FIITJEE

Solutions to JEE (Main)-2021

JEE–Main–2021 –Feb–26–Second–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

(PHYSICS)

Answers

Section-A

1.	A	**2.**	B	**3.**	C	**4.**	A
5.	C	**6.**	D	**7.**	A	**8.**	D
9.	D	**10.**	A	**11.**	B	**12.**	D
13.	A	**14.**	D	**15.**	B	**16.**	A
17.	B	**18.**	A	**19.**	C	**20.**	C

Section-B

1.	7	**2.**	60	**3.**	1	**4.**	150
5.	9	**6.**	25	**7.**	3	**8.**	9
9.	243	**10.**	4				

SECTION – A

Sol1.

$mg - T = ma$ .... (1)

$T \times R = I\,\alpha$ .... (2)

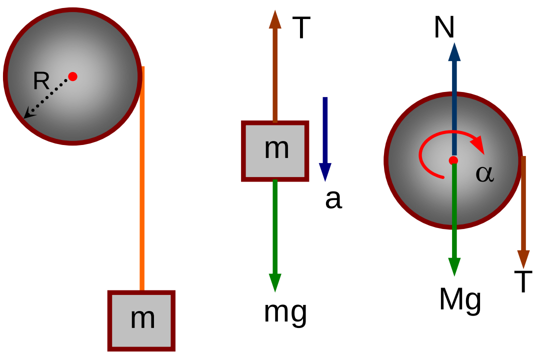
$a = \alpha R$ ....(3)

With the help of equations (1) , (2) and (3), we get

$$\Rightarrow a = \frac{mg}{m + \frac{I}{R^2}}$$

$$v = \sqrt{2ah} = \omega R$$

$$\Rightarrow \omega^2 = \frac{2\,mgh}{I + mR^2}$$



Sol2. $y = \alpha x - \beta x^2$

$$\Rightarrow \frac{dy}{dx} = \alpha - 2\beta x = 0$$

$$\Rightarrow x = \frac{\alpha}{2\beta}$$

$$y_{max} = \alpha \times \frac{\alpha}{2\beta} - \beta \times \left(\frac{\alpha}{2\beta}\right)^2 = \frac{\alpha^2}{4\beta}$$

$$H = \frac{\alpha^2}{4\beta} = \frac{u^2 \sin^2\theta}{2g}$$

& On comparing with

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$\Rightarrow \tan\theta = \alpha$

$\Rightarrow \theta = \tan^{-1}(\alpha)$

Sol3. Stress = Y× strain

$\Rightarrow \quad \frac{T_1}{A} = Y\times\frac{(\ell_1-\ell)}{\ell}$ (i)

$\frac{T_2}{A} = Y\times\frac{(\ell_2-\ell)}{\ell}$ (ii)

$\Rightarrow \quad \frac{T_1}{T_2} = \frac{\ell_1-\ell}{\ell_2-\ell}$

$$\Rightarrow \ell = \frac{T_1\ell_2 - T_2\ell_1}{T_1-T_2} = \frac{T_2\ell_1-T_1\ell_2}{T_2-T_1}$$

Sol4. $A = A_0e^{-\lambda t_1}$ [Radio active decay law]

$\frac{A}{5} = A_0\,e^{-\lambda t_2}$

$\Rightarrow \ln 5 = \lambda(t_2 - t_1)$

$\Rightarrow$ Average life $= \frac{1}{\lambda} = \frac{(t_2-t_1)}{\ell n\,5}$

Sol5. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\Rightarrow \quad Z = \sqrt{(120)^2 + (10-100)^2} = 150\Omega$

$I_0 = \frac{V_0}{Z} = \frac{30}{150} = 0.2\,A$

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-1}\times10^{-4}}} = \sqrt{10^5}$

$\because \quad \omega = 2\pi f$

$\Rightarrow \quad f = \frac{10^3}{2\pi\sqrt{10}} = 50\,Hz$

Sol6. Frequency increases on filing. So, initial frequency of A is 335 Hz.

$f = 340 - 5 = 335$ Hz

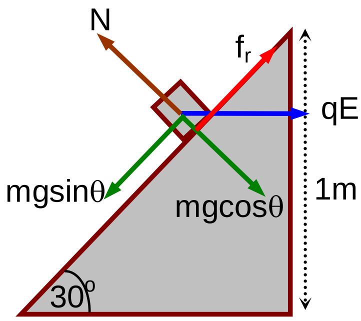
Sol7.

$N = mg\cos 30^\circ + qE\sin 30^\circ$

$a = \dfrac{mg\sin\theta - qE\cos\theta - \mu N}{m} = 2.30\, m/s^2$

$S = ut + \dfrac{1}{2}at^2$

$\Rightarrow \quad t = \sqrt{\dfrac{2\ell}{a}} = 1.31$ sec



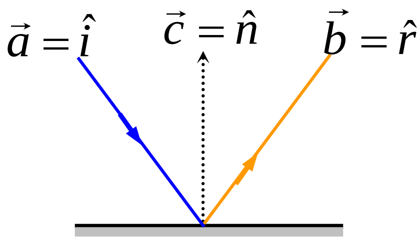
Sol8.

$\hat{r} - \hat{i} = 2\cos\theta\,\hat{n}$ .....(1)

$\hat{i}\,.\,\hat{n} = -\cos\theta$ ......(2)

$\Rightarrow \hat{r} = \hat{i} - 2(\hat{i}\,.\,\hat{n})\hat{n}$

$\Rightarrow \vec{b} = \vec{a} - 2(\vec{a}\,.\,\vec{c})\,\vec{c}$



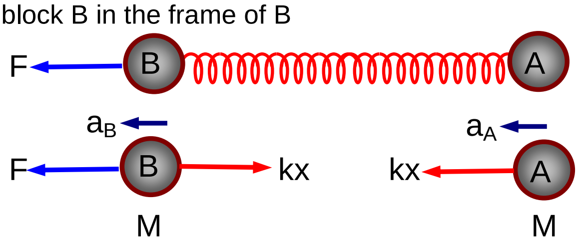
Sol9. Let the spring is in extension state and $a_B > a_A$, $\vec{a}_{AB} = -a_A\hat{i} - (-a_B\hat{i}) = (-a_A + a_B)\hat{i}$

Hence we can say that block moves away from block B in the frame of B

$F - kx = Ma_B$ ....(1)

$kx = Ma$ ....(2)

$\Rightarrow a_B = \dfrac{F}{M} - a$



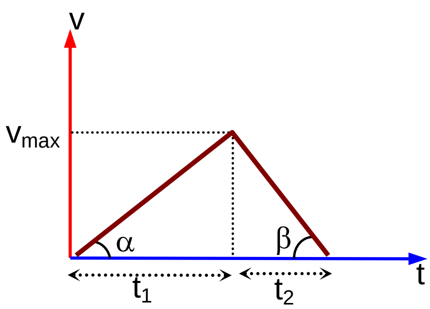
Sol10. $v = \omega\sqrt{A^2 - x^2} \Rightarrow \dfrac{v^2}{A^2\omega^2} + \dfrac{x^2}{A^2} = 1 \Rightarrow$ Path is ellipse.

Sol11.

