FIITJEE

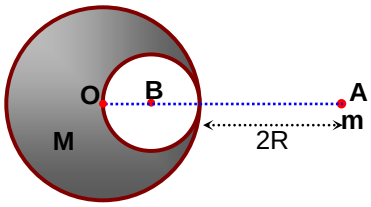
Solutions to JEE (Main)-2021

JEE–Main–2021 –Feb–25–First–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

(PHYSICS)

SECTION -A

Q1. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1 . Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2 . The value of F_1: F_2 is:

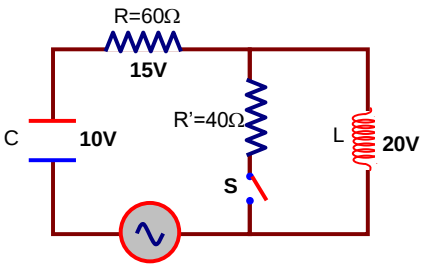


(A) 36 : 25 (B) 50 : 41
(C) 25 : 36 (D) 41 : 50

Q2. An α particle and a proton are accelerated from rest by a potential difference of 200 V. After this, their de Broglie wavelengths are λ_α and λ_P respectively. The ratio $\frac{\lambda_P}{\lambda_\alpha}$ is :

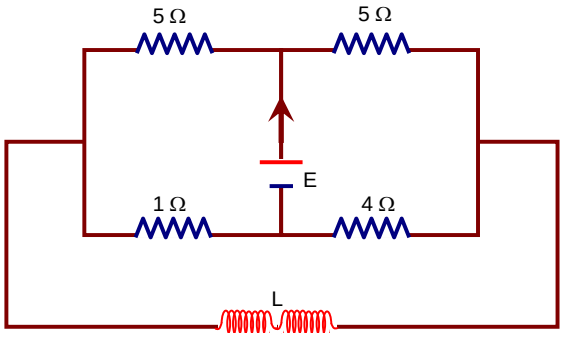
(A) 3.8 (B) 7.8
(C) 2.8 (D) 8

Q3. The angular frequency of alternating current in a L-C- R circuit is 100 rad/ s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



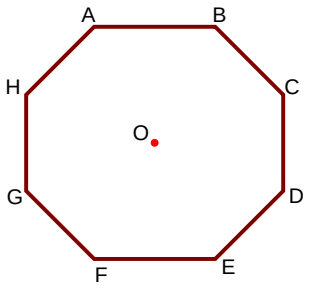
(A) 0.8 H and 150 μ F
(B) 1.33 H and 250 μ F
(C) 0.8 H and 250 μ F
(D) 1.33 H and 150 μ F

Q4. The current (i) at time t = 0 and $t = \infty$ respectively for the given circuit is :



(A) $\frac{5E}{18}, \frac{18E}{55}$
(B) $\frac{18E}{55}, \frac{5E}{18}$
(C) $\frac{5E}{18}, \frac{10E}{33}$
(D) $\frac{10E}{33}, \frac{5E}{18}$

Q5. In an octagon ABCDEFGH of equal side, what is the sum of $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$,if, $\overrightarrow{AO}=2\hat{i}+3\hat{j}-4\hat{k}$



(A) $16\hat{i}+24\hat{j}+32\hat{k}$

(B) $16\hat{i}-24\hat{j}+32\hat{k}$

(C) $-16\hat{i}-24\hat{j}+32\hat{k}$

(D) $16\hat{i}+24\hat{j}-32\hat{k}$

Q6. Two coherent light sources having intensity in the ratio 2x produce an interference pattern.

