FIITJEE

Solutions to JEE (Main)-2021

JEE–Main–2021 –Feb–25–First–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

(PHYSICS)

Answers

Section-A

1.	B	**2.**	C	**3.**	C	**4.**	C
5.	D	**6.**	C	**7.**	A	**8.**	A
9.	A	**10.**	D	**11.**	B	**12.**	B
13.	C	**14.**	B	**15.**	C	**16.**	A
17.	C	**18.**	A	**19.**	B	**20.**	B

Section-B

1.	50	**2.**	1	**3.**	15	**4.**	10
5.	128	**6.**	144	**7.**	1	**8.**	3.6
9.	5	**10.**	1				

SECTION – A

Sol1. $F_1 = \frac{GMm}{9R^2}$

$$F_2 = \frac{GMm}{9R^2} - \frac{G\frac{M}{8}m}{\left(\frac{5R}{2}\right)^2} = \frac{GMm}{R^2}\left(\frac{1}{9} - \frac{1}{50}\right)$$

$$\frac{F_1}{F_2} = \frac{1}{1-\frac{9}{50}} = \frac{50}{41}$$

Sol2. $K = qV$

$$p = \sqrt{2mK} = \sqrt{2mqV}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}.\frac{q_\alpha}{q_p}} = \sqrt{4\times 2} = 2\times 1.4 = 2.8$$

Sol3. Key is open.

$$i_{rms} = \frac{15}{60} = \frac{1}{4}\,A$$

$$20 = \frac{1}{4}\times 100\times L \Rightarrow L = 0.8H$$

$$10 = \frac{1}{4}.\frac{1}{100C} \Rightarrow C = 2.5\times 10^{-4}\,F = 250\,\mu F$$

Sol4. At t = 0, no current flows through inductor.

$$i = \frac{E}{\frac{6\times 9}{6+9}} = \frac{5E}{18}$$

At $t=\infty$, current flows through circuit as if inductor is shorted.

$$i = \frac{E}{\frac{5}{2}+\frac{4}{5}} = \frac{10E}{33}$$

Sol5. $\overrightarrow{AB}+\overrightarrow{AH}= \overrightarrow{AO}$

$\overrightarrow{AC}+\overrightarrow{AG}=2\overrightarrow{AO}$

$\overrightarrow{AD}+\overrightarrow{AF}=3\overrightarrow{AO}$

$\overrightarrow{AE}=2\overrightarrow{AO}$

Adding all,

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}= 8\overrightarrow{AO}= 16\hat{i}+24\hat{j}-32\hat{k}$

Sol6. $$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{\frac{I_1}{I_2}}+1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}}-1\right)^2} \Rightarrow \frac{I_{max}-I_{min}}{I_{max}+I_{min}} = \frac{2.\sqrt{\frac{I_1}{I_2}}}{\frac{I_1}{I_2}+1} = \frac{2\sqrt{2x}}{2x+1}$$

Sol7. $\left(v_e\right)_A = \left(v_e\right)_B \quad \Rightarrow \frac{M_1}{R_1}=\frac{M_2}{R_2}$

R is not correct.

Sol8. Side band frequencies are 1MHz $\pm$ 1 kHz

Sol9. LC = $\frac{1}{100}$ mm $= 0.01$ mm

Zero error = + 0.08 mm.

Diameter = 1 + 72 × 0.01 – 0.08 = 1.64 mm

Radius = 0.82mm

Sol10. Thermal stress is developed on heating when expansion of rod is hindered.

Sol11. D_1 is in forward bias and D_2 is in reverse bias.

Current , $i = \frac{5-0.7}{10} = 0.43\,A$

Sol12. At constant pressure,

$dU = nC_V\,dT = n.\frac{5}{2}R\,dT$

$dQ = nC_P dT = n.\frac{7}{2}R\,dT$

$dW = nR\,dT$

$dU : dQ : dW = 5:7:2$

Sol13. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow 2 = 2\pi\sqrt{\frac{2}{g}}$

$\Rightarrow g = 2\pi^2 \ m/s^2$

Sol14. Number of half lives of Y = 3
Number of half lives of X = 6 [As half life of X is half of that of Y].

$\frac{N_1}{2^6} = \frac{N_2}{2^3} \Rightarrow \frac{N_1}{N_2} = 8$

Sol15. Magnetic field on the axis of a circular coil at distance x from centre , $B = \frac{\mu_0 Nir^2}{2(r^2 + x^2)^{3/2}}$

$$\frac{(r^2 + (0.2)^2)^{3/2}}{(r^2 + (0.05)^2)^{3/2}} = 8 \Rightarrow \frac{r^2 + (0.2)^2}{r^2 + (0.05)^2} = 4 \Rightarrow r^2 = 0.01 \Rightarrow r = 0.1\,m.$$

Sol16. For first resonance,

$\ell + 0.3d = \frac{v}{4f}$

$\Rightarrow \ell + 0.3 \times 6 = \frac{336 \times 100}{4 \times 504} \Rightarrow \ell = 14.8\,cm$

Sol17. $[h] = ML^2T^{-1}$

$[E] = ML^2T^{-2}$

$[V] = ML^2T^{-2}C^{-1}$

$[P] = MLT^{-1}$

Sol18. $v = \frac{p}{m} \Rightarrow v_p : v_d : v_\alpha = 1 : \frac{1}{2} : \frac{1}{4} = 4 : 2 : 1$

$F_{mag} = qvB \Rightarrow F_p : F_d : F_\alpha = 1 \times 4 : 1 \times 2 : 2 \times 1 = 2 : 1 : 1$

Sol19. $T = 2\pi\sqrt{\frac{r^3}{GM_e}}$

$$T_B - T_A = \frac{2\pi}{\sqrt{GM_e}}\left(r_B^{3/2} - r_A^{3/2}\right) = \frac{2 \times 3.14}{\sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24}}}\left[(8 \times 10^6)^{3/2} - (7 \times 10^6)^{3/2}\right] = 1.3 \times 10^2\,s.$$

Sol20. Assume the length of train be l and its acceleration be a.

$v^2 = u^2 + 2al \Rightarrow al = \frac{v^2 - u^2}{2}$

Velocity when middle point crosses the post,

$$V_m = \sqrt{u^2 + 2a\frac{\ell}{2}.} = \sqrt{u^2 + \frac{v^2 - u^2}{2}} = \sqrt{\frac{u^2 + v^2}{2}}$$

SECTION – B

Sol1. $P = kV^3 \Rightarrow PV^{-3} = k \Rightarrow$ Polytropic Process with index m = - 3

$$W = \frac{nR\,(T_2 - T_1)}{1 - m} = \frac{nR(300 - 100)}{1 - (-3)} = 50nR.$$

Sol2. $\frac{E_1}{E_2} = \frac{380}{760} = \frac{1}{2}$

Sol3. Image is virtual , when an object is placed at distance 10cm from the lens and image is real when object is placed at 20cm from the lens.

