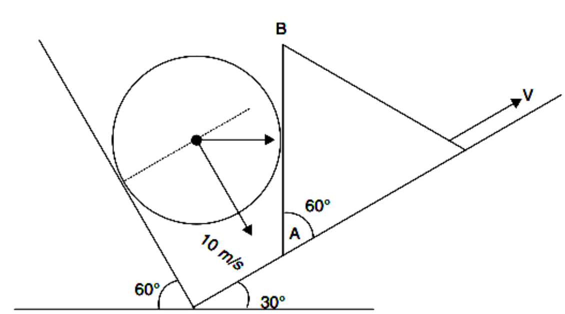
$\mu(mg\cos\theta - m\omega^2 r\sin\theta) = mg\sin\theta + m\omega^2 r\cos\theta$

$\therefore \frac{\mu g\sin\theta + \omega^2 r\cos\theta}{g\cos\theta - \omega^2 r\sin\theta}$

$= \frac{10\times 0.6 + 10^2\times 0.06\times 0.8}{10\times 0.8 - 10^2\times 0.06\times 0.6} = \frac{0.6+0.48}{0.8-0.36} = 2.45$

18. 00005.77

Sol. Wall AB of the wedge is vertical. For ball to remain in contact with the wedge the velocity component of the ball perpendicular to the wall AB must be equal to velocity component of the wedge in horizontal direction (i.e., perpendicular to wall AB)



$\therefore 10\cos 60^\circ = V\cos 30^\circ$

$5 = V\frac{\sqrt{3}}{2}$

$V = \frac{10}{\sqrt{3}}$ m/s

Chemistry

PART – II

SECTION – A

19. A, B, C, D

Sol. (A) 2-D-hexagonal packing

$$\text{Packing efficiency} = \frac{\pi R^2 + 6 \times \frac{\pi R^2}{3}}{6 \times \frac{\sqrt{3}(2R)^3}{4}}$$

(B) simple cubic $= \frac{1 \times \frac{4}{3}\pi R^3}{a^3}$

$a = 2R$

$$\text{H.C.P.} = (3D) = \frac{6 \times \frac{4}{3}\pi R^3}{24\sqrt{2}R^3}$$

$$\text{F.C.C.} = (3D) = \frac{4 \times \frac{4}{3}\pi R^3}{(2\sqrt{2}R)^3}$$

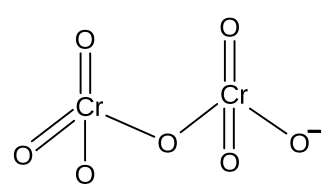
20. A, C, D

Sol. $KE = \frac{3}{2}RT(1\,\text{mole})U_{rms} = \sqrt{\frac{3RT}{M_w}}$

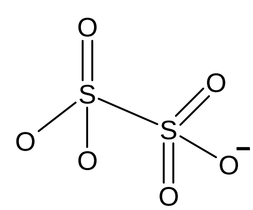
$d = \frac{PM_w}{RT}$

21. A, B, C, D

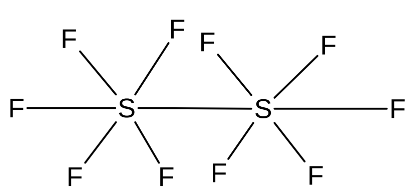
Sol. (A)



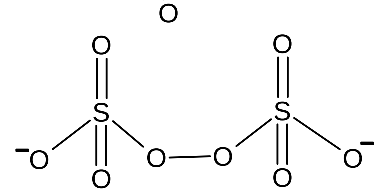
(B)



(C)



(D)



22. A, B, C

Sol. Factual

23. A, C

Sol. $H_2A \rightleftharpoons H^+ + HA^-$; $\because$ y is very less so $\underset{x-y=x}{x + y} \approx x$ $\therefore$ $K_{a_1} = \frac{x^2}{c - x}$

at equilibrium C-X x+y x-y $K_{a_2} = y = 10^{-12}; pK_{a_2} = 12$

$HA^{-1} \rightleftharpoons H^{+} + A^{2-}$

at equilibrium x-y x+y y

$pK_{a_2} - pK_{a_1} = 12 - 5 = 7$

$pH = \frac{1}{2}[pK_{a_1} - \log c]$

$3 = \frac{1}{2}[pK_{a_1} + 1] \quad pK_{a_1} = 5$

24. C

Sol. As ΔV is positive w is negative

$W = -P\Delta V$

So work is done on the system.

