FIITJEE

ALL INDIA TEST SERIES

FULL TEST – II

JEE (Advanced)-2021

PAPER –1

TEST DATE: 04-01-2021

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. D

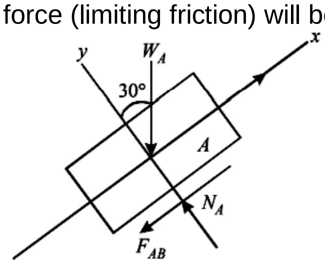
Sol. Since $0 = (v\sin\theta)t + \frac{1}{2}(-a)t^2 \Rightarrow t = \frac{2v\sin\theta}{a}$

Also, $h = (v\cos\theta)t + \frac{1}{2}gt^2$

$\Rightarrow h = \frac{2v^2}{a}\sin\theta\left(\cos\theta + \frac{g}{a}\sin\theta\right)$

2. A

Sol. For the impending motion, block A must slip up and block C down the inclined plane. Since the normal force between A and B is less than that between block B and C, the maximum frictional force (limiting friction) will be reached first between A and B while B and C will stay together.



Writing equilibrium equations:

$\Sigma F_y = 0$;

$N_A - W_A\cos 30° = 0$

$N_A = W_A\cos 30°$

$N_A = 20\sqrt{3}N$

Also, for impending motion if F_{AB} is frictional force between blocks A and B, then

$$F_{AB} = \mu_s . N_A = 20\sqrt{3}\mu_s N \quad \ldots(1)$$

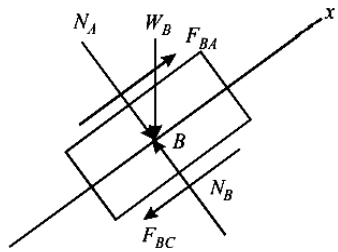
$$\Sigma F_x = 0;$$

$$T - W_A \sin 30^\circ - F_{AB} = 0$$

$$T - 40\frac{1}{2} - 20\sqrt{3}\mu_s = 0$$

$$T = 20(1+\sqrt{3}\mu_s)$$

From FBD of block B and C combined



Writing equilibrium equation

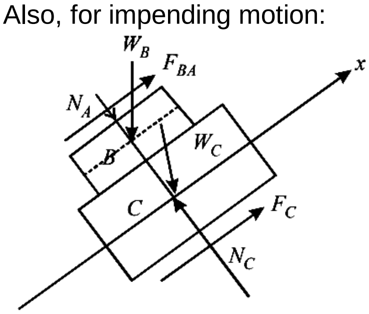
$$\Sigma F_y = 0;$$

$$N_C - N_A - (W_B + W_C)\cos 30^\circ = 0$$

$$N_C - 20\sqrt{3} - 110\frac{\sqrt{3}}{2} = 0$$

$$N_C = 75\sqrt{3}N$$

Also, for impending motion:



$$F_C = \mu_s . N_C = 75\sqrt{3}\mu_s \quad \ldots(3)$$

For $\Sigma F_x = 0$, we have

$$T_A + (F_{BA} + F_C) - (W_B + W_C)\sin 30^\circ = 0$$

$$T + [20\sqrt{3} + 75\sqrt{3}\mu_s] - \frac{110}{2} = 0$$

$$T = (55 - 95\sqrt{3}\mu_s)$$

Since tension is same, so from (2) and (4), we get

$$20(1+\sqrt{3}\mu_s) = (55 - 95\sqrt{3}\mu_s)$$

Solving for μ_s we get, $115\sqrt{3}\mu_s = 35$

or $\quad \mu_s = \frac{35}{115\sqrt{3}} = 0.1757$

$\therefore \quad$ Minimum μ_s = 0.1757

3. D

Sol. The velocity of particle after falling through height h

$$u = \sqrt{2gh} \qquad \ldots(i)$$

$$v^2 = u^2 - 2gh$$

$\therefore$ $$0 = e^2u^2 - 2gh_1$$

or $$h_1 = \frac{e^2u^2}{2g} = \frac{e^2 2gh}{2} = e^2h \quad \text{[From Eq. (i)]}$$

$$0 = e^4u^2 - 2gh_2$$

or $$h_2 = e^4h$$

$$= h + 2h_1 + 2h_2 + \ldots\infty$$

$$= h + 2e^2h + 2e^2h + \ldots\infty$$

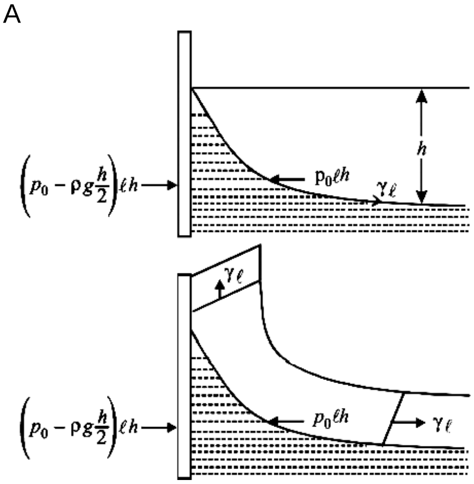
$$= h + 2e^2h(1 + e^2 + e^4 + \ldots\infty)$$

$$= h + 2e^2h\left(\frac{1}{1-e^2}\right)$$

$$= h\left(1 + \frac{2e^2}{1-e^2}\right) = \left(\frac{1+e^2}{1-e^2}\right)h$$

4. A

Sol.