The ratio $\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$ will be :

(A) $\frac{\sqrt{2x}}{x+1}$ (B) $\frac{\sqrt{2x}}{2x+1}$

(C) $\frac{2\sqrt{2x}}{2x+1}$ (D) $\frac{2\sqrt{2x}}{x+1}$

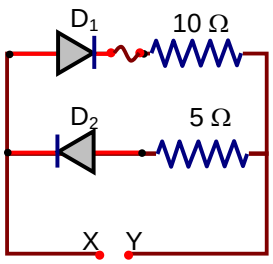
Q7. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.
Reason R: The product of their mass and radius must be same. $M_1 R_1 = M_2 R_2$
In the light of the above statements, choose the most appropriate answer from the options given below :
(A) **A** is correct but **R** is not correct
(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**
(C) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(D) **A** is not correct but **R** is correct

Q8. Given below are two statements :
Statement I : A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The bandwidth requirement for the signal is 4 kHz.
Statement II : The side band frequencies are 1002 kHz and 998 kHz.
In the light of the above statements, choose the correct answer from the options given below:
(A) Both **Statement I** and **Statement II** are true (B) **Statement I** is true but **Statement II** is false
(C) **Statement I** is false but **Statement II** is true (D) Both **Statement I** and **Statement II** are false

Q9. The pitch of the screw gauge is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72^{nd} division on circular scale coincides with the reference line, The radius of the wire is :
(A) 0.82 mm (B) 1.64 mm
(C) 0.90 mm (D) 1.80 mm

Q10. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A: When a rod lying freely is heated, no thermal stress is developed in it.
Reason R : On heating, the length of the rod increases.
In the light of the above statements, choose the correct answer from the options given below :
(A) Both **A** and **R** are true and **R** is the correct explanation of **A**
(B) **A** is false but **R** is true
(C) **A** is true but **R** is false
(D) Both **A** and R are true but **R** is NOT the correct explanation of A

Q11. A 5 V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery if the +ve terminal of the battery is connected to point X.



(A) ~ 0.5 A (B) ~ 0.43 A
(C) ~ 0.86 A (D) ~ 1.5 A

Q12. A diatomic gas, having $C_P = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$, is heated at constant pressure. The ratio dU : dQ : dW :

(A) 5 : 7 : 3
(B) 5 : 7 : 2
(C) 3 : 5 : 2
(D) 3 : 7 : 2

Q13. If the time period of a two meter long simple pendulum is 2s, the acceleration due to gravity at the place where pendulum is executing S.H.M. is :

(A) 9.8 ms^{-2}
(B) 16 m/s^2
(C) $2\pi^2$ ms^{-2}
(D) π^2 ms^{-2}

Q14. Two radioactive substances X and Y originally have N_1 and N_2 nuclei respectively. Half life of X is half of the life of Y. After three half lives of Y, number of nuclei of both are equal.

The ratio $\frac{N_1}{N_2}$ will be equal to :

(A) $\frac{3}{1}$
(B) $\frac{8}{1}$
(C) $\frac{1}{3}$
(D) $\frac{1}{8}$

Q15. Magnetic fields at two points on the axis of a circular coil at a distance of 0.05m and 0.2m from the centre are in the ratio 8 : 1. The radius of coil is ----------.

(A) 0.15 m
(B) 1.0 m
(C) 0.1 m
(D) 0.2 m

Q16. A student is performing the experiment of resonance column. The diameter of the column tube is 6cm. The Frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is :

(A) 14.8 cm
(B) 16.6 cm
(C) 18.4 cm
(D) 13 cm

Q17. Match List – I with List – II :

List - I	List – II
(A) h (Planck's constant)	(i) [M L T^{-1}]
(B) E (kinetic energy)	(ii) [M L^2 T^{-1}]
(C) V (electric potential)	(iii) [M L^2T^{-2}]
(D) P (liner momentum)	(iv) [M L^2 I^{-1} T^{-3}]

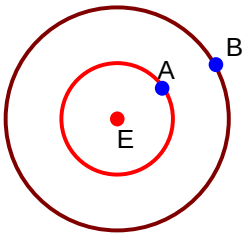
Choose the correct answer from the options given below :

(A) (a) $\rightarrow$ (i) (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (iii)
(B) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (i)
(C) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (i)
(D) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (i)

Q18. A proton, a deuteron and an α particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is ---------- and their speed is ----------in the ratio.

