FIITJEE

Solutions to JEE (Main)-2021

JEE–Main–2021 –Feb–26–First–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

(PHYSICS)

Answers

Section-A

1.	D	**2.**	C	**3.**	B	**4.**	B
5.	D	**6.**	A	**7.**	A	**8.**	A
9. Q. dropped by NTA		**10.**	C	**11.**	D	**12.**	A
13.	A	**14.**	D	**15.**	A	**16.**	D
17.	B	**18.**	C	**19.**	D	**20.**	C

Section-B

1.	26	**2.**	20	**3.**	300	**4.**	33
5.	500	**6.**	3.3	**7.**	282.84	**8.**	492
9.	137	**10.**	1215				

SECTION – A

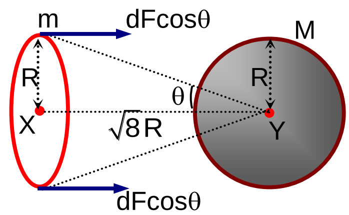
Sol1. As height of image is less than height of image and has same orientation as that of object, so mirror must be convex.

Sol2. Method-I:

$$F = M_{sphere}\ E_{ring} = M \times \frac{Gmx}{(R^2+x^2)^{3/2}} = M \times \frac{Gmx\sqrt{8}R}{(R^2+8R^2)^{3/2}} = \frac{\sqrt{8}\,GMm}{27R^2}$$

Method-II:

$$F = \int dF\cos\theta = \cos\theta\int(dm)\frac{GM}{(R^2+8R^2)} = \frac{\sqrt{8}GMm}{27R^2}$$

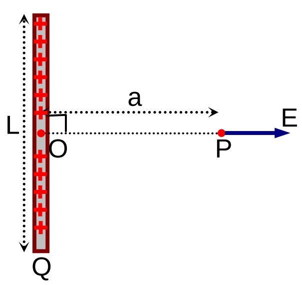


Sol3.

$$E = \frac{\lambda}{4\pi\varepsilon_0 a}[\sin\alpha + \sin\alpha] = \frac{\lambda\sin\alpha}{2\pi\varepsilon_0 a}$$

$$\Rightarrow E = \frac{Q}{L\ 2\pi\varepsilon_0 \times \left(\frac{\sqrt{3}L}{2}\right)} \times \frac{1}{2} = \frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$$

$$\tan\alpha = \frac{L/2}{\sqrt{3}L/2} \Rightarrow \alpha = \frac{\pi}{6} \Rightarrow \sin\alpha = \frac{1}{2}$$



Sol4. $W = \alpha^2\beta \; e^{-\frac{\beta x^2}{kT}}$

$$[\beta x^2] = [kT] = [ML^2T^{-2}] \qquad \left[E = \frac{3}{2}kT\right]$$

$$\Rightarrow [\beta][L^2] = [ML^2T^{-2}]$$

$$\beta = MT^{-2}$$

$$[W] = [\alpha^2\beta] \Rightarrow [ML^2T^{-2}] = [\alpha^2][MT^{-2}] \Rightarrow [\alpha^2] = [L^2] \Rightarrow [\alpha] = [L]$$

Sol5. $F = \frac{k}{R^3} = \frac{mv^2}{R} \Rightarrow v = \sqrt{\frac{k}{mR^2}} = \sqrt{\frac{k}{m}} \times \frac{1}{R}$

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{k}{m}} \times \left(\frac{1}{R}\right)} \Rightarrow T \; \alpha \; R^2$$

Sol6. mass m will acquire velocity 2u. Total momentum of system will be conserved but total kinetic energy is not conserved during collision

Sol7.

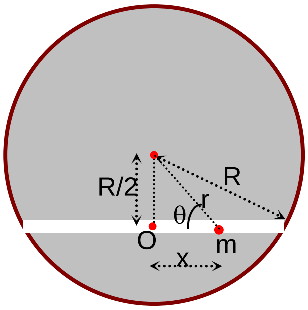
$$F = mE\cos\theta$$

$$= m\left(\frac{GM}{R^3} \times r\right) \times \frac{x}{r}$$

$$= \frac{m\,gR^2}{R^3} \times x \left[g = \frac{GM}{R^2}\right]$$

$$ma = \frac{mg}{R}\,x \Rightarrow a = \frac{g}{R}\,x$$

$$\Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$



O⇒Mean Position

Sol8. From volume conservation

$$n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R^3 = nr^3 \; .... (1)$$

Decrease in surface area $= n \times 4\pi r^2 - 4\pi R^2$

$$(\Delta A) = 4\pi[nr^2 - R^2] = 4\pi\left[\frac{n \times r^3}{r} - R^2\right] = 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Energy released (W)= $T \times \Delta A = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$

Heat produced (Q) $= \frac{W}{J} = \frac{4\pi TR^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$

Now, $Q = ms\,\Delta\theta$

$$\Rightarrow \frac{4\pi R^3 T}{J}\left[\frac{1}{r} - \frac{1}{R}\right] = \left(\frac{4}{3}\pi R^3\right) \times |x| \times \Delta\theta \Rightarrow \Delta\theta = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Sol9. According question , we can write

$$\frac{C_1 \times C_2}{C_1 + C_2} = \frac{15}{4}(C_1 + C_2) \Rightarrow 4C_1C_2 = 15(C_1^2 + C_2^2 + 2C_1C_2)$$

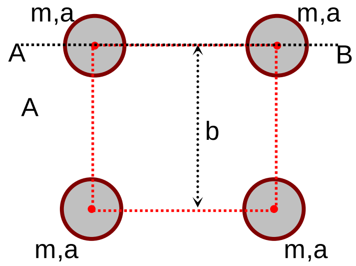
$$\Rightarrow 4\left(\frac{C_2}{C_1}\right) = 15\left(\frac{C_2}{C_1}\right)^2 + 30\left(\frac{C_2}{C_1}\right) + 15 \Rightarrow 15x^2 + 26x + 15 = 0, \text{ where } x = \frac{C_2}{C_1}$$

$$\Rightarrow x = \frac{-26 \pm \sqrt{676 - 900}}{30}$$

Since x cannot be real , so this Question has been dropped by NTA

Sol10.

$$I_{AB} = \frac{2}{5}ma^2 \times 2 + \left(\frac{2}{5}ma^2 + mb^2\right) \times 2 = \frac{8ma^2}{5} + 2mb^2$$

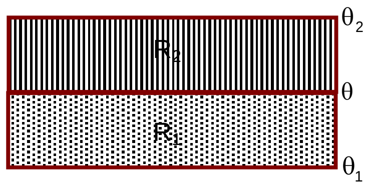


Sol11. $\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.9 \times 1.6 \times 10^{-19}} = 654\,nm$ (red)