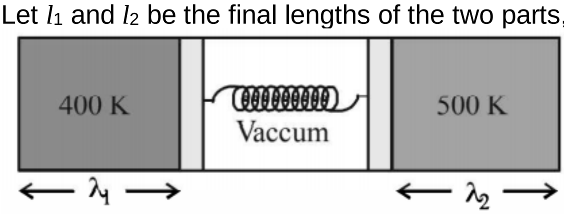


Balancing forces in horizontal direction

$$\left(p_0 - \rho g\frac{h}{2}\right)\ell h + \gamma\ell = p_0\ell h \Rightarrow h = \sqrt{\frac{2\gamma}{\rho g}}$$

5. A

Sol. Let l_1 and l_2 be the final lengths of the two parts, the from gas equation



$$\frac{P_0A\ell_0}{T_0} = \frac{PA\ell_1}{T_1} = \frac{PA\ell_2}{T_2} \qquad \ldots(i)$$

$$P_0A = kx_0 \text{ and } PA = kx$$

or $\frac{P}{P_0} = \frac{x}{x_0}$ …(ii)

∴ $x - x_0 = \ell_1 + \ell_2 - 2\ell_0$ …(iii)

From equation (i),

$$\ell_1 = \frac{P_0\ell_0T_1}{PT_0} \text{ and } \ell_2 = \frac{P_0\ell_0T_2}{PT_0}$$

From equation (ii),

$$\ell_1 = \frac{x_0\ell_0T_1}{xT_0} \text{ and } \ell_2 = \frac{x_0\ell_0T_2}{xT_0}$$

Putting these in equation (iii),

$$x - x_0 = \frac{x_0\ell_0}{xT_0}[T_1 + T_2] - 2\ell_0$$

Substituting the values and solving for x, we get

$x \approx 1.3$ m

6. A

Sol. $f = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$;

In air : $T = mg = \rho Vg$

$$\therefore f = \frac{1}{2\ell}\sqrt{\frac{\rho Vg}{m}} \quad \text{…(1)}$$

In water : $T = mg - \text{upthrust}$

$$= V\rho g - \frac{V}{2}\rho_\omega g = \frac{Vg}{2}(2\rho - \rho_\omega)$$

$$\therefore f' = \frac{1}{2\ell}\sqrt{\frac{\frac{Vg}{2}(2\rho - \rho_\omega)}{m}}$$

$$= \frac{1}{2\ell}\sqrt{\frac{Vg\rho}{m}}\sqrt{\frac{(2\rho - \rho_\omega)}{2\rho}}$$

$$\frac{f'}{f} = \sqrt{\frac{2\rho - \rho_\omega}{2\rho}}$$

$$f' = f\left(\frac{2\rho - \rho_\omega}{2\rho}\right)^{1/2}$$

$$300\left(\frac{2\rho - 1}{2\rho}\right)^{1/2} \text{ Hz}$$

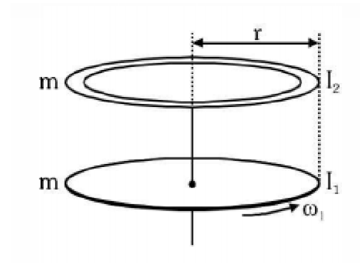
7. A, B, D

Sol. Let ω_1 = the initial angular velocity of the disc.

ω_2 = the final common angular velocity of the disc and the ring.

For the disc, $I_1 = \frac{1}{2}mr^2$

For the ring, $I_2 = mr^2$



By conservation of angular momentum,

$L = I_1\omega_1 - (I_1 + I_2)\omega_2$

or $\omega_2 = \dfrac{I_1\omega_1}{I_1 + I_2} = \omega_1/3$

Initial kinetic energy = $E_1 = \dfrac{1}{2}I_1\omega_1^2$

Final kinetic energy = $E_2 = \dfrac{1}{2}(I_1 + I_2)\omega_2^2$

Heat produced = loss in kinetic energy = $E_1 - E_2$

Ratio of heat produced to initial kinetic energy = $\dfrac{E_1 - E_2}{E_1} = \dfrac{2}{3}$

8. A, B, C

Sol. Potential on innermost shell is zero

$\dfrac{q_1}{r} + \dfrac{q_2}{2r} + \dfrac{q_3}{3r} = 0$

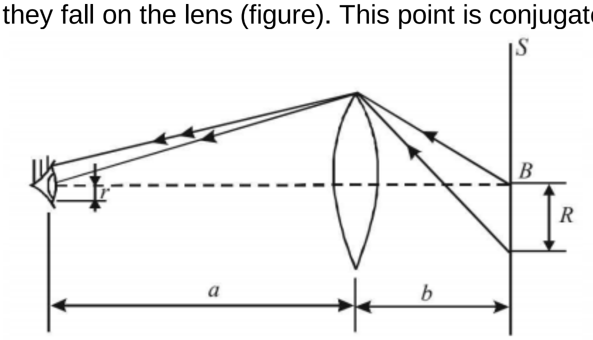
$\Rightarrow 6q_1 + 3q_2 + 2q_3 = 0$ …(1)

Potential on outermost shell is zero

$\dfrac{q_1}{3r} + \dfrac{q_2}{3r} + \dfrac{q_3}{3r} = 0 \Rightarrow q_1 + q_3 = -q_2$ …(2)

9. A, C

Sol. Let us first neglect the size of pupil, assuming that it is point-like. Obviously, only those of the beams passing through the lens will get into the eye which have passed through point B before they fall on the lens (figure). This point is conjugate to the point at which the pupil is located.



