

FIITJEE

Solutions to JEE (Main)-2021

JEE–Main–2021 –Feb–24–Second–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

Answers

Section-A

1. B	2. A	3. B	4. D
5. B	6. C	7. A	8. D
9. C	10. C	11. C	12. B
13. D	14. C	15. B	16. B
17. D	18. C	19. B	20. A

Section-B

1. 400	2. 226	3. 8	4. 2
5. 900	6. 8	7. 667	8. 5
9. 8	10. 2		

SECTION – A

Sol1. According to KTG, the gas exerts pressure because its molecules suffer change in momentum when impinge on the walls of container.

Sol2.

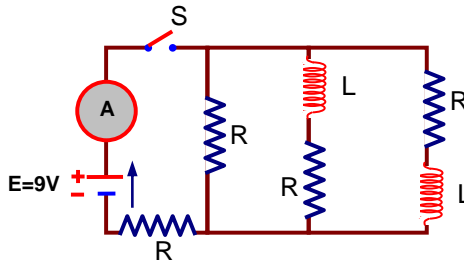
$$R = 2\Omega$$

$$L = 2 \text{ mH}$$

$$E = 9\text{V}$$

$$i = \frac{\varepsilon}{2R} = \frac{9\text{V}}{4\Omega} = 2.25\text{A}$$

Just after the switch 'S' is closed, the inductor acts as open circuit.



Sol3. $T = 2\pi\sqrt{\frac{L}{g}}$ or $T^2 = 4\pi^2\left(\frac{L}{g}\right)$

$$\Rightarrow g = 4\pi^2\left(\frac{L}{T^2}\right)$$

$$\Rightarrow \frac{\Delta g}{g}\% = \left(\frac{\Delta L}{L} + \frac{2\Delta T}{T}\right)\% = \left[\frac{1\text{mm}}{1\text{m}} + \frac{2 \times 0.01}{1.95}\right] \times 100\% = (0.001 + 0.0102) \times 100 = 1.13\%$$

Sol4. Zener break down occurs in p-n junction having p and n both: Heavily doped and have narrow depletion layer.

Sol5. As we go from pole to equator, acceleration due to gravity decreases. So, weight of the body will also reduce. Thus, weight on equator will be slightly smaller than 49N. ie, 48.83 N.

Sol6. When a soft ferromagnetic substance is placed in external magnetic field, the size of domain lying in the opposite direction of external magnetic field increases while size of domain lying in the opposite

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direction of field decreases if field is weak. However, if field is strong then the domain rotate in the direction of external magnetic field due to strong torque.

Sol7. A transistor is formed by doping a semi conducting wafer from one side by N-type dopant (high concentration) while comparatively lower concentration on other side thus forming NPN transistor & vice-versa.

So, statement (I) is false & statement (II) is true.

Sol8. $F = -\alpha x^2 \Rightarrow \frac{mvdv}{dx} = -\alpha x^2$

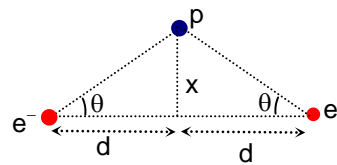
$$\Rightarrow \int_{v_0}^0 vdv = -\frac{\alpha}{m} \int_0^x x^2 dx \Rightarrow \frac{0^2}{2} - \frac{v_0^2}{2} = \frac{-\alpha}{3m} x^3$$

$$\Rightarrow \frac{-v_0^2}{2} = -\frac{\alpha}{3m} x^3 \Rightarrow x = \left(\frac{3v_0^2 m}{2\alpha} \right)^{1/3}$$

Sol9.

$$F_{net} = \frac{2ke^e}{(d^2 + x^2)} \sin\theta = \left(\frac{2ke^2}{(d^2 + x^2)} \right) \theta$$

[for small displacement i.e, x is small so, $\sin\theta \approx \theta \approx \tan\theta$]



$$= \frac{2ke^2}{(d^2 + x^2)} \cdot \frac{x}{d} = 2 \cdot \frac{1}{4\pi\epsilon_0} q^2 \frac{x}{d^3} \quad [q = e]$$

$$= \frac{q^2}{2\pi\epsilon_0 d^3} x = m\omega^2 x$$

$$\therefore \omega = \sqrt{\frac{q^2}{2\pi\epsilon_0 d^3 m}}$$

Sol10. $\Delta E = hv \rightarrow \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$

(1) $n_1 = 3, n_2 = 2$

$$\left(\frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{9-4}{36} = \frac{5}{36}$$

