FIITJEE

Solutions to JEE (Main)-2021

JEE–Main–2021 –Feb–25–Second–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

(PHYSICS)

Answers

Section-A

1.	C	**2.**	C	**3.**	A	**4.**	A
5.	C	**6.**	C	**7.**	B	**8.**	B
9.	C	**10.**	None	**11.**	B	**12.**	C
13.	D	**14.**	None	**15.**	A	**16.**	B
17.	D	**18.**	A	**19.**	A	**20.**	A

Section-B

1.	20	**2.**	1	**3.**	208	**4.**	2
5.	2	**6.**	10	**7.**	10	**8.**	180
9.	36	**10.**	2				

SECTION – A

Sol1. As we know that maximum acceleration will act on the particle when it is at extreme position.

$$\omega = \frac{\frac{\pi}{6}}{0.1} = \frac{10\pi}{6}$$

$$f = m\omega^2 x$$

$$\frac{f}{m} = \omega^2 A = \frac{1000\pi^2}{36} \times 0.36 = \pi^2 = 9.87\text{N}$$

Sol2. Since equation of the gas process is given so we can convert it into T & V form.

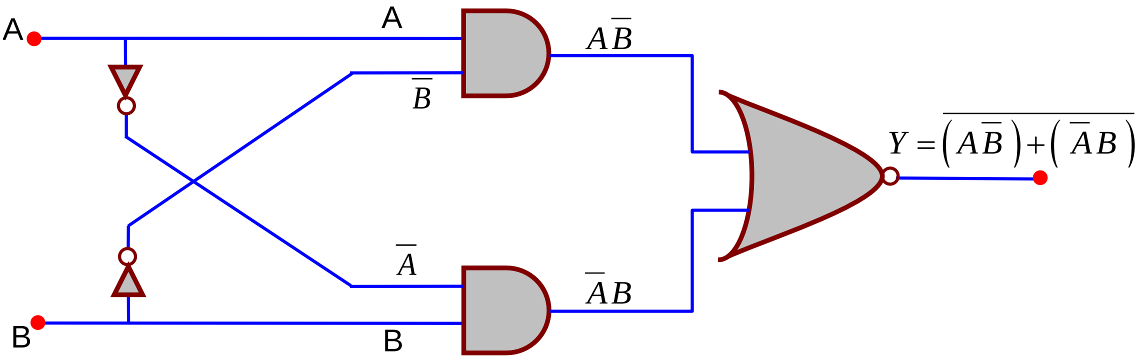
$$PV^{1/2} = C$$

$$\Rightarrow \frac{T}{\sqrt{V}} = C$$

$$\Rightarrow \frac{T_1}{\sqrt{V_1}} = \frac{T_2}{\sqrt{V_2}}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{1}{\sqrt{2}}$$

Sol3. Using truth table of logic gates



$$Y=\overline{(A\bar{B})+(\bar{A}B)}=\left(\overline{(A\bar{B})}\right)\left(\overline{(\bar{A}B)}\right)$$

$$Y=\left(\bar{A}+\bar{\bar{B}}\right)\left(\bar{\bar{A}}+\bar{B}\right)=\left(\bar{A}+B\right)\left(A+\bar{B}\right)=AB+\bar{A}\bar{B}$$

Sol4. Stopping potential defined in terms of wavelength as:

$$eV=\frac{hc}{\lambda}-\phi\Rightarrow 0.710\,eV=\frac{hc}{491}-\phi\Rightarrow 1.43\text{ eV}=\frac{hc}{\lambda}-\phi$$

Using above two equation we can calculate:

$\lambda = 382\,nm$

Sol5. Fermi level of p-type semiconductors will go downward

Sol6. $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}=\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{r^2}\times\frac{r^2}{\frac{hc}{\lambda}\times\lambda}=\frac{Fr\times r}{E\times\lambda}$

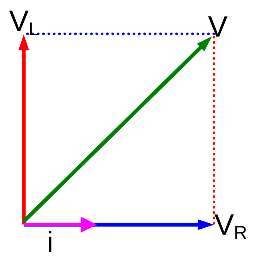
Dimension of 'Fr' and 'E' are same.

$\Rightarrow$ dimensionless

Sol7.

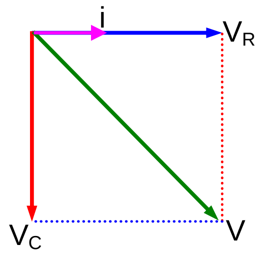
When capacitor is removed.

$\tan 45^0=\frac{X_L}{X_R}\Rightarrow\omega L=R$



When inductor is removed.

$\tan 45^0=\frac{X_C}{X_R}\Rightarrow\frac{1}{\omega C}=R$



$$i_0=\frac{V_0}{Z}=\frac{V_0}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}=\frac{V_0}{R}=\frac{220}{110}=2A$$

Sol8.

When second stone is released
Equation for first ball :

$$x + 20 = 10t + \frac{1}{2}gt^2$$

Equation for second ball:

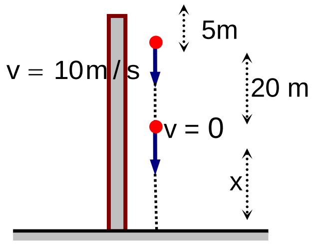
$$x = \frac{1}{2}gt^2$$

Using these two equation

$$20 = 10t \Rightarrow t = 2\sec$$

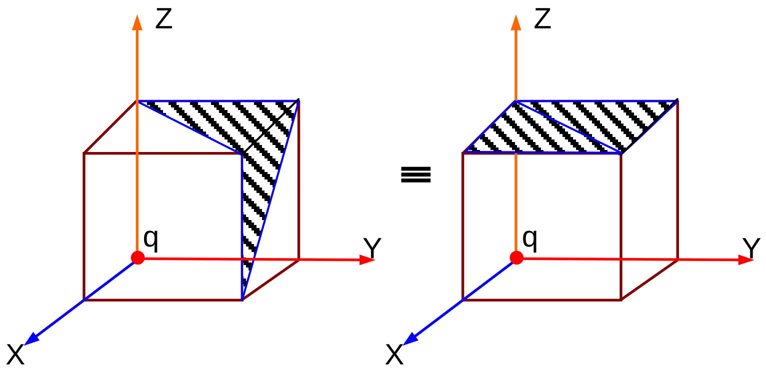
$$x = \frac{1}{2} \times 10 \times 4 = 20m$$

$$H = 20 + 20 + 5 = 45m.$$



Sol9. Translational kinetic energy will be equal to rotational kinetic energy corresponds to each degree of freedom

Sol10.