$\dfrac{1}{F} = \dfrac{1}{a} + \dfrac{1}{b}, b = \dfrac{aF}{a - F} = 12 \text{ cm}$

$R = \dfrac{b}{a}r \simeq 0.5\text{mm},$ and the screen must be placed in the plane S with its centre at point B.

10. A, B, D

Sol. $F(r) = -\dfrac{k}{r^n}$

$\Rightarrow U(r) = -\int F(r)dr = -\dfrac{k}{(n-1)} \cdot \dfrac{1}{r^{n-1}}$

Kinetic energy $= \dfrac{L^2}{2I} = \dfrac{L^2}{2mr^2} = K(r)$

Since total energy $E(r) = U(r) + K(r)$

$$\Rightarrow E(r) = -\frac{k}{(n-1)}.\frac{1}{r^{n-1}} + \frac{L^2}{2mr^2}$$

$$\Rightarrow \left.\frac{\partial E}{ar}\right|_{r=r_0} = 0 \text{ and } \left.\frac{\partial^2 E}{\partial r^2}\right|_{r=r_0} > 0,$$

Using both conditions, $(3-n)\frac{L^2}{m} > 0$

11. A, B, C, D

Sol. $I = \frac{\varepsilon}{\frac{2R}{3}} = \frac{3\varepsilon}{2R}$

$= \frac{3}{2R} \times \frac{1}{2}B\omega\ell^2 = \frac{3B\omega\ell^2}{4R}$

Magnetic force $F = \frac{3B\omega\ell^2}{4R} \times \ell \times B$

$= \frac{3B^2\omega\ell^2}{4R}$

$\tau = \frac{3B^2\omega\ell^3}{4R} \times \frac{\ell}{2} = \frac{3B^2\omega\ell^4}{8R}$

$\therefore$ Force to be applied at the end $= \frac{3B^2\omega\ell^3}{8R}$.

12. A, B, C, D

Sol. For maxima d = nλ

For minima d = (n + 1/2)λ

For intensity $\frac{3}{4}$ th of maximum $d = \left(n \pm \frac{1}{3}\right)\frac{\lambda}{2}$

SECTION – C

13. 00000.78

Sol. From conservation of energy

$m_n c^2 = m_p c^2 + k_p + m_e c^2 + k_e + m_v c^2 + k_v$

939.5656 = 938.2723 + 0.5109 + 0.0004 + (k_P + k_e)

[$\because$ $m_v c^2 + k_v$ = 0.0004 MeV]

$\Rightarrow$ $k_P + k_e$ = 0.0004 MeV

$$\Rightarrow \frac{P^2}{2m_P} + \frac{P^2}{2m_e} = 0.7820 \text{ MeV}$$

$$\Rightarrow \frac{P^2}{2m_e}\left[1 + \frac{m_e}{m_p}\right] = 0.7820$$

$$k_e = \left(\frac{m_P}{m_P + m_e}\right) \times 0.7820 \simeq 0.7820 \text{ MeV}$$

14. 00024.62

Sol. $h_{app} = \mu h$

$h_{app} = 50 - d = 40$ m and $m = \mu = 4/3$

$\therefore$ $h = 30$ m

Now, $h_{app} = \mu h$

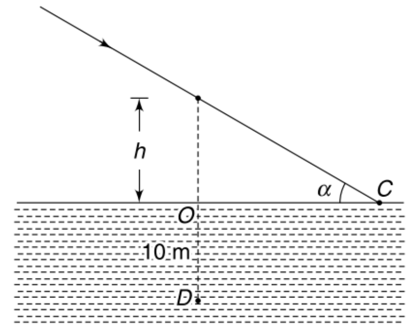
$V_{y\,app} = \mu V_y$

$V_{x\,app} = V_x$

Given $\frac{V_{yapp}}{V_{xapp}} = \tan 45° = 1$

$\therefore$ $\frac{\mu V_y}{V_x} = 1 \Rightarrow \tan\alpha = \frac{1}{\mu} = \frac{3}{4}$

$\alpha = 37°$



α is the true angle that the line of motion of the bird makes with horizontal.