(A) 2 : 1 : 1 and 4 : 2 : 1
(B) 4 : 2 : 1 and 2 : 1 : 1
(C) 1 : 2 : 4 and 1 : 1 : 2
(D) 1 : 2 : 4 and 2 : 1 :1

Q19. Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively.
If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:
[Given : radius of earth = 6400 km, mass of earth = 6 $\times 10^{24}$kg]



(A) 3.33 × 10^2s
(B) 1.33 ×10^2s
(C) 4.24 ×10^2s
(D) 4.24 × 10^3s

Q20. An engine of a train, moving with uniform acceleration, passes the signal – post with velocity u and the last compartment with velocity υ. The velocity with which middle point of the train passes the signal post is :

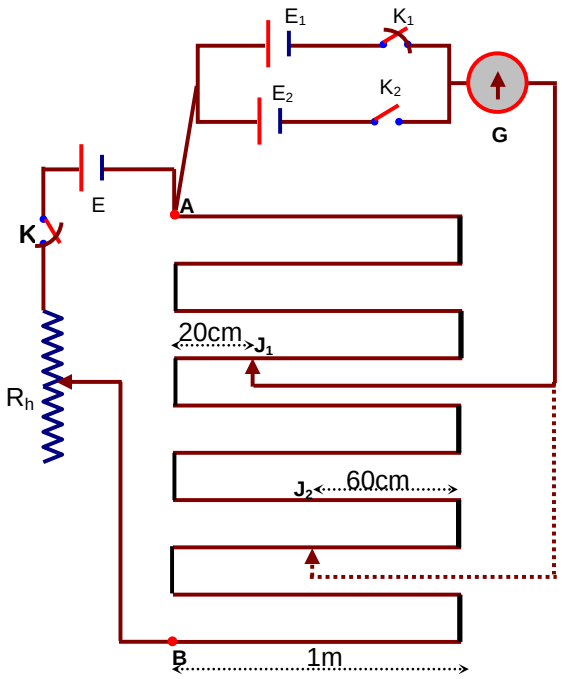
(A) $\frac{u+\upsilon}{2}$ (B) $\sqrt{\frac{\upsilon^2+u^2}{2}}$

(C) $\frac{\upsilon-u}{2}$ (D) $\sqrt{\frac{\upsilon^2-u^2}{2}}$

SECTION B

Q1. In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from $100^\circ C$ to $300^\circ C$ will be ..... nR, where n denotes numbers of moles of a gas.

Q2. In the given circuit of potentiometer, the potential difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 so that there is no deflection in the galvanometer. Now the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = ...



Q3. The same size images are formed by a convex lens by a convex lens when the object is placed at 20 cm or at 10cm from the lens. The focal length of convex lens is ..... cm.

Q4. A transmitting station releases waves of wavelength 960m. A capacitor of $2.56\,\mu F$ is used in the resonant circuit. The self inductance of coil necessary for resonance is ..... $\times 10^{-8}$ H.

Q5. 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is ..... V.

Q6. A coil of inductance 2 H having negligible resistance is connected to a source of supply whose voltage is given by V = 3t volt. (where t is in second). If the voltage is applied when t = 0, the energy stored in the coil after 4 s is ..... J.

Q7. The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as

$$U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$$

Where, α and β are positive constants. The equilibrium between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$, where a = ...

Q8. A mono-atomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped then change in temperature of the gas

(R = gas constant) is $\frac{x}{3R}$. Value of x is …….

Q9. A small bob tied at one end of a thin string of length 1m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is ……. m/s. (Take g = 10 m/s^2)

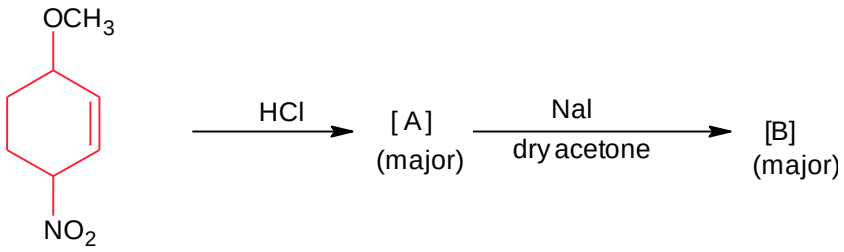
Q10. The electric field in a region is given by $\vec{E} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)\frac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area 0.2m^2 (parallel to y – z plane) to that of the surface of area 0.3 m^2 (parallel to x – z plane) is a : b, where a = …….