(2) $n_1 = 4, n_2 = 3$

$$\left(\frac{1}{3^2} - \frac{1}{4^2} \right) = \frac{16-9}{144} = \frac{7}{144}$$

(3) $n_1 = 2, n_2 = 1$

$$\left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{4-1}{4} = \frac{3}{4}$$

(4) $n_1 = 5, n_2 = 4$

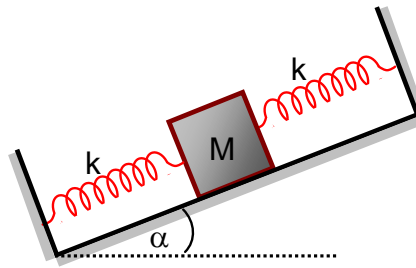
$$\left(\frac{1}{4^2} - \frac{1}{5^2} \right) = \frac{25-16}{400} = \frac{9}{400}$$

Sol11.

Here, $K_{eq} = 2k$

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$



Sol12. $V = 1.24 \times 10^6$ volt

$\lambda_{min} = ?$

$$\lambda_{min} = \frac{hc}{eV} = \frac{1242\text{nm}}{e(1.24 \times 10^6)} = \frac{1000 \times 10^{-9}}{10^6} \Rightarrow \lambda_{min} = 10^{-3} \text{ nm}$$

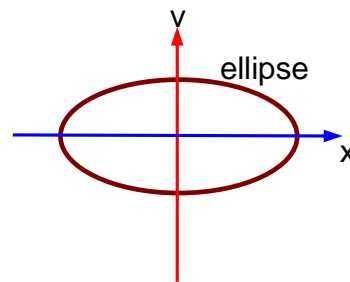
Sol13.

Let $x = A \sin \omega t$ & $v = A\omega \cos \omega t$

$$\therefore v = \omega \sqrt{A^2 - x^2}$$

$$v^2 = \omega^2 x^2 - \omega^2 A^2$$

$$\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$$



Sol14.

$n=1$ mole

$$W_{AB} = nRT \ln \frac{v_2}{v_1} = 1 \times RT \ln \frac{2v_1}{v_1} = RT \ln 2$$

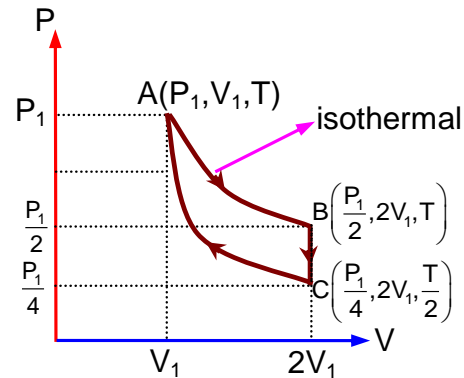
$W_{BC} = 0$

$$W_{CA} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{\frac{P_1}{4} \times 2V_1 - P_1 \times V_1}{\gamma}$$

$$\Rightarrow W_{CA} = \frac{\frac{P_1 V_1}{2} - P_1 V_1}{\gamma - 1} = \frac{-\frac{P_1 V_1}{2}}{2(\gamma - 1)} = \frac{-1RT}{2(\gamma - 1)}$$

$$\Rightarrow W_{CA} = \frac{-RT}{2(\gamma - 1)}$$

$$\therefore W_{total} = RT \left[\ln 2 - \frac{1}{2(\gamma - 1)} \right]$$



- Sol15. (A) Source of microwave frequency
 (B) Source of infrared frequency
 (C) Source of Gamma rays
 (D) Source of x-rays

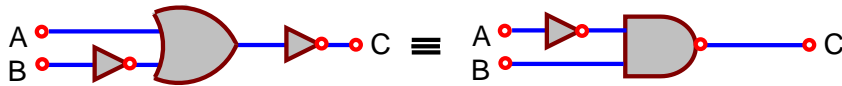
- \Rightarrow Magnetron
 \Rightarrow Vibration of atoms and molecules
 \Rightarrow Radioactive decay of nucleus
 \Rightarrow inner shell electrons.

Sol16. $\lambda_{Red} > \lambda_{violet}$

$$\beta = \frac{\lambda D}{d} \Rightarrow \text{Fringe width}$$

If the source of light used in a Young's double slit experiment is changed from red to violet: consecutive fringe lines will come closer.

Sol17.