Thus, $m_1 = -m_2 \Rightarrow \frac{f}{f-10} = \frac{-f}{f-20} \Rightarrow f = 15\,cm$

Sol4. $2\pi f = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{\lambda^2}{4\pi^2 Cc^2} = \frac{960^2}{4 \times 3.14^2 \times 2.56 \times 10^{-6} \times 9 \times 10^{16}} = 10^{-7} = 10 \times 10^{-8}\,H$

Sol5. Let the charge of on each drop be q and radius of each drop be r.

$\frac{kq}{r} = 2$

When all drops are joined, radius,

$r' = (512)^{1/3}\,r = 8r$

Potential of the new drop,

$V = \frac{k.\,512q}{8r} = 64\,\frac{kq}{r} = 128\,V$

Sol6. $L\frac{di}{dt} = V \Rightarrow 2di = 3t\,dt \Rightarrow 2\int_0^i di = 3\int_0^4 t\,dt \Rightarrow i = 12\,A$

$U = \frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times 12^2 = 144\,J$

Sol7. For equilibrium ,

$\frac{dU}{dr} = 0 \Rightarrow \frac{-10\alpha}{r^{11}} + \frac{5\beta}{r^6} = 0 \Rightarrow r = \left(\frac{2\alpha}{\beta}\right)^{1/5}$

Sol8. $\Delta U + \Delta KE = 0$

$\Rightarrow \frac{f}{2}nR\Delta T = \frac{1}{2}mv^2 \Rightarrow \Delta T = \frac{m}{n}\frac{v^2}{fR} = 4 \times 10^{-3} \times \frac{30^2}{3R} = \frac{3.6}{3R}\,K.$

Sol9. Let v is velocity at highest position.

$T_{max} = 5\,T_{min} \Rightarrow mg + \frac{m\,(v^2 + 4g\ell)}{\ell} = 5\left(\frac{mv^2}{\ell} - mg\right) \Rightarrow 4.\frac{v^2}{\ell} = 10g$

$\Rightarrow v = \sqrt{\frac{5}{2}g\ell}. = \sqrt{\frac{5}{2} \times 10 \times 1} = 5\,m/s$

Sol10. $\phi_1 = \frac{3}{5}E_0(0.2)\ Nm^2C^{-1}$, and $\phi_2 = \frac{4}{5}E_0(0.3)\ Nm^2C^{-1}$

$\Rightarrow \frac{\phi_1}{\phi_2} = \frac{3 \times 0.2 \times 5}{5 \times 0.3 \times 4} = \frac{1}{2}$

PART –B (CHEMISTRY)

Answers

Section-A

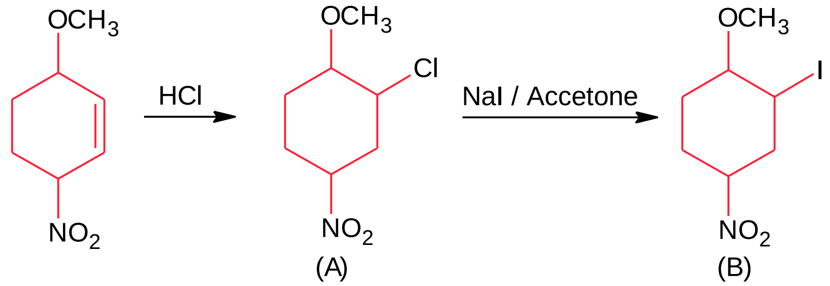
1.	A	**2.**	A	**3.**	B	**4.**	A
5.	D	**6.**	C	**7.**	A	**8.**	D
9.	D	**10.**	D	**11.**	C	**12.**	B
13.	D	**14.**	A	**15.**	D	**16.**	C
17.	B	**18.**	A	**19.**	B	**20.**	A

Section-B

1.	526	**2.**	173	**3.**	1	**4.**	4
5.	741	**6.**	375	**7.**	70	**8.**	5576
9.	4	**10.**	7				

SECTION – A

Sol1.



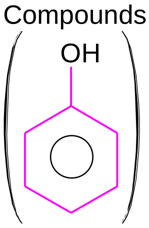
Sol2. For 3S number of radial nodes are
$= n - \ell - 1 = 3 - 0 - 1 = 2$

Sol3. Buna-S required nascent oxygen
Butadiene + styrene $\xrightarrow{[O]}$ (Buna-S)

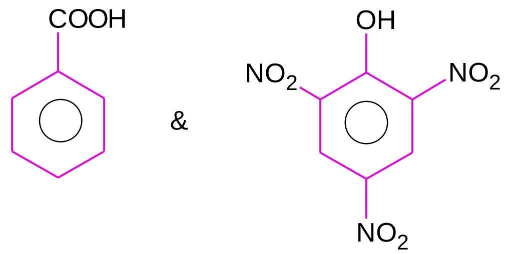
Sol4. Primary pollutants are oxides of N & C and classical smog is due to SO_2 & particulates of fossil

Sol5. Ellingham diagram is in between ΔG & temperature for the reducing behavior of oxides

Sol6. Compounds which are more acidic than phenol



Gives CO_2 with$NaHCO_3$

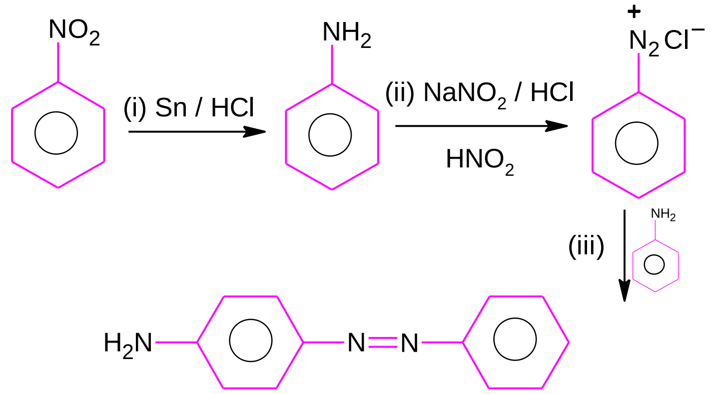


Sol7. % of C $= \frac{\text{Wt of } CO_2}{\text{Wt of comp.}} \times \frac{12}{44} \times 100 = \frac{2.64}{1.8} \times \frac{12}{44} \times 100 = 40\%$

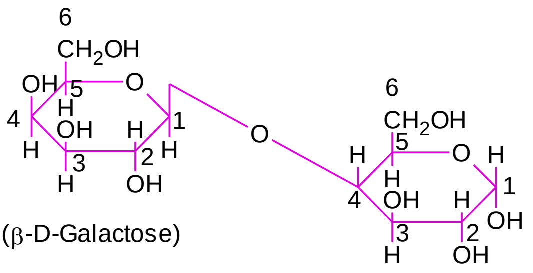
% of H = $\frac{\text{Wt of } H_2O}{\text{Wt of comp.}} \times \frac{2}{18} \times 100 = \frac{1.08}{1.8} \times \frac{2}{18} \times 100 = 6.67\%$

% of O = 100– (40+6.67) = 53.33%

Sol8.