Sol12. taking $\theta_1 > \theta_2$

Heat current $= \frac{\theta_1 - \theta}{R_1} = \frac{\theta - \theta_2}{R_2} \Rightarrow \frac{\theta_1}{R_1} + \frac{\theta_2}{R_2} = \theta\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$$\Rightarrow \theta = \frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$$



Sol13. $T_1 = 2\pi\sqrt{\frac{m}{k}}$

$$k_{eq} = \frac{k \times k}{k + k} = \frac{k}{2}$$

$$T_2 = 2\pi\sqrt{\frac{m}{(k/2)}} = 2\pi\sqrt{\frac{2m}{k}}$$

$$\Rightarrow \frac{T_2}{T_1} = \sqrt{2} \text{ and } \frac{k_2}{k_1} = \frac{1}{2}$$

Sol14. In elliptical orbit, areal velocity is constant

Sol15. $I = I_1 \sin\omega t + I_2 \cos\omega t = \sqrt{I_1^2 + I_2^2}\left[\left(\frac{I_1}{\sqrt{I_1^2 + I_2^2}}\right)\sin\omega t + \frac{I_2}{\sqrt{I_1^2 + I_2^2}}\cos\omega t\right]$

$$= \sqrt{I_1^2 + I_2^2}\sin(\omega t + \alpha) = I_0 \sin(\omega t + \alpha)$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{\sqrt{I_1^2 + I_2^2}}{\sqrt{2}}$$

Sol16. Lyman Series $n_f = 1, \quad n_i = 2, 3.....$

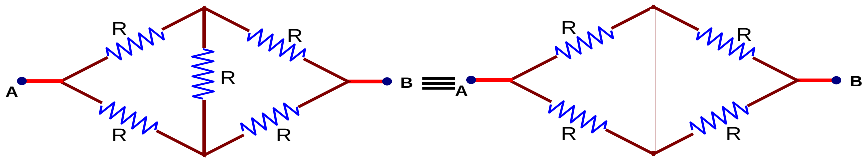
Paschen series $n_f = 3, \quad n_i = 4, 5, 6 - - -$

$$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\frac{1/\lambda_1}{1/\lambda_2} = \frac{1 - \frac{1}{16}}{\frac{1}{9} - \frac{1}{16}} \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{\frac{15}{16}}{\frac{7}{9 \times 16}} = \frac{15 \times 9}{7} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{7}{135}$$

Sol17. Given circuit can be re-drawn and it becomes case of balanced Wheatstone bridge



$$R_{AB} = \frac{(2R)(2R)}{2R+2R} = R$$

Sol18. fact based

Sol19. From definition of bulk modulus

$$K = -\frac{P}{(dV/V)} \text{ ....(1)}$$

$$m = \rho v = \text{constant}$$

$$\Rightarrow \rho dV + V d\rho = 0$$

$$\Rightarrow \frac{dV}{V} = -\frac{d\rho}{\rho} \text{ .... (2)}$$

With the help of equations (1) and (2) ,we can write

$$K = -\frac{P}{\left(-\frac{d\rho}{\rho}\right)} = \frac{\rho P}{d\rho} \Rightarrow d\rho = \frac{\rho P}{K}$$

Sol20. $\beta = \frac{\lambda D}{d} = \frac{500\times 10^{-9}\times 1}{2\times 10^{-3}} = 250\times 10^{-6}\text{m} = 250\times 10^{-3}\text{ mm} = 0.25\text{mm}$

Section-B

Sol1. $P = \frac{P_1V_1+P_2V_2}{V_1+V_2} = \frac{2\times 4.5+3\times 5.5}{10} = \frac{9+16.5}{10} = 25.5\times 10^{-1}\text{atm}$

Sol2. D_1 is forward biased & D_2 is reverse biased.

$$I = \frac{6}{120+130+50} = \frac{6}{300} = 2\times 10^{-2}\text{A} = 20\times 10^{-3}\text{A} = 20\text{ mA}$$

Sol3. $W = q\,\Delta V = 20\times 15 = 300\text{J}$

Sol4. Modulation Index $(\mu) = \frac{A_{max}-A_{min}}{A_{max}+A_{min}} = \frac{16-8}{16+8} = \frac{1}{3} = 0.33 = 33\times 10^{-2}$

Sol5. $a_x = \frac{F_x}{m} = \frac{20}{2} = 10\text{m/s}^2$

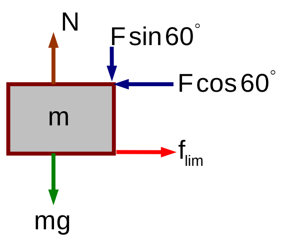
$$S_x = \frac{1}{2}a_x t^2 = \frac{1}{2}\times 10\times 10^2$$

Sol6. $N = mg + F\sin 60^\circ$

$f_{\ell im} = F\cos 60^\circ \Rightarrow \mu N = F\cos 60^\circ$

$\Rightarrow \mu(mg + F\sin 60^\circ) = F\cos 60^\circ$

$\Rightarrow \mu mg = F[\cos 60^\circ - \mu\sin 60^\circ]$



$$F = \frac{\mu mg}{\cos 60^\circ - \mu \sin 60^\circ} = \frac{\frac{1}{3\sqrt{3}} \times \sqrt{3} \times 10}{\frac{1}{2} - \frac{1}{3\sqrt{3}} \times \frac{\sqrt{3}}{2}} = \frac{10/3}{\frac{1}{2} - \frac{1}{6}} = \frac{10/3}{1/3} = 10\text{N} = 3 \times \left(\frac{10}{3}\right)\text{N}$$

Sol7. As we know that $Q = \frac{\omega L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

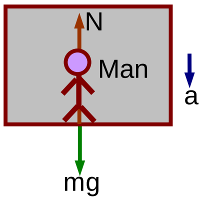
$$\Rightarrow \quad \frac{Q'}{Q} = \left(\sqrt{\frac{L'}{L}}\right) \times \left(\frac{R}{R'}\right) = 2 \times \sqrt{2}$$

$$\Rightarrow \quad Q' = 2\sqrt{2}\,Q = 2\sqrt{2} \times 100 = 282.84$$

Sol8.