Truth Table

A	B	C
0	0	0
0	1	1
1	0	0
1	1	0

Sol18.

$$\lambda = \frac{h}{p} \text{ or } \lambda = \frac{h}{mv}$$

$$\lambda_p = \lambda_\alpha \quad [\text{Given}]$$

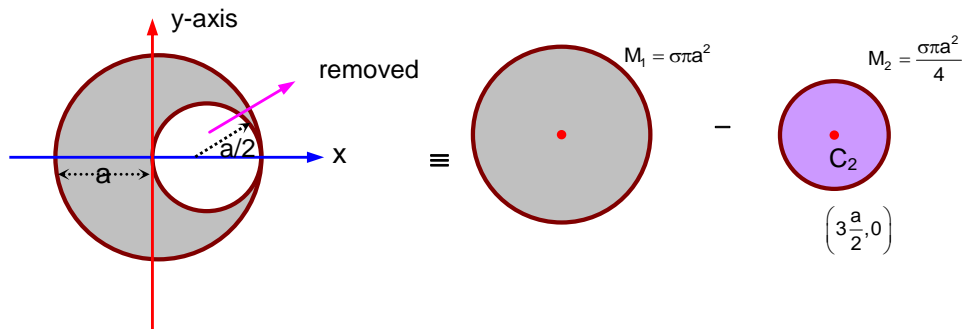
$$\frac{h}{m_p v_p} = \frac{h}{m_\alpha v_\alpha} \Rightarrow \frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p}$$

$$\Rightarrow \frac{v_p}{v_\alpha} = \frac{4m}{m} = \frac{4}{1} \quad \text{or } 4:1$$

Sol19. Traveling wave is represented by $f\left(t \pm \frac{x}{v}\right)$

Thus, $y = A \sin(15x - 2t)$ represents a travelling wave

Sol20.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{\sigma a^2 \times a - \frac{\sigma a^2}{4} \times \frac{3a}{2}}{\sigma a^2 - \frac{\sigma a^2}{4}} = \frac{\frac{5}{8} \sigma a^3}{\frac{3}{4} \sigma a^2} = \frac{5}{8} \sigma a^2 \times \frac{4}{3} \sigma a^2 = \frac{5}{6} a$$

SECTION - B

Sol1. $T = 27^\circ\text{C}$, $P = 1\text{atm} = 10^5\text{N/m}^2$, $V_{rms} = 200\text{m/s}$

As we know

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow \frac{v_{rms}}{v'_{rms}} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{3RT'}{M}}}$$

$$\frac{200}{v'_{rms}} = \sqrt{\frac{T}{T'}} = \sqrt{\frac{300}{400}}$$

$$V'_{rms} = 200 \times \frac{2}{\sqrt{3}} = \frac{x}{\sqrt{3}}$$

$$x = 400$$

Sol2. Flux through the 6 sides of square (i.e cube)

$$\phi_T = \frac{q_{in}}{\epsilon_0} = \frac{12 \times 10^{-6} C}{\epsilon_0}$$

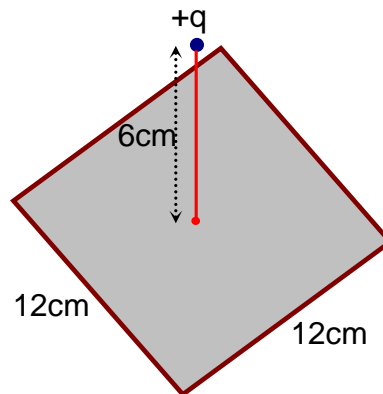
∴ Flux through a square

$$\phi = \frac{\phi_T}{6} = \frac{12 \times 10^{-6}}{\epsilon_0 \times 6} = \frac{2 \times 10^{-6}}{\epsilon_0}$$

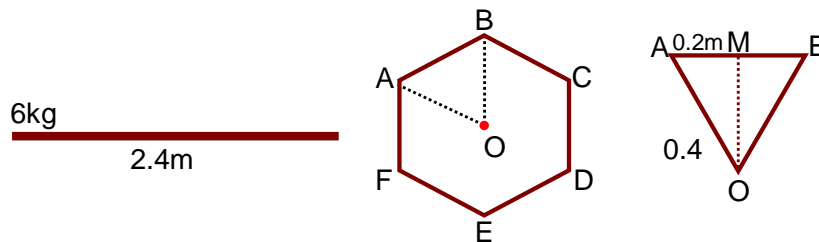
$$\Rightarrow \phi = \frac{2 \times 10^{-6} C}{8.85 \times 10^{-12}} Nm^2 / C^2$$

$$\Rightarrow \phi = 0.22598 \times 10^6$$

$$\Rightarrow \phi = 225.98 \times 10^3 Nm^2 / C \approx 226 Nm^2 / C$$



Sol3.