Sol9.

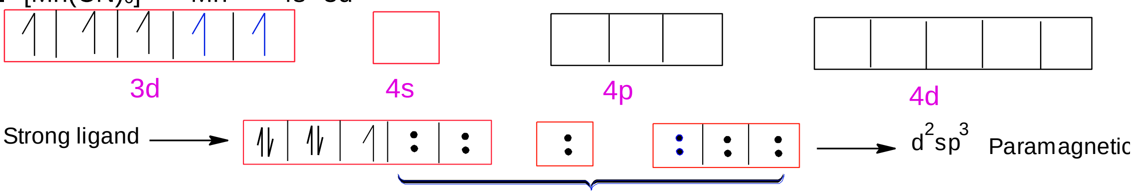


(β-D-Galactose)

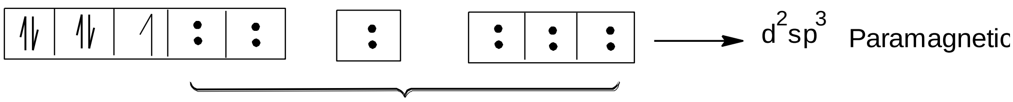
(β-D-Glucose)

Glycosidic linkage is present in lactose between C_1 of galactose & C_4 of glucose.

Sol10. $[Mn(CN)_6]^{-4} = Mn^{+2} = 4s^o\ 3d^5$



$[Fe(CN)_6]^{-3} = Fe^{+3} = 4s^o3d^5$



Sol11.

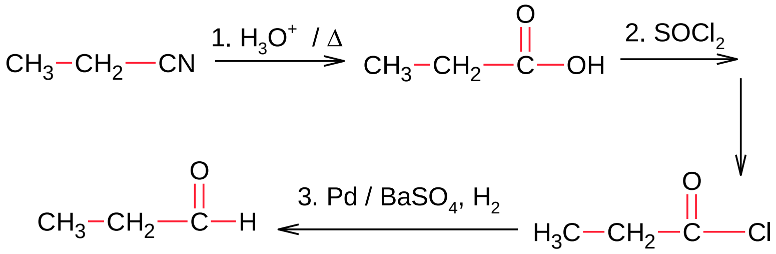
$$CH_2{=}CH_2 + O_2 \xrightarrow[H_2O]{\text{Pd (II) / Cu (II)}} CH_3{-}\overset{O}{\overset{||}{C}}{-}H$$

$$CH_3{-}CH_2{-}OH \xrightarrow{\text{Cu / 573 k}} H_3C{-}\overset{O}{\overset{||}{C}}{-}H$$

$$CH_3{-}CH_2{-}OH \xrightarrow{CrO_3 / H_2SO_4} CH_3{-}\overset{O}{\overset{||}{C}}{-}OH$$

$$CH_3{-}CN \xrightarrow[(2)\ H_2O]{(1)\ \text{DIBAL - H}} CH_3{-}\overset{O}{\overset{||}{C}}{-}H$$

Sol12.



Sol13. $AgCN_{(s)} \rightleftharpoons Ag^+_{(aq)} + CN^-_{(aq)}$

S S

$$CN^- + H^+ \rightleftharpoons HCN$$

	CN^-	H^+	HCN
Before reaction	S	10^{-3}	0
After reaction	0	10^{-3}	S

$\therefore$ HCN $\rightleftharpoons H^+ + CN^-$

$$K_a = \frac{[H^+][CN^-]}{[HCN]} \qquad [CN^-] = K_a\frac{[HCN]}{[H^+]} = \frac{6.2\times10^{-10}}{10^{-3}}[HCN] = S\times6.2\times10^{-7}$$

$K_{sp} = [Ag^+] \times [CN^-]$

$K_{sp} = S\times[CN^-]$

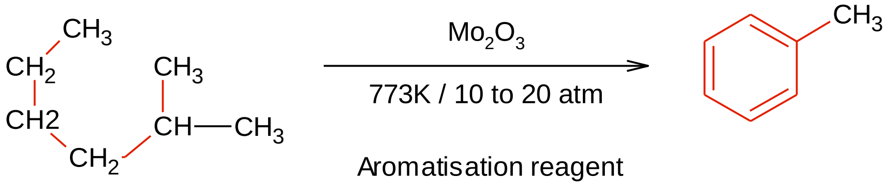
$2.2\times10^{-16} = S\times S\times6.2\times10^{-7}$

$$S^2 = \frac{2.2\times10^{-16}}{6.2\times10^{-7}} = \frac{2.2}{6.2}\times10^{-9}$$

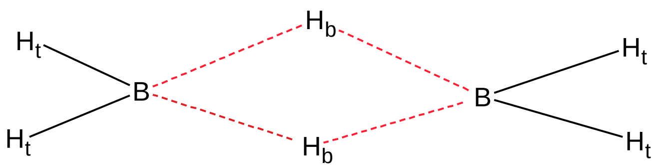
$$S = \sqrt{\frac{2.2}{6.2}\times10^{-9}} = \sqrt{\frac{22}{6.2}\times10^{-10}}$$

$$= \sqrt{3.54\times10^{-10}} = 1.88\times10^{-5} = 1.9\times10^{-5}$$

Sol14.



Sol15.



Angle of $H_t - B - H_t$ > angle of $H_b - B - H_b$

Bond angle ∝ % S-Character $\propto \frac{1}{\%\text{of P} - \text{character}}$

$\angle H_t B H_t$ is more so % P-Character is less.

Sol16. (1) He_2^- $\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^1$ $B.O = \frac{3-2}{2} = \frac{1}{2}$

(2) He_2^+ $\sigma_{1s}^2\sigma_{1s}^{*1}$ $B.O = \frac{2-1}{2} = \frac{1}{2}$

(3) Be_2 $\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}$ $B.O = \frac{4-4}{2} = 0$

So Be_2 does not exits

Sol17. In Ce & Eu (+3) oxidation state is more stable

$\therefore$ $Ce^{+4} + e^{-} \longrightarrow Ce^{+3}$ (Reduction) ; so CeO_2 is oxidising agent

$Eu^{+2} \longrightarrow Eu^{+3} + e-$ (oxidation) ; so $EuSO_4$ is reducing agent.

Sol18. $I^{-} + H_2O_2 + 2H^{+} \longrightarrow I_2 + 2H_2O$

(-1) oxidation (0)

Here, I^{-} is reducing agent

$\therefore$ H_2O_2 behaves like oxidizing agent

Sol19. Freundlich adsorption Isotherm

$$\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$$

At moderate pressure; $\frac{x}{m}$ varies non-linearly with P.