Upper shaded part is symmetric w.r.t to charge but right side shaded part is not symmetric. Hence flux is less then the $\frac{q}{24\varepsilon_0}$

Sol11. $\lambda_{air} = \frac{c}{f_c}$

Sol12. Using diffraction formula of circular hole:

$$r \propto \frac{1}{D}$$

$\Rightarrow$ size will decrease.

Sol13. Using de-Broglie equation for wave nature of particle:

$$\lambda = \frac{hc}{mv} \Rightarrow \lambda \alpha \frac{1}{m}$$

$$\Rightarrow \frac{\lambda_e}{\lambda_P} = \frac{\frac{1}{m_e}}{\frac{1}{m_P}} = \frac{m_P}{m_e} = 1836$$

Sol14.

For Pos 1.

Initial angular momentum

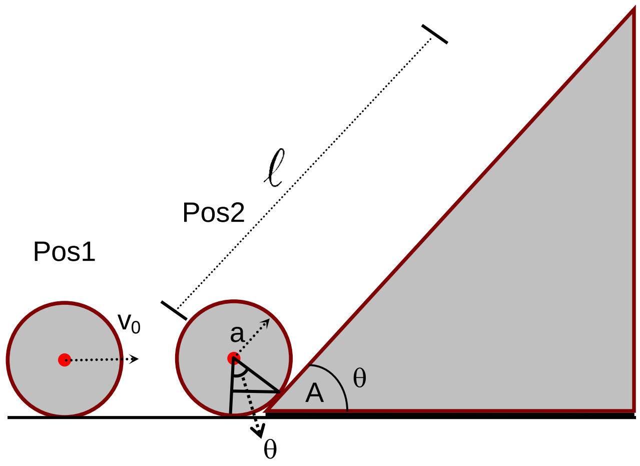
$$mv_0R\cos\theta + \frac{2}{5}mR^2\omega$$

For Pos 2.

$$mva + \frac{2}{5}mR^2\omega$$

From energy conservation

$$\frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mR^2\omega^2 = mg\ell\sin\theta$$



Sol15. Using Bohr's theory:

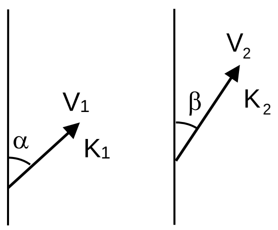
$$\lambda = \frac{hc}{E_2 - E_1} = \frac{1242\,ev-nm}{(-3.4)-(-13.6)} = 121.8\,nm$$

Sol16. In ferromagnetic material, below Curie's temperature, a domain is defined as macroscopic region with saturation magentisation.

Sol17. since force will act in horizontal direction so vertical component of speed will be same

$$v_1\cos\alpha = v_2\cos\beta$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{v_1}{v_2}\right)^2 = \frac{\cos^2\beta}{\cos^2\alpha}$$



Sol18. Using general equation of SHM:

$$y = A\sin(\omega t + \phi_0) = \frac{A}{2}$$

$$\Rightarrow \omega t + \phi_0 = \frac{\pi}{6}, \frac{5\pi}{6}$$

At t = 0

$$\Rightarrow \phi_0 = \frac{\pi}{6}, \frac{5\pi}{6}$$

Since it is moving in $-x$ direction:

$$\phi = \frac{5\pi}{6}$$

Sol19. Fact based

Sol20. Since both spring applied force in same direction.

Equivalent spring constant $= 4k$

$$T = 2\pi\sqrt{\frac{m}{4k}}$$

$$T = \pi\sqrt{\frac{m}{k}}$$

Section-B

Sol1.

$$T\sin\theta = \frac{kq^2}{r^2}$$

$$T\cos\theta = mg$$

$$\tan\theta = \frac{kq^2}{mgr^2}$$

Using the above equation find the value of 'q'

Sol2. $P = \sqrt{2Km}$

$$\frac{n}{2} = \sqrt{\frac{m_1}{m_2}} = \frac{1}{2}$$

$n = 1$

Sol3. Efficiency in the Carnot engine is defined as:

$$\eta = \frac{\text{work}}{\text{heat}} = \frac{1}{4}$$

$$\eta = 1 - \frac{T_c}{T_h},\ 2\eta = 1 - \frac{T_c - 52}{T_h} \Rightarrow T_h = 208K$$

Sol4. $i_2 = 2i_1$

$\Rightarrow i_1 + i_2 = 6$

$\Rightarrow i_1 = 2A.$

Sol5. $v = \sqrt{\frac{T}{\mu}}$

new $v' = \sqrt{\frac{1.04\,T}{\mu}}$

$$\%\text{ change} = \left(\sqrt{1.04} - 1\right) \times 100$$

$$= 2\%$$

Sol6. Energy corresponding to a particle is mc^2

$$\frac{hc}{\lambda} = mc^2 \Rightarrow \frac{hc}{\lambda} = \left(\frac{x}{3}h\right)c^2 \Rightarrow x = \frac{3}{\lambda c} = \frac{3}{10 \times 10^{-10} \times 3 \times 10^{8}} = \frac{1}{10^{-1}} = 10$$

Sol7. Using conservation of energy :

$$\Rightarrow \frac{1}{2}mv_i^2 - \frac{Gmm}{R} = 0 - \frac{Gmm}{11R}$$

$$\Rightarrow v_i^2 = \frac{20}{11}\frac{Gm}{R}$$

Escape velocity is defined as $\sqrt{\frac{2GM}{R}}$

$$v_i = \sqrt{\frac{10}{11}}v_e$$

Sol8. If angle between them is 180^0 then

$$\vec{P} \times \vec{Q} = \vec{q} \times \vec{P} = 0$$

Sol9. After contact, charges an each sphere will be

$$\frac{q_1 + q_2}{2} = 1\,nc \Rightarrow F = \frac{k q_1 q_2}{r^2} = 36 \times 10^{-9}\,N$$

Sol10. $I = \frac{1}{2} c\varepsilon_0 E_0^2 \Rightarrow \frac{8}{4\pi(10)^2} \times \frac{1}{2} = \frac{1}{4} \times c \times \frac{1}{c^2 \mu_0} \times E_0^2 \Rightarrow E_0 = \frac{2}{10}\sqrt{\frac{\mu_0 c}{\pi}} \Rightarrow x = 2$

PART –B (CHEMISTRY)