$\therefore$ $OC = h\tan a = 30.\frac{3}{4}$

$= 22.5$ m

$\therefore$ $DC = \sqrt{(22.5)^2 + 10^2}$

$= 24.62$ m

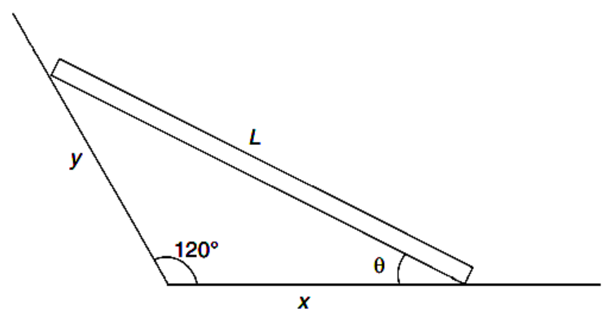
15. 00002.26

Sol. Sine Rule

$$\frac{x}{\sin(60-\theta)} = \frac{y}{\sin\theta} = \frac{L}{\sin 120°}$$

$$\therefore x = \frac{2L}{\sqrt{3}}\sin(60-\theta)$$

$$\therefore \frac{dx}{dt} = \frac{2L}{\sqrt{3}}\cos(60-\theta)\left(-\frac{d\theta}{dt}\right)$$



Note $-\frac{d\theta}{dt} = \omega =$ angular speed.

[θ is decreasing, hence a negative sign]

When $\theta = 20°, \frac{dx}{dt} = 1.5\text{ m/s}.$

$$\therefore \omega = \frac{1.5 \times 1.732}{2 \times 0.75 \times 0.766} = 2.26 \text{ rad/s}$$

16. 00001.15

Sol. Let tension in BC & BD be T_1 and that in string BA be T_2

The string CB and DB make an angle of $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ with vertical because the diagonal of a cube makes $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ angle with a side.

Line BM makes $\beta = 45°$ with vertical.

$$CB = \frac{\sqrt{3}a}{2}, \quad CM = \frac{a}{2}$$

$\therefore \sin\theta = \frac{1}{\sqrt{3}}$

Resultant of tension in CB and DB is along BM equal to

$2T_0 = 2T_1\cos\theta = 2T_1\sqrt{\frac{2}{3}}$

Vertical component of T_0 balance Mg and its horizontal component is equal to T_2.

$\therefore T_0\cos\beta = Mg$

$T_0\sin\beta = T_2$

$\Rightarrow 2\sqrt{\frac{2}{3}}T_1\sin\beta = T_2$

$\Rightarrow 2\sqrt{\frac{2}{3}}T_1\frac{1}{\sqrt{2}} = T_2$

$\Rightarrow \frac{T_2}{T_1} = \frac{2}{\sqrt{3}}$

17. 00024.00

Sol. $C = \frac{K\epsilon_0 A}{d} =$ a constant

For A to be minimum, d must be minimum. The separation between the plates is limited by the breakdown strength of the dielectric.

For air capacitor $\frac{V}{d_{min}} = E_{air}$ [E_{air} = Breakdown field for air]

$\therefore \quad d_{min} = \frac{V}{E_{air}}$

Now $\frac{\epsilon_0 A_{min}}{d_{min}} = C$

$\Rightarrow \quad A_{min} = \frac{C}{\epsilon_0}\frac{V}{E_{air}}$

$\therefore \quad A_1 = \frac{CV}{\epsilon_0 E_{air}}$

With dielectric, similar calculation gives

$A_2 = \frac{CV}{K\epsilon_0 E_{dielec}}$

$\therefore \quad \frac{A_1}{A_2} = \frac{KE_{dielec}}{E_{air}} = 3\times 8 = 24$

18. 00051.70

Sol. $q_{conv} = h(T - T_0) = 6(80 - 20) = 360\ Wm^{-2}$

For 1m length of the pipe

$Q_{conv} = q_{conv}A = q_{conv} \times 2\pi r$

$= 360 \times 2 \times 3.14 \times 0.01 = 22.6\ Wm^{-1}$

$q_{rad} = \sigma(T^4 - T_0^4) = 5.67 \times 10^{-8}(353^4 - 293^4) = 462\ Wm^{-2}$

For 1m length of the pipe

$Q_{rad} = q_{rad}A = 462 \times 2 \times 3.14 \times 0.01 = 29.1\ Wm^{-1}$

$\therefore \quad Q_{conv} + Q_{rad} = 22.6 + 29.1 = 51.7\ Wm^{-1}$

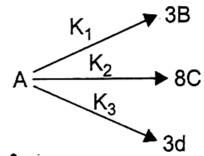
Chemistry

PART – II

SECTION – A

19. B

Sol. Mole of A remain after 45 days



$K_1 : K_2 : K_3 = 4 : 2 : 1$

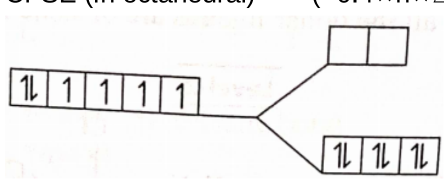
$$\frac{N_0}{2^n} = \frac{N_0}{2^{\frac{T}{t_{1/2}}}}\frac{1}{2^{45/15}} = \frac{1}{2^3} = \frac{1}{8}$$

Moles of A convert into product $\frac{7}{8}$mol

Moles of $[C] = \frac{K_2}{K_1 + K_2 + K_3} \times \frac{7}{8} \times \frac{8}{1} = 2$

20. C

Sol. CFSE (In octahedral) = $= (-0.4 \times n \times \Delta_0) + 2PE$



21. D

Sol. $H_2S + Ba(OH)_2 \rightarrow \underset{\text{(Water soluble)}}{BaS} + 2H_2O$

22. D

Sol. $T_b = 80.26, \quad \Delta T_b = 0.16;$

$$0.16 = 2.53 \times \frac{0.26 / M}{11.20} \times 1000; M \approx 367$$

That is almost molar mass of $C_{20}H_{16}Fe_2$.