$mg - N = ma$

$N = m(g - a) = 60 \times (10 - 1.8) = 60 \times 8.2 = 492\text{ N}$



Sol9. $I = \frac{1}{2} c\,\varepsilon_0 E_0^2 = \eta \times \left(\frac{P}{4\pi r^2}\right)$ Here η is the efficiency

$$E_0 = \sqrt{\frac{\eta P}{2\pi r^2 c\,\varepsilon_0}} = \sqrt{\frac{1.25}{100} \times \frac{1000}{2 \times 3.14 \times 4 \times 3 \times 10^8 \times 8.85 \times 10^{-12}}}$$

$$\Rightarrow E_0 = \sqrt{\frac{12.5}{8 \times 3.14 \times 3 \times 8.85 \times 10^{-4}}} = 13.69\text{ V/m} = 136.9 \times 10^{-1}\text{V/m}$$

Sol10. $v = \frac{\omega}{k} = \frac{30}{1} = 30\text{ m/s}$

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow T = \mu v^2 = \frac{0.135 \times 10^{-3}}{10^{-2}} \times (30)^2 = 12.15\text{ N} = 1215 \times 10^{-2}\text{ N}$$

PART –B (CHEMISTRY)

Answers

Section-A

1.	A	**2.**	C	**3.**	A	**4.**	B
5.	A	**6.**	C	**7.**	D	**8.**	D
9.	B	**10.**	B	**11.**	B	**12.**	D
13.	D	**14.**	A	**15.**	B	**16.**	D
17.	A	**18.**	B	**19.**	C	**20.**	C

Section-B

1.	0	**2.**	1	**3.**	25	**4.**	50
5.	73	**6.**	6	**7.**	2	**8.**	24
9.	8	**10.**	200				

SECTION – A

Sol1. **List – I** **Elements** / **List – II** **$\Delta_i H$**

(a) $1s^2\ 2s^2$ (Be) — (i) 801 KJ / mol
(b) $1s^2\ 2s^2\ 2p^4$(O) — (ii) 899 KJ/ mol
(c) $1s^2\ 2s^2\ 2p^3$ (N) — (iii) 1314 KJ / mol
(d) $1s^2 2s^2 2p^1$ (B) — (iv) 1402 KJ / mol

N > O > Be > B

Due to half filled configuration N has more I.E than oxygen and due to fully filled configuration Be has more I.E than B.

(a)→ (ii) (b) → (iii) (c) →(iv) (d) →(i)

Sol2. Ce, Pr, Nd, Tb and Dy are the only lanthanoids which shows +4 O.S, so can form MO_2, but Yb does not shows +4 O.S so can't form MO_2.

Sol3.

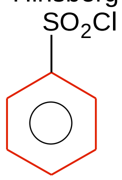
$$\underset{}{K_2Cr_2O_7} + H_2SO_4 + \underset{\text{Gas X}}{SO_2} \longrightarrow \underset{\substack{\text{Green}\\ \text{Y}}}{Cr_2(SO_4)_3} + K_2SO_4 + H_2O$$

Sol4. Not all dipole- dipole interactions are responsible for hydrogen bonding, so assertion A is false.
F is most E.N atom and also hydrogen bonds in HF are symmetrical.
So; reason R is true.

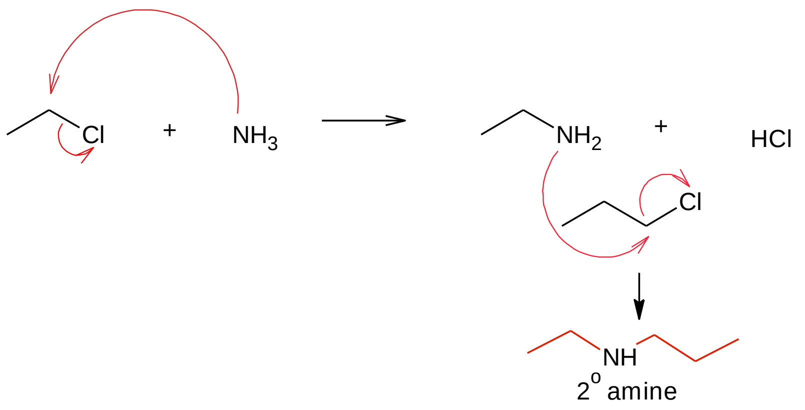
Sol5. Boiling points of chloroform is 334 K and aniline is 457 K. Here difference in b.pt is more so they can be separated by simple distillation.
Aniline is insoluble (immiscible in water), the mixture of insoluble substance in water boils close to but below 373 K. so, aniline boils below its b.pt

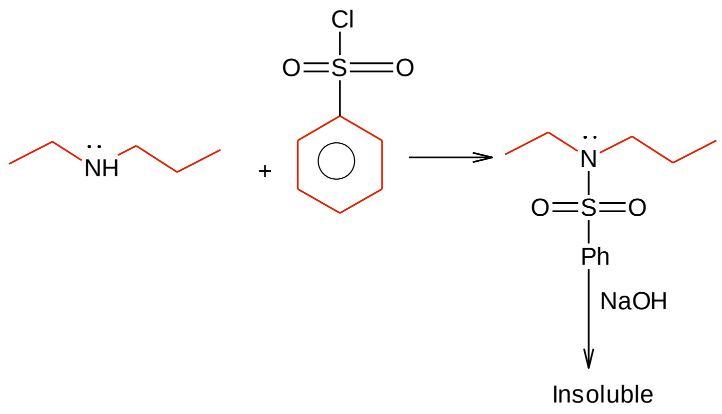
Sol6. Vitamin E is helpful in delaying blood clotting while vitamin K help blood clotting.

Sol7. Hinsberg test for amines



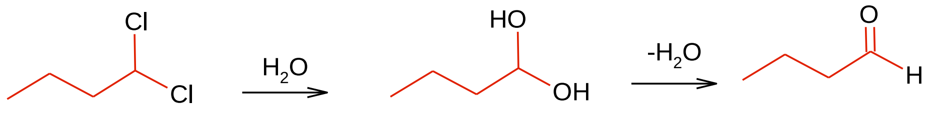
Hinsberg's reagent (benzene sulphonyl chloride





Sol8.