Answers

Section-A

1.	B	**2.**	D	**3.**	C	**4.**	D
5.	C	**6.**	C	**7.**	D	**8.**	B
9.	C	**10.**	A	**11.**	A	**12.**	C
13.	C	**14.**	A	**15.**	B	**16.**	B
17.	D	**18.**	A	**19.**	B	**20.**	D

Section-B

1.	4	**2.**	52	**3.**	9076	**4.**	15
5.	3	**6.**	6	**7.**	1	**8.**	181
9.	1	**10.**	2				

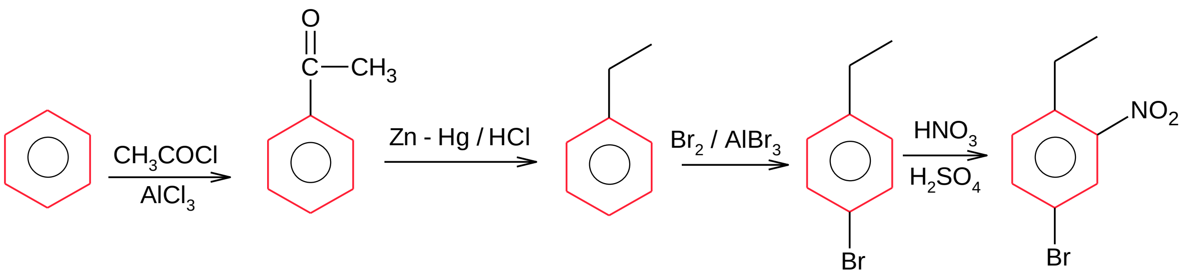
SECTION – A

Sol1. Statement I- True and Statement II- False
At room temperature stable form of sulphur is octahedral or Rhombic or α- sulphur

Sol2. Viscosity of the hydrophilic sol is always higher than H_2O, because dispersed phase particle attracted by dispersion medium.

Sol3. SF_4 has sp^3d hybridization and one type is axial while other type is equatorial
SiF_4 has sp^3 (all bonds are equal)
BF_4^- has sp^3 (all bonds are equal)
XeF_4 has sp^3d^2 and shape is sq. planar (All bonds are equal)

Sol4. Conversion from



Sol5. $Ca(OH)_2 \rightleftharpoons Ca^{++} + 2OH^-$
S S 2S
$\therefore K_{sp} = 4S^3$

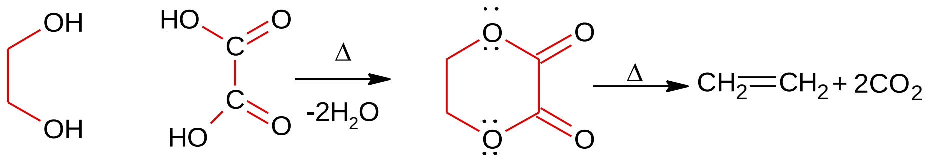
$$S = \sqrt[3]{\frac{k_{sp}}{4}} = \sqrt[3]{\frac{5.5\times10^{-6}}{4}} = 1.11\times10^{-2}$$

Sol6. In acidic medium Aniline gets protonated and formed anilinium ion which is the meta- directing group, but A is major product of 51% .

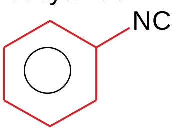
Sol7. Indium and other semi conducting materials behaves different characteristic of impurities and pure particle at high temperature zone. Hence in metallurgical process it is refined by the zone refining.

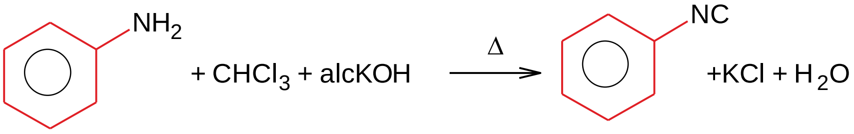
Sol8. I > III > IV > II
$[FeF_6]^{3-} \rightarrow Fe^{3+} - 3d^5$ – all are unpaired (n =5)
$[Co(NH_3)_6]^{3+} \rightarrow Co^{3+} - 3d^6$ – all are paired (n = 0)
$[NiCl_4]^{2-} \rightarrow Ni^{2+} - 3d^8 \rightarrow$ (n= 2)
$[Cu(NH_3)_4]^{2+} \rightarrow Cu^{++} \rightarrow 3d^9 \rightarrow$ (n= 1)

Sol9.



Sol10. Primary amine, whether aliphatic or aromatic when warmed with $CHCl_3$, alcoholic (KOH) it forms isocyanide





(Carbyl Amine Reaction)

Sol11. Bond dissociation enthalpy is the energy required to break 1 mole of bond. According to experimental data

Bond energy ; $Cl_2 > Br_2 > F_2 > I_2$

Sol12. NaCN converted into HCN and Na_2S converted into H_2S

$NaCN + HNO_3 \rightarrow NaNO_3 + HCN$

$Na_2S + 2HNO_3 \rightarrow NaNO_3 + H_2S$

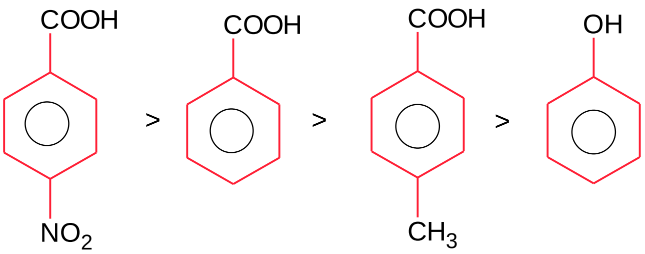
Sol13. **Cu, Zn, Ni**

Composition of german silver [Cu-50%, Zn – 20% , Ni-30%]

Sol14. $CH_3CH_2CH_2CH_2CHO$

This is the industrial process of aldehyde prepration from alkene, called oxo process

Sol15. Acidic order



Simple concept of conjugate base stability

Sol16. Both statements are true when pH decreases from 5.6 then it becomes more acidic hence it is called Acid rain.

Sol17. α- anomer of maltose is 1,4 combination.(C_1–C_4) glycosidic linkage

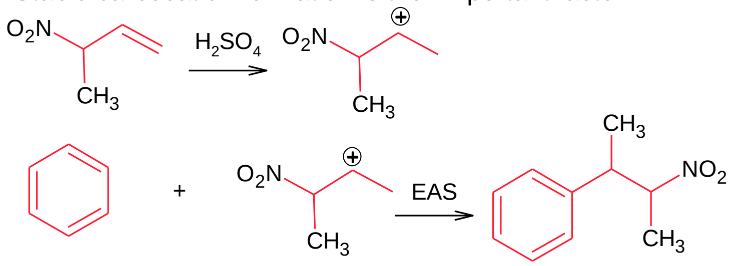
Sol18. Statement – I True

Statement – II False

Both statements are true

Sol19. With CO_2 gas water forms carbonic acid

Sol20. Stable carbocation formation is the important factor.