[Here $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z – axes respectively]

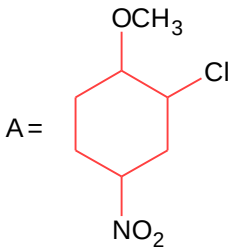
PART –B (CHEMISTRY)

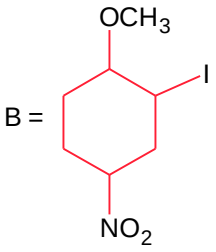
SECTION A

Q1. Identify A and B in the chemical reaction.

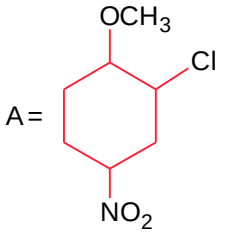


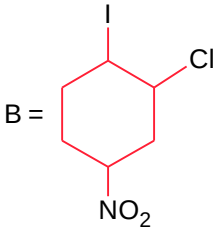
(A)



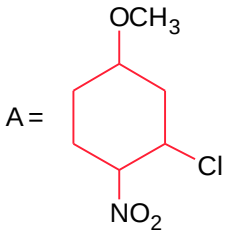


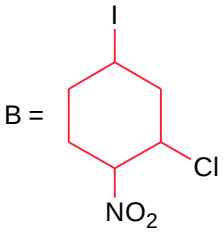
(B)



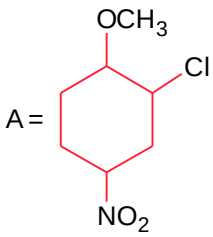


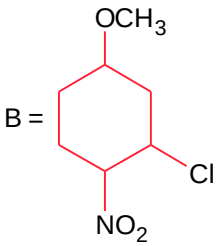
(C)



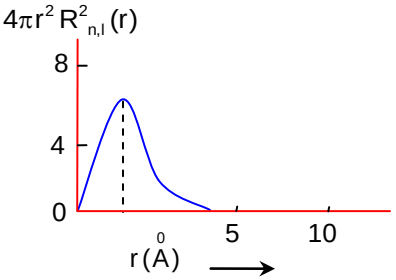


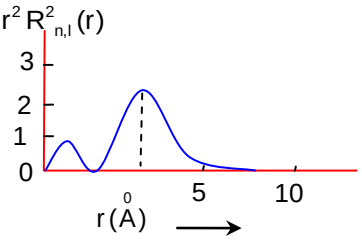
(D)

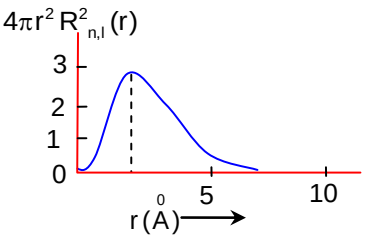


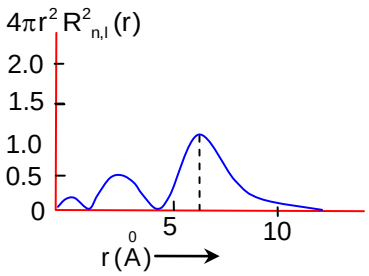


Q2. The Plots of radial distribution functions for various orbitals of hydrogen atom against 'r' are given below :

(A) 

(B) 

(C) 

(D) 

The correct plot for 3s orbital is :
(A) (D)
(B) (B)
(C) (A)
(D) (C)

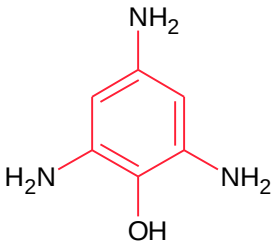
Q3. Which statement is correct ?
(A) Buna – S is a synthetic and linear thermosetting polymer.
(B) Synthesis of Buna – S needs nascent oxygen.
(C) Neoprene is an addition copolymer used in plastic bucket manufacturing.
(D) Buna – N is a natural polymer.