Let $m_{AB} = m = 1 \text{ kg}$

$$AB = 0.4 \text{ m} = \ell$$

$$d = OM = \sqrt{0.16 - 0.04} = \sqrt{0.12}$$

$$d = 2\sqrt{3} \times 10^{-1}$$

$$I_{AB \text{ about } O} = \frac{m\ell^2}{12} + md^2$$

$$\therefore I_{\text{hexagon, about } O} = 6 \left[\frac{m\ell^2}{12} + md^2 \right] = \frac{m\ell^2}{2} + 6md^2$$

$$= \frac{1 \times (0.4)^2}{2} + 6 \times 1 \times (2\sqrt{3} \times 10^{-1})^2 = \frac{0.16}{2} + 6 \times 4 \times 3 \times 10^{-2}$$

$$= 0.08 + 0.72 = 0.8 \text{ kgm}^2 = 8 \times 10^{-1} \text{ kgm}^2$$

Sol4. As we know that

$$Y = \frac{FL}{A\Delta L}$$

$$\Delta L = 0.04 \text{ m} = \frac{FL}{AY} \dots\dots\dots(i)$$

If length and diameter both are doubled

$$\Delta L' = \frac{F \cdot 2L}{4A \cdot Y} = \frac{FL}{2Y} = 0.02 \text{ m} = 2 \text{ cm}$$

Sol5. $\omega_0 = 10^5 \text{ rad / sec}$

$P = 16 \text{ w}$, 120 v at resonance

$$P = \frac{v^2}{R} \Rightarrow 16 = \frac{(120)^2}{R} \Rightarrow R = \frac{14400}{16} = 900 \Omega$$

Sol6. Sound level is given in dB = $10 \log \left(\frac{P_0}{P_i} \right)$ or $10 \log \left(\frac{I}{I_0} \right)$

As sound level decreases 5dB every km,

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So, in 20 km sound level will decrease by $20 \times 5 = 100\text{dB}$.

$$\Delta\beta = \beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1} \right)$$

$$-100 = 10 \log \left(\frac{I_2}{I_1} \right)$$

$$-10 = \log \left(\frac{I_2}{I_1} \right)$$

$$10^{-10} = \frac{I_2}{I_1} \Rightarrow I_2 = 10^{-10} I_1$$

$$P_2 = 10^{-10} P_1$$

$$P_2 = 10^{-10} \times (0.1 \times 10^3) \Rightarrow P_2 = 10^{-8} \text{W} = 10^{-x} \text{W}$$

$$\therefore x = 8$$

Sol7. $f = 3\text{GHz} = 3 \times 10^9 \text{ Hz}$

$$\lambda_{\text{vacuum}} = \frac{v}{f} = \frac{3 \times 10^8 \text{ m/s}}{3 \times 10^9 / \text{s}} = 10^{-1} \text{ m} = 0.1 \text{ m}$$

$$\lambda_{\text{vacuum}} = \frac{\lambda_{\text{medium}}}{\mu_{\text{medium}}} = \frac{0.1}{\mu_{\text{medium}}}$$

$$\mu_{\text{medium}} = \sqrt{\mu_r \epsilon_r} = \sqrt{1 \times 2.25} = 1.5$$

$$\therefore \mu_{\text{medium}} = \frac{0.1 \text{ m}}{1.5} = 0.0667 \text{ m} = 6.67 \text{ cm} = 667 \times 10^{-2} \text{ cm}$$

Sol8. radius $r = 0.5 \text{ mm}$

$$\sigma = 5 \times 10^7 \text{ s/m}$$

$$\vec{E} = 10 \times 10^{-3} \text{ V/m} = 10^{-2} \text{ V/m}$$

As we know

$$J = \sigma E$$

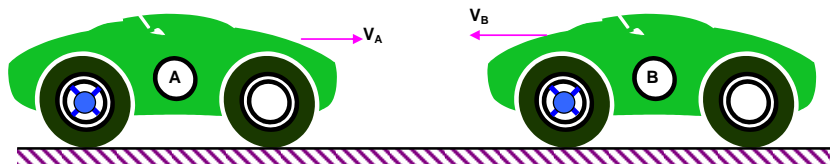
$$J = \frac{i}{A} = \sigma E$$

$$\therefore i = \sigma EA = 5 \times 10^7 \times 10^{-2} \times \pi \times (0.5 \times 10^{-3})^2 = 1.25 \pi \times 10^{-1} \text{ A} = 125 \pi \text{ mA} = x^3 \pi \text{ mA}$$