So, $x = \frac{1}{n}$

Sol20. (1) $Cr^{+1} = 4s^{o}3d^{5}$ $Mn^{+2} = 4s^{o}3d^{5}$

(2) $Ni^{+2} = 4s^{o}3d^{8}$ $Cu^{+} = 4s^{o}3d^{10}$

(3) $Fe^{+2} = 4s^{o}3d^{6}$ $Co^{+} = 4s^{1}3d^{7}$

(4) $V^{+2} = 4s^{o}3d^{3}$ $Cr^{+} = 4s^{o}3d^{5}$

SECTION- B

Sol1. $\log K = \log A - \frac{Ea}{2.303RT}$

Slope = $-\frac{Ea}{2.303R}$ = - 10,000K

$\frac{Ea}{2.303R} = 10^{4}$

$\log\left(10^{-5}\right) = \log A - 10^{4} \times \frac{1}{500}$ (at 500 K temperature)

$\log A = -5 + \frac{100}{5} = -5 + 20 = 15$, $\therefore$ log A = 15

$\log 10^{-4} = 15 - \frac{10^{4}}{T} \Rightarrow -4 = 15 - \frac{10^{4}}{T}$

$\frac{10^{4}}{T} = 19$

$\therefore T = \frac{10^{4}}{19} K = \frac{100}{19} \times 10^{2} K = 5.2631 \times 10^{2} K = 526.31 K \approx 526 K$

Sol2.

$$\underset{(+6)}{CrO_4^{-2}} + \underset{(+2)}{S_2O_3^{-2}} \longrightarrow \underset{(+3)}{Cr(OH)_4^{-}} + \underset{(+6)}{SO_4^{-2}}$$

O.N.C =4

n factor of $S_2O_3^{--}$ = 8

n factor of CrO_4^{--} = 3

m.e CrO_4^{-2} = m.e $S_2O_3^{-2}$

$V \times (0.154 \times 3) = 40 \times (0.25 \times 8)$ $\therefore v = \frac{80}{0.462}$

$= 173.16 \text{ ml} \approx 173 \text{ ml}$

Sol3. $BF_3 + 3H_2O \longrightarrow H_3BO_3 + 3HF$

$SiCl_4 + 4H_2O \longrightarrow H_4SiO_4 + 4HCl$

$PCl_5 + H_2O \longrightarrow POCl_3 + 2HCl$

$SF_6 + H_2O \longrightarrow X$

Due to steric crowding SF_6 does not participate in hydrolysis

Sol4.

$NaOH + Na_2CO_3$

(1) When Hph is added

$$\text{m.e NaOH} + \frac{1}{2}\text{m.e } Na_2CO_3 = \text{m.e HCl} = 17.5 \times \frac{1}{10} = 1.75 \qquad (\text{m.e} = \text{milli equivalents})$$

(2) When MeOH is added after Hph

$$\frac{1}{2}\text{m.e } Na_2CO_3 = \text{m.e HCl} = 1.5 \times \frac{1}{10} = 0.15$$

$\therefore$ m.e NaOH = 1.75 – 0.15 = 1.6

. m.e Na_2CO_3 = 0.15 × 2 = 0.3

$$\frac{W_{Na_2CO_3}}{E_{Na_2CO_3}} \times 1000 = 0.3$$

$$W_{Na_2CO_3} = \frac{0.3 \times 53}{1000} \qquad \left[E_{Na_2CO_3} = 53\right]$$

Weight % of Na_2CO_3= $\left(\dfrac{\frac{0.3\times 53}{1000}}{0.4}\right) \times 100 = \dfrac{0.3 \times 53}{10 \times 0.4} = \dfrac{15.9}{4} = 3.975\% \approx 4\ \%$.

Sol5. ΔU = -742.24 kJ/ mole $\qquad \Delta H_{298}$ = ?

$$NH_2CN(s) + \frac{3}{2}O_2(g) \rightarrow N_2(g) + O_2(g) + H_2O(l)$$

$$\Delta H = \Delta U + \Delta n_g RT \quad \Delta n_g = 2 - \frac{3}{2} = \frac{1}{2}$$

$$= -742.24 + \frac{1}{2} \times \frac{8.314}{1000} \times 298$$

= -742.24 + 1.238 = -741.002 ≈ -741kJ/ mol

So, magntitude of ΔH = 741 kJ/mol.

Sol6. $A_2B_3 \rightleftharpoons 2A^{+3} + 3B^{-2}$

$1-\alpha \qquad 2\alpha \qquad 3\alpha$

$\therefore$ i = 1 + 4α = 1+4×0.6 = 1 +2.4=3.4

$\Delta T_b = i\, k_b m$ = 3.4×0.52 × 1 = 1.768 ≈1.77K

T_b- 373 = 1.77K

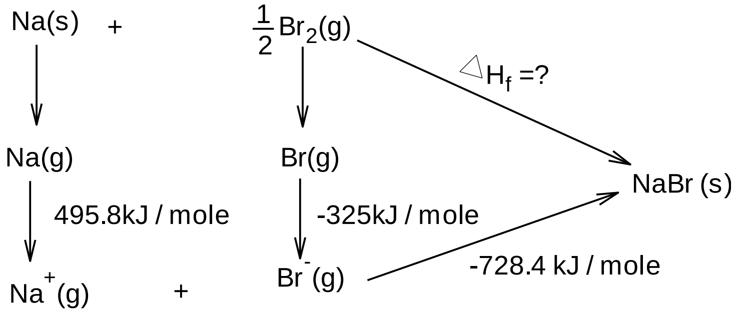
T_b = 374.77K ≈ 375 K.

Sol7. $\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2} \Rightarrow \dfrac{35}{300} = \dfrac{40}{T_2}$

$$\therefore T_2 = \frac{40 \times 300}{35} = 342.85\text{K}$$

T_2 (°C) = 69.707 ≈70°C

Sol8.