23. B

Sol. $X(s) \rightleftharpoons \underset{x}{A(g)} + \underset{(2x+2y);}{2B(g)}$

$K_{P_1} = P_A . P^2_{B(total)}$

$Y(s) \rightleftharpoons \underset{y}{C(g)} + \underset{(2y+2x);}{2B(g)}$

$K_{p_2} = P_C . P^2_{B\ (total)}$

$$\frac{K_{P_1}}{K_{P_2}} = \frac{x}{y} \Rightarrow x = 2y$$

$K_{p_1} = x(2x + 2y)^2$

$\Rightarrow x = 0.1$ atm;

$\therefore$ y = 0.05 atm

Total pressure of gases $= P_A + P_B + P_C$

$= 3(x + y)$

$= 0.45$ atm.

24. A

Sol. Balanced reaction is

$2F_2O_3 + 3C \rightarrow 4Fe + 3CO_2$

Number of moles of Fe_2O_3

$= \left(\frac{120 \times 1000}{2 \times 56 + 48}\right) \times \frac{90}{100}$

Mass of 80% pure iron produced

$= \frac{120 \times 1000 \times 0.9}{2 \times 56 + 48} \times \frac{2 \times 56}{0.8}$

$= 94500$ gram or 94.5 kg

25. A, C

Sol. Apply Le chatlier principle.

26. B

Sol. (ii) and (iii)

2^{nd} ion is aromatic.

27. A, B, C

Sol. $\Delta H = \Delta U + P.\Delta V + V.\Delta P$

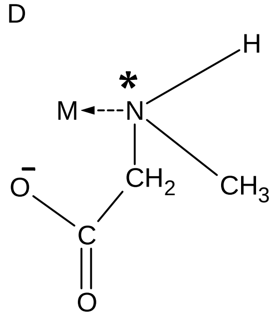
is correct relation.

28. A, C, D

Sol. Factual

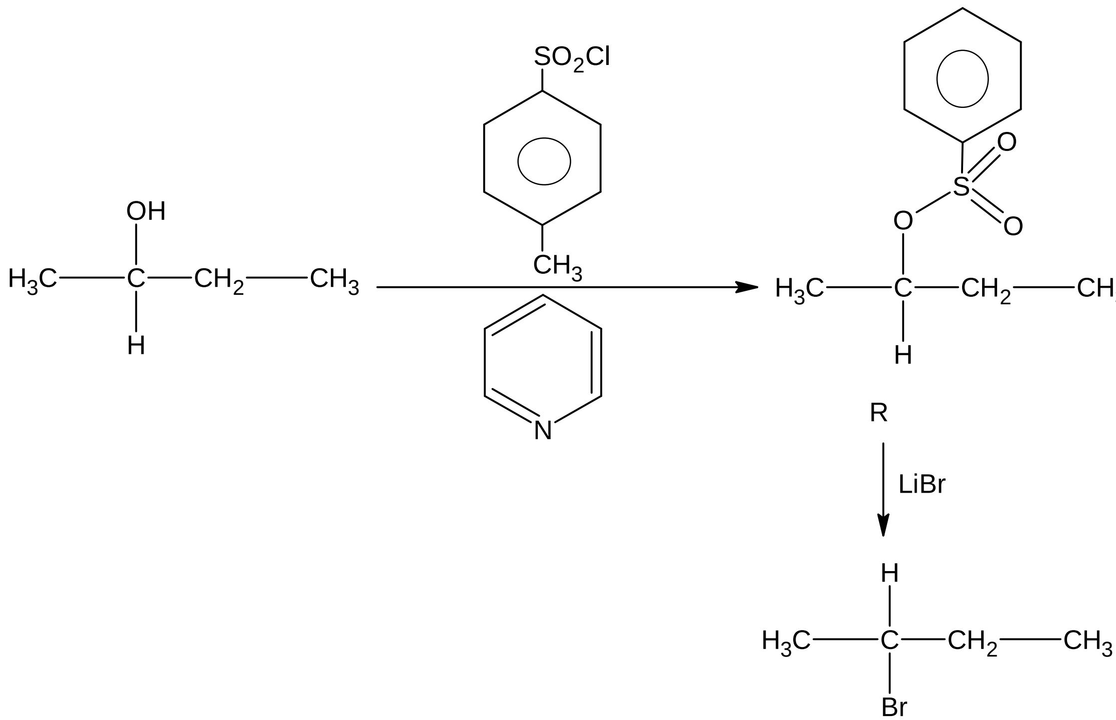
29. D

Sol.