SECTION – B

25. 3

Sol. Fraction of edge unoccupied $= \frac{a - 2R}{a}$

$a = 2\sqrt{2}R \qquad X = \frac{2(\sqrt{2} - 1)}{2\sqrt{2}}$

$X = \frac{0.414}{1.414} = 0.293$

$Z = \frac{X}{0.097} = \frac{0.293}{0.097} = 3$

26. 00007.00

Sol. alk. $KMnO_4$ & $Na_2S_2O_3$ will not

React with kI to give I_2

27. 8

Sol. Process AC = polytropic process (P = KV)

Molar Heat capacity $c_m = c_V + R/2 = 2R$

Process AB = Isobaric

$c_m = c_p = 5R/2$

$$\frac{q_{AC}}{q_{AB}} = \frac{\int_{T_A}^{T_C} nC_m.dT}{\int_{T_A}^{T_B} n.C_{p,m*}dT} = \frac{2R}{\frac{5}{2}R} = 0.8$$

$$\frac{q_{AC}}{q_{AB}} = \times 10 = 0.8 \times 10 = 8$$

28. 3

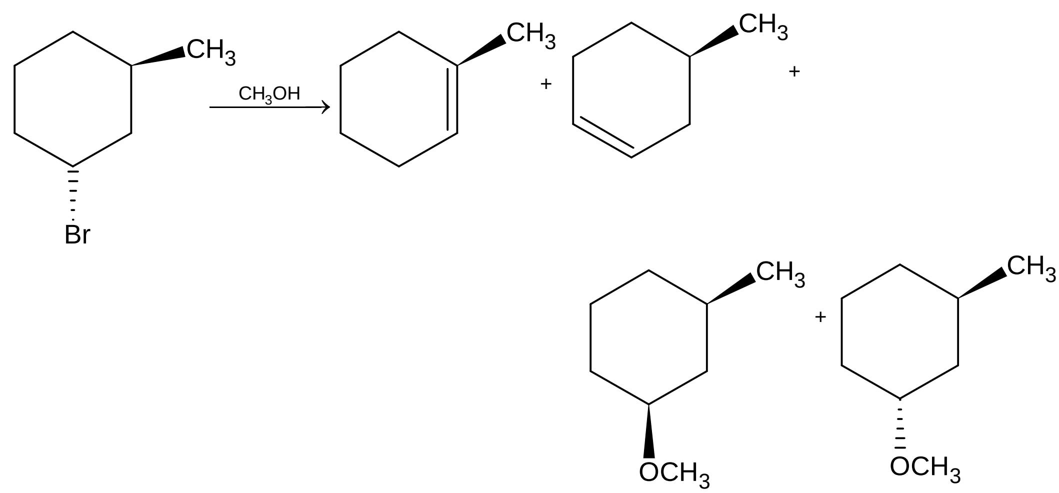
Sol. Ca, CaH_2 and produce H_2 gas with cold water.

29. 4

Sol. $\underset{\text{(Metal oxide basic)}}{MO} + \underset{\text{(Acedic)}}{SiO_2} \rightarrow \underset{\text{(slag)}}{MSiO_3}$

30. 4

Sol.



SECTION – C

31. 00024.06

Sol. $k = \frac{2.303}{10}\log\frac{0.04}{0.03}$

$t_{1/2} = \frac{0.693}{K}$

32. –0.24

Sol. $-0.12 - \frac{0.0591}{2}\log\left(\frac{1}{x}\right) = -0.24$

$\log\frac{1}{x} = \frac{0.12\times 2}{0.06} = 4$

$x = 10^{-4}$

33. 00000.10

Sol. $\alpha = \sqrt{\frac{Ka}{C}}$

34. 00000.50

Sol. $\frac{\alpha_1}{\alpha_2} = \sqrt{\frac{C_2}{C_1}} \Rightarrow \frac{0.01}{\alpha_2} = \sqrt{\frac{0.4}{0.1}}$

$\alpha_2 = 0.005$ or $\%\,\alpha_2 = 0.5$

35. 27419.25

Sol. $\bar{V} = R\times\left[\frac{1}{2^2} - \frac{1}{\alpha^2}\right]$

36. 00004.52

Sol. $K_c = \frac{[\text{complex}]}{[H_3BO_3][\text{glycerine}]}$.