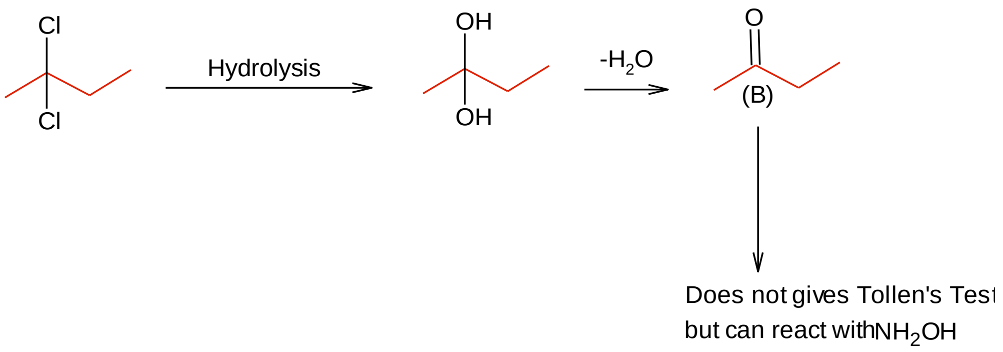
$$\underset{(C_4H_8Cl_2)}{\overset{A}{}} \xrightarrow[373K]{\text{Hydrolysis}} \underset{(C_4H_8O)}{\overset{B}{}}$$



(A) (B)

(1, 1- dichloro butane) (butanal but it gives Tollen's test)

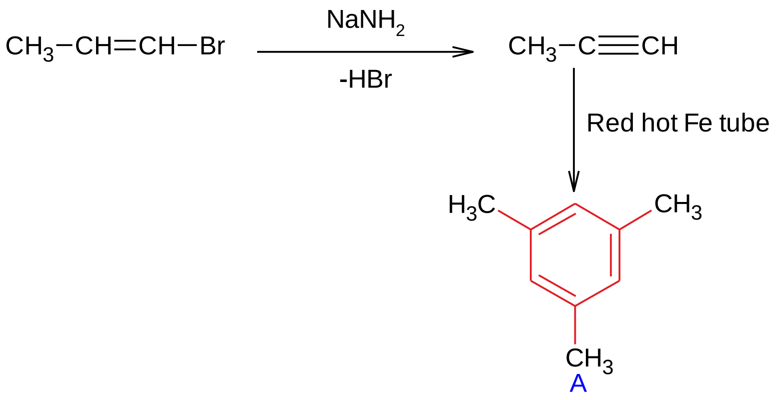
But actual answer



(2,2 – dichloro butane) ⇒ A

(butane – 2- one) ⇒ (B)

Sol9.



Sol10. Radial node $= n - \ell - 1$ (5d)
$= 5 - 2 - 1 = 2$
Angular node $= \ell$
$= 2$

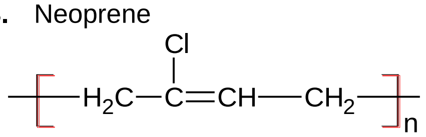
Sol11. $\underset{A}{NH_3} + H_2O + CO_2 \longrightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + CO_2 \longrightarrow \underset{B}{NH_4HCO_3}$

$NH_4HCO_3 + NaCl \longrightarrow NH_4Cl + \underset{C}{NaHCO_3}$

Sol12. PbO_2 is used in lead storage battery and is amphoteric as well as strong oxidizing agent since Pb(II) is more stable than Pb(IV).

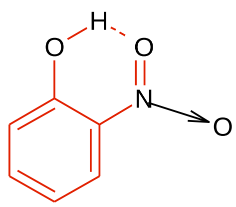
Sol13. Neoprene



Sol14. Option (A) is false statement because estimation of nitrogen in an organic compound is done by Kjeldahl's method.

Sol15. Kernite ($Na_2B_4O_7.4H_2O$) $\rightarrow$ it is ore of boron
Cassiterite (SnO_2) $\rightarrow$ it is ore of tin
Calamine ($ZnCO_3$) $\rightarrow$ it is ore of zinc
Cryolite (Na_3AlF_6) $\rightarrow$ it is ore fluorine

Sol16.



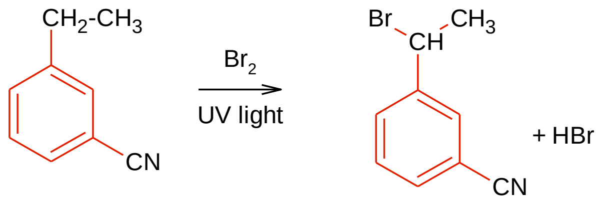
o – Nitrophenol is more volatile due to intramolecular hydrogen bonding,
So I statement is correct but statement (II) is false

Sol17. O_3 generally exist in stratosphere and they acts as protector , because it protect us from harmful UV radiations but sometimes O_3 formed in troposphere and it acts as a pollutants.
O_3 in stratosphere : - Protector
O_3 in Troposphere :- Pollutant (photochemical smog)

Sol18. Heavy water (D_2O) is used in organic reaction to know the kinetics of reaction
Heavy water D_2O is obtained by repeated electrolysis of H_2O.
Heavy water has b.pt 101.4°C and ordinary water has b.pt 100°C
(b.pt of D_2O > b.pt of H_2O)

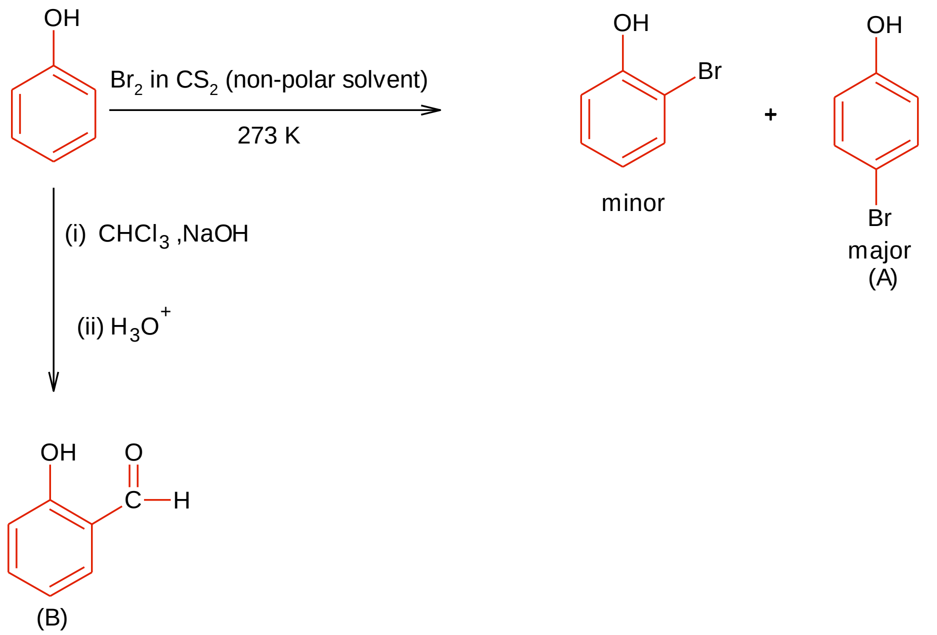
D_2O has more molecular mass than H_2O, so greater degree of association and hence greater b.pt and viscosity.

Sol19.



This is free radical mechanism, bromination takes place at benzylic carbon (major product).