$$\therefore x = 5$$

Sol9. $7.2 \times \frac{5}{18} = 2 \text{ m/s}$

[$V_A = V_B = 2 \text{ m/s}$]



$$f'_B = f_A \left(\frac{v + v_B}{v - v_A} \right)$$

↓

App freq heard by car B

$$f'_B = 676 \left[\frac{340 + 2}{340 - 2} \right] = 676 \times \frac{342}{338} = 684 \text{ Hz}$$

$$\text{Similarly, } f'_A = f_B \left(\frac{v + v_B}{v - v_B} \right) = 684 \text{ Hz}$$

$$\therefore \text{Beat frequency heard by both} = 684 - 676 = 8 \text{ Hz}$$

Sol10. A \rightarrow $m_A = 1\text{ kg}$, $P_A = P$
B \rightarrow $m_B = 2\text{ kg}$, $P_B = P$

$$\therefore kE = \frac{P^2}{2m}$$

$$\therefore \frac{kE_A}{kE_B} = \frac{\left(\frac{P^2}{2m_A}\right)}{\left(\frac{P^2}{2m_B}\right)} = \frac{m_B}{m_A} = \frac{2}{1}$$

PART – B (CHEMISTRY)

Answers

Section-A

1. A	2. A	3. D	4. A
5. D	6. D	7. B	8. C
9. A	10. A	11. A	12. D
13. C	14. C	15. C	16. C
17. A	18. A	19. A	20. C

Section-B

1. 3	2. 81	3. 141	4. 8
5. 243	6. 5	7. 3776	8. 855
9. 1	10. 1		

SECTION – A

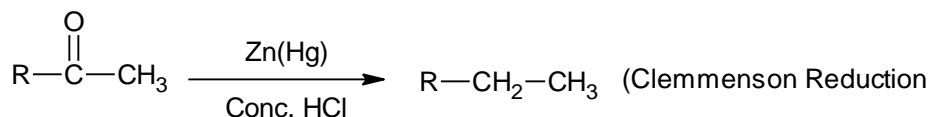
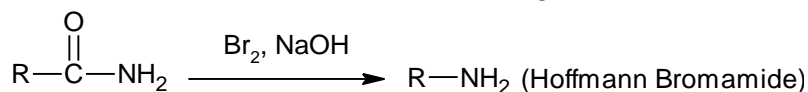
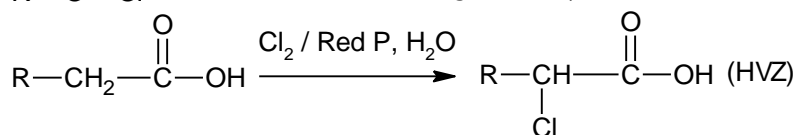
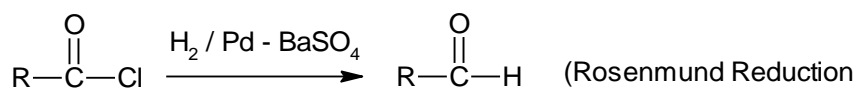
Sol1. Density = $\frac{\text{mass}}{\text{Volume}}$
Cu > Co > Fe > Cr > Zn

Sol2. Valium – Tranquilizer
Morphine – Analgesic
Norethindrone – Antifertility drug
Vitamin B₁₂ – Pernicious anaemia

Sol3. Blood – Negatively charged colloid.
According to Hardy – Schulze Rule
FeCl₃ is more efficient for blood clotting

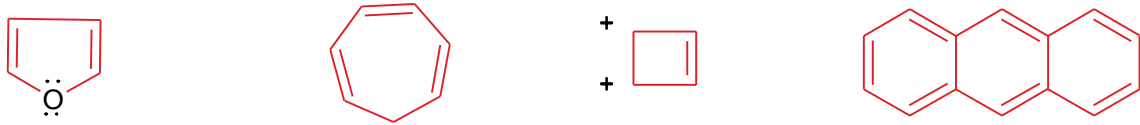
Sol4. Aluminium – Kaolinite
Iron – Siderite
Copper – Malachite
Zinc – Calamine

Sol5.