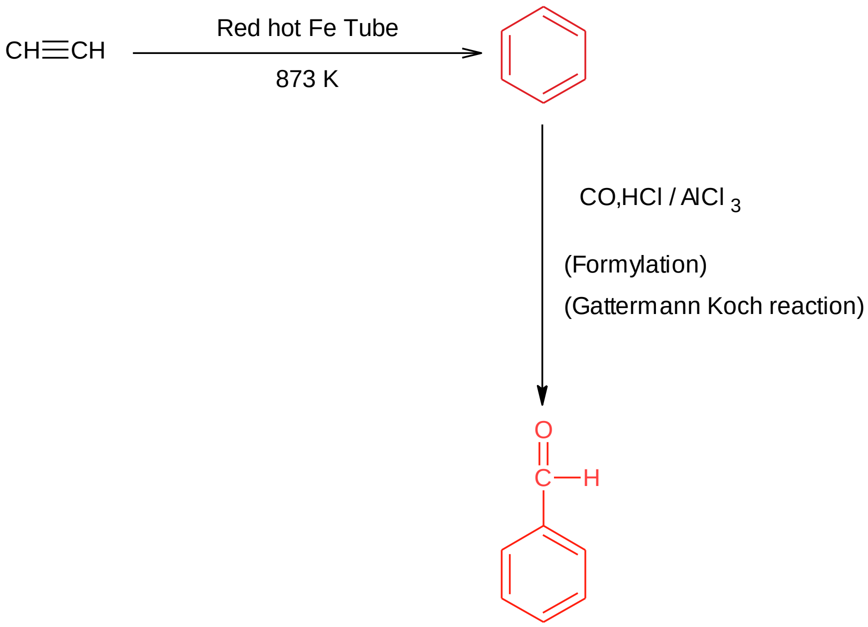


ΔH_{sub} of Na metal = 496 kJ / mole

Bond dissociation energy of Br_2 = 228 kJ / mole
ΔH_f = 495.8 + –325–728.4 = –557.6 kJ / mole= –5576 $\times 10^{-1}$kJ / mole
Note: In question $\Delta_a H^o$ of Na, $\Delta_{vap} H^o$ of $Br_2(l)$ and $\Delta_{bond} H^o$ of $Br_2(g)$ is neglected.

Sol9. $R_f \text{value} = \dfrac{\text{Distance moved by substance from base line}}{\text{Distance moved by the solvent from base line}} = \dfrac{2}{5} = 0.4 = 4.0 \times 10^{-1}$

Sol10.



PART–C (MATHEMATICS)

Answers

SECTION A

1.	A	**2.**	C	**3.**	B	**4.**	A
5.	B	**6.**	C	**7.**	A	**8.**	C
9.	D	**10.**	D	**11.**	B	**12.**	A
13.	B	**14.**	A	**15.**	B	**16.**	B
17.	D	**18.**	B	**19.**	B	**20.**	B

SECTION B

1.	32	**2.**	64	**3.**	2	**4.**	13
5.	12	**6.**	7	**7.**	9	**8.**	2
9.	144	**10.**	21				

SECTION A

1. As per questions

$$\frac{dy}{dx}=\frac{x^2-4x+y+8}{x-2}$$

$$\frac{dy}{dx}=\frac{(x-2)^2+(y+4)}{(x-2)}$$

$$\frac{dy}{dx}=(x-2)+\frac{y+4}{x-2}\text{......(i)}$$

Let $\frac{y+4}{x-2}=t$

$(y+4)=t(x-2)$

$$\frac{dy}{dx}+0=(x-2)\frac{dt}{dx}+t.1$$

$$\frac{dy}{dx}=(x-2)\frac{dt}{dx}+t$$

Putting in equation (i)

$$(x-2)\frac{dt}{dx}+t=(x-2)+t$$

$$\frac{dt}{dx}=1$$

$dt=dx$

Integrating on both the sides $t=x+c$

$$\frac{y+4}{x-2}=x+c$$

Passing through origin C = -2

$\therefore$ equation of curve $\frac{y+4}{x-2}=x-2$

2. $f:N\to N,$

$f(n+1)=f(n)+f(1)$

Let $f(1)=a, a\in N$

$f(2) = f(1) + f(1) = 2a$

$f(3) = f(2) + f(1) = 3a$

and so on

$\Rightarrow f(m) = ma, m, a \in N$

$\Rightarrow$ f is one – one, $\Rightarrow$ option (2) is true.

If f is onto, a = 1 $\Rightarrow$ $f(m) = m \Rightarrow f(n) = n \forall n \in N$

Suppose g is onto but it is many – one,

Then for $x_1 \neq x_2, g(x_1) = g(x_2) = \ell(\text{say})$

$f(g(x_1)) = f(g(x_2)) = f(\ell)$

$\Rightarrow$ $f(g(x))$ is not one-one

Suppose $f(g(x))$ is one – one,

then $f(g(x_1)) \neq f(g(x_2))$ for $x_1 \neq x_2$

$\Rightarrow g(x_1) \neq g(x_2)$ (as f is one – one)

$\Rightarrow g$ is one – one

3. $$\int_{-1}^{1} x^2 e^{[x^3]} dx = \int_{-1}^{0} x^2 . e^{-1} dx + \int_{0}^{1} x^2 . e^{0} dx = \frac{1}{e}\int_{-1}^{0} x^2 dx + \int_{0}^{1} x^2 dx$$

$$= \frac{1}{e}\left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1} = \frac{1}{e}\left[0 - \left(\frac{-1}{3}\right)\right] + \left[\frac{1}{3} - 0\right] = \frac{1}{3e} + \frac{1}{3} = \frac{e+1}{3e}$$

4. $f(x) = x^3 - ax^2 + bx - 4$

$f(1) = f(2)$

$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$

$\Rightarrow 3a - b = 7$............(i)

$f'(x) = 3x^2 - 2ax + b$

$$f'(4/3) = 3\left(\frac{4}{3}\right)^2 - 2a\left(\frac{4}{3}\right) + b$$

$$\frac{16}{3} - \frac{8a}{3} + b = 0$$

$16 - 8a + 3b = 0$

$8a = 16 + 3b$………(ii)

(i) & (ii) $\Rightarrow (a, b) = (5, 8)$

5. $$\frac{x^2}{a} + \frac{y^2}{b} = 1, \frac{x^2}{c} - \frac{y^2}{(-d)} = 1$$

Ellipse and Hyperbola are orthogonal so these will be confocal.

$\sqrt{a - b} = \sqrt{c + (-d)}$

$a - b = c - d$

6. $A \to (B \to A)$

$\sim A \vee (B \to A)$

$\sim A \vee (\sim B \vee A)$

$(\sim A \vee \sim B) \vee (\sim A \vee A) = t$ $\quad$ $A \to (A \cup B)$

7. $2x + y = 1$

$m_1 = \frac{-2}{1} = -2$ and $m_1 m_2 = -1$

$-2.m_2 = -1$

$m_2 = \frac{1}{2}$

$y^2 = 6x$

$y^2 = 4(3/2)x$

$y = mx + \frac{a}{m}$

$y = \frac{1}{2}x + \frac{\frac{3}{2}}{\frac{1}{2}}$

$y = \frac{x}{2} + 3$

8. $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+.......+\frac{1}{n}}{n^2}\right)^n$ limit is in the form of 1^∞

$\ell = \exp\left(\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}}{n^2}\right)$

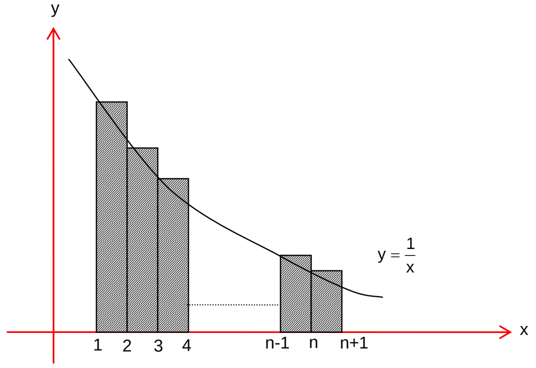
$0 \le 1+\frac{1}{2}+\frac{1}{3}+......+\frac{1}{n} \le 1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+.....+\frac{1}{\sqrt{n}} \le 2\sqrt{n}-1$