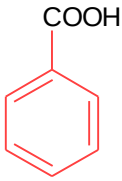
Q4. Given below are two statements :
Statement I : An allotrope of oxygen is an important intermediate in the formation of reducing smog.
Statement II : Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the formation of photochemical smog.
In the light of the above statements, choose the correct answer from the options given below :

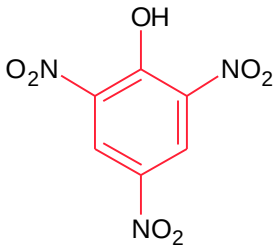
(A) Both **Statement I** and **Statement II** are false
(B) **Statement I** is false but **Statement II** is true
(C) **Statement I** is true but **Statement II** is false
(D) Both **Statement I** and **Statement II** are true

Q5. Ellingham diagram is a graphical representation of :
(A) ΔG vs P
(B) ΔH vs T
(C) $(\Delta G - T\Delta S)$ vs T
(D) ΔG vs T

Q6. Compound (s) which will liberate carbon dioxide with sodium bicarbonate solution is/ are :

A = 

B = 

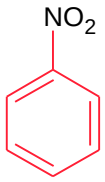
C = 

(A) A and B only
(B) B only
(C) B and C only
(D) C only

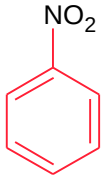
Q7. Complete combustion of 1.80 g of an oxygen containing compound ($C_x H_y O_z$) gave 2.64 g of CO_2 and 1.08g of H_2O. The percentage of oxygen in the organic compound is

(A) 53.33
(B) 50.33
(C) 63.53
(D) 51.63

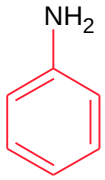
Q8. Which of the following reaction / s will not give p – aminoazobenzene ?

A - 

i) Sn / HCl
ii) HNO_2
iii) Aniline

B - 

i) Na BH_4
ii) NaOH
iii) Aniline

C - 

i) HNO_2
ii) Aniline, HCl

(A) C only
(B) A and B
(C) A only
(D) B only

Q9. Which of the glycosidic linkage between galactose and glucose is present in lactose ?

(A) C–1 of glucose and C– 4 of galactose
(B) C–1 of glucose and C-6 of galactose
(C) C-1 of galactose and C– 6 of glucose
(D) C–1 of galactose and C– 4 of glucose

Q10. The hybridization and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$, respectively are :
(A) sp^3d^2 and diamagnetic
(B) sp^3d^2 and paramagnetic
(C) d^2sp^3 and diamagnetic
(D) d^2sp^3 and paramagnetic

Q11. Which one of the following reactions will not form acetaldehyde ?

(A) $CH_2{=}CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$

(B) $CH_3CH_2OH \xrightarrow[573\ K]{Cu}$

(C) $CH_3CH_2OH \xrightarrow{CrO_3 - H_2SO_4}$

(D) $CH_3CN \xrightarrow[ii)\ H_2O]{i)\ DIBAL - H}$

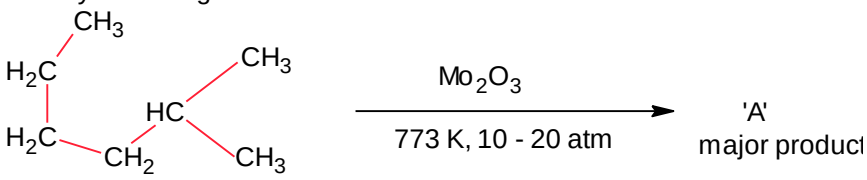
Q12. The major product of the following chemical reaction is :

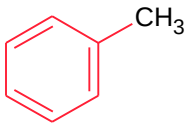
$$CH_3CH_2CN \xrightarrow[3)\ Pd/BaSO_4, H_2]{1)\ H_3O^+, \Delta \\ 2)\ SOCl_2} ?$$