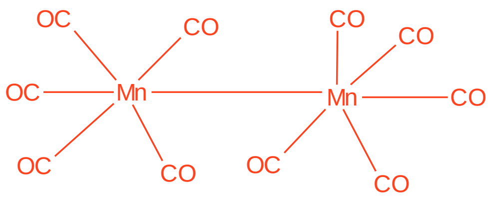
Sol20.



SECTION-B

Sol1.

$[Mn_2(CO)_{10}]$



Bridging ligand (CO) is (0)

Sol2. $P(V_m - b) = RT$

$PV_m - Pb = RT$

$$\frac{PV_m}{RT} - \frac{Pb}{RT} = 1$$

$$Z = 1 + \frac{Pb}{RT}$$

$$\left(\frac{\partial z}{\partial P}\right)_T = \frac{b}{RT}$$

So, comparing with $\frac{xb}{RT}$

$x = 1$

Sol3. $MnO_4^- + 8H + 5e^- \longrightarrow Mn + 4H_2O \quad E^o = 1.51V$

Quantity of electricity required to reduce 1 mole of MnO_4^- is 5F

So, for 5 mole MnO_4^- 25F electricity is required.

Sol4. $X \rightarrow Y$

$E_{a_f} - E_{a_b} = -20$

$30 - E_{a_b} = -20$

$E_{a_b} = 50\ kJ / mole$

Sol5.

	$AB_2(g)$	$\rightleftharpoons$ $A(g)$	+	$2B(g)$
Initial	1 mole	-		-
At equilibrium	1-x mole	x mole		2x mole

V = 25 L and
T = 300 K.
At equilibrium, P= 1.9 atm
Total moles at equilibrium, n = 1 + 2x
V =25 L
T = 300 K
Using, PV = nRT

$1.9 \times 25 = (1+2x) \times 0.08206 \times 300$

x=0.465

Now; Partial pressure of AB_2 at equilibrium = $\frac{0.535}{1.93} \times 1.9\ atm$

Partial pressure of A at equilibrium = $\frac{0.465}{1.93} \times 1.9\ atm$

Partial pressure of B at equilibrium = $\frac{2 \times 0.465}{1.93} \times 1.9\ atm$

Using ; $K_p = \frac{P_A.P_B^2}{P_{AB_2}}$

$$K_p = \frac{\left(\frac{0.465 \times 1.9}{1.93}\right)\left(\frac{2 \times 0.465}{1.93} \times 1.9\right)^2}{\frac{0.535}{1.93} \times 1.9} = 0.728$$

$K_p = 0.73$

$K_p = 73 \times 10^{-2}$

So; x = 73

Sol6. $Cr_2O_7^{2-} + 2OH^- \longrightarrow 2CrO_4^{2-} + H_2O$

Dichromate ion converted to chromate ion in basic medium and oxidation number of Cr in CrO_4^{2-} is +6.

Sol7. PV = nRT

$1 \times V = \frac{3.12}{32} \times 0.0821 \times 300$

V = 2.4 litre

Vol of O_2 adsorbed per gm = 2.4 / 1.2 = 2 litre

Sol8. Moles of SO_2 $= \frac{224 \times 10^{-3}}{22.4}$

= 0.01 mole

Moles of NaOH = 0.1×0.1

= 0.01 mole

SO_2	+	NaOH $\rightarrow$	$NaHSO_3$
0.01 mole		0.01 mole	-
-	-		0.01 mole

Non-volatile solute is $NaHSO_3$

Moles of water = $\frac{36}{18} = 2$

Using ; relative lowering in V.P

$\frac{P^o - P}{P^o} = ix_B$

Where ; $\Delta P = P^o - P$ is lowering in V.P

$\Delta P = P^o ix_B$

i for $NaHSO_3 = 2$

here ; $x_B = \frac{n_B}{n_A}$ since solution is dilute

$\Delta P = 24 \times 2 \times \frac{0.01}{2} = 0.24$

$\Delta P = 24 \times 10^{-2}$ mm Hg

So; x = 24

Sol9. 50000.020×10^{-3}

The significant figure in the given number is 8.

Sol10. $\Delta_r H^o = 80\,kJmol^{-1}$

$\Delta_r S^o = 2T\,kJmol^{-1}$

For a reaction to be spontaneous;

$\Delta_r S^o > \frac{\Delta_r H^o}{T}$

$T > \frac{\Delta_r H^o}{\Delta_r S^o}$

$T > \frac{80 \times 10^3}{2T}$

$T^2 > 40000K$

$T > 200K$

So, minimum T at which reaction will be spontaneous is 200K.

PART–C (MATHEMATICS)

Answers

Section-A

1.	B	**2.**	A	**3.**	B	**4.**	A
5.	A	**6.**	C	**7.**	D	**8.**	D
9.	B	**10.**	A	**11.**	C	**12.**	C
13.	A	**14.**	D	**15.**	D	**16.**	B
17.	C	**18.**	A	**19.**	C	**20.**	B

Section-B

1.	**Q. dropped by NTA**			**2.**	1	**3.**	1
4.	1	**5.**	8	**6.**	11	**7.**	3
8.	45	**9.**	2	**10.**	2		

SECTION – A

Sol1. $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}=\lambda$

$\therefore \frac{\sin^{-1}x}{a}=\lambda$

$\therefore \sin^{-1}x=a\lambda$

$x=\sin a\lambda$...........(i)

$\therefore \frac{\cos^{-1}x}{b}=\lambda$

$\cos^{-1}x=b\lambda$

$x=\cos b\lambda$...........(ii)

$\therefore \frac{\tan^{-1}y}{c}=\lambda$

$\tan^{-1}y=c\lambda$

$y=\tan c\lambda$...........(iii)

From (i) & (ii), we get, $\sin a\lambda=\cos b\lambda=\sin\left(\frac{\pi}{2}-b\lambda\right)$

$\therefore a\lambda=\frac{\pi}{2}-b\lambda \Rightarrow (a+b)\lambda=\frac{\pi}{2}\Rightarrow \lambda=\frac{\pi}{2(a+b)}$..........(iv)

From (iii) $y=\tan c\lambda$

$\cos c\lambda=\frac{1}{\sqrt{1+y^2}}$

$\cos^2 c\lambda=\frac{1}{1+y^2}\Rightarrow 2\cos^2 c\lambda=\frac{2}{1+y^2}$

$1+\cos 2c\lambda=\frac{2}{1+y^2}\Rightarrow \cos 2c\lambda=\frac{1-y^2}{1+y^2}$

$\Rightarrow \cos\left(\frac{2c\pi}{2(a+b)}\right)=\frac{1-y^2}{1+y^2}\Rightarrow \cos\left(\frac{\pi c}{a+b}\right)=\frac{1-y^2}{1+y^2}$