Taking limit $(n\to\infty)$

$\ell = \exp(0)$ (from sandwich)

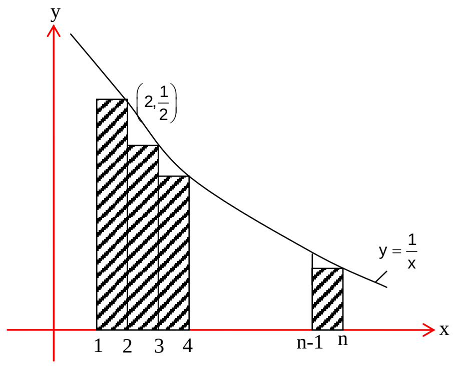
$\ell = 1$

Second Method:



$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n} \ge \int_n^{n+1}\frac{1}{x}dx$

$1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n} \ge \ln(n+1)$ ....(i)



$$1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}\le 1+\int_1^n \frac{1}{2}dx \le 1+\ln n \text{ ....(ii)}$$

From (i) & (ii)

$$\ln(n+1)\le 1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{n}\le 1+\ln n,\ \forall n\in N,\ n\ge 2$$

$$\frac{\ln(n+1)}{n}\le\frac{1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{n}}{n}\le\frac{1+\ln n}{n}$$

As $\lim\limits_{n\to\infty}\frac{\ln(n+1)}{n}=0$ and $\lim\limits_{n\to\infty}\frac{1+\ln(n)}{n}=0$

$\therefore$ from sandwich theorem

$$\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{n}}{n}=0$$

$\therefore\ e^0=1$

9. Let

A : Missile hit the target

B : Missile intercepted

$P(B)=\frac{1}{3}\quad P(A/\bar{B})=\frac{3}{4}$

$P(\bar{B})=\frac{2}{3}$

$P(\bar{B}\cap A)/P(\bar{B})=\frac{3}{4}$

$\Rightarrow P(\bar{B}\cap A)=\frac{3}{4}\times\frac{2}{3}=\frac{1}{2}$

Required probability $=\frac{2}{3}\times\frac{3}{4}\times\frac{2}{3}\times\frac{3}{4}\times\frac{2}{3}\times\frac{3}{4}=\frac{1}{8}$

10. $x=\sum_{n=0}^{\infty}\cos^{2n}\theta=\cos^0\theta+\cos^2\theta+\cos^4\theta+........=1+\cos^2\theta+\cos^4\theta+........$

$a=1, r=\cos^2\theta$

$x=S_\infty=\frac{a}{1-r}=\frac{1}{1-\cos^2\theta}=\frac{1}{\sin^2\theta}$

$x=\frac{1}{\sin^2\theta}\Rightarrow\frac{1}{x}=\sin^2\theta$

Similarly, $y=\frac{1}{\cos^2\theta}\Rightarrow\frac{1}{y}=\cos^2\theta$

$z=\frac{1}{1-\sin^2\theta.\cos^2\theta}$

$\therefore z = \dfrac{1}{1-\dfrac{1}{x}\dfrac{1}{y}}$

$z = \dfrac{xy}{xy-1} \Rightarrow xyz - z = xy$ ………..(i)

Also, $\dfrac{1}{x} + \dfrac{1}{y} = 1 \Rightarrow x + y = xy$ ……….(ii)

(i) & (ii) $\Rightarrow xyz = xy + z \Rightarrow (x+y)z = xy + z$

11. $(2-i)z = (2+i)\bar{z}$, put $z = x + iy$

$y = \dfrac{x}{2}$ ……….(i)

(ii) $(2+i)z + (i-2)\bar{z} - 4i = 0$

$x + 2y = 2$

(iii) $iz + \bar{z} + 1 + i = 0$

Equation of tangent $x - y + 1 = 0$

Solving (i) and (ii)

$x = 1.\ y = \dfrac{1}{2} \Rightarrow$ centre $\left(1, \dfrac{1}{2}\right)$

Perpendicular distance of point $\left(1, \dfrac{1}{2}\right)$ from $x - y + 1 = 0$ is $p = r \Rightarrow \left|\dfrac{1-\dfrac{1}{2}+1}{\sqrt{2}}\right| = r$

$r = \dfrac{3}{2\sqrt{2}}$

12. $\sin 2\theta + \tan 2\theta > 0$

$\dfrac{2\tan\theta}{1+\tan^2\theta} + \dfrac{2\tan\theta}{1-\tan^2\theta} > 0$

Let $\tan\theta = x$

$\dfrac{2x}{1+x^2} + \dfrac{2x}{1-x^2} > 0$

$\dfrac{2x\left(1-x^2\right) + 2x\left(1+x^2\right)}{\left(1+x^2\right)\left(1-x^2\right)} > 0$

$\dfrac{4x}{\left(1+x^2\right)\left(1-x^2\right)} > 0$

$\dfrac{4x}{1-x^2} > 0$

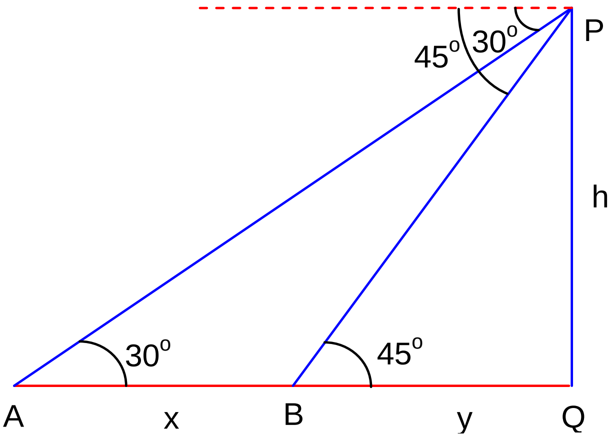
$\dfrac{4x}{(1-x)(1+x)} > 0$

$\dfrac{4x}{(x-1)(x+1)} < 0$

$\therefore x \in (-\infty, -1) \cup (0, 1)$

$\tan\theta < -1$ or $0 < \tan\theta < 1$

$\theta \in \left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right) \cup \left(\pi, \dfrac{5\pi}{4}\right) \cup \left(\dfrac{3\pi}{2}, \dfrac{7\pi}{4}\right)$



13. $\therefore$ In ΔAQP

$\tan 30^o = \frac{PQ}{AQ}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+y}$

$x + y = \sqrt{3}h$........(i)

$\therefore$ In ΔBQP

$\tan 45^o = \frac{h}{y}$

$1 = \frac{h}{y}$

$h = y$.........(ii)

(i) & (ii) $x + y = \sqrt{3}y$

$\Rightarrow x = \left(\sqrt{3}-1\right)y$.....(iii)