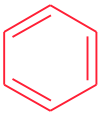
(A) $CH_3CH_2CH_2OH$
(B) CH_3CH_2CHO
(C) $(CH_3CH_2CO)_2O$
(D) $CH_3CH_2CH_3$

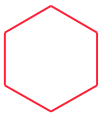
Q13. The solubility of AgCN in a buffer solution of pH = 3 is x. The value of x is :
[Assume : No cyano complex is formed ; K_{sp} (AgCN) = 2.2×10^{-16} and K_a (HCN) = 6.2×10^{-10}]
(A) 1.6×10^{-6} (B) 0.625×10^{-6}
(C) 2.2×10^{-16} (D) 1.9×10^{-5}

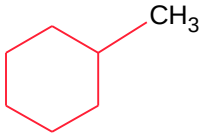
Q14. Identify A in the given chemical reaction.



(A)


(B)


(C)


(D)


Q15. The correct statement about B_2H_6 is :
(A) All B – H – B angles are of 120° .
(B) The two B - H – B bonds are not of same length.
(C) Its fragment, BH_3, Behaves as a Lewis base.
(D) Terminal B – H bonds have less p – character when compared to bridging bonds.

Q16. According to molecular orbital theory, the species among the following that does not exist is :
(A) He_2^- (B) He_2^+
(C) Be_2 (D) O_2^{2-}

Q17. Given below are two statements :
Statement I : CeO_2 can be used for oxidation of aldehydes and ketones.
Statement II : Aqueous solution of $EuSO_4$ is a strong reducing agent.
In the light of the above statements, choose the correct answer from the options given below :
(A) Statement I is true but Statement II is false (B) Both Statement I and Statement II are true
(C) Statement I is false but Statement II is true (D) Both Statement I and Statement II are false

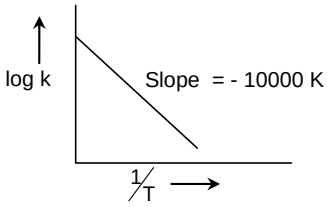
Q18. Which of the following equation depicts the oxidizing nature of H_2O_2 ?
(A) $2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$
(B) $Cl_2 + H_2O_2 \rightarrow 2HCl + O_2$
(C) $KIO_4 + H_2O_2 \rightarrow KIO_3 + H_2O + O_2$
(D) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Q19. In Freundlich adsorption isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to P^x. The value of x is :
(A) zero (B) $\frac{1}{n}$
(C) 1 (D) ∞

Q20. In which of the following pairs, the outer most electronic configuration will be the same ?
(A) Cr^+ and Mn^{2+} (B) Ni^{2+} and Cu^+
(C) Fe^{2+} and Co^+ (D) V^{2+} and Cr^+

SECTION B

Q1. For the reaction aA+ bB $\rightarrow$ cC + dD, the plot of log K vs $\frac{1}{T}$ is given below :



The temperature at which the rate constant of the reaction is 10^{-4} s^{-1} is ---------- K.
(Rounded – off to the nearest integer)
[Given : The rate constant of the reaction is 10^{-5} s^{-1} at 500 K]

Q2. In basic medium CrO_4^{2-} oxidises $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^-$.
The volume of 0.154 M CrO_4^{2-} required to react with 40mL of 0.25 M $S_2O_3^{2-}$ is ---------- mL.
(Rounded – off to the nearest integer)

Q3. Among the following, the number of halide(s) which is / are inert to hydrolysis is ----------.
(A) BF_3 (B) $SiCl_4$
(C) PCl_5 (D) SF_6

Q4. 0.4 g mixture of NaOH, Na_2CO_3 and some inert impurities was first titrated with $\frac{N}{10}$ HCl using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After this methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is ----------. (Rounded – off to the nearest integer)

Q5. The reaction of cyanamide, $NH_2CN_{(s)}$ with oxygen was run in a bomb calorimeter and ΔU was found to be -742.24 kJ mol^{-1}. The magnitude of ΔH_{298} for the reaction