$\tan\alpha = \dfrac{v_{max}}{t_1} = a_1$

$\tan\beta = \dfrac{v_{max}}{t_2} = a_2$

$\Rightarrow \dfrac{a_1}{a_2} = \dfrac{t_2}{t_1} \Rightarrow \dfrac{t_1}{t_2} = \dfrac{a_2}{a_1}$



Sol12. $\lambda = \dfrac{c}{v} = \dfrac{Q^3}{W^2} = \dfrac{I^3T^3}{M^2L^4T^{-4}} \Rightarrow [\lambda] = [M^{-2}L^{-4}T^7I^3]$

Sol13. $U = 3PV + 4$

$\Rightarrow \quad nC_vT = 3PV + 4$

$\Rightarrow \quad n \times \dfrac{fRT}{2} = 3PV + 4$

$\Rightarrow \quad \dfrac{f}{2} \times PV = 3PV + 4$

$\Rightarrow \quad f = 6 + \frac{8}{PV}$

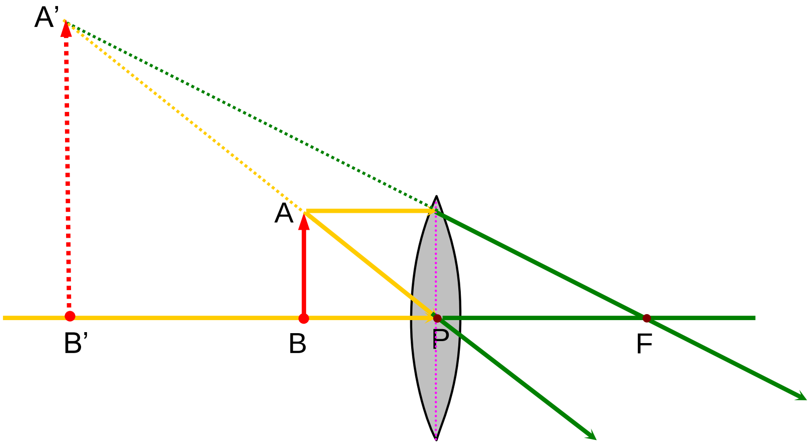
$\Rightarrow \quad f > 6 \;\Rightarrow$ poly atomic

Sol14. $B_v = B \sin 60^\circ$

$\Rightarrow \quad B_v = 2.5 \times 10^{-4} \times \frac{\sqrt{3}}{2}$

$\text{Emf} = B_v \times v \times \ell = 2.5 \times 10^{-4} \times \frac{\sqrt{3}}{2} \times 180 \times \frac{5}{18} \times 1 = 108.25 \times 10^{-3}$ volts

Sol15. Fact based



Sol16. $R_i = \frac{\rho\ell}{A}$

$R_f = \frac{\rho\,(1.25\,\ell)}{(A/1.25)} = (1.25)^2 \times \frac{\rho\,\ell}{A}$

$\Rightarrow \quad R_f = 1.5625 \times R_i$

$\Rightarrow \quad \frac{R_f - R_I}{R_I} \times 100 = 56.25\%$

Sol17. Time period of second pendulum is 2 seconds.

Sol18. $\Delta E = 13.6\left(\frac{1}{1^2} - \frac{1}{5^2}\right) = 13.6 \times \frac{24}{25}$ eV

$\Rightarrow \frac{hc}{\lambda} = 13.6 \times \frac{24}{25}$ eV ....(1)

With the help of conservation of linear momentum , we can write

$\frac{h}{\lambda} = m_H v_H \Rightarrow \frac{hc}{\lambda} = c m_H v_H \Rightarrow v_H = \frac{\frac{hc}{\lambda}}{c m_H} = \frac{13.6 \times \frac{24}{25} \times 1.6 \times 10^{-19}}{3 \times 10^{8} \times 1.67 \times 10^{-27}} = 4.17 \text{ m/s}$

Sol19. All the charge given to a conducting sphere resides on outer surface.

Sol20. $y = \overline{\overline{A}+B} = A.\overline{B}$

Section – B

Sol1. Total oscillation = $\frac{1}{2}+\frac{1}{8}=\frac{5}{8}$

$\Rightarrow$ time taken = $\frac{T}{2}+\frac{T}{12}=\frac{7T}{12}$

$\Rightarrow \alpha = 7$

Sol2. $V = kT^{2/3}$

$\Rightarrow TV^{-3/2} = K$

$\Rightarrow \gamma - 1 = -3/2$

$\Rightarrow \gamma = -1/2$

Work done = $\frac{nR\Delta T}{-y+1} = \frac{1\times R\times 90}{3/2} = 60\,R$

$\Rightarrow$ **n = 60**

Sol3. Case I : - $2\phi - \phi = \frac{1}{2}mv_1^2$ .....(i)

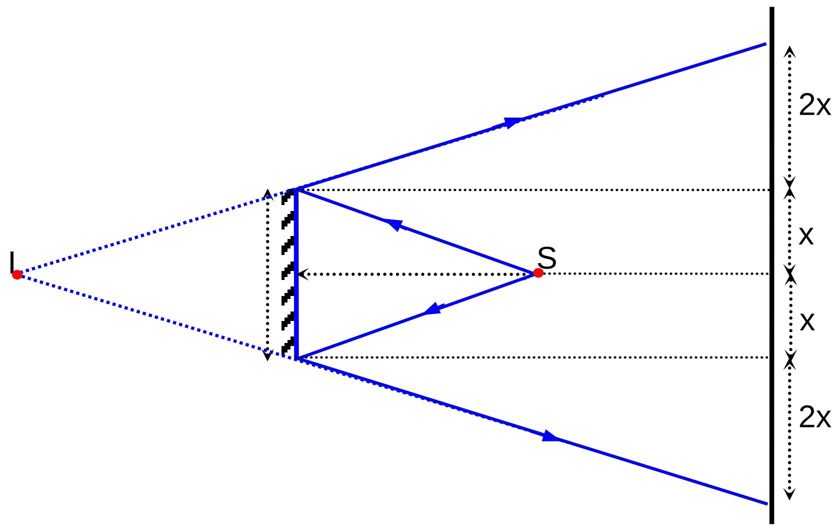
Case II : - $10\phi - \phi = \frac{1}{2}mv_2^2$ ......(ii)

$\Rightarrow \frac{1}{9} = \frac{v_1^2}{v_2^2}$

$\Rightarrow v_1 : v_2 = 1 : 3$

$\Rightarrow$ **x = 1**

Sol4.



$\Rightarrow$ Maximum Distance $= 3(x+x) = 6x = 3\times 2x = 3\times 50 = 150\,cm$

Sol5. Band Width $= 2 \times n \times$ Highest modulation frequency

$$\Rightarrow n = \frac{90kHz}{2 \times 5kHz} = 9$$

Sol6. $\Delta Q = \Delta U + \Delta W$

$$\Rightarrow Q = \Delta U + \frac{Q}{5} \Rightarrow \Delta U = \frac{4Q}{5} = n\,C_v\,\Delta T \Rightarrow \frac{4Q}{5} = \frac{5R}{2}\,\Delta T \Rightarrow \Delta T = \frac{8Q}{25R}$$

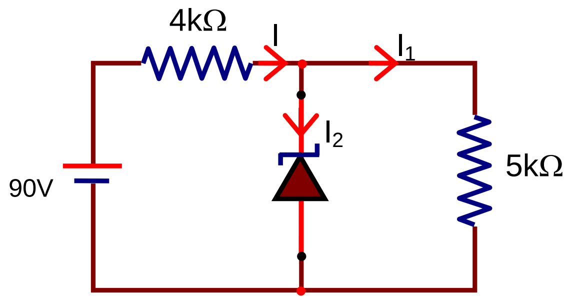
$$Q = nc\Delta T = 1 \times C \times \frac{80}{25R} \Rightarrow C = \frac{25\,R}{8} \Rightarrow x = 25$$