Let the speed be S

$\frac{x}{S} = 20$

$x = 20.S$

from (iii)

$20S = \left(\sqrt{3}-1\right)y$

$\frac{y}{S} = \frac{20}{\sqrt{3}-1}$

$\frac{y}{S} = 10\left(\sqrt{3}+1\right)$

14. $xyz = 24$

$24 = 2^3 \times 3$

Let's distribute 2,2,2,3 among 3 variables. No. of positive integral solution =

No. of ways to distribute = ${}^{3+3-1}C_{3-1} \times {}^{1+3-1}C_{3-1} = {}^5C_2 \times {}^3C_2 = 10 \times 3 = 30$

15. $l + m - n = 0$

$l + m = n$ .....(i)

$l^2 + m^2 = n^2$

Now from (i)

$l^2 + m^2 = (l + m)^2$

$\Rightarrow 2lm = 0$

$\Rightarrow lm = 0$

$\Rightarrow lm = 0$

$l = 0$ or $m = 0$

$\Rightarrow m = n \Rightarrow l = n$

$l^2 + m^2 + n^2 = 1$

$m = \pm\frac{1}{\sqrt{2}}$ $\qquad \ell = \pm\frac{1}{\sqrt{2}}$

D.C's - $0, \frac{1}{\sqrt{2}}, \frac{+1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

$0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$ and $\frac{-1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}$

If we take direction cosine of line

$0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

$\cos\alpha = \frac{1}{2}$

$\sin^4\alpha + \cos^4\alpha = \left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{1}{2}\right)^4 = \frac{9}{16} + \frac{1}{16} = \frac{10}{16} = \frac{5}{8}$

16. Let $\sin\theta = t$

$$\int \frac{\sin\theta(2\sin\theta.\cos\theta)\left(\sin^6\theta + \sin^4\theta + \sin^2\theta\right)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{2\sin^2\theta}d\theta$$

$\sin\theta = t$

$\cos\theta \,.\, d\theta = dt$

$\int\left(t^6 + t^4 + t^2\right)\sqrt{2t^4 + 3t^2 + 6}\ dt$

$\int\left(t^5 + t^3 + t\right)\sqrt{2t^6 + 3t^4 + 6t^2}\ dt$

Let

$2t^6 + 3t^4 + 6t^2 = u$

$\left(12t^5 + 12t^3 + 12t\right) = \frac{du}{dt}$

$\left(t^5 + t^3 + t\right).dt = \frac{du}{12}$

$$\int\frac{u^{1/2}}{12}du = \frac{u^{3/2}}{18} + C = \frac{\left(2t^6 + 3t^4 + 6t^2\right)^{3/2}}{18} + C = \frac{\left(2\sin^6\theta + 3\sin^4\theta + 6\sin^2\theta\right)^{3/2}}{18} + C$$

17. $ax^2 + bx + c = 0$

$D = b^2 - 4ac$

$D = 0$

$b^2 - 4ac = 0$

$b^2 = 4ac$

(i) $ac = 1,\ b = 2\ (1,2,1)$ is one way

(ii) $ac = 4,\ b = 4$

$$\left.\begin{matrix} a = 4 & c = 1 \\ a = 2 & c = 2 \\ a = 1 & c = 4 \end{matrix}\right\}\text{3 ways}$$

(iii) $ac = 9,\ b = 6, a = 3, c = 3$ is one way

$1 + 3 + 1 = 5$ way

Required probability = $\frac{5}{216}$

18. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = r$

For 'B', $x = 2r + 1, y = 3r - 1, z = -2r + 1$

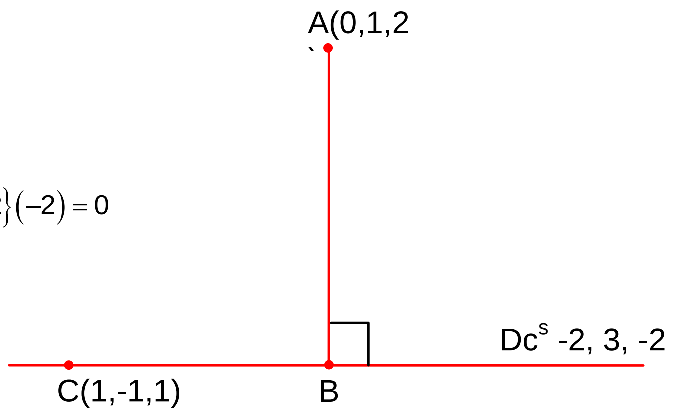
As AB is perpendicular to the line,

$\{(2r+1) - 0\}2 + \{(3r-1) - 1\}3 + \{(-2r+1) - 2\}(-2) = 0$

$r = \frac{2}{17} \Rightarrow B\left(\frac{2}{17}, \frac{-11}{7}, \frac{13}{17}\right) \Rightarrow$

direction ratios of AB are

$(2r + 1, 3r - 2, -2r - 1)$



$= \left(\frac{21}{17}, \frac{-28}{17}, \frac{-21}{17}\right)$

$= (-3, 4, 3)$

Equation of AB

$\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

19. $x^2 - 2(3k-1)x + 8k^2 - 7 > 0 \Rightarrow a > 0, \& D < 0,$

$a = 1 > 0$ and $D < 0$

$4(3k-1)^2 - 4\left(8k^2 - 7\right) < 0$

$\left(9k^2 + 1 - 6k - 8k^2 + 7 < 0\right)$

$k^2 - 6k + 8 < 0$

$k^2 - 4k - 2k + 8 < 0$

$k(k-4) - 2(k-4) < 0$

$k \in (2, 4)$

K = 3

20. Equation of given line x – y + 1 = 0...........(i), equation of perpendicular line PP' is $-x - y + \lambda = 0$

As line passing through (3, 5)

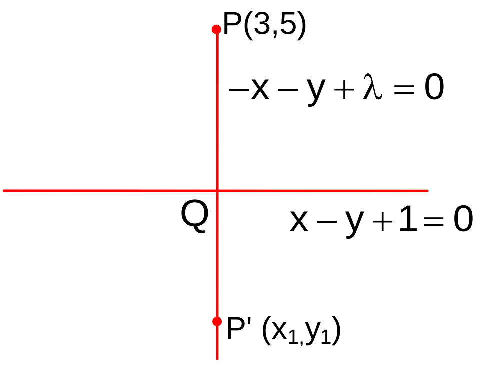
$\therefore -3 - 5 + \lambda = 0$

$\lambda = 8$

Equation of line PP' is $-x - y + 8 = 0$..........(ii)

Solving (i) and (ii)

$\left.\begin{array}{r} -2y = -9 \\ y = \frac{9}{2} \end{array}\right\} Q = \left(\frac{7}{2}, \frac{9}{2}\right)$