Sol2. $\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\(a+2)(a+3) & a+3 & 1\\(a+3)(a+4) & a+4 & 1\end{vmatrix}=\begin{vmatrix}(a+1)(a+2) & a & 1\\(a+2)(a+3) & a & 1\\(a+3)(a+4) & a & 1\end{vmatrix}+\begin{vmatrix}(a+1)(a+2) & 2 & 1\\(a+2)(a+3) & 3 & 1\\(a+3)(a+4) & 4 & 1\end{vmatrix}$, [using properties]

$=0+(a+1)(a+2)(-1)+2(a+3)(a+4)-(a+2)(a+3)+4((a+2)(a+3)-3(a+3)(a+4))$

$=-(a+1)(a+2)-(a+3)(a+4)+2(a+2)(a+3)$

$=(a+2)(a+3)-(a+1)(a+2)+(a+2)(a+3)-(a+3)(a+4)$

$=2(a+2)-2(a+3)=2(a+2-a-3)=-2$

Sol3. $I=\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^x}dx$...........(i)

Using properties, $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$I=\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^{-x}}dx=\int_{-\pi/2}^{\pi/2}\frac{3^x\cos^2 x}{1+3^x}dx$..........(ii)

Adding (i) and (ii), we get

$2I=\int_{-\pi/2}^{\pi/2}\frac{(1+3^x)\cos^2 x}{1+3^x}dx=\int_{-\pi/2}^{\pi/2}\cos^2 x dx=2\int_0^{\pi/2}\cos^2 x dx$

$=\int_0^{\pi/2}(1+\cos 2x)dx=\left[x+\frac{\sin 2x}{2}\right]_0^{\pi/2}=\frac{\pi}{2}$

$\therefore I=\frac{\pi}{4}$

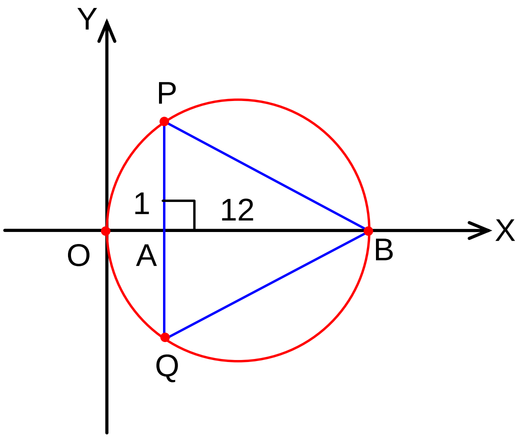
Sol4. By properties,

$OA.AB=PA\,.\,AQ,\ OA=1,\ AB=12$

$12=a^2$

$a=2\sqrt{3}$

$\therefore$ Area of PQB $=\frac{1}{2}\times 12\times 2.2\sqrt{3}=24\sqrt{3}$ sq. units



Sol5.

1	1	1	1	1	2	3

2	2	2	1	1	1	1

No. of ways = $\frac{7!}{5!}=42$

No. of ways = $\frac{7!}{3!4!}=35$

Total number of required ways= 42 + 35 = 77

Sol6. The points on the curve are (0, 0), (2, 2) and $\left(3, \frac{21}{2}\right)$

$$\therefore \frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$$

at $(0,0)$, $\frac{dy}{dx} = -19$, at $(2,2)$, $\frac{dy}{dx} = 9$ at $\left(3, \frac{21}{2}\right)$, $\frac{dy}{dx} = 8$

Hence maximum slope at (2, 2) is 9.

Sol7. $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx = \sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx$

$$= \int_0^1 e^{x-[x]}dx + \int_1^2 e^{x-[x]}dx + \int_2^3 e^{x-[x]}dx + \ldots\ldots + \int_{99}^{100} e^{x-[x]}dx$$

$$= \int_0^1 e^x dx + \int_1^2 e^{x-1}dx + \int_2^3 e^{x-2}dx + \ldots\ldots + \int_{99}^{100} e^{x-99}$$

$$= e - 1 + \frac{1}{e}\left(e^2 - e\right) + \frac{1}{e^2}\left(e^3 - e^2\right) + \ldots\ldots + \frac{1}{e^{99}}\left(e^{100} - e^{99}\right)$$

$$= e - 1 + (e-1) + (e-1) + \ldots\ldots + (e-1), 100 \text{ times}$$

$$= 100(e-1)$$

2nd method

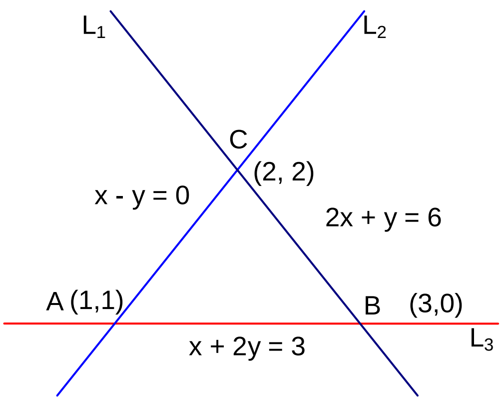
$$\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx = \sum_{n=1}^{100} \int_{n-1}^{n} e^{\{x\}}dx = \sum_{n=1}^{100}\left(\int_0^1 e^x dx\right) = \sum_{n=1}^{100}(e-1) = 100(e-1)$$

Sol8. $\underset{h\to 0}{\text{Lt}} \dfrac{2\times 2\left[\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}+h\right) - \frac{1}{2}\cos\left(\frac{\pi}{6}+h\right)\right]}{2\sqrt{3}h\left[\frac{\sqrt{3}}{2}\cos h - \frac{1}{2}\sin h\right]}$

$$= \underset{h\to 0}{\text{Lt}}\ 4\frac{\sin\left(\frac{\pi}{6}+h-\frac{\pi}{6}\right)}{2\sqrt{3}h\ \sin\left(\frac{\pi}{3}-h\right)} = \frac{2}{\sqrt{3}} \times 1 \times \frac{1}{\frac{\sqrt{3}}{2}} = \frac{4}{3}$$

Sol9.

x - y = 0..........(i)
x + 2y = 3........(ii)
2x + y = 6........(iii)



Solving (i) & (ii), we get (1, 1)
Solving (ii) & (iii), we get (3, 0)
Solving (iii) & (i), we get (2, 2)

$AB = \sqrt{5}, BC = \sqrt{5}, CA = \sqrt{2}$

$\therefore AB = BC$ Isoceles triangle

Hence, option (B) is correct answer.