$NH_2CN_{(S)} + \frac{3}{2}O_{2(g)} \rightarrow N_{2(g)} + O_{2(g)} + H_2O_{(l)}$ is ---------- kJ. (Rounded off to the nearest integer)

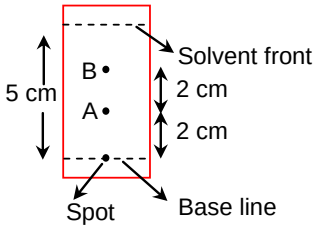
[Assume ideal gases and R = 8.314 J mol^{-1} K^{-1}]

Q6. 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionised. The boiling point of the solution at 1 atm is ---------- K. (Rounded – off to the nearest integer)
[Given K_b for (H_2O) = 0.52 K kg mol^{-1}]

Q7. A car tyre is filled with nitrogen gas at 35 psi at 27 °C. It will burst if pressure exceeds 40 psi. The temperature in °C at which the car tyre will burst is ----------. (Rounded – off to the nearest integer)

Q8. The ionization enthalpy of Na^+ formation from $Na_{(g)}$ is 495.8 kJ mol^{-1}, while the electron gain enthalpy of Br is – 325.0 kJ mol^{-1}. Given the lattice enthalpy of NaBr is – 728. 4 kJ mol^{-1} . The energy for the formation of NaBr ionic solid is (-) ---------- × 10^{-1}kJ mol^{-1}.

Q9. Using the provided information in the following paper chromatogram :
Fig : Paper chromatography for compounds A and B. The calculated R_f value of A ----------× 10^{-1}.



Q10. Consider the following chemical reaction.

$$HC\equiv CH \xrightarrow[2)\ CO,\ HCl,\ AlCl_3]{1)\ Red\ hot\ Fe\ tube,\ 873\ K} Product$$

The number of sp^2 hybridized carbon atoms (s) present in the product is ----------.

PART–C (MATHEMATICS)

1. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point :

(A) (5, 5) (B) (4, 4)
(C) (4, 5) (D) (5, 4)

2. Let f , g : N $\to$ N such that f (n+1) = f (n) + f (1) $\forall$ n $\in$ N and g be any arbitrary function. Which of the following statements is NOT true ?

(A) If *fog* is one – one, then g is one – one
(B) f is one – one
(B) If g is onto, then *fog* is one – one
(D) If f is onto, then f (n) = n $\forall$ n $\in$ N

3. The value of $\int_{-1}^{1} x^2 e^{\left[x^3\right]} dx$, where [t] denotes the greatest integer $\leq$ t, is :

(A) $\frac{e-1}{3e}$ (B) $\frac{e+1}{3e}$
(C) $\frac{1}{3e}$ (D) $\frac{e+1}{3}$

4. If Rolle's theorem holds for the function f (x) = $x^3 - ax^2 + bx - 4$, x $\in$ [1, 2] with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

(A) (5, 8) (B) (5, - 8)
(C) (-5, 8) (D) (- 5, -8)

5. If the curves, $\frac{x^2}{a}+\frac{y^2}{b} = 1$ and $\frac{x^2}{c}+\frac{y^2}{d} = 1$ intersect each other at an angle of 90°, then which of the following relations is TRUE ?

(A) ab = $\frac{c+d}{a+b}$ (B) a – b = c – d
(C) a+ b = c + d (D) a – c = b + d

6. The statement A $\to$ (B $\to$ A) is equivalent to :

(A) A $\to$ (A $\leftrightarrow$ B) (B) A $\to$ (A $\to$ B)
(C) A $\to$ (A $\vee$ B) (D) A $\to$ (A $\wedge$ B)

7. A tangent is drawn to the parabola y^2 =6x which is perpendicular to the line 2x + y = 1. Which of the following points does NOT lie on it ?