Sol7. $v = \omega\sqrt{A^2 - s^2} \Rightarrow \frac{A\omega}{2} = \omega\sqrt{A^2 - s^2} \Rightarrow s = \frac{\sqrt{3}\,A}{2} \Rightarrow x = 3$

Sol8.

$$I = \frac{90 - 30}{4000} = 15\text{ mA}$$

$$I_1 = \frac{30}{5000} = 6\text{mA} \;\&\; I_2 = 9\text{ mA}$$



Sol9. With the help of conservation of Volume , we can write

$$27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R = 3r \text{ ....(1)}$$

With the help of conservation of Charge , we can write

$$Q = 27q \text{ ....(2)}$$

Potential energy of single drop $= U_1 = \frac{q^2}{8\pi\varepsilon_0 r}$

Potential energy of bigger drop $= U_2 = \frac{Q^2}{8\pi\varepsilon_0 R} = \frac{27 \times 27 \times q^2}{8\pi\varepsilon_0 (3r)} = 243\left(\frac{q^2}{8\pi\varepsilon_0 r}\right) = 243U_1$

$$\Rightarrow \frac{U_2}{U_1} = 243$$

Sol10. $\frac{GM}{(3R/2)^2} = \frac{GM}{R^3} \times r$

$$\Rightarrow OA = \frac{4R}{9} = r$$

$$AB = R - \frac{4\,R}{9} = \frac{5R}{9} \Rightarrow OA : AB = 4 : 5 = x : y \Rightarrow x = 4$$

PART –B (CHEMISTRY)

Answers

Section-A

1.	B	**2.**	A	**3.**	B	**4.**	B
5.	D	**6.**	D	**7.**	D	**8.**	D
9.	C	**10.**	C	**11.**	B	**12.**	A
13.	D	**14.**	B	**15.**	A	**16.**	C
17.	B	**18.**	D	**19.**	B	**20.**	A

Section-B

1.	309	**2.**	1	**3.**	6	**4.**	2
5.	14	**6.**	1	**7.**	7	**8.**	3
9.	147	**10.**	13				

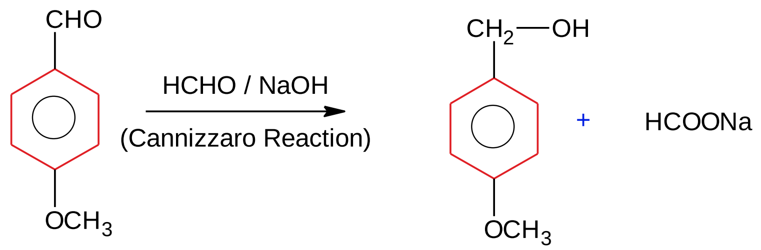
SECTION – A

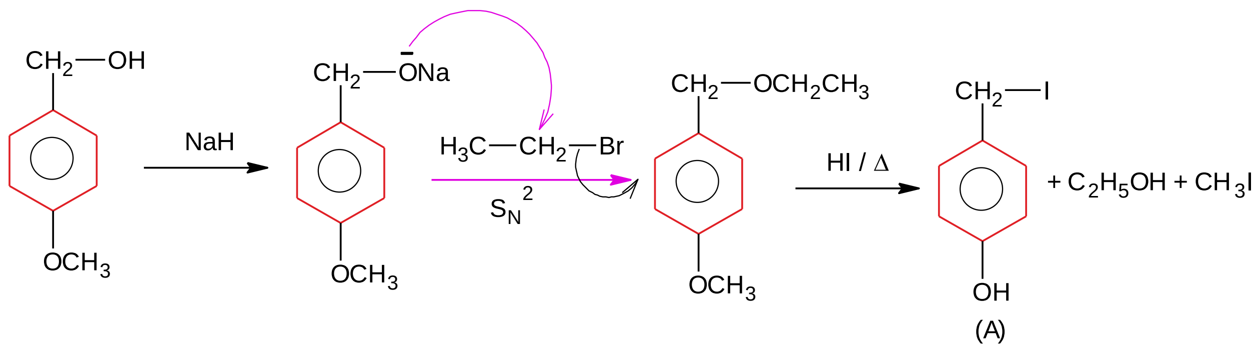
Sol1. Siderite – $FeCO_3$ (ore of iron)
Calamine – $ZnCO_3$ (ore of zinc)
Malachite – $CuCO_3.Cu(OH)_2$ (ore of copper)
Cryolite – Na_3AlF_6 (ore of aluminium)

Sol2. Since $SOCl_2$ is used to convert aliphatic (R-OH) into chlorides. It will not react with aromatic alcohol

Sol3. Tritium is radioactive and it decays into He^3 during emission of β- radiation
$${}_1T^3 \longrightarrow {}_2He^3 + {}_{-1}e^o$$

Sol4.



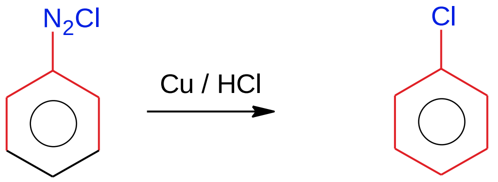


Sol5. $FeCl_3 \xrightarrow{\text{hydrolysis}} Fe(OH)_3 \downarrow \xrightarrow[\text{Adsorption}]{Fe^{3+}} Fe(OH)_3 \mid Fe^{3+}$ (colloidal particle)

Sol6.

(a) 

(Sandmeyer Reaction)

(b) 

(Gattermann Reaction)

(c) $2CH_3CH_2Cl \xrightarrow[\text{ether}]{2Na} C_2H_5\text{-}C_2H_5 + 2NaCl$ (Wurtz Reaction)

(d) $2C_6H_5Cl \xrightarrow[\text{ether}]{2Na} C_6H_5\text{-}C_6H_5 + 2NaCl$ (Wittig Reaction)

Sol7. Ceric ammonium nitrate is used to test alcohol while $CHCl_3$ / alc. KOH is used to test 1º amine

Sol8. It does not contains 2nd most abundant element by weight in earth crust because that is Si

Calgon $\longrightarrow Na_2[Na_4(PO_3)_6] \xrightarrow{\text{Water soluble}} 2Na^+ [Na_4(PO_3)_6]^{2-}$.

$\xrightarrow{Ca^{2+}} 2Na^+[Na_2Ca(PO_3)_6]^{2-}$

Sol9.