SECTION- B

Sol1. $NO_3^- + 4H^+ + 3e^- \longrightarrow NO + 2H_2O$; $E^o_{NO_3^-/NO} = 0.96V$

$NO_3^- + 2H^+ + e^- \longrightarrow NO_2 + H_2O$; $E^o_{NO_3^-/NO_2} = 0.79V$

Let $[HNO_3] = y\ M$

$[H^+] = y\ M$, $[NO_3^-] = y\ M$

For same thermodynamic tendency of reduction

$E_{NO_3^-/NO} = E_{NO_3^-/NO_2}$

$$E^o_{NO_3^-/NO} - \frac{0.059}{3}\log\frac{p_{NO}}{y \times y^4} = E^o_{NO_3^-/NO_2} - \frac{0.059}{1}\log\frac{p_{NO_2}}{y \times y^2}$$

$$0.96 - \frac{0.059}{3}\log\frac{p_{NO}}{y^5} = 0.79 - 0.059\log\frac{p_{NO_2}}{y^3}$$

Assume $(p_{NO} = p_{NO_2} = 1bar)$

$$0.17 = -\frac{0.0591}{1}\log\frac{p_{NO_2}}{y^3} + \frac{0.059}{3}\log\frac{p_{NO}}{y^5}$$

$$0.17 = -\frac{0.059}{3}\log\frac{p^3{}_{NO_2}}{y^9} + \frac{0.059}{3}\log\frac{p_{NO}}{y^5}$$

$$= \frac{0.059}{3}\left[\log\frac{p_{NO}}{y^5} - \log\frac{p^3{}_{NO_2}}{y^9}\right] = \frac{0.059}{3} \times \log\left[\frac{p_{NO}}{y^5} \times \frac{y^9}{p^3{}_{NO_2}}\right] = \frac{0.059}{3}\log y^4$$

$$\log y^4 = \frac{0.17 \times 3}{0.059} \Rightarrow y = 10^{2.16}$$

Comparing x = 2.16 and 2x = 4.32

Nearest integer is 4.

Sol2. $$\log\left(\frac{k_2}{k_1}\right) = \frac{Ea}{2.303 \times 8.314}\left[\frac{1}{300} - \frac{1}{325}\right]$$

$$\log(5) = \frac{Ea}{2.303 \times 8.314}\left[\frac{1}{300} - \frac{1}{325}\right]$$

$$0.7 = \frac{Ea}{2.303 \times 8.314}\left[\frac{25}{300 \times 325}\right]$$

$$\therefore Ea = \frac{0.7 \times 2.303 \times 8.314 \times 300 \times 325}{25} = 52271.7$$

Ea in kJ / mole $= \frac{52271.7}{1000}$ = 52.2 kJ / mol

Nearest integer is 52.

Sol3. Density = $\frac{M \times z}{N_A a^3}$

$$d = \frac{63.5 \times 4}{6.02 \times 10^{23} \times \left(3.596 \times 10^{-10}\right)^3 \times 1000} = 9076 \text{ kg / m}^3$$

Sol4. If $\Delta E = 0$

Q= –W

$W = -\text{Pext}(\Delta V) = -4.3\ \ nRT\left[\frac{1}{P_2} - \frac{1}{P_1}\right]$

$= -4.3 \times 5 \times 8.314 \times 293\left[\frac{1}{1.3} - \frac{1}{2.1}\right]$

$= -4.3 \times 5 \times 8.314 \times 293\left[\frac{2.1 - 1.3}{2.1 \times 1.3}\right] = -15347.70 \text{ K} = -15.3 \text{ kJ}$

Q = 15 kJ

Sol5. $AB \rightleftharpoons A^+ + B^-$

1-x x x $\therefore i = 1 + x$

$\Delta T_b = i \times k_b \times m$

$2.5 = (1+x) \times 0.52$ m

$2.5 = 1.75 \times 0.52 \times m$

$\therefore$ molality = $\frac{2.5}{1.75 \times 0.52} = 2.74$

Nearest integer is 3.

Sol6. Meq of NaOH = Meq $H_2C_2O_4$

$4.44 \times N = 1.25 \times 2 \times 10$

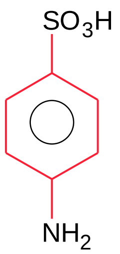
$\therefore N = \frac{1.25 \times 2 \times 10}{4.44} = 5.63$

$\therefore$ N = M = 5.63

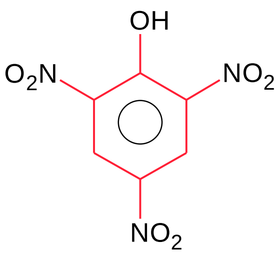
Nearest integer is 6

Sol7.

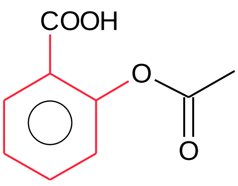
Sulphanilic Acid



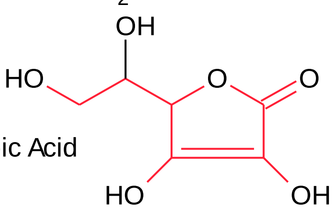
Picric Acid



Aspirin



Ascorbic Acid



Sol8. $E = \frac{hc}{\lambda} = \left\{\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{663 \times 10^{-9}}\right\}$

$= 0.03 \times 10^{-17} = 3.0 \times 10^{-19}$ J / atom

$= (3.0 \times 10^{-19} \times 6.02 \times 10^{23} \times 10^{-3})$KJ / mol

$= 18.06 \times 10^1$ kJ / mole= 180.6 ≈ 181KJ / mole

Sol9. Cs is used in photoelectric cell

Sol10. $Cu^{++} = 3d^9\ 4s^0$

n = 1

$\mu = \sqrt{1\times(1+2)} = \sqrt{3}$ = 1.73BM

Nearest integer is 2.