30. D

Sol.



S – 2 – Butylbromide

SECTION – C

31. 00069.50

Sol. M-eq. Of $FeSO_4.7H_2O$ in 1 litre

$= 20 \times 0.02 \times 5 \times \left(\frac{1000}{25}\right) = 80$

$\therefore \frac{W}{278} \times 1 \times 1000 = 80 \Rightarrow W = 22.24$

Mass & of $FeSO_4.7H_2O$ in given sample

$= \frac{22.24}{32} \times 100 = 69.5$

32. 00024.63

Sol. $P_{H_2O} = 760 - 722 = \frac{38}{760}\text{atm};\ n_{H_2O} = \frac{0.9}{18}$

$V = \frac{0.9}{18} \times \frac{0.821 \times 300}{38} \times 760; V = 24.63L$

33. 00039.21

Sol. $-q_{reaction} = q_{bomb} + q_{water}$

$q_{reacton} = (C(bomb) + (m_{water} \times c))\Delta T$

$= (652 + 500 \times 4.18) \times 14.3$

$= 39210J$ or 39.21 kJ

34. 00000.60

Sol. $\overset{+7}{Mn}O_4^- + \overset{+2}{S_2}O_3^{2-} \rightarrow \overset{+4}{Mn}O_2 + \underset{X}{\overset{+6}{SO_4^{2-}}}$

Equivalents of MnO_4^- = equivalents of SO_4^{2-}

Moles of $MnO_4^- \times n$-factor = moles of $SO_4^{2-} \times n-\text{factor}$

$8 \times 3 = X \times 4$

$X = 6$

$\frac{X}{10} = 0.6$

35. 00148.14

Sol. No. of equivalent of aluminium, $\frac{W}{E} = \frac{I \times \eta \times t}{9600}$

$\frac{24 \times 5}{27} \times 3 = \frac{9650 \times 0.9 \times t}{96500}$

$t = 148.14$ sec

36. 00000.02

Sol. $P \propto n,$ initial mole $n_1 = \frac{PV}{RT} = 0.1\,\text{mole}$

$\frac{P_1}{P_2} = \frac{n_1}{n_2}$

Mathematics

PART – III

SECTION – A

37. C

Sol. Correct prediction can be given in 1 way, and Incorrect prediction can be in 2 ways

$\therefore$ Required number of ways = $^{10}C_7\,(1)^7\,(2)^3 = 960$.

38. A

Sol. $\int \frac{1}{\sqrt[3]{x^2}\,\sqrt[3]{(2+3x)^4}}dx$

$= \int \frac{1}{x^{2/3}(2+3x)^{4/3}}dx$

Let $\frac{2+3x}{x} = t \Rightarrow \frac{-2}{x^2}dx = dt$

$= -\frac{1}{2}\int \frac{1}{t^{4/3}}dt = \frac{3}{2}t^{-1/3} = \frac{3}{2}\left[\frac{x}{2+3x}\right]^{1/3} + c$.

39. A

Sol. If the external bisector of $\angle BAC$ meets BC at E, then E divides BC externally in the ratio of AB : AC.

AB = 7, AC = 3.

$\therefore$ Coordinates of E are $\left(\frac{-15}{4}, 7, \frac{11}{4}\right)$

$\therefore$ The direction ratio of line AE are $\frac{-15}{4}+1,\ 7-2,\ \frac{11}{4}+3 = -\frac{11}{4},\ 5,\ \frac{23}{4}$

= – 11, 20, 23

40. A

Sol. $|z-1| + |z-i| = 4$ represents ellipse with foci (1, 0) and (0, –1) and length of major axis = 4.

Centre of ellipse is $\left(\frac{1}{2}, -\frac{1}{2}\right)$ and $\left(\frac{1-i}{2}\right)$

$2a = 4$, $2ae = \sqrt{2} \Rightarrow e = \frac{1}{2\sqrt{2}}$

From $b^2 = a^2(1-e^2)$, we get $b = \sqrt{\frac{7}{2}}$

Now $|2z-1+i| = \sqrt{14}$

$\Rightarrow \left|z-\frac{1}{2}+\frac{i}{2}\right| = \sqrt{\frac{7}{2}}$

$\Rightarrow \left|z-\left(\frac{1-i}{2}\right)\right| = \sqrt{\frac{7}{2}}$ represents a circle having centre at $\frac{1-i}{2}$ and radius $= \sqrt{\frac{7}{2}}$.

Since radius of circle = minor axis of ellipse.

Hence number of solution of given equation = 2.

41. A

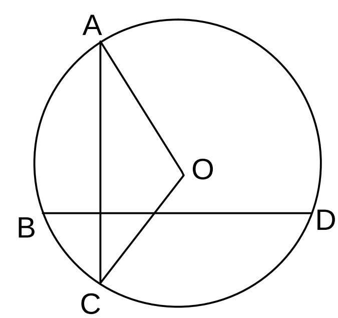
Sol. We know $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{\sin A + \sin B + \sin C}{a+b+c} = \frac{1}{2R}$

$$\frac{\sum \sin A}{\sum a(\cos B + \cos C)} = \frac{1}{2R} \Rightarrow \frac{\sum \sin A}{\sum 2a\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)} = \frac{1}{2R}$$

$$\Rightarrow \frac{\sum \sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)}{\sum \sin A} = R \Rightarrow n = 1.$$

42. C

Sol. As OA = OC



so $\overrightarrow{OA} + \overrightarrow{OC}$ will be perpendicular to AC

$\therefore$ $\overrightarrow{OA} + \overrightarrow{OC} = \lambda\overrightarrow{BD}$ …..(1)

where λ is the real number.