Sol6. VOSO₄ → V⁴⁺ Reducing agent
Cr₂O₃ → Amphoteric oxide
Red colour of ruby due to Cr³⁺ present in Al₂O₃
Ru⁸⁺ → oxidizing agent

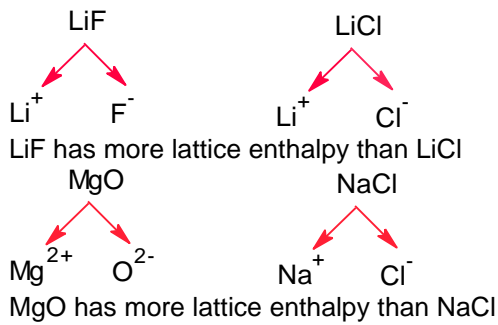
Sol7.



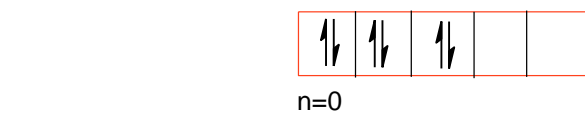
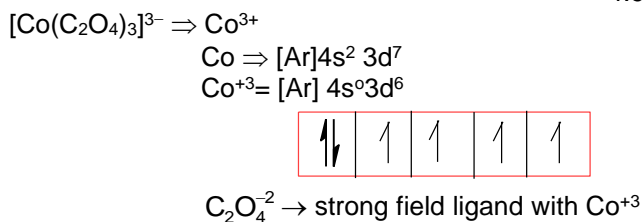
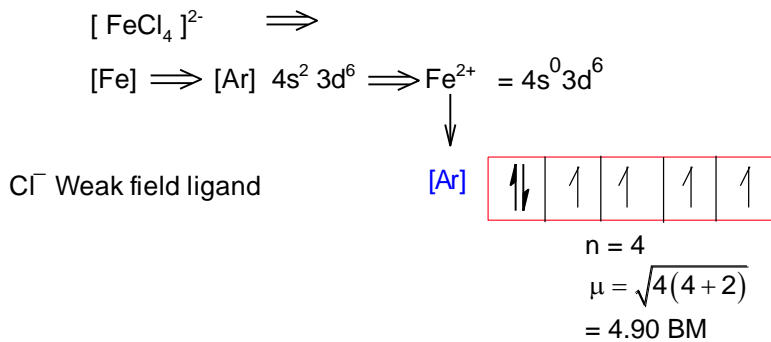
- | | | | |
|---|---|--|---|
| <ul style="list-style-type: none"> → Cyclic → Planar → Follow Huckel Rule → Conjugation | <p>→ Conjugation
Is not present
in whole ring</p> | <ul style="list-style-type: none"> → Cyclic → Planar → Huckel Rule → Conjugation | <ul style="list-style-type: none"> → Cyclic → Planar → Follow Huckel Rule → Conjugation |
|---|---|--|---|

Sol8. Hydrogen is most abundant element in earth crust and lightest element. But in troposphere nitrogen gas is most abundant element

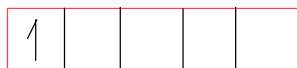
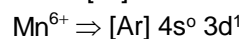
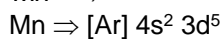
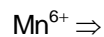
Sol9.



Sol10.



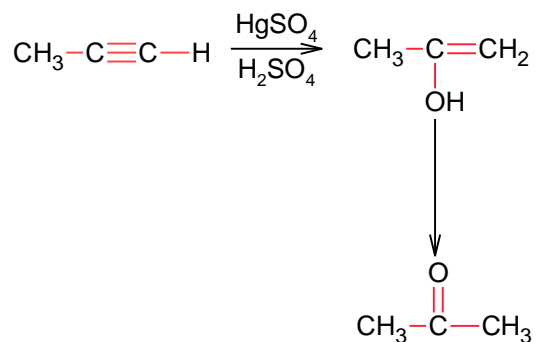
μ = 0 BM



n=1

$\mu = \sqrt{n(n+2)} = \sqrt{1(3)} = 1.73\text{BM}$

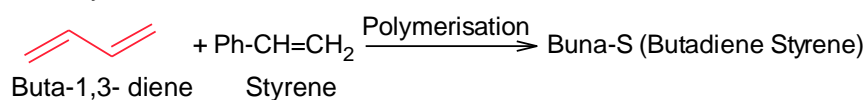
Sol11.