PART–C (MATHEMATICS)

Answers

SECTION – A

1.	A	**2.**	A	**3.**	B	**4.**	B
5.	D	**6.**	C	**7.**	B	**8.**	D
9.	B	**10.**	C	**11.**	B	**12.**	D
13.	A	**14.**	C	**15.**	A	**16.**	D
17.	B	**18.**	D	**19.**	D	**20.**	A

SECTION – B

1.	44	**2.**	5	**3.**	45	**4.**	9
5.	1	**6.**	1	**7.**	19	**8.**	5
9.	4	**10.**	2				

SECTION – A

Sol1. $\alpha^2-6\alpha-2=0,\ \beta^2-6\beta-2=0$

$\Rightarrow \alpha^2-2=6\alpha, \beta^2-2=6\beta$

$$\frac{a_{10}-2a_8}{3a_9}=\frac{\alpha^{10}-\beta^{10}-2\left(\alpha^8-\beta^8\right)}{3\left(\alpha^9-\beta^9\right)}=\frac{\alpha^8\left(\alpha^2-2\right)-\beta^8\left(\beta^2-2\right)}{3\left(\alpha^9-\beta^9\right)}$$

$$\frac{\alpha^8.\,6\alpha-\beta^8.6\beta}{3\left(\alpha^9-\beta^9\right)}=2$$

Sol2. 2x + 3y + 2z = 9 ………(i)
3x + 2y + 2z = 9 ………(ii)
x – y + 4z = 8 …………(iii)
(ii) – (i) $\Rightarrow$ x = y
Then (iii) $\Rightarrow$ z = 2
(i) $\Rightarrow 5x+4=9\Rightarrow\left(x=1, y=1, z=2\right)$ unique solution

Sol3. $I_n=\int_{\pi/4}^{\pi/2}\cot^n x\,dx=\int_{\pi/4}^{\pi/2}\cot^{n-2}x\left(\operatorname{cosec}^2x-1\right)dx$

$$=\left[\frac{-\cot^{n-1}x}{n-1}\right]_{\pi/4}^{\pi/2}-I_{n-2}=\frac{1}{n-1}-I_{n-2}$$

$$\Rightarrow I_n+I_{n-2}=\frac{1}{n-1}$$

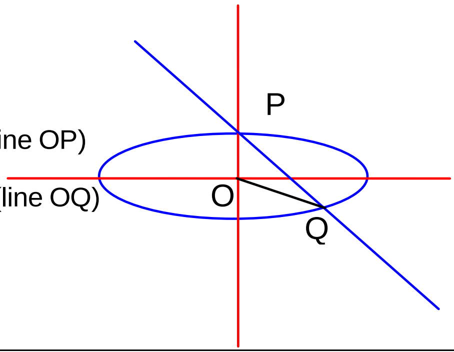
$$\left.\begin{aligned}n=4\Rightarrow I_4+I_2&=\frac{1}{3}\\ n=5\Rightarrow I_5+I_3&=\frac{1}{4}\\ n=6\Rightarrow I_6+I_4&=\frac{1}{5}\end{aligned}\right\}\Rightarrow \frac{1}{I_2+I_4},\frac{1}{I_3+I_5},\frac{1}{I_4+I_6}\text{ are in A.P.}$$

Sol4.

combined equation of pair of lines OP and OQ is

$x^2+2y^2=2(x+y)^2$

$\Rightarrow x^2+4xy=0 \Rightarrow x(x+4y)=0 \Rightarrow \begin{cases} x=0 \text{ (line OP)} \\ y=-\dfrac{x}{4} \text{ (line OQ)} \end{cases}$

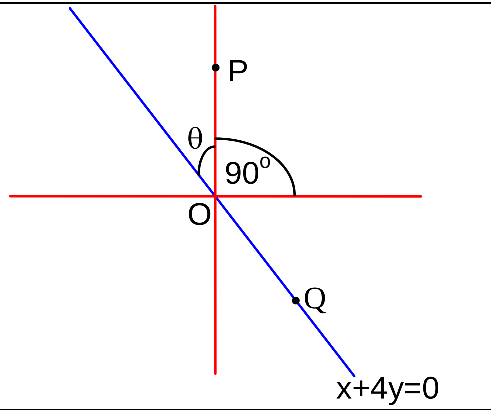


$\tan(90^o+\theta)=-\dfrac{1}{4}$

$\Rightarrow -\cot\theta=-\dfrac{1}{4} \Rightarrow \tan\theta=4$

$\theta=\tan^{-1}4=\cot^{-1}\dfrac{1}{4}=\dfrac{\pi}{2}-\tan^{-1}\dfrac{1}{4}$

$\angle POQ=\pi-\theta=\dfrac{\pi}{2}+\tan^{-1}\dfrac{1}{4}$



Sol5. $z^2+\alpha z+\beta=0, \quad \alpha,\beta\in R$

roots : $1-2i,\ 1+2i$

Sum of roots = $2=-\alpha$ and product of roots = 5 = β

$\alpha-\beta=-2-5=-7$

Sol6. $x,y\in(0,\pi)$

$\cos x+\cos y-\cos(x+y)=\dfrac{3}{2}$

$2\cos\dfrac{x+y}{2}.\cos\dfrac{x-y}{2}-\left(2\cos^2\dfrac{x+y}{2}-1\right)=\dfrac{3}{2}$

$\Rightarrow 2\cos^2\dfrac{x+y}{2}-2\cos\dfrac{x-y}{2}.\cos\dfrac{x+y}{2}+\dfrac{1}{2}=0$

$\Rightarrow \cos^2\dfrac{x+y}{2}-\cos\dfrac{x-y}{2}.\cos\dfrac{x+y}{2}+\dfrac{1}{4}=0$

$D=\cos^2\dfrac{x-y}{2}-4.1.\dfrac{1}{4}\geq 0$ as $\cos\left(\dfrac{x+y}{2}\right)\in R$

$\Rightarrow -\sin^2\dfrac{x-y}{2}\geq 0 \Rightarrow \sin^2\dfrac{x-y}{2}=0$

$\Rightarrow \sin\dfrac{x-y}{2}=0$ & $\cos\dfrac{x+y}{2}=\dfrac{\cos\dfrac{x-y}{2}\pm 0}{2}$

As, $x,y\in(0,\pi)$

$\dfrac{-\pi}{2}<\dfrac{x-y}{2}<\dfrac{\pi}{2}$

$\sin\dfrac{x-y}{2}=0 \Rightarrow \dfrac{x-y}{2}=0 \Rightarrow x=y$

then equation : $\cos x+\cos y-\cos(x+y)=\dfrac{3}{2}$

$\Rightarrow 2\cos x-\cos 2x=\dfrac{3}{2} \Rightarrow 2\cos x-\left(2\cos^2 x-1\right)=\dfrac{3}{2} \Rightarrow 2\cos^2 x-2\cos x+\dfrac{1}{2}=0$

$$\cos x = \frac{2 \pm \sqrt{4-4}}{4} = \frac{1}{2}$$

$$x = y = \frac{\pi}{3} \Rightarrow \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$$