Sol10. $\left|f(x)-f(y)\right| \le \left|(x-y)^2\right|$

$$\left|\frac{f(x)-f(y)}{x-y}\right| \le x-y$$

Taking the limit $y \to x$ on both sides

$$\underset{y\to x}{\text{Lt}}\left|\frac{f(x)-f(y)}{x-y}\right| \le \underset{y\to x}{\text{Lt}}(x-y)$$

$$\left|f'(x)\right| \le 0$$

Hence, modulus cannot be zero. Hence f'(x) = 0. Integrating, we get f(x) = c

at x = 0, f(0) = c = 1

$\therefore f(x) = 1 > 0, \forall x \in R$

Hence, option (A) is correct option.

Sol11. All the points $A(1,5,35), B(7,5,5), C(1,\lambda,7), D(2\lambda,1,2)$ are coplanar. Hence

$$\overrightarrow{AB}\times\overrightarrow{AC}\,.\,\overrightarrow{AD} = \begin{vmatrix} 6 & 0 & 30 \\ 0 & \lambda-5 & 28 \\ 2\lambda-1 & -4 & 33 \end{vmatrix} = 0$$

$\Rightarrow 5\lambda^2 - 44\lambda + 95 = 0$

$\Rightarrow$ sum of roots $= \dfrac{44}{5}$

Sol12. $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ is symmetric. So, q = r.

$$AA = A^2 = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} p^2+qr & pq+qs \\ rp+rs & rq+s^2 \end{bmatrix}$$

Sum of diagonal elements =

$p^2+qr+rq+s^2 = 1 \Rightarrow p^2+2r^2+s^2 = 1, (\text{as } q=r), p=0, r=0 \text{ and } s=\pm1 \text{ or } r=0, s=0 \text{ and } p=\pm1$

Total number of matrices = 4.

Sol13. $\dfrac{dv}{dt} \propto V \;\Rightarrow \dfrac{dv}{dt} = \lambda V \Rightarrow \displaystyle\int_{1000}^{1200}\frac{dv}{v} = \lambda\int_0^2 dt \Rightarrow \lambda = \frac{1}{2}\ell n\frac{6}{5}$

$$\int_{1000}^{2000}\frac{dv}{v} = \frac{1}{2}\ell n\left(\frac{6}{5}\right)\int_0^T dt \Rightarrow T = \frac{2\ell n2}{\ell n\left(\frac{6}{5}\right)} \Rightarrow k = 2\ln 2$$

Sol14. put $x = \dfrac{1}{3}$

$S = 1+2x+7x^2+12x^3+17x^4+22x^5+........$

$xS = x+2x^2+7x^3+12x^4+17x^5+........$

Subtracting,

$(1-x)S = 1+x+5x^2+5x^3+5x^4+5x^5+........$

$= 1-4x+5x+5x^2+5x^3+.........$

$$= 1-4x+\frac{5x}{1-x} = \frac{1-x-4x(1-x)+5x}{1-x}$$

$$= \frac{1-x-4x+4x^2+5x}{1-x} = \frac{4x^2+1}{1-x}$$

$S = \frac{4x^2+1}{(1-x)^2}$ put $x = \frac{1}{3}$ we get $s = \frac{\frac{4}{9}+1}{\frac{4}{9}} = \frac{13}{4}$

Sol15. Let n be the number of times.

$p = \frac{1}{2}, q = \frac{1}{2}$

According to question,

${}^{n}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^{n}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$

$\Rightarrow {}^{n}C_7 = {}^{n}C_9 \Rightarrow n-7=9 \Rightarrow n=16$

$p(x=2) = {}^{16}C_2\left(\frac{1}{2}\right)^2.\left(\frac{1}{2}\right)^{14} = \frac{15}{2^{13}}$

Sol16. $T_{r+1} = {}^{10}C_r\left(tx^{\frac{1}{5}}\right)^{10-r}\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$

$= {}^{10}C_r x^{\frac{10-r}{5}}.(1-x)^{\frac{r}{10}}.\, t^{10-r-r}$

According to question, $10-2r=0 \Rightarrow r=5$

$\therefore T_6 = {}^{10}C_5 x(1-x)^{\frac{1}{2}}$

T_6 is maximum, when $f(x) = x(1-x)^{\frac{1}{2}}$ is maximum.

$f'(x) = (1-x)^{\frac{1}{2}} - \frac{x}{2\sqrt{1-x}} = \frac{2(1-x)-x}{2\sqrt{1-x}}$

For maximum, $f'(x) = 0 \Rightarrow x = \frac{2}{3}$

$\therefore T_6 = {}^{10}C_5\frac{2}{3}.\left(\frac{1}{3}\right)^{\frac{1}{2}} = \frac{2(10!)}{3\sqrt{3}(5!)^2}$

Sol17. $\vec{a}\times(\vec{a}\times\vec{b}) = (\vec{a}.\vec{b})\vec{a} - |\vec{a}|^2\vec{b} = 0 - |\vec{a}|^2\vec{b}$ (as $\vec{a}.\vec{b}=0$ given)

$\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b})) = -|\vec{a}|^2\vec{a}\times\vec{b}$

$\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b}))) = -|\vec{a}|^2\vec{a}\times(\vec{a}\times\vec{b}) = -|\vec{a}|^2\left(-|\vec{a}|^2\vec{b}\right) = |\vec{a}|^4\vec{b}$

Sol18. $P_1 : 3x+15+21z = 9$

$P_2 : x-3y-z = 5$

$P_3 : 2x+10y+14z = 5$

Ratio of the direction cosines of P_1 and P_2

$\frac{3}{2} = \frac{15}{10} = \frac{21}{14}$

Hence, P_1 and P_3 are parallel.

Sol19. $R = \{(P,Q)$ | P and Q are at the same distance from the origin}.

at (1, -1), $x^2+y^2 = (1)^2+(-1)^2 = 1+1 = 2$

Sol20. Let $a, ar, ar^2, ...$ an increasing G.P then $r > 1 \& a > 0$

given : $ar + ar^5 = \frac{25}{2}$.........(i)

and $ar^2\ ar^4 = 25 \Rightarrow a^2r^6 = 25 \Rightarrow \left(ar^3\right)^2 = 25$

$\Rightarrow ar^3 = 5$........(ii)

From (i) & (ii), $\frac{ar\left(1+r^4\right)}{ar^3} = \frac{25}{2\times 5}$

$\Rightarrow 2 + 2r^4 = 5r^2 \Rightarrow 2r^4 - 5r^2 + 2 = 0$

$\Rightarrow \left(2r^2 - 1\right)\left(r^2 - 2\right) = 0$

$\Rightarrow r^2 = \frac{1}{2}, 2 \quad \therefore r^2 = 2 \Rightarrow r = \sqrt{2}$