Mathematics

PART – III

SECTION – A

37. A, D

Sol. $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$ …(1)

$\Rightarrow \quad a_0 + a_2 + a_4 \ldots = \frac{3^n+1}{2}$

$a_1 + a_3 + a_5 \ldots = \frac{3^n-1}{2}$

Put $x = i$ in equation (1) where $i = \sqrt{-1}$

If $n = 4m+1$

$\Rightarrow \quad a_1 - a_3 + a_5 \ldots = 1$

$a_0 - a_2 + a_4 \ldots = 0$

So, $a_1 + a_5 + a_9 \ldots = \frac{3^n+1}{4} = S_2$

$a_3 + a_7 + a_{11} \ldots = \frac{3^n-3}{4} = S_4$

Similarly if $n = 4m+3$

Then, $S_2 = \frac{3^n-3}{4}$ and $S_4 = \frac{3^n+1}{4}$

38. A, B, C, D

Sol. $f_1(x) = \frac{\pi}{2} - \sin^2 x, \ \ f_2(x) = \frac{\pi}{2} - \cos^2 x$

$f_3(x) = \frac{\pi}{2} - \cos^2 x, \ \ f_4(x) = \frac{\pi}{2} - \sin^2 x$

39. A, B

Sol. $c^2 = a^2 + b^2 - 2ab\cos C$

$= (a-b)^2 + 2ab(1-\cos C)$

$\Delta = \frac{1}{2}ab\sin C$

or $2ab = \frac{4\Delta}{\sin C}$

here $c^2 = (a-b)^2 + 4\Delta\left(\frac{1-\cos C}{\sin C}\right)$

$= (a-b)^2 + 4\Delta\tan\frac{C}{2}$

$4\Delta\tan\frac{C}{2}$ is constant so for c^2 to be minimum a = b thus $2ab = 2a^2 = \frac{4\Delta}{\sin C}$

$\Rightarrow a = b = \sqrt{\frac{2\Delta}{\sin C}}$.

40. B, D

Sol. $f(x) = x^5 - 10a^3 x^2 + b^4 x + c^5$

$f'(x) = 5x^4 - 20a^3 x + b^4$

$f''(x) = 20 x^3 - 20a^3$

If $x = \alpha$ be a root that is repeated three times,

$\Rightarrow f''(\alpha) = 0,\ f'(\alpha) = 0,\ f(\alpha) = 0$

$\Rightarrow \alpha = a,\ 5a^4 - 20 a^4 + b^4 = 0,\ a^5 - 10a^5 + ab^4 + c^5 = 0$

$\Rightarrow \alpha = a,\ b^4 = 15a^4,\ c^5 + ab^4 - 9a^5 = 0 \Rightarrow c^5 + 15a^5 - 9a^5 \Rightarrow 6a^5 + c^5 = 0.$

41. A, D

Sol. Since $\int_a^b f(x)dx = (b-a)\int_0^1 f\{(b-a)x + a\}dx$,

$$\int_1^2 \sin x^2 dx = \int_0^1 \sin(x+1)^2 dx = \int_0^1 \sin(x^2 + 2x + 1)dx$$

$$\int_{-4}^4 \cos x^2\, dx = 8\int_0^1 \cos(8x-4)^2\, dx$$

$$= 8\int_1^0 \cos 16(2x-1)^2\, dx.$$

42. B, C

Sol. $f(x) = 0$

If $\sin x \neq \pm 1$ $(1 \leq 1 + \sin^2 x < 2,\ \sin^2 = 2)$

$= \pi/3$ If $\sin x = \pm 1$

So f(x) is not continuous at the points, where $\sin x = \pm 1$ i.e. x is an odd multiple of $\pi/2$

SECTION – B

43. 6

Sol. AB = 22 = AC

Now AP + PR + AR = (AP + PQ) + (QR + AR)

= AB + AC = 44.