Similarly,

$\overrightarrow{OB} + \overrightarrow{OD} = \mu\overrightarrow{AC}$ ….(2)

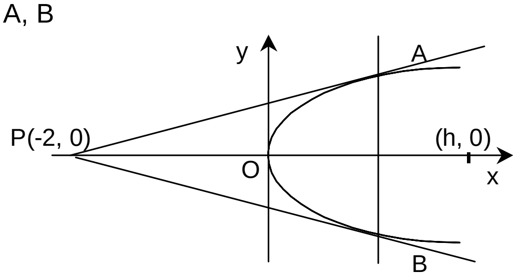
where μ is the real number.

Now using (1) and (2), we get

$\left(\overrightarrow{OA} + \overrightarrow{OC}\right)\left(\overrightarrow{OB} + \overrightarrow{OD}\right) = \mu.\lambda\ \overrightarrow{AC}\cdot\overrightarrow{BD} = 0.$

43. A, B

Sol.



Point 'P' clearly lies on the directrix of $y^2 = 8x$.

Thus slope of PA and PB are 1 and -1 respectively.

Equation of PA : y =x +2, equation of PB : y = -x –2, equation of AB : x = 2.

Let the centre of the circle be (h, 0) and radius be 'r'

$\Rightarrow \frac{|h+2|}{\sqrt{2}} = \frac{|h-2|}{1} = r$

$\Rightarrow$ $h^2 + 4 + 4h = 2(h^2 + 4 - 4h) \Rightarrow h^2 - 12h + 4 = 0$

$h = \frac{12 \pm 8\sqrt{2}}{2} = 6 \pm 4\sqrt{2} \Rightarrow |h-2| = 4(\sqrt{2}-1), 4(\sqrt{2}+1).$

44. B, D

Sol. $a + \bar{c} \neq 0$

$(a+\bar{c})w_1^2 + (b+\bar{b})w_1 + (\bar{a}+c) = 0$

$(\bar{a}+c)\bar{w}_1^2 + (\bar{b}+b)\bar{w}_1 + (a+\bar{c}) = 0$

$\because \bar{w}_1 \neq 0$ $\therefore (a+\bar{c})\frac{1}{\bar{w}_1^2} + (b+\bar{b})\frac{1}{\bar{w}_1} + (\bar{a}+c) = 0$

$\therefore \frac{1}{\bar{w}_1}$ is also a root but $w_2 \neq \frac{1}{\bar{w}_1}$

$\therefore w_1 = \frac{1}{\bar{w}_1} \Rightarrow |w_1| = 1$ and $|w_2| = 1$

45. A, B, C, D

Sol. At $x = -\frac{\pi}{2}, \frac{3\pi}{2}; [1+\sin x] = 0, [1-\cos x] = 1$

$\therefore$ $\sin x = 0 + 1 \Rightarrow -1 = 1$ (absurd)

At x = 0, $[1+\sin x] = 1, [1-\cos x] = 0$

$\therefore$ $\sin x = 1 + 0 \Rightarrow 0 = 1$ (absurd)

At $x = \frac{\pi}{2}$, $[1+\sin x] = 2, [1-\cos x] = 1$

$\therefore$ sin x = 2 + 1 = 3 (absurd)

At x = π, [1+ sin x]= 1, [1 – cos x] = 2

$\therefore$ sin x = 1 +2 = 3 (absurd)

In $\left(-\frac{\pi}{2}, 0\right), [1+\sin x] = 0, [1-\cos x] = 0$

$\therefore$ sin x = 0 + 0 = 0 (absurd)

In $\left(0, \frac{\pi}{2}\right), [1+\sin x] = 1, [1-\cos x] = 0$

$\therefore$ sin x = 1 + 0 = 1 (absurd)

In $\left(\frac{\pi}{2}, \pi\right), [1+\sin x] = 1, [1-\cos x] = 1$

$\therefore$ sin x = 1 + 1 = 2 (absurd)

In $\left(\pi, \frac{3\pi}{2}\right), [1+\sin x] = 0, [1-\cos x] = 1$

$\therefore$ $\sin x = 0 + 1 = 1$ (absurd)

46. A, D

Sol. We have $a^4 + b^4 + c^4 = 2a^2\left(b^2 + c^2\right)$

$a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2 = 2b^2c^2$

$\left(b^2 + c^2 - a^2\right)^2 = 2b^2c^2$

$b^2 + c^2 - a^2 = \sqrt{2}bc$ or $b^2 + c^2 - a^2 = -\sqrt{2}bc$, $\frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{\sqrt{2}}$

or $\frac{b^2 + c^2 - a^2}{2bc} = -\frac{1}{\sqrt{2}}$

$\cos A = \frac{1}{\sqrt{2}} = \cos 45°$

$\cos A = -\frac{1}{\sqrt{2}} = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