(A) (5, 4) (B) (-6, 0)
(C) (0, 3) (D) (4, 5)

8. $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+.....+\frac{1}{n}}{n^2}\right)^n$ is equal to :

(A) 0 (B) $\frac{1}{2}$
(C) 1 (D) $\frac{1}{e}$

9. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then probability that all three hit the target, is :

(A) $\frac{3}{4}$ (B) $\frac{1}{27}$

(C) $\frac{3}{8}$ (D) $\frac{1}{8}$

10. If $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta . \sin^{2n}\phi$ then :

(A) xy + yz + zx = z (B) xy – z = (x+y) z

(C) xyz = 4 (D) xy + z = (x+y) z

11. Let the lines $(2 - i)z = (2+ i)\bar{z}$ and $(2+ i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is :

(A) $\frac{3}{\sqrt{2}}$ (B) $\frac{3}{2\sqrt{2}}$

(C) $\frac{1}{2\sqrt{2}}$ (D) $3\sqrt{2}$

12. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

(A) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$ (B) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(C) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$ (D) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

13. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in second) by the boat from B to reach the base of the tower is :

(A) $10(\sqrt{3} - 1)$ (B) $10(\sqrt{3} + 1)$

(C) 10 (D) $10\sqrt{3}$

14. The total number of positive integral solutions (x, y, z) such that xyz = 24 is :

(A) 30 (B) 24

(C) 36 (D) 45

15. Let α be the angle between the lines whose direction cosines satisfy the equations l + m – n = 0 and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is:

(A) $\frac{1}{2}$ (B) $\frac{5}{8}$

(C) $\frac{3}{8}$ (D) $\frac{3}{4}$

16. The value of the integral

$$\int \frac{\sin\theta \sin 2\theta (\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$$

(where c is a constant of integration)

(A) $\frac{1}{18}\left[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta\right]^{\frac{3}{2}} + c$

(B) $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}}+c$

(C) $\frac{1}{18}\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\frac{3}{2}}+c$

(D) $\frac{1}{18}\left[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{\frac{3}{2}}+c$

17. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is :

(A) $\frac{1}{72}$ (B) $\frac{1}{54}$

(C) $\frac{1}{36}$ (D) $\frac{5}{216}$

18. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ is :

(A) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$ (B) $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$

(C) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$ (D) $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$

19. The integer 'k', for which the inequality $x^2 – 2(3k – 1) x + 8k^2 -7 > 0$ is valid for every x in R, is :

(A) 4 (B) 3

(C) 0 (D) 2

20. The image of the point (3, 5) in the line x- y + 1 = 0, lies on :

(A) $(x - 4)^2 + (y + 2)^2 = 16$ (B) $(x – 2)^2 + (y- 4)^2 = 4$

(C) $(x - 2)^2 + (y – 2)^2 = 12$ (D) $(x - 4)^2 + (y – 4)^2 = 8$

SECTION B

1. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is …….

2. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to …….

3. The number of points, at which the function $f(x) = |2x+1|- 3|x + 2| + |x^2 + x – 2|$, $x \in R$ is not differentiable, is …….

4. If $A=\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2-A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 +b^2)$ is equal to …….

5. Let $\vec{a}=\hat{i}+2\hat{j}-\hat{k}, \vec{b}=\hat{i}-\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}.\vec{b}=0$, then $\vec{r}.\vec{a}$ is equal to …….

6. Let $A=\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that x + y + z > 0 and xyz = 2 If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is …….

7. Let $A_1, A_2, A_3, \ldots\ldots$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12cm then smallest value of an for which area of A_n is less than one, is ……

8. The locus of the point of intersection of the lines $(\sqrt{3})\, kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}\, x - y - 4(\sqrt{3})\, k = 0$ is a conic, whose eccentricity is ……..

9. Let f (x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = -1 and x = 1. If $\lim\limits_{x \to 0} \dfrac{f(x)}{x^3} = 1$, then 5 f(2) is equal to …….

10. If the system of equations

$$kx + y + 2z = 1$$
$$3x - y - 2z = 2$$
$$-2x - 2y - 4z = 3$$

has infinitely many solutions, then k is equal to …….