44. 3

Sol. $|z| + |z-1| + |2z-3| = |z| + |z-1| + |3-2z| \geq |z + z - 1 + 3 - 2z| = 2$

$\therefore$ $|z| + |z-1| + |2z-3| \geq 2$

$\therefore$ $\lambda = 2$

then $2[x] + 3 = 3[x - \lambda]$

$= 3[x - 2]$

$\Rightarrow$ $2[x] + 3 = 3([x] - 2)$

or $[x] = 9$, then $y = 2.9 + 3 = 21$

$\therefore$ $[x + y] = [x + 21] = [x] + 21 = 9 + 21 = 30$

45. 2

Sol. No. of fights = ${}^{24}C_2 - {}^{x}C_2 - {}^{x+1}C_2 - {}^{23-2x}C_2$

f(x) = $-3x^2 + 45x + 23$.

$f(x) = 0 \Rightarrow x = \dfrac{15}{2} = 7.5$

but x can't be a fraction.

So x = 7 (nearest integer)

$\therefore$ f(x) = 191 Ans.

46. 2

Sol. $k=1 \quad \frac{1}{3}\left({}^1C_0 + {}^1C_0\right) = \frac{1}{3}\times 2 = \frac{2}{3}$

$k=2 \quad \frac{1}{3^2}\left({}^2C_0 + {}^2C_1 + {}^2C_2\right) = \frac{2^2}{3^2} = \left(\frac{2}{3}\right)^2$

$k=3 \quad \frac{1}{3^3}\left({}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3\right) = \frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$

– –

– –

$\therefore$ Reg. Sum = $\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + ...\infty$

$= \frac{\frac{2}{3}}{1-\frac{2}{3}} = 2$

47. 4

Sol. equation has four positive real root.

$\Rightarrow \quad \frac{\frac{1}{x_1}+\frac{2}{x_2}+\frac{3}{x_3}+\frac{4}{x_4}}{4} \ge \left(\frac{24}{x_1, x_2, x_3, x_4}\right)^{1/4} = 2.$

$\Rightarrow \quad \frac{1}{x_1}+\frac{2}{x_2}=\frac{2}{x_3}=\frac{4}{x_4}=k$

$\Rightarrow \quad k=2 \quad \therefore \textit{roots are } \frac{1}{2}, 1, \frac{3}{2} \textit{ and } 2$

$\Rightarrow \quad \frac{x_4}{x_1}=4$

48. 2

Sol. Since $\alpha 2 - 2\alpha + 3 = 0$ & $\beta 2 - 2\beta + 3 = 0$

$\therefore \quad \alpha 3 - 3\alpha 2 + 5\alpha - 2 = \alpha 3 - 2\alpha 2 + 3\alpha - \alpha 2 + 2\alpha - 3 + 1$

$= \alpha(\alpha 2 - 2\alpha + 3) - (\alpha 2 - 2\alpha + 3) + 1 = 1$

And $\beta 3 - \beta 2 + \beta + 5 = \beta 3 - 2\beta 2 + 3\beta + \beta 2 - 2\beta + 5 = \beta(\beta 2 - 2\beta + 3) + (\beta 2 - 2\beta + 3) + 2 = 2$

So the ref. equation is

x2 – (2 + 1) x + 2.1 = 0

x2 – 3x + 2 = 0

SECTION – C

49. 58060.80

Sol. 10 IIT students $T_1, T_2, \ldots\ldots T_{10}$ can be arranged in 10 ! ways. Now the number of ways in which two PET student can be placed will be equal to the number of ways in which 3 consecutive IIT students can be taken i.e. in 8 ways and can be arranged in two ways Þ (10 !) (8) (2 !).

Alternatively 3 IIT students can be selected in ${}^{10}C_3$ ways. Now each selection of 3 IIT and 2 PET students in $P_1\ T_1\ T_2\ T_3\ P_2$ can be arranged in (2 !) (3 !) ways. Call this box X. Now this X and the remaining IIT students can be arranged in 8 ! ways

$\Rightarrow$ Total ways ${}^{10}C_3$ (2 !) (3 !) (8 !)]