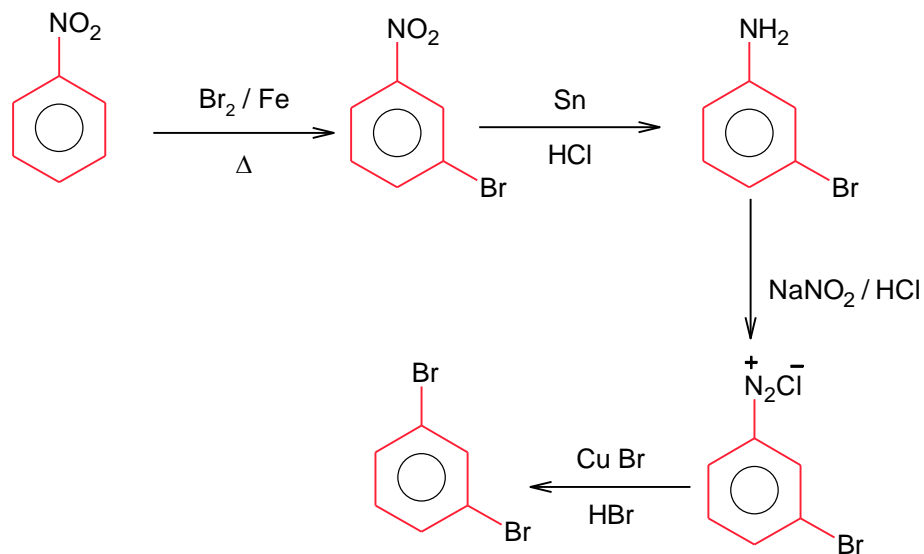
Hence

 $\text{CH}_3-\text{CH}_2-\text{CHO}$ can't be formed.

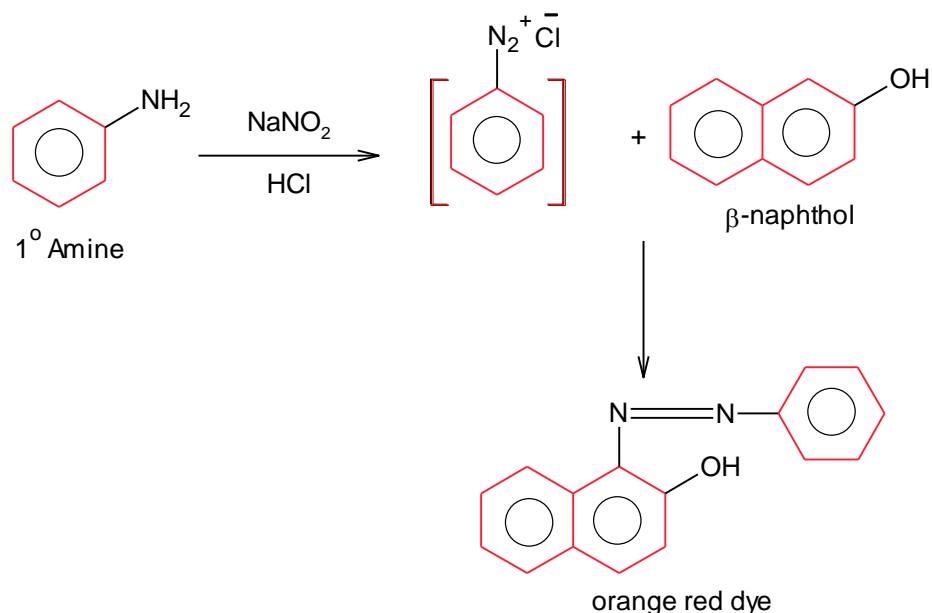
In case of unsymmetrical alkynes, more positive charge stabilizing carbon attacked by H_2O and finally converted into carbonyl.

Sol12. S \rightarrow styrene

Sol13.

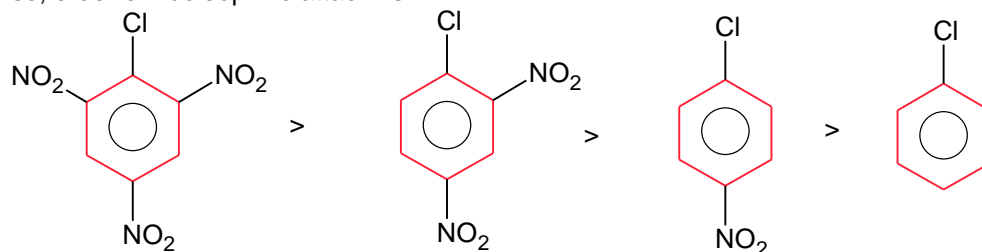


Sol14.