Sol7. $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$16 = 25\left(1 - e^2\right)$

$\Rightarrow e = \frac{3}{5}$

$OF_1 = 5\left(\frac{3}{5}\right) = 3$

For Hyperbola : $e' = \frac{5}{3}$

$a = 3$

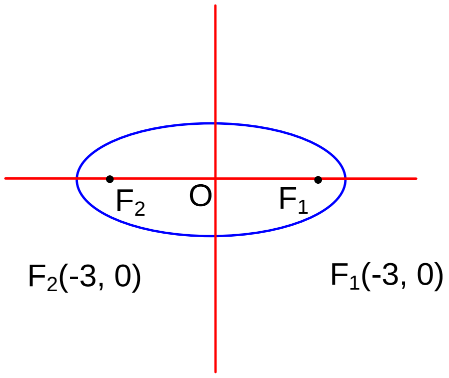
$e' = \frac{5}{3}$

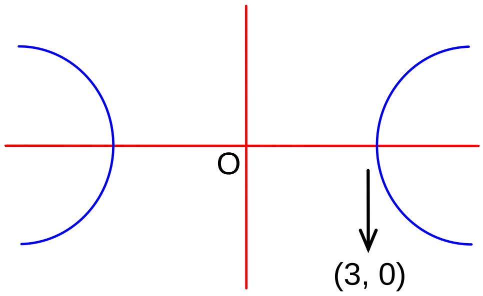
$b^2 = a^2\left(e'^2 - 1\right)$

$b^2 = 9\left(\frac{25}{9} - 1\right) = 16$

$b = 4$

Hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$





Sol8. Statement : "If you will work, you will earn money"
Contrapositive: If you will not earn money, you will not work.

Sol9. Let $A = \{a, b, c\}, B = \{1, 2, 3, 4, 5\}$ $n(A \times B) = 15$

x = number of one-one functions from A to B.

$= {}^5C_3 . 3! = 60$

y = number of one-one functions for A to $(A \times B)$

$= {}^{15}C_3 . 3! = 15 \times 14 \times 13 = 2730$

$\frac{Y}{x} = \frac{2730}{60} \Rightarrow 2y = 91x$

Sol10. $\operatorname{cosec}\left(2\cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)\right)$

Let $\cot^{-1}(5) = \theta$ and $\cos^{-1}\left(\frac{4}{5}\right) = \alpha$

$\Rightarrow \cot\theta = 5$ $\qquad$ $\cos\alpha = \frac{4}{5}$

$0 < \theta < \frac{\pi}{2}$ $\qquad$ $0 < \alpha < \frac{\pi}{2}$

$= \operatorname{cosec}(2\theta + \alpha) = \frac{1}{\sin(2\theta + \alpha)}$

$= \frac{1}{\sin 2\theta \cos\alpha + \cos 2\theta \sin\alpha} = \frac{1}{\frac{5}{13} . \frac{4}{5} + \frac{12}{13} . \frac{3}{5}}$

as
$$\begin{cases} \sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{2\left(\dfrac{1}{5}\right)}{1+\dfrac{1}{25}} = \dfrac{5}{13} \\ \cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\dfrac{1}{25}}{1+\dfrac{1}{25}} = \dfrac{12}{13} \end{cases}$$

Sol11. $I = \int \frac{8x^3+20x^2}{x^4+5x^3-7x^2}dx = 4\int \frac{2x+5}{x^2+5x-7}dx = 4\ln\left|x^2+5x-7\right| + c$

Sol12. $2x + 3y + 2z = 9$ ....(i)
$3x + 2y + 2z = 9$ ....(ii)
$x - y + 4z = 8$ ....(iii)
(ii) – (i) $\Rightarrow x = y$
Then (iii) $\Rightarrow z = 2$
(i) $\Rightarrow 5x + 4 = 9 \Rightarrow (x = 1, y = 1, z = 2)$ unique solution

Sol13. $P\left(\frac{\text{Smoker and non-vegetarian}}{\text{dead}}\right)$

$$= \frac{\frac{16}{40}\times\frac{35}{100}}{\frac{16}{40}\times\frac{35}{100}+\frac{10}{40}\times\frac{15}{100}+\frac{14}{40}\times\frac{10}{100}}$$

$$= \frac{16\times35}{16\times35+10\times15+14\times10}$$

$$= \frac{560}{560+150+140}$$

$$= \frac{56}{85}$$

Sol14. $\frac{a^{a^x}+a^{1-a^x}}{2} \geq \sqrt{a^{a^x}.a^{1-a^x}}$ $\qquad (AM \geq GM)$

$\Rightarrow a^{a^x}+a^{1-a^x} \geq 2\sqrt{a}$

Sol15. $A_{3\times3}$

$\det(A) = 4 \qquad 2A = \begin{bmatrix} 2a_1 & 2a_2 & 2a_3 \\ 2b_1 & 2b_2 & 2b_3 \\ 2c_1 & 2c_2 & 2c_3 \end{bmatrix}$

For $R_2 \to 2R_2 + 5R_3$

$$B = \begin{bmatrix} 2a_1 & 2a_2 & 2a_3 \\ 4b_1+10c_1 & 4b_2+10c_2 & 4b_3+10c_3 \\ 2c_1 & 2c_2 & 2c_3 \end{bmatrix}$$

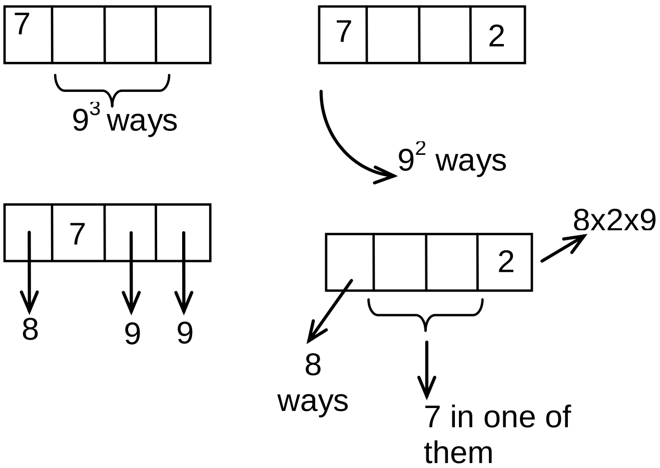
$$\det(B) = \begin{vmatrix} 2a_1 & 2a_2 & 2a_3 \\ 4b_1+10c_1 & 4b_2+10c_2 & 4b_3+10c_3 \\ 2c_1 & 2c_2 & 2c_3 \end{vmatrix} = \begin{vmatrix} 2a_1 & 2a_2 & 2a_3 \\ 4b_1 & 4b_2 & 4b_3 \\ 2c_1 & 2c_2 & 2c_3 \end{vmatrix} = 16\det(A) = 64$$

$(R_2 \to R_2 - 5R_3)$

Sol16. $\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}$

$$\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{n}.\frac{1}{\left(1+\frac{r}{n}\right)^2}=\int_0^1\frac{1}{(1+x)^2}=\left[-\frac{1}{1+x}\right]_0^1=-\frac{1}{2}-(-1)=\frac{1}{2}$$

Sol17.