$\therefore a^2 . 2^3 = 25$

$a = \frac{5}{2^{3/2}}$

$t_4 = ar^3 = \frac{5}{2^{3/2}} . 2^{3/2} = 5$

$t_6 = ar^5 = \frac{5}{2^{3/2}} . 2^{5/2} = 5.2 = 10$

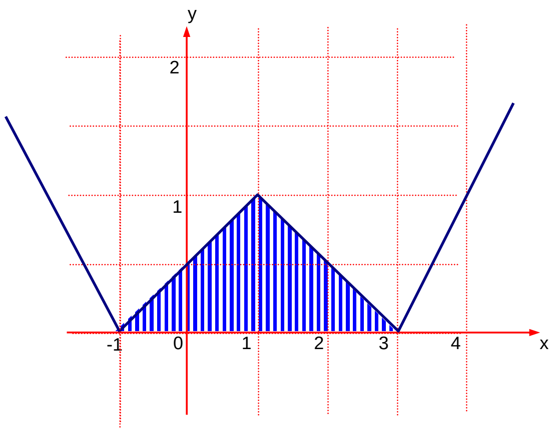
$t_8 = ar^7 = \frac{5}{2^{3/2}} . 2^{7/2} = 5.2^2 = 20$

$\therefore t_4 + t_6 + t_8 = 5 + 10 + 20 = 35$

Section B

Sol1. Information missing. The question was dropped by NTA. $y = \left|\left|x-1\right| - 2\right|$

Area bounded region $= \frac{1}{2}\times 4 \times 2 = 4$



Sol.2 $\sqrt{3}\cos^2 x = \left(\sqrt{3} - 1\right)\cos x + 1$

$\sqrt{3}\cos^2 x - \left(\sqrt{3} - 1\right)\cos x - 1 = 0$

$\left(\sqrt{3}\cos x + 1\right)\left(\cos x - 1\right) = 0$

$\therefore \cos x = -\frac{1}{\sqrt{3}}$ (rejected)

Hence, cosx = 1 $\Rightarrow$ x = 0 one solution

Sol3. $\log_4\left(x-1\right) = \log_2\left(x-3\right)$

$\Rightarrow \frac{1}{2}\log_2\left(x-1\right) = \log_2\left(x-3\right)$

$\Rightarrow \log_2\left(x-1\right) = \log_2\left(x-3\right)^2$

$\Rightarrow x-1=\left(x-3^2\right)$
$\Rightarrow x-1=x^2-6x+9$
$\Rightarrow x^2-7x+10=0$
$\Rightarrow (x-5)(x-2)=0$
$\Rightarrow x=2,5$
also $x-1>0$ and $x-3>0$
$x>1$ & $x>3$
Hence, x = 5 possible. Only one solution.

Sol4. $e^{\sin y}\cos y\dfrac{dy}{dx}+e^{\sin y}\cos x=\cos x$
Put $e^{\sin y}=t$
$e^{\sin y}\cos y\dfrac{dy}{dx}=\dfrac{dt}{dx}$
$\therefore \dfrac{dt}{dx}+t\cos x=\cos x$
I.F = $e^{\int \cos x\,dx}=e^{\sin x}$
$\therefore t\,e^{\sin x}=\int e^{\sin x}\cos x\,dx$
Put $\sin x=u, \cos x dx=du$
$e^{\sin y}e^{\sin x}=\int e^u du=e^u+c=e^{\sin x}+c$
$\therefore \text{put} x=0,\ y(0)=0, 1=1+c\Rightarrow c=0$
$\therefore e^{\sin x}e^{\sin y}=e^{\sin x}$
$\therefore e^{\sin x}\neq 0,\ e^{\sin y}=1=e^0$
$\sin y=0\Rightarrow y=0$
$\therefore 1+0+0+0=1$

Sol5. Let $A(-2,-21,29), B(-1,-16,23), P(\lambda,2,1), Q(4,-2,2)$
Given $\overrightarrow{AB}\perp\overrightarrow{PQ}$
$\therefore \overrightarrow{AB}.\overrightarrow{PQ}=0$
$\left(\hat{i}+5\hat{j}-6\hat{k}\right).\left((4-\lambda)\hat{i}-4\hat{j}+\hat{k}\right)=0$
$4-\lambda-20-6=0$
$\lambda=-22$
$\therefore \left(\dfrac{\lambda}{11}\right)^2+\left(\dfrac{-4\lambda}{11}\right)-4=4+8-4=8$

Sol6. $3\sin x+4\cos x=k+1, \cos\alpha=\dfrac{3}{5},\ \sin\alpha=\dfrac{4}{5}$
$5\sin(x+\alpha)=k+1,$
$\therefore -5\le k+1\le 5\Rightarrow -6\le k\le 4$
$\therefore$ total number of integral values of k is 11.

Sol7. $(x-1)\left(x^2-x+1\right)=0$
$x=1, x=\dfrac{1\pm\sqrt{3}i}{2}=\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i, \dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i=e^{i\frac{\pi}{3}}, e^{-i\frac{\pi}{3}}$
Sum of 162^{th} power of roots = $1+e^{i54\pi}+e^{-i54\pi}=1+1+1=3$

Sol8. $30\ {}^{30}C_0 + 29\ {}^{30}C_1 + ... + 2\ {}^{30}C_{28} + 1.\ {}^{30}C_{29} = n\ .\ 2^m$

$$\sum_{r=0}^{2.9}(30-r)\,{}^{30}C_r = \sum_{r=0}^{29}(30-r)\frac{30!}{r!(30-r)!}$$

$$= \sum_{r=0}^{29}\frac{30(29!)}{r!(29-r)!} = 30\sum_{r=0}^{29}{}^{29}C_r = 30\ .\ 2^{29} = 15.2^{30}$$

$\therefore n = 15, m = 0$

$\therefore n + m = 15 + 30 = 45$

Sol9. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$

$2yy_1 = a$

$$y^2 = 2yy_1\left(x + \frac{\sqrt{2yy_1}}{2}\right)$$

$y = 2y_1x + y_1\sqrt{2yy_1}$

$(y - 2y_1x)^2 = y_1^2.2yy_1 = 2yy_1^3$

$\therefore$ order $= 1$, degree $= 3$

Hence, degree – order = 3 – 1 = 2

Sol10. $\int_0^{\pi}|\sin 2x|dx$

$$= 2\int_0^{\pi/2}\sin 2x\ dx$$

$$= 2\left[\frac{-\cos 2x}{2}\right]_0^{\pi/2} = 2\left(\frac{1}{2} - \left(-\frac{1}{2}\right)\right) = 2$$