Sol15. Clean water would have BOD Value less than 5ppm, highly polluted water could have BOD value of 17ppm or more.

Sol16. If we add $-\text{NO}_2$ group on ortho and para position, then electron deficiency increases in benzene ring so, order of nucleophilic attack is



Sol17. According to Bohr's theory,

$$\text{K.E} = 13.6 \frac{Z^2}{n^2} \quad \text{frequency } (\nu) \text{ of revolution of electron } \propto \frac{Z^2}{n^3}$$

$$\nu = 2.18 \times 10^6 \frac{Z}{n}$$

$$F = \frac{kq_1q_2}{r^2} = \frac{Kze^2}{r^2} \left(r \propto \frac{n^2}{Z} \right)$$

$$F \propto \frac{Z^3}{n^4}$$

Sol18. Li \rightarrow Crimson Red
 Na \rightarrow Yellow
 Rb \rightarrow Red violet
 Cs \rightarrow Blue

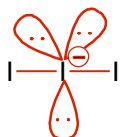
VIBGYOR

Wavelength

Red violet > Crimson Red > Yellow > Blue

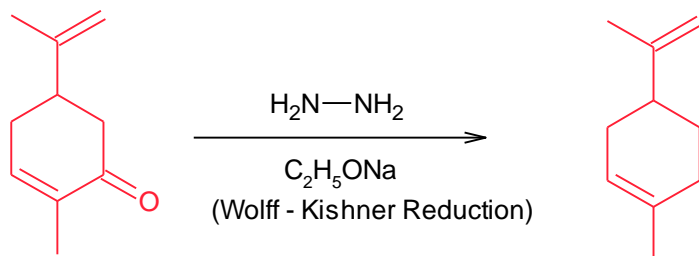
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 780nm 670.8nm 589.2nm 455.5nm

Sol19.



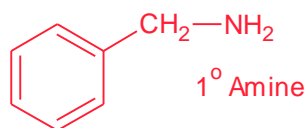
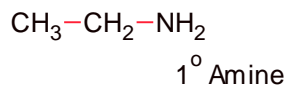
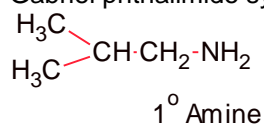
Linear (180°)

Sol20.



SECTION - B

Sol1. Gabriel phthalimide synthesis is used for 1° Aliphatic / alicyclic amine



Sol2. $\text{C}_{12}\text{H}_{22}\text{O}_{11} + \text{H}_2\text{O} \rightarrow \text{C}_6\text{H}_{12}\text{O}_6 + \text{C}_6\text{H}_{12}\text{O}_6$
 Glucose + Fructose

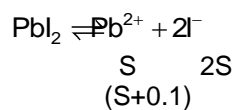
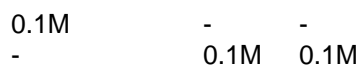
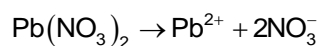
$$K = \frac{2.303}{t} \log \frac{a}{a-x}$$

$$\frac{kt}{2.303} = \log \frac{a}{a-x}$$

$$\frac{\ln 2 \times 9}{\frac{10}{3} \times 2.303} = \log \left(\frac{1}{f} \right)$$

$$\log \left(\frac{1}{f} \right) = 81.24 \times 10^{-2} = 81 \times 10^{-2}$$

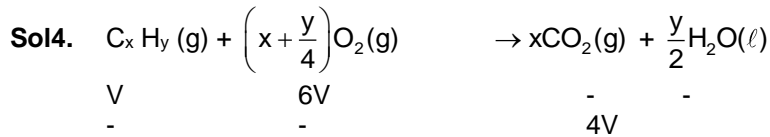
Sol3. $\text{PbI}_2 \rightleftharpoons \text{Pb}^{2+} + 2\text{I}^- \quad K_{sp} = 8.0 \times 10^{-9}$



$$K_{sp} = 8 \times 10^{-9}; \text{ Using } K_{sp} = [\text{Pb}^{2+}] [\text{I}^-]^2$$

$$8 \times 10^{-9} = 0.1 \times (2S)^2$$

$$S = 141 \times 10^{-6} \text{M}$$



$$\text{Volume of } CO_2 = 4 \times V_{C_x H_y}$$

$$Vx = 4V$$

$$x = 4$$

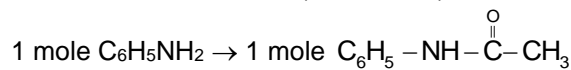