$y = \frac{9}{2}$

$\therefore x - \frac{9}{2} + 1 = 0$

$x = \frac{7}{2}$

$\therefore \frac{3 + x_1}{2} = \frac{7}{2}$

$x_1 = 4$

$P' = (4, 4)$

$\frac{5 + y_1}{2} = \frac{9}{2}$

$y_1 = 4$

SECTION B

1. Number divisible by 3;

(a) Sum of the digit must be divisible by 3

(i)

1	2	3

$\longrightarrow$ 3! = 6

(ii) 1,3,5 $\rightarrow$ 3! = 6

(iii) 2,3,4 $\rightarrow$ 3! = 6

(iv) 3,4,5 = 3! = 6

Total = 24

(b) Divisible by 5

		5

$\longrightarrow$ 4x3=12

(c) Now common divisible by both

1	3	5

$\longrightarrow$ 2! = 2

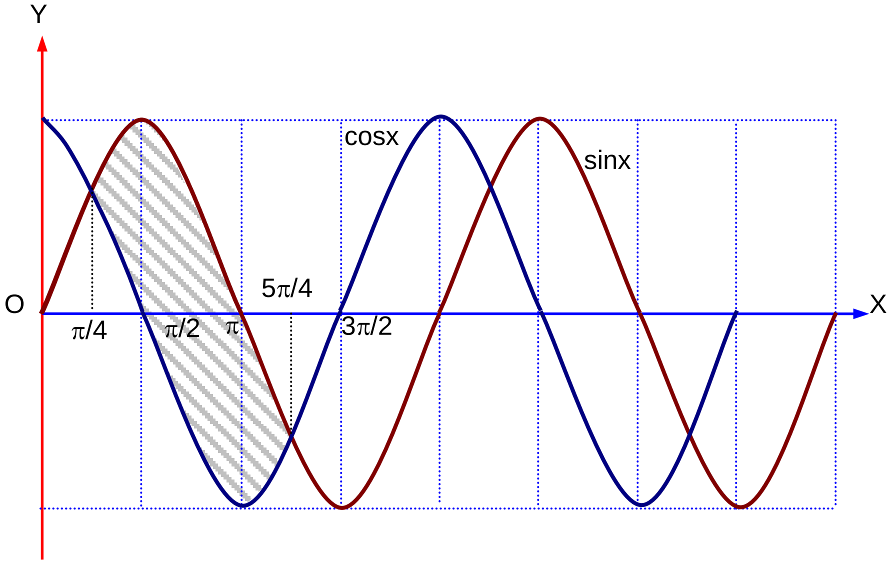
For 3,4

		5

$\longrightarrow$ 2! = 2

$\therefore$ Total ways = 24 + 12 – 4 = 32

2.



$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx = [-\cos x - \sin x]_{\pi/4}^{5\pi/4} = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$A = 2\sqrt{2}$

$A^4 = 64$

3. $f(x)|2x+1| - 3|x+2| + |(x+2)(x-1)|$

Critical point of function are $x = \frac{-1}{2}, 1 \text{and} -2$ but x = -2 is making zero.

$\therefore$ non differentiable at $x = \frac{-1}{2}, 1$

4. $\begin{bmatrix} 0 & -\tan\theta/2 \\ \tan\theta/2 & 0 \end{bmatrix}, I_2 + A = \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}, I_2 - A = \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}$

$(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\left|(I_2 + A)(I_2 - A)^{-1}\right| = a^2 + b^2$

$a^2 + b^2 = \left|(I_2 + A)(I_2 - A)^{-1}\right| = \sec^2 \theta/2 \times \cos^2 \theta/2 = 1$

$\therefore 13\left(a^2 + b^2\right) = 13 \times 1 = 13$

5. $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} - \hat{j}, \vec{c} = \hat{i} - \hat{j} - \hat{k}$

$\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{a}$

Now, $0 = \vec{b}.\vec{c} + \lambda\vec{a}\,.\,\vec{b}$ as $\vec{r}.\vec{b} = 0$

$\lambda = \frac{-\vec{b}.\vec{c}}{\vec{a}.\vec{b}} = 2$

$\therefore \vec{r}.\vec{a} = \vec{a}.\vec{c} + 2a^2 = 12$

6. $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, $|A| = 3xyz - \left(x^3 + y^3 + z^3\right) = -(x + y + z)\left[(x + y + z)^2 - 3(xy + yz + zx)\right]$

$A^2 = I$

$A.A' = I$ (as $A = A'$)

$\therefore x^2 + y^2 + z^2 = 1$ and $xy + yz + zx = 0$

$(x + y + z)^2 = 1 + 2 \times 0 \Rightarrow x + y + z = 1(\text{as } x + y + z > 0)$

$x^3 + y^3 + z^3 = 3 \times 2 + 1 \times (1 - 0) = 7$

7. Let a_n be the side of square A_n

$a_n = \sqrt{2}a_{n+1}$

$a_1 = 12$

$a_n = 12 \times \left(\frac{1}{\sqrt{2}}\right)^{n-1}$

$(a_n)^2 < 1$

$\Rightarrow \frac{144}{2^{(n-1)}} < 1$

$\Rightarrow 2^{(n-1)} > 144$

$\Rightarrow n - 1 \geq 8$

$\Rightarrow n \geq 9$

8. $K = \frac{4\sqrt{3}}{\sqrt{3}x + y}$

$K = \frac{\sqrt{3}x - y}{4\sqrt{3}}$

$\therefore = \frac{4\sqrt{3}}{\sqrt{3}x + y} = \frac{\sqrt{3}x - y}{4\sqrt{3}}$

$3x^2 - y^2 = 48$

$\frac{x^2}{16} - \frac{y^2}{48} = 1$

$48 = 16\left(e^2 - 1\right)$

$e = \sqrt{4} = 2$

9. Let $f(x) = x^6 + ax^5 + bx^4 + ax^3 + dx^2 + ex + f$

$\lim_{x \to 0} \frac{f(x)}{x^3} = 1$ Non zero finite

So, d = e = f = 0

$f(x) = x^6 + ax^5 + bx^4 + cx^3$

$\lim_{x \to 0} \frac{f(x)}{x^3} = 1 = c$

$f'(x) = 0$ at $x = 1$ and $x = -1$

$\lim_{x \to 0} \frac{f(x)}{x^3} = 1$ Non zero finite

$f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

$f'(1) = 0$

$6 + 5a + 4b + 3 = 0$

$5a + 4b = -9$........(i)

$f'(-1) = 0$

$-6 + 5a - 4b + 3 = 0$...........(ii)

Solving (i) and (ii)

$a = -3/5, b = -3/2$

$\therefore f(x) = x^6 + \left(\frac{-3}{5}\right)x^5 + \left(\frac{-3}{2}\right)x^4 + x^3$

$\therefore 5.f(2) = 144$

10. $kx + y + 2z = 1$............(i)

$3x - y - 2z = 2$............(ii)

$-2x - 2y - 4z = 3$...........(iii)

$(ii) \times 5 - (i) \equiv (iii) \times 3 \Rightarrow \left(15 - k\right) = -6$

$K = 21$