47. A, C

Sol. $\log_{\sqrt{3}} \tan\theta\left[\sqrt{\frac{\log_{\sqrt{3}} 3}{\log_{\sqrt{3}} \tan\theta}+\frac{\log\left(\sqrt{3}\right)^3}{\log\sqrt{3}}}\right]=-1$

$\Rightarrow \quad \log_{\sqrt{3}} \tan\theta\left[\sqrt{\frac{2}{\log_{\sqrt{3}} \tan\theta}+3}\right]=-1$

Let $\log_{\sqrt{3}} \tan\theta = y$

$\Rightarrow \quad y\sqrt{\frac{2}{y}+3}=-1 \Rightarrow \sqrt{\frac{2}{y}+3}=-\frac{1}{y} \Rightarrow \frac{2}{y}+3=\frac{1}{y^2}$ or $y^2(2+3y)=y \Rightarrow y\left[3y^2+2y-1\right]=0$

$\because \quad y < 0$

$y(3y-1)(y+1)=0$

$y = -1$ ($\because$ y cannot be positive)

$\Rightarrow \quad \log_{\sqrt{3}} \tan\theta = -1$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\therefore \quad \theta = \frac{\pi}{6}$ and $\frac{7\pi}{6}$

$\therefore$ There are two value of θ in $[0, 2\pi]$

48. A, C

Sol. Let the drawer contains *p* balls of which 'm' are red.

Probability of drawing two red balls at random is $\frac{^mC_2}{^pC_2}=\frac{1}{2}$

$\Rightarrow 2m(m-1)=p(p-1)$

$\Rightarrow 2m^2-2m-p^2+p=0$

$\Rightarrow m=\frac{2\pm\sqrt{4-8(p-p^2)}}{4}=\frac{1\pm\sqrt{1-2p+2p^2}}{2}$

$\Rightarrow 1-2p+2p^2$ should be an odd perfect square.

i.e., $p = 21, 4$ but $p \neq 3$

when 3 balls out of 4 are red.

15 balls out of 21 are red.

SECTION – C

49. 00002.25

Sol. Let the coordinates of a point lying on the straight line 3x + 4y = 24 is $\left(t, \frac{24-3t}{4}\right)$

Equation of the chord of contact is $tx + \frac{y}{16}(24-3t) = 1$

$\Rightarrow (24y - 16) + t(16x - 3y) = 0$

$\Rightarrow$ this line always passes through the fixed point which is the point of intersection of the lines $24y - 16 = 0$ and $16x - 3y = 0$

fixed point $\equiv \left(\frac{1}{8}, \frac{2}{3}\right)$, which lies on $16x - 3y = 0$, $9y^2 = 32x$ and $24x + 24y = 19$.

50. 00900.50

Sol.
$$\text{S. D} = \frac{\begin{vmatrix} 10 & 2 & 3 \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}}{\sqrt{8^2+8^2+4^2}} = \frac{108}{12} = 9$$

51. 00007.00

Sol. Let general point of line be A (2λ + 1, 4λ + 3, 3λ + 2). Let this point lies at the same distance as the point p (3, 8, 2) from the plane 3x + 2y – 2z + 15 = 0

Therefore, $\frac{3.3+2.8-2.2+15}{\sqrt{17}} = \frac{3(2\lambda+1)+2(4\lambda+3)-2(3\lambda+2)+15}{\sqrt{17}}$

$\Rightarrow 36 = 8\lambda + 20 \Rightarrow \lambda = 2$

Therefore, A is (5, 11, 8)

PA = $\sqrt{(5-3)^2+(11-8)^2+(8-2)^2} = \sqrt{4+9+36} = 7$

52. 00000.50

Sol. f(–x) = – f(x) = g(x)

$\therefore$ f(x) . g(x) = – (f(x))^2 or f(1) g(1) = – (f(1))^2 = $-\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 3 & 1 \end{vmatrix}^2 = -4$

$\Rightarrow$ λf(1) g(1) = – 2 $\Rightarrow$ λ(–4) = –2 $\Rightarrow$ λ = $\frac{1}{2}$

53. 00125.00

Sol.
$$|\text{adjB}| = |\text{adj}(\text{adjA})| - |\text{A}|^{(3-1)^2} = |\text{A}|^4$$
$$\because |\text{n}| = 5$$
$$= 125$$

54. 00001.00

Sol. $f_p(\alpha) = e^{\frac{i\alpha}{p^2}(1+2+\ldots+p)} = e^{\frac{i\alpha}{2}\left(1+\frac{1}{p}\right)}$

$\lim_{n\to\infty} f_n(\pi) = \lim_{n\to\infty} e^{\frac{i\alpha}{2}\left(1+\frac{1}{n}\right)} = e^{\frac{i\alpha}{2}}$

$\left|\lim_{n\to\infty} f_n(\pi)\right| = \left|e^{\frac{i\alpha}{2}}\right| = 1$