$8 \times 3 \times 9^2 = 24 \times 81$

Total = n(A) = 9^3 + 24 x 81 = 81 x 33

In favourable events, the last digit of the number must be either 2 or 7.

Favourable cases = $8\times 9^2+9^2+8\times 9\times 2=873$, probability $=\frac{873}{81\times 33}=\frac{97}{297}$

Sol18.

Slope = 1

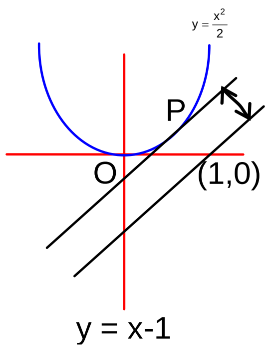
$\frac{dy}{dx}=x=1$

$P\left(1,\frac{1}{2}\right)$

Equation of tangent at P : $y-\frac{1}{2}=1(x-1)$

$\Rightarrow y=x-\frac{1}{2}$

dist. = $\frac{\frac{1}{2}}{\sqrt{2}}=\frac{1}{2\sqrt{2}}$



Sol19. $1+\alpha^2=1\Rightarrow\alpha=0$

$\&\ \alpha^2+\beta^2=1\Rightarrow\beta^2=\pm1$

$\&\ \alpha-\alpha\beta=0\Rightarrow\alpha(1-\beta)=0\Rightarrow\alpha=0$ or $\beta=1$

$\alpha=0\ \&\ \beta=1$ or $\alpha=0\ \&\ \beta=-1$

$\alpha^4+\beta^4=1$

Sol20. $f(x)=\frac{5^x}{5^x+5}$

$$f(2-x)=\frac{5^{2-x}}{5^{2-x}+5}=\frac{25/5^x}{\frac{25}{5^x}+5}=\frac{25}{25+5^{x+1}}=\frac{5}{5+5^x}$$

$f(x)+f(2-x)=1$

$S=\sum_{k=1}^{39} f\left(\frac{k}{20}\right)$........(i)

$S=\sum_{k=1}^{39} f\left(\frac{40-k}{20}\right)$

$\Rightarrow S=\sum_{k=1}^{39} f\left(2-\frac{k}{20}\right)$..........(ii)

$(i)+(ii)\Rightarrow 2S=\sum_{k=1}^{39}\left(f\left(\frac{k}{20}\right)+f\left(2-\frac{k}{20}\right)\right)$

$2S=\sum_{k=1}^{39} 1=39$

$S=\frac{39}{2}$

SECTION – B

Sol1. $\ell_1:\frac{x-3}{1}=\frac{y+1}{2}=\frac{z-4}{2}=t$

$\ell_2:\frac{x-3}{2}=\frac{y-3}{2}=\frac{z-2}{1}=s$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 2\\ 2 & 2 & 1\end{vmatrix}=\left(-2\hat{i}+3\hat{j}-2\hat{k}\right)$$

$\ell:\vec{r}=\vec{0}+\lambda\left(-2\hat{i}+3\hat{j}-2\hat{k}\right)$

$\ell\ \&\ \ell_1\rightarrow$

$$\left.\begin{array}{c}3+t=-2\lambda\\ \&\\ -1+2t=3\lambda\end{array}\right\}\Rightarrow 7=-7\lambda\Rightarrow\lambda=-1$$

Point of intersection $\ell\ \&\ \ell_1$ is $P(2,-3,2)$

Point Q on ℓ_2 is (3 + 2s, 3 + 2s, 2 + s).

$PQ=\sqrt{17}$

$17=(1+2s)^2+(6+2s)^2+s^2\Rightarrow 17=(1+2s)^2+(6+2s)^2+s^2\Rightarrow 17=9s^2+28s+37$

$9s^2+28s+20=0\Rightarrow s=\frac{-28\pm 8}{18}=\frac{-20}{18},-2$

As Q is in first octant $s=\frac{-20}{18}$ s =

$Q\left(3-\frac{20}{9},3-\frac{20}{9},2-\frac{10}{9}\right)=\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)=(a,b,c),\ 18(a+b+c)=44$

Sol2. $\lim_{x\to 0}\frac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}=b$, use of L'Hospital rule implies

$\lim_{x\to 0}\frac{a-4e^{4x}}{a\left(e^{4x}-1\right)+ax\left(4e^{4x}\right)}$

$=\frac{a-4}{0}\Rightarrow a=4$

$$\Rightarrow \text{the limit} = \lim_{x\to 0}\frac{4\left(1-e^{4x}\right)}{4\left(e^{4x}-1\right)+16xe^{4x}}\left(\frac{0}{0}\right)$$

$$\Rightarrow \lim_{n\to 0}\frac{4\left(-4.e^{4x}\right)}{4.4e^{4x}+16e^{4x}+16x.4e^{4x}}$$

$$=\frac{-16}{16+16}=-\frac{1}{2}=b$$

$$a-2b=4-(-1)=5$$

Sol3. digit at one's place in the value of 3^n is 3,9,7,1,3,9,7,1,…..
digit at one's place in the value of 7^n is 7,9,3,1,7,9,3,1,…...
For $3^n + 7^n$ to be a multiple of 10, digits at one's place in the value of 3^n and 7^n should be 3 and 7 or 7 and 3 or 9 and 1 or 1 and 9 respectively.
Case I : For digits 7 and 3, $n = 3,7,11,15,......$

$$11+(k-1)4\le 99\Rightarrow k\le 23$$

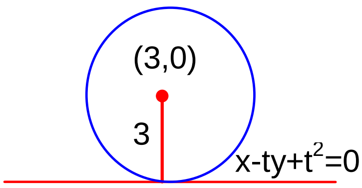
Case II : For digits 3 and 7, n = 1,5,9,13,…..

$$13+(k-1)4\le 99\Rightarrow k\le 22$$

Case III : For digits 9 and 1 or 1 and 9, there is no value of n.
Total number $23+22=45$

Sol4.