50. 08855.00

Sol. x + y + z + w < 25

let x + y + z + w + a = 25 such that a > 0 ....(i)

$x = -1 + t_1 \quad \therefore \quad t_1 \ge 0$

$y = 2 + t_2 \quad \therefore \quad t_2 \ge 0$

$z = t_3 \quad \therefore \quad t_3 \ge 0$

$w = 4 + t_4 \quad \therefore \quad t_4 \ge 0$

$a = 1 + t_5 \quad \therefore \quad t_5 \ge 0$

put in equation (i)

$(-1 + t_1) + (2 + t_2) + t_3 + (4 + t_4) + (1 + t_5) = 25$

$t_1 + t_2 + t_3 + t_4 + t_5 = 19$

by fictitious partition solution ${}^{19+\overline{5-1}}C_{19} = {}^{23}C_{19}$

51. 01002.00

Sol. $x^3 - 3xy^2 = 2005 \Rightarrow \left[\left(\frac{x}{y}\right)^3 - 3\left(\frac{x}{y}\right) = \frac{2005}{y^3}\right] \times 2004$ ....(1)

$y^3 - 3x^2y = 2004 \Rightarrow \left[1 - 3\left(\frac{x}{y}\right)^2 = \frac{2004}{y^3}\right] \times 2004$ ....(2)

Subtract (1) & (2) & put $\frac{x}{y} = t$

$2004t^3 + 6015t^2 - 6012t - 2005 = 0$ → t_1, t_2, t_3

$$\frac{y_1.y_2.y_2}{(y_1 - x_1)(y_2 - x_2)(y_3 - y_3)} = \frac{1}{(1-t_1)(1-t_2)(1-t_3)}$$

$$= \frac{1}{1 + (t_1t_2 + t_2t_3 + t_3t_1) - (t_1 + t_2 + t_3) - t_1t_2t_3}$$

put values

= 1002

52. 00010.20

Sol. $$\left(1 - \frac{2}{3}\right)^{-\frac{1}{2}} = 1 + \frac{1}{2}.\frac{2}{3} + \frac{\frac{1}{2}.\frac{3}{2}}{\underline{|2}}.\left(\frac{2}{3}\right)^2 + \frac{\frac{1}{2}.\frac{3}{2}.\frac{5}{2}\left(\frac{2}{3}\right)^3}{\underline{|3}}$$

$$\Rightarrow \left(\frac{1}{3}\right)^{-\frac{1}{2}} - = \frac{1}{3} + \frac{1.3}{(3)^2\underline{|2}} + \frac{1.3.5}{(3)^3\underline{|3}} + \text{.........}\infty$$

$\sqrt{3} - 1 = z \qquad \Rightarrow z^2 + 2z - 2 = 0$

$\because \quad z = \frac{-2 \pm \sqrt{4+8}}{2}$

$\because \quad z = -2 \pm \sqrt{3},\ but\ z \ne -1 - \sqrt{3}$

53. 00003.50

Sol. Let $f(x) = ax^2 + bx + c$, $\because \int_{\alpha}^{\beta} f(x)dx > 0 \quad \forall \alpha, \beta \in R$

$a, b, c \in I^+ \qquad f(x) > 0 \Rightarrow D < 0$

$f''(x) = 2a \qquad \Rightarrow g(t) = 2a(at^2 + bt + c) = 2a^2t^2 + 2abt + 2ac.$

$g(0) = 2ac = 12 \Rightarrow \qquad ac = 6.$

$$\left.\begin{array}{ll} a = 6, & c = 1 \\ a = 1 & c = 6 \\ a = 3 & c = 2 \\ a = 2 & c = 3 \end{array}\right] \Rightarrow b^2 < 24; \qquad b = 1, 2, 3, 4$$

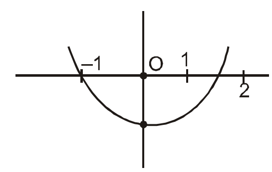
$\therefore$ 16 quadratic are possible

54. 00001.00

Sol. Let $f(x) = ax^2 + (a - 2)x - 2$

$f(0) = -2$

and $f(-1) = 0$



Since the quadratic expression is negative for exactly two integral values

$\Rightarrow \quad f(1) < 0 \quad$ and $\quad f(2) \geq 0$

$\Rightarrow \quad a + a - 2 - 2 < 0 \quad$ and $\quad 4a + 2a - 4 - 2 \geq 0$

$\Rightarrow \quad a < 2 \quad$ and $\quad a \geq 1$

$\therefore \quad a \in [1, 2)$