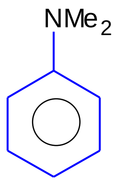
$$\underset{sp^2}{\overset{1}{CH_2}}=\underset{sp}{\overset{2}{C}}=\underset{sp^2}{\overset{3}{CH}}-\underset{sp^3}{\overset{4}{CH_3}}$$

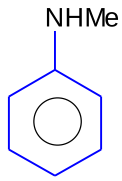
Sol10. (a) Sucrose $\rightarrow$ α-D glucose and β-D fructose
(b) Lactose $\rightarrow$ β-D- galactose and β-D glucose
(c) Maltose $\rightarrow$ α-D glucose and α-D- glucose

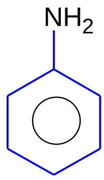
Sol11. Both (A) and (B) are correct but R is not correct explanation of A. In both $T\ell I_3$ and CsI_3 oxidation state of metal is +1, also both have similar lattice structure. $T\ell = 4f^{14}5d^{10}6s^2 6p^1$

Sol12. S > Se > Te > O

Sol13. A > B > C > D

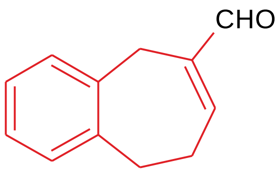
(A) $Ph-CH_2-\ddot{N}H_2$ (B) 

(C) 

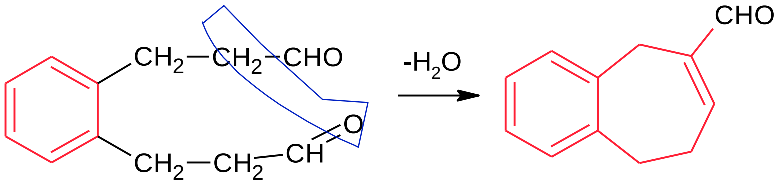
(D) 

Lone pair is localized in (A) while all 3 have delocalized lone pair but they can be compared by 3º > 2º > 1º because methyl group increases the basicity.

Sol14.



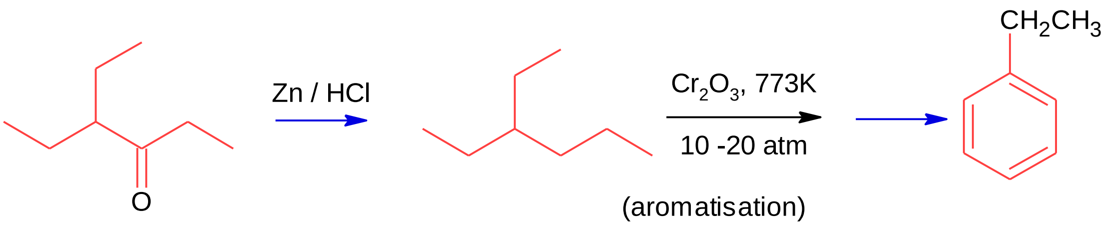
It is an intramolecular aldol condensation



Sol15. (a) $Na_2CO_3 \rightarrow$ Solvay
(b) Ti $\rightarrow$ Van-Arkel
(c) $Cl_2 \rightarrow$ Deacon
(d) NaOH $\rightarrow$ Castner- kellner

Sol16. 2,4- DNP (Brady's Reagent) is used to test carbonyl compound

Sol17.



Sol18.

Species	B.O
Ne_2	0
N_2	3
F_2	1
O_2	2

Sol19.

Oxide	Nature
CaO	Basic
B_2O_3	Acidic
SiO_2	Acidic
BaO	Basic
N_2O	Neutral

Sol20. Seliwanoff's test is used to distinguish carbohydrates while xanthoproteic test is used to distinguish proteins

SECTION-B

Sol1. $S(g) + 6F(g) \longrightarrow SF_6(g)$
$\Delta H^o_R = \Delta H^o_f(SF_6) - \Delta H^o_f(S) - 6\times\Delta H^o_f(F)$
$=(-1100) - (275) - 6(80) = -1855$
$\Delta H^o_{S-F} = \frac{1855}{6} = 309.16$ KJ/mol $\quad \cong 309$

Sol2. Effective no. of octahedral voids in a lattice = n
Effective no. of lattice point in a lattice = n
$\therefore$ Ratio = n/n = 1

Sol3. $MnO_4^- + S_2O_3^{2-} \xrightarrow{OH^-} MnO_2 + SO_4^{2-}$
O.S of S in $SO_4^{2-} = +6$

Sol4. $\Delta T_f = K_f \times m \times i$
$0.93 = 1.86 \times 1 \times i$
$i = ½$

$i = 1 + \left(\frac{1}{n} - 1\right)$

$\therefore$ n=2

Sol5. Fraction of molecules having enough energy to form product = $e^{-Ea/RT}$

Fraction of molecules having enough energy to form product = $e^{-\frac{80.9\times10^3}{8.314\times700}}$

$= e^{-13.8} \cong e^{-14}$

So, x = 14

Sol6. Uncertainty in speed = $90 \times \frac{5}{100} = 4.5 m/s = \Delta V$

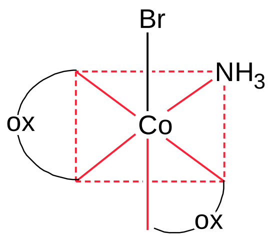
$\Delta x = \frac{h}{4\pi m \Delta V} = \frac{6.63\times10^{-34}}{4\times3.14\times10^{-2}\times4.5} = 1.173\times10^{-33}$

Nearest integer is 1

Sol7. pH = $\frac{1}{2}\left[pK_w + pK_a - pK_b\right]$

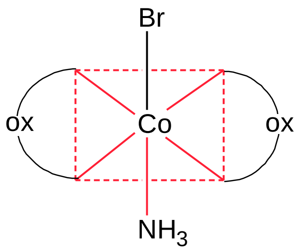
$= \frac{1}{2}[14 + 5.23 - 4.75]$ = 7.24 ≈ 7

Sol8.



Cis (optically active)

d & l form (2)



Trans (optically inactive)

(1)

So, total 3 isomerism.

Sol9. $E^o_{Cell} = E^o_{Ag^+/Ag} - E^o_{Zn^{+2}/Zn}$ = 0.8 +0.76 =1.56 V

Anode : $Zn(s) \rightarrow Zn^{2+}(aq) + 2e^-$

Cathode: $2Ag^+(aq) + 2e^- \rightarrow 2Ag(s)$

$Zn(s) + 2Ag^+(aq) \rightarrow Zn^{2+}(aq) + 2Ag(s)$

$E_{cell} = E^o_{Cell} - \frac{0.0591}{n}\log\frac{\left[Zn^{2+}\right]}{\left[Ag^+\right]^2} = 1.56 - \frac{0.0591}{2}\log\left(\frac{0.1}{10^{-4}}\right)$

= 1.56 – 0.088 = 1.472 V= 147 × 10^{-2}V

x = 147

Sol10. Mass of Na^+ in 50 ml = 70 × 50 = 3500mg

23000 mg of Na^+ is present in 85000 mg of $NaNO_3$(1 mole $NaNO_3$ contains 1 mole Na^+)

$\therefore$ 3500 mg Na^+ will be present in $\frac{85000}{23000}\times3500$

=12934.78 mg

=12.93478 gm

≅13

PART–C (MATHEMATICS)

Answers

Section-A

1.	A	**2.**	C	**3.**	C	**4.**	B
5.	A	**6.**	B	**7.**	C	**8.**	B
9.	C	**10.**	C	**11.**	D	**12.**	A
13.	A	**14.**	B	**15.**	C	**16.**	B
17.	B	**18.**	A	**19.**	B	**20.**	B

Section-B

1.	3	**2.**	10	**3.**	1000	**4.**	324
5.	48	**6.**	2	**7.**	4	**8.**	1
9.	9	**10.**	4				

SECTION – A

Sol1. Given $f(x) = \int_e^x e^t f(t)\, dt + e^x$............(i)

using Leibniz rule then

$f'(x) \quad = e^x f(x) + e^x$

$\Rightarrow \frac{dy}{dx} = e^x y + e^x$ where $y = f(x)$ then $\frac{dy}{dx} = f'(x)$

$\Rightarrow \frac{dy}{dx} - e^x y = e^x$

$P = -e^x, Q = e^x$

Solution be $y.(\text{I. F.}) = \int Q\,(\text{I.F.})\, dx + c$

$\text{I. f.} = e^{\int -e^x dx} = e^{-e^x}$

$\Rightarrow y.(e^{-e^x}) = \int e^x . e^{-e^x} dx + c$

Put $e^{-e^x} = t$

$e^{-e^x}(-e^x)dx = dt$

$y.e^{-e^x} = -\int dt + c = -t + c = -e^{-e^x} + c$ ....(ii)

Put x = 0, in (i) f (0) = 1

From (ii), $\frac{1}{e} = -\frac{1}{e} + c$ given $c = \frac{2}{e}$

From (ii), $y.e^{-e^x} = -e^{-e^x} + \frac{2}{e}$

Hence $f(x) = 2.e^{(e^x - 1)} - 1$

Sol2. $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ & $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$

Given $\vec{a}_1$ & $\vec{a}_2$ are collinear then $\vec{a}_1 = \lambda \vec{a}_2$

$\Rightarrow (x\hat{i} - \hat{j} + \hat{k}) = \lambda(\hat{i} + y\hat{j} + z\hat{k})$

$\Rightarrow (x - \lambda)\hat{i} - (1 + \lambda y)\hat{j} + (1 - \lambda z)\hat{k} = 0$

Since $\hat{i}, \hat{j}$ & $\hat{k}$ are not collinear so

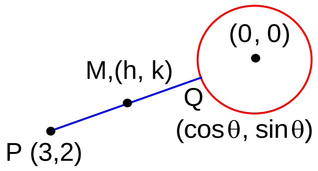
$x - \lambda = 0$, $1 + \lambda y = 0$ & $1 - \lambda z = 0$

$x = \lambda$ $y = \frac{-1}{\lambda}$ and $z = \frac{1}{\lambda}$

So $x\hat{i} + y\hat{j} + z\hat{k} = \lambda\hat{i} - \frac{1}{\lambda}\hat{j} + \frac{1}{\lambda}\hat{k}$

Hence possible unit vector parallel to it be $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ for $\lambda = 1$

Sol3. $h = \frac{\cos\theta + 3}{2}$



$k = \frac{\sin\theta + 2}{2}$

$\Rightarrow \cos\theta = 2h - 3$ & $\sin\theta = 2k - 2$

$\cos^2\theta + \sin^2\theta = 1$ gives $(2h-3)^2 + (2k-2)^2 = 1$

$\Rightarrow (h - 3/2)^2 + (k-1)^2 = \frac{1}{4}$

$\therefore$ circle of radius $r = \frac{1}{2}$

Sol4. $g(2) = \lim\limits_{x\to 2} g(x) = \lim\limits_{x\to 2} \frac{x^2 - x - 2}{2x^2 - x - 6} = \lim\limits_{x\to 2} \frac{(x-2)(x+1)}{2x(x-2) + 3(x-2)}$

$= \lim\limits_{x\to 2} \frac{(x-2)(x+1)}{(2x+3)(x-2)} = \lim\limits_{x\to 2} \frac{x+1}{2x+3} = \frac{3}{7}$

Now fog $= f(g(x)) = \sin^{-1} g(x) = \sin^{-1}\left(\frac{x^2 - x - 2}{2x^2 - x - 6}\right)$

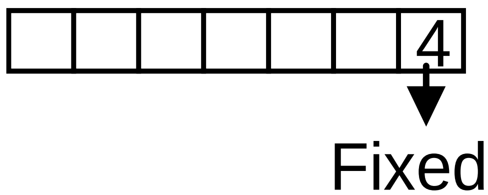
$\Rightarrow$ for domain $-1 \le \frac{x^2 - x - 2}{2x^2 - x - 6} \le 1$ $\frac{x^2 - x - 2}{2x^2 - x - 6} + 1 \ge 0$ & $\frac{x^2 - x - 2}{2x^2 - x - 6} - 1 \le 0$

$\frac{3x^2 - 2x - 8}{2x^2 - x - 6} \ge 0$ & $\frac{-x^2 + 4}{2x^2 - x - 6} \le 0$

On solving we get $x \in (-\infty, -2) \cup \left[\frac{-4}{3}, \infty\right)$

As x = 2 also lies in domain since $g(2) = \lim\limits_{x\to 2} g(x)$

Sol.5. 3, 3, 4, 4, 4, 5, 5



In remaining six places you have to arrange
3, 3, 4, 4, 5, 5

So no. of ways $= \frac{6!}{2!\,2!\,2!}$

Total no. of seven digits nos. $= \frac{7!}{2!\,3!\,2!} \times 1$

Hence Req. prob. $\frac{\frac{6!}{2!\,2!\,2!}}{\frac{7!}{2!\,3!\,2!}} = \frac{6!\,3!}{2!\,7!} = \frac{3}{7}$

Sol6. $\lim\limits_{x\to a} \frac{x\,f(a) - a\,f(x)}{x - a}\left(\frac{0}{0}\right)$

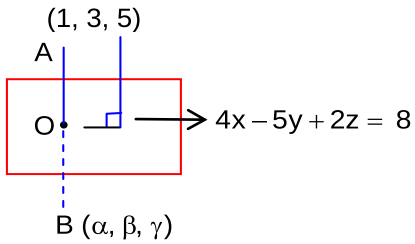
By L'hospital Rule

$= \lim\limits_{x\to a} \frac{f(a) - af'(x)}{1} = f(a) - af'(a)$

$= 4 - 2a$

Now equation of line OA be

Sol7. Now equation of line OA be



$$\frac{x-1}{4} = \frac{y-3}{-5} = \frac{z-5}{2} = \lambda$$

direction cosines of plane are 4, -5, 2

Equation of any point on OA be

$O\,(4\lambda+1,\, -5\lambda+3,\, 2\lambda+5)$

Since O lies on given plane so

$$4(4\lambda+1)-5(-5\lambda+3)+2(2\lambda+5)=8$$

$(45\lambda = 9)$ gives $\lambda = \frac{1}{5}$

So, $O(9/5, 2, 27/5)$. Hence by mid-point formula

$$B\left(\frac{13}{5}, 1, \frac{29}{5}\right) \Rightarrow 5(\alpha+\beta+\gamma) = 47$$

Sol8. $$\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$$

put $2n+1 = r,\quad r = 3, 5, 7, \ldots..$

so $n = \frac{r-1}{2}$

Now $$\sum_{r=(3,5,7,\ldots..)}^{\infty} \frac{n^2+6n+10}{(2n+1)!} = \sum_{r=(3,5,7,\ldots..)}^{\infty} \left(\frac{\frac{(r-1)^2}{4}+3r-3+10}{r!}\right) = \sum_{r=(3,5,7,\ldots.)}^{\infty} \left(\frac{r^2+10r+29}{4.r!}\right)$$

Now $$\sum_{r=3,5,7,\ldots.}^{\infty} \frac{r^2+10r+29}{4.r!} = \frac{1}{4}\sum_{r=3,5,7,\ldots.}^{\infty} \frac{r(r-1)+11r+29}{r!}$$

$$= \frac{1}{4}\sum_{r=3,5,7,\ldots.}^{\infty}\left(\frac{1}{(r-2)!}+\frac{11}{(r-1)!}+\frac{29}{r!}\right)$$

$$= \frac{1}{4}\left[\left(\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots.\right)+11\left(\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots.\right)+29\left(\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+\ldots\right)\right]$$

$$= \frac{1}{4}\left\{\frac{e-\frac{1}{e}}{2}+11\left(\frac{e+\frac{1}{e}-2}{2}\right)+29\left(\frac{e-\frac{1}{e}-2}{2}\right)\right\}$$

$$= \frac{1}{8}\left\{41e-\frac{19}{e}-80\right\} = \frac{41}{8}e-\frac{19}{8}e^{-1}-10$$

Sol9. Given $f(k) = \begin{cases} k+1, & k \text{ is odd} \\ k, & k \text{ is even} \end{cases}$

$\because g : A \to A$ such that $g(f(x)) = f(x)$

Case I : If x is even then $g(x) = x$........(i)

Case II : If x is odd then $g(x+1) = x+1$........(ii)

From (i) & (ii), $g(x) = x$, when x is even

So total no. of functions $= 10^5 \times 1 = 10^5$

Sol10. Given $\frac{dy}{dx} = \frac{xy^2+y}{x} = y^2 + \frac{y}{x}$

OR $\frac{dy}{dx} - \frac{y}{x} = y^2$ OR $\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{x}.\frac{1}{y} = 1$……….(i)

put $\frac{1}{y} = t$ then $-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

From (i) $\frac{-dt}{dx} - \frac{1}{x}.t = 1$ OR $\frac{dt}{dx} + \frac{t}{x} = -1$

$P = \frac{1}{x}$, $Q = -1$

I.F. = $e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

Solution be $t.x = \int -1.(x)\,dx + c = \frac{-x^2}{2} + c$

$\Rightarrow \frac{x}{y} = -\frac{x^2}{2} + c$…………(ii)

Since curve intersect $x + 2y = 4$ at $x = -2$ then y = 3 so

From (ii) $\frac{-2}{3} = -2 + c$ OR $c = 2 - \frac{2}{3} = \frac{4}{3}$

From (ii) $\frac{x}{y} = \frac{-x^2}{2} + \frac{4}{3}$

put $x = 3$, then $\frac{3}{y} = \frac{-9}{2} + \frac{4}{3} = \frac{-19}{6}$

$\Rightarrow y = \frac{-18}{19}$

Sol11. Given $n = 2^x.3^y.5^z$……….(i)

$y + z = 5$ & $\frac{1}{y} + \frac{1}{z} = \frac{5}{6}$

On solving we get $y = 3, z = 2$

So, $n = 2^x.3^3.5^2$

So that no. of odd divisor $= (3+1)(2+1) = 12$

Hence no. of divisors including $1 = 12$

Sol12. Given $x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$

Here $\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix} = (42-22) - 2(14+11) - 3(-4-6) = 20 - 50 + 30 = 0$

$\Delta_1 = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix} = a(42-22) - 2(7b+11c) - 3(-2b-6c) = 20a - 8b - 4c$

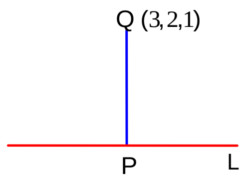
$\Delta_2 = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix} = (7b+11c) - a(14+11) - 3(2c-b) = 10b - 25a + 5c$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix} \begin{matrix} = (6c+2b) - 2\,(2c-b) + a\,(-4-6) \\ = 2c + 4b - 10a \end{matrix}$$

For infinite solution $\Delta_1 = \Delta_2 = \Delta_3$

$20a - 8b - 4c = 0 \Rightarrow 5a = 2b + c$

Sol13. For line of intersection of two planes



put $z = \lambda$ then

$x + 2y = 6 - \lambda$ & $y = 4 - 2\lambda \Rightarrow x + 2(4 - 2\lambda) = 6 - \lambda$

$\Rightarrow x = 3\lambda - 2$

$\Rightarrow \dfrac{x+2}{3} = \dfrac{y-4}{-2} = \dfrac{z-0}{1} = \lambda$ symmetrical form of line

Let P be $(3\lambda - 2, -2\lambda + 4, \lambda)$

So, $\overrightarrow{PQ} = (3\lambda - 5)\,\hat{i} + (-2\lambda + 2)\,\hat{j} + (\lambda - 1)\,\hat{k}$

Parallel vector to given line L is

$\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$

Now $\vec{a}.\overrightarrow{PQ} = 0$ gives $9\lambda - 15 + 4\lambda - 4 + \lambda - 1 = 0$

$14\lambda = 20 \Rightarrow \lambda = \dfrac{10}{7}$

So, $P\left(\dfrac{16}{7}, \dfrac{8}{7}, \dfrac{10}{7}\right) = P(\alpha, \beta, \gamma)$

$\Rightarrow 21(\alpha + \beta + \gamma) = 21 \times \dfrac{34}{7} = 102$

Sol14. Given $f(x) = \displaystyle\int_1^x \frac{\log_e t}{(1+t)}\,dt$ ....(i)

So $f\left(\dfrac{1}{x}\right) = \displaystyle\int_1^{1/x} \frac{\lambda og_e t}{1+t}\,dt$ ....(ii)

put $t = \dfrac{1}{z}$ then $dt = -\dfrac{1}{z^2}\,dz$

$$f\left(\frac{1}{x}\right) = \int_1^x \frac{\log_e \frac{1}{z}}{1 + \frac{1}{z}} \times \left(-\frac{1}{z^2}\right) dz = \int_1^x \frac{\log_e z}{z(1+z)} dz$$

OR $f\left(\dfrac{1}{x}\right) = \displaystyle\int_1^x \frac{\log_e t}{t(1+t)}\,dt$ ....(iii)

(i) + (iii), $f(x) + f\left(\dfrac{1}{x}\right) = \displaystyle\int_1^x \left(\frac{\log_e t}{1+t} + \frac{\log_e t}{t(1+t)}\right) dt$

$$= \int_1^x \frac{(\log_e t)(1+t)}{t\,(1+t)}\,dt = \int_1^x \frac{\log_e t}{t}\,dt$$

$$= \left[\frac{(\log_e t)^2}{2}\right]_1^x = \frac{(\log_e x)^2}{2}$$

Hence $f(e) + f\left(\dfrac{1}{e}\right) = \dfrac{1}{2}$

Sol15. Given

$$(x)=\begin{cases} 2\sin\left(\frac{-\pi x}{2}\right) & , \quad x<-1 \\ \left|ax^2+x+b\right| & , \quad -1\le x\le 1 \\ \sin\pi x & , \quad x>1 \end{cases}$$

If f (x) is continuous for all $x\in R$ then it should be continuous at $x=1$ & $x=-1$.

At $x=-1$, L. H. L = R.H.L, $\Rightarrow 2=|a+b-1|$

$\Rightarrow a+b-3=0$ OR $a+b+1=0$ ....(i)

At $x=1$, L.H.L = R.H.L gives $|a+b+1|=0$

$\Rightarrow a+b+1=0$ ....(ii)

(i) & (ii), $a+b=-1$

Sol16. $PA^2+PB^2=\cos^2\theta+(\sin\theta-3)^2+\cos^2\theta+(\sin\theta+6)^2$

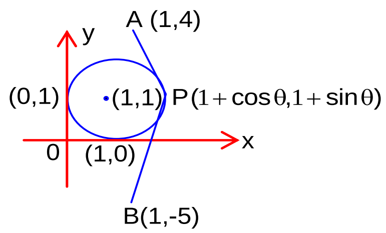
$=2\cos^2\theta+2\sin^2\theta+6\sin\theta+45$

$=6\sin\theta+47$

for maximum of PA^2+PB^2, $\sin\theta=1$

then P (1, 2)

Hence P, A & B will lie on a straight line.



Sol17. $A_1+A_2=\int_0^{\pi/2}\cos x\,dx$

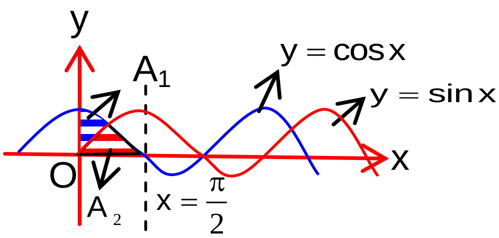
$=(\text{Sin } x)_0^{\pi/2}=1$

$A_1=\int_0^{\pi/4}(\cos x-\sin x)\,dx=(\sin x+\cos x)_0^{\pi/4}$

$=\frac{2}{\sqrt{2}}-1=\sqrt{2}-1$

So $A_2=1-\left(\sqrt{2}-1\right)=2-\sqrt{2}=\sqrt{2}\left(\sqrt{2}-1\right)$

Now $\frac{A_1}{A_2}=\frac{1}{\sqrt{2}}$



Sol18. Given $\tan^{-1}a+\text{ta}^{-1}b=\pi/4$

$\tan^{-1}\left(\frac{a+b}{1-ab}\right)=\pi/4$

$\Rightarrow \frac{a+b}{1-ab}=1$...........(i)

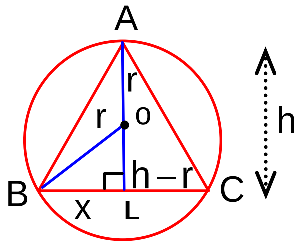
OR $a+b=1-ab$ ....(ii)

Now, $(a+b)-\left(\frac{a^2+b^2}{2}\right)+\left(\frac{a^3+b^3}{3}\right)-\left(\frac{a^4+b^4}{4}\right)+$........

$\left(a-\frac{a^2}{2}+\frac{a^3}{3}+\frac{a^3}{3}-\frac{a^4}{4}+.....\right)+\left(b-\frac{b^2}{2}+\frac{b^3}{3}-\frac{b^4}{4}+.....\right)$

$\log(1+a)+\log(1+b)=\log(1+a)(1+b)=\log\{1+a+b+ab\}=\log_e 2$

Sol19. From option let it be isosceles where AB = AC then



$x = \sqrt{r^2 - (h-r)^2}$

$= \sqrt{r^2 - h^2 - r^2 + 2rh}$

$x = \sqrt{2hr - h^2}$ ....(i)

Now ar $(\Delta ABC) = \Delta = \frac{1}{2} BC \times AL$

$\Delta = \frac{1}{2} \times 2\sqrt{2hr - a^2} \times h$

For Δ to be maximum, Δ^2 also should be maximum so

$P = \Delta^2 = h^2\left(2hr - h^2\right) = 2h^3r - h^4$

Now, $\frac{dp}{dh} = 6h^2r - 4h^3$

For maxima / minima , $\frac{dp}{dh} = 0$ gives $2h^2(3r - 2h) = 0$

$\Rightarrow \quad h = \frac{3r}{2}$

then $x = \sqrt{2 \times \frac{3r}{2} \times r - \frac{9r^2}{4}} = \frac{\sqrt{3}}{2} r$ from (i)

$\Rightarrow \quad BC = \sqrt{3}r$

So $AB = \sqrt{h^2 + x^2} = \sqrt{\frac{9r^2}{4} + \frac{3r^2}{4}} = \sqrt{3}r$

Hence Δ be equilateral having each side of length $\sqrt{3}r$.

Sol20. By truth table

A	B	C	~A	~B	~C	$A \vee B$	$A \wedge \sim B$	$\sim C \wedge (A \vee B)$	$(A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$
T	T	T	F	F	F	T	F	F	F
T	T	F	F	F	T	T	F	T	T
T	F	T	F	T	F	T	T	F	T
T	F	F	F	T	T	T	T	T	T
F	T	T	T	F	F	T	F	F	T
F	T	F	T	F	T	T	F	T	T
F	F	T	T	T	F	F	F	F	T
F	F	F	T	T	T	F	F	F	T

So $F_1(A,B,C)$ is not a tautology

Now again by truth table

A	B	~A	$A \vee B$	$B \to \sim A$	$(A \vee B) \vee (B \to \sim A)$
T	T	F	T	F	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	F	T	T

So F_2 (A, B) be a tautology.

SECTION – B

Sol1. Given curves $\frac{x^2}{9} + \frac{y^2}{4} = 1$..............(i)

& $x^2 + y^2 = \frac{31}{4}$..............(ii)

Equation of any tangent to (i) be $y = mx + \sqrt{9m^2 + 4}$ ....(iii)

For common tangent (iii) also should be tangent to (ii) so by condition of common tangency

$9m^2 + 4 = \frac{31}{4}(1+m^2)$

OR $36m^2 + 16 = 31 + 31m^2$

$\Rightarrow m^2 = 3$

Sol2. Given sequence is $-16, 8, -4, 2, ...$

are in G. P. with first term $a = -16$ & common ratio $r = \frac{-1}{2}$

Now $t_p = ar^{P-1} = -16\left(-\frac{1}{2}\right)^{p-1}$ & $t_q = ar^{q-1} = -16\left(-\frac{1}{2}\right)^{q-1}$

So A.M $= -8\left[\left(-\frac{1}{2}\right)^{p-1} + \left(\frac{-1}{2}\right)^{q-1}\right] = \alpha(\text{let})$

$G.M = \sqrt{256.\left(-\frac{1}{2}\right)^{p+q-2}} = 16\left(-\frac{1}{2}\right)^{\frac{p+q-2}{2}} = \beta$ (Let)

Given equation is $4x^2 - 9x + 5 = 0$ gives $x = 1, \frac{5}{4}$

From roots we get possible value of $\beta = 1$ so

$16\left(-\frac{1}{2}\right)^{\frac{P+q-2}{2}} = 1$ OR $\left(-\frac{1}{2}\right)^{\frac{p+q-2}{2}} = \frac{1}{16} = \left(-\frac{1}{2}\right)^4$

$\Rightarrow \frac{p+q-2}{2} = 4 \Rightarrow p+q = 10$

Sol3. $18 = 3^2 \times 2$

For G.C.D to be 3. no. of four digits should be only multiple of 3, but not multiple of 9 & also should not be even.

As we know no. of the form

$9k \rightarrow 1000$

$9k+1 \rightarrow 1000$

$9k+2 \rightarrow 1000$

$9k+3 \rightarrow 1000 \rightarrow$ Total no. = 2000

$9k+4 \rightarrow 1000$

$9k+5 \rightarrow 1000$

$9k+6 \rightarrow 1000$

$9k+7 \rightarrow 1000$

$9k+8 \rightarrow 1000$

In which half will be even & half be odd so Required no. $= 1000$

Sol4. Given $P_n = \alpha^n + \beta^n$, $P_{n-1} = 11$ & $P_{n+1} = 29$

$P_n = \alpha^{n-2}.\alpha^2 + \beta^{n-2}.\beta^2$ ....(i)

Now quadratic equation having roots α & β will be $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

i e $x^2 - x - 1 = 0$, put $x = \alpha$ and put $x = \beta$

So $\alpha^2 = \alpha + 1$ & $\beta^2 = \beta + 1$

(i) $P_n = \alpha^{n-2}(\alpha+1) + \beta^{n-2}(\beta+1)$

$P_n = \alpha^{n-1} + \alpha^{n-2} + \beta^{n-1} + \beta^{n-2}$

$P_n = P_{n-1} + P_{n-2}$

So $P_{n+1} = P_n + P_{n-1}$

$$\Rightarrow \quad P_n = P_{n+1} - P_{n-1}$$
$$= 29 - 11 = 18$$
$$\Rightarrow \quad P_n^{\ 2} = 324$$

Sol5. Given $|z+5| \le 4$ ....(i)

$\rightarrow$ Represent a circle $(x+5)^2 + y^2 \le 16$

$= z(1+i) + \bar{z}\,(1-i) \ge -10$ ....(ii)

$\rightarrow$ Represent a line $x - y \ge -5$

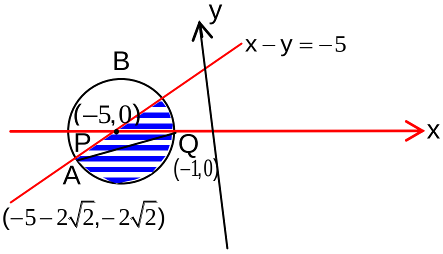
So max $|z+1|^2 =$ AQ^2

$= \left(-4-2\sqrt{2}\right)^2 + 8$

$= 32 + 16\sqrt{2}$

$= \alpha + \beta\sqrt{2}$

Hence $\alpha + \beta = 48$



Sol6. Given $f(x) = 2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$

$f(-1) = 3 > 0$ & $f(-2) = -34 < 0$

So at least one root will lie in $(-2, -1)$

now $f'(x) = 10x^4 + 20x^3 + 30x^2 + 20x + 10$

$= 10\left[x^4 + 2x^3 + 3x^2 + 2x + 1\right]$

$= 10\left[x^2 + 2x + 3 + \frac{2}{x} + \frac{1}{x^2}\right]x^2$

$= 10x^2\left[\left(x^2 + \frac{1}{x^2}\right) + 2\left(x + \frac{1}{x}\right) + 3\right]$

$= 10x^2\left[\left(x + \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) + 1\right]$

$= 10x^2\left(x + \frac{1}{x} + 1\right)^2 > 0 \quad \forall \ x \in R$

So, f (x) be purely increasing function so exactly one root of f (x) that will lie in (-2,1). Hence $|a| = 2$

Sol7. Given $\sum_{i=1}^{18}(x_i - \alpha) = 36$ ie $\sum_{i=1}^{18} x_i - 18\alpha = 36$ ....(i)

& $\sum_{i=1}^{18}\left(x_i - \beta\right)^2 = 90$ ie $\sum_{i=1}^{18} x_i^2 - 2\beta \sum_{i=1}^{18} x_i + 18\beta^2 = 90$ ....(ii)

(i) & (ii) $\sum_{i=1}^{18} x_i^{\ 2} = 90 - 18\beta^2 + 36\beta\,(\alpha + 2)$ ....(iii)

Now variance $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = 1$ given

i.e. $\frac{1}{18}(90 - 18\beta^2 + 36\beta\,(\alpha+2)) - \left(\frac{18(\alpha+2)}{18}\right)^2 = 1$

$\Rightarrow \quad (\alpha - \beta)\ (\alpha - \beta + 4) = 0$

Since $\alpha \neq \beta$ so $|\alpha - \beta| = 4$

Sol8. Given $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}dx$ ....(i)

put $1-x = t \begin{cases} x=0, t=1 \\ x=1, t=0 \end{cases}$

$dx = -dt$

From (i) $I_{m,n} = \int_1^0 (1-t)^{m-1}.\ t^{n-1}(-dt)$

$I_{m,n} = \int_0^1 t^{n-1}(1-t)^{m-1}dt = \int_0^1 x^{n-1}(1-x)^{m-1}dx$ ....(ii)

Put $x = \frac{1}{1+y}$ in (i) then $dx = -\frac{1}{(1+y)^2}\ dy$

(i) $I_{m,n} = \int_\infty^0 \frac{1}{(1+y)^{m-1}}\left(1-\frac{1}{1+y}\right)^{n-1}\left(\frac{-dy}{(1+y)^2}\right) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}\ dy$ ....(iii)

Similarly by (ii) $I_{m,n} = \int_0^\infty \frac{y^{m-1}}{(y+1)^{m+n}}\ dy$ ....(iv)

Adding (iii) & (iv) $2I_{m,n} = \int_0^\infty \frac{y^{n-1}+y^{m+1}}{(y+1)^{m+n}}$

$= \int_0^1 \frac{y^{n-1}+y^{m-1}}{(y+1)^{m+n}}$

$= \int_0^1 \frac{y^{n-1}+y^{m-1}}{(y+1)^{m+n}}dy + \int_1^\infty \frac{y^{n-1}+y^{m-1}}{(y+1)^{m+n}}dy$

Put $y = \frac{1}{z}\begin{cases} y=1, z=1 \\ y=\infty, z=0 \end{cases} \Rightarrow dy = \frac{-1}{z^2}dz$

$2I_{m,n} = \int_0^1 \frac{y^{n-1}+y^{m-1}}{(y+1)^{m+n}} + \int_0^1 \frac{z^{n-1}+z^{m-1}}{(z+1)^{m+n}}\ dz + \int_0^1 \frac{y^{n-1}+y^{m-1}}{(y+1)^{m+n}}\ dy$

Hence $I_{m,n} = \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx = \alpha Im,n$

$\Rightarrow \alpha = 1$

Sol9. Let the equation of normal is $Y - y = -\frac{1}{m}(X-x)$

where m is slope of tangent to the given curve then

$Y - y = -\frac{dx}{dy}(X-x)$

It passes through (a, b) so $b - y = \frac{-dx}{dy}(a-x)$

$\Rightarrow (a-x)dx = (y-b)dy$

On integration $ax - \frac{x^2}{2} = \frac{y^2}{2} - by + c$ ....(i)

(ii) passes through $(3,-3)$ & $(4,-2\sqrt{2})$ then

$3a - 3b - c = 9$ ....(ii)

& $4a - 2\sqrt{2}b - c = 12$ ....(iii)

also given $a - 2\sqrt{2}b = 3$ ....(iv)

Solve (ii), (iii) & (iv) $b = 0, a = 3$

Hence $a^2 + b^2 + ab = 9$

Sol10. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^3 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^3 & 0 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Similarly we get $A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}$ & $A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} \alpha & 0 & 0 \\ 0 & 2^{19}.\alpha & 0 \\ 3\alpha & 0 & -\alpha \end{bmatrix} + \begin{bmatrix} \beta & 0 & 0 \\ 0 & 2\beta & 0 \\ 3\beta & 0 & -\beta \end{bmatrix}$

$= \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20}+2^{19}\alpha+2\beta & 0 \\ 3\alpha+3\beta & 0 & 1-\alpha-\beta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad 1+\alpha+\beta = 1$ gives $\alpha+\beta = 0$ …………(i)

$2^{20} + 2^{19}.\alpha + 2\beta = 4$

$2^{20} + (2^{19} - 2)\alpha = 4$ from (i)

$\Rightarrow \alpha = \frac{4 - 2^{20}}{2^{19} - 2} = \frac{4(1 - 2^{18})}{-2(1 - 2^{18})} = -2$

So, $\beta = 2$

Hence $\beta - \alpha = 4$