A tangent to $y^2=4x$ is $x-ty+t^2=0$



$$\frac{3+t^2}{\sqrt{1+t^2}}=3$$

$$\Rightarrow \left(3+t^2\right)^2=9\left(1+t^2\right)$$

$$\Rightarrow 9+t^4+6t^2=9+9t^2$$

$$t^4-3t^2=0$$

$$\Rightarrow t^2=0,3,$$

$$\Rightarrow t=0,\pm\sqrt{3}$$

$$x-\sqrt{3}y+3=0$$

$$y=\frac{x}{\sqrt{3}}+\sqrt{3}$$

Point of contact $\left(3,2\sqrt{3}\right)=(a,b)$

$$x-\sqrt{3}y+3=0$$

$$\sqrt{3}x+y-3\sqrt{3}=0$$

$$4x-6=0$$

$$x=\frac{3}{2},y=\frac{\sqrt{3}}{2}+\sqrt{3}$$

$$\&\left(\frac{3}{2},\frac{\sqrt{3}}{2}+2\right)=(c,d),\ 2(a+c)=2\left(3+\frac{3}{2}\right)=9$$

Sol5. $\left(2xy^2-y\right)dx+xdy=0$

$$\Rightarrow \frac{dy}{dx}+2y^2-\frac{y}{x}=0$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}-\frac{1}{x}\left(\frac{1}{y}\right)=-2$$

Put $\frac{1}{y} = z$

Then $-\frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx}$

$\frac{dz}{dx} + \frac{1}{x}z = 2$

$IF = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$zx = \int 2x dx$

$\frac{x}{y} = x^2 + c$

Point $(2,1) \Rightarrow c = 2-4 = -2 \Rightarrow y = \frac{x}{x^2-2}$

$|y(1)| = 1$

Sol6. $x = 4k+3; \quad k \in I$

$\frac{(2020+x)^{2022}}{8}$

$= \frac{(2020+4k+3)^{2022}}{8}$

$(2023+4k)^{2022}$

$= {}^{2022}C_0 (2023)^{2022}.(4k)^0 + {}^{2022}C_1 (2023)^{2021}.4k + ...$

$= (2023)^{2022} + 8m$

$= (253 \times 8 - 1)^{2022} + 8m = 8\ell + 8m + 1$

Required remainder = 1

Sol7. $I = 3\int_{-2}^{2} |x^2 - x - 2| dx$

$= 3\left(\int_{-2}^{-1}(x^2-x-2)dx - \int_{-1}^{2}(x^2-x-2)dx\right)$

$= 3\left[\left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)_{-2}^{-1} - \left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)_{-1}^{2}\right]$

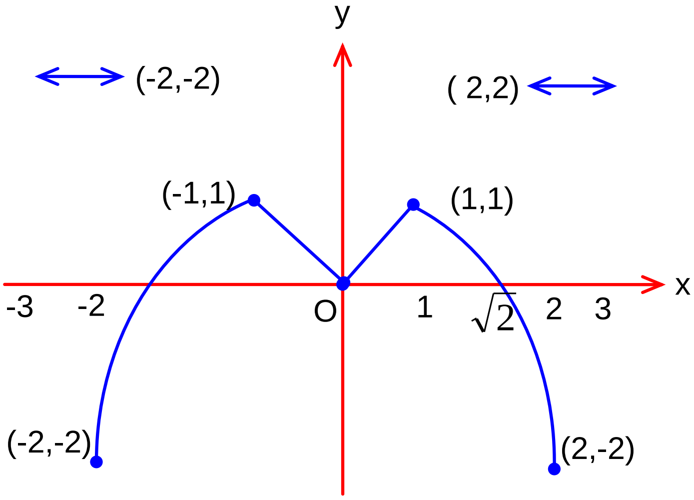
$= 3\left[\left(\frac{-1}{3} - \frac{1}{2} + 2\right) - \left(\frac{-8}{3} - 2 + 4\right) - \left\{\left(\frac{8}{3} - 2 - 4\right) - \left(\frac{-1}{3} - \frac{1}{2} + 2\right)\right\}\right]$

$= 3\left[\left(\frac{7}{6} + \frac{2}{3}\right) - \left\{-\frac{10}{3} - \frac{7}{6}\right\}\right]$

$= 3\left(\frac{11}{6} + \frac{27}{6}\right) = \frac{38}{2} = 19$

Sol8. $f(x) = \begin{Bmatrix} \min\{|x|, 2-x^2\}, & -2 \le x \le 2 \\ [|x|] & , 2 < |x| \le 3 \end{Bmatrix}$

Number of points where f is not differentiable = 5



Number of points where f is not differentiable = 5.

Sol9. $x = y^4$, $xy = k$

$\frac{dy}{dx} = \frac{1}{4y^3}$, $\frac{dy}{dx} = \frac{-k}{x^2}$

$P(x_1, y_1)$

where $x_1 = y_1^4$ & $x_1 y_1 = k$

$\Rightarrow y_1 = k^{1/5}, x_1 = k^{4/5}$

$m_1 = \frac{1}{4 \cdot k^{3/5}}, m_2 = \frac{-k}{k^{8/5}} = \frac{-1}{k^{3/5}}$

$m_1 m_2 = -1$

$m_1 = \frac{1}{4 \cdot k^{3/5}}, m_2 = \frac{-k}{k^{8/5}} = \frac{-1}{k^{3/5}}$

$m_1\ m_2 = -1$

$\Rightarrow \frac{-1}{4.k^{6/5}} = -1 \Rightarrow k^{6/5} = \frac{1}{4} \Rightarrow k^6 = \frac{1}{4^5} = \frac{1}{1024}$

$(4k)^6 = 2^{12} . \frac{1}{2^{10}} = 2^2 = 4$

Sol10. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \alpha & 3 \\ 3 & -\alpha & 1 \end{vmatrix} = \left(4\alpha\hat{i} + 8\hat{j} - 4\alpha\hat{k}\right)$

$|\vec{a} \times \vec{b}| = \sqrt{32\alpha^2 + 64} = 8\sqrt{3}$

$\Rightarrow 32\alpha^2 + 64 = 192$

$\Rightarrow \alpha^2 = \frac{128}{32} = 4$

$\vec{a} . \vec{b} = 3 - \alpha^2 + 3 = 6 - \alpha^2 = 6 - 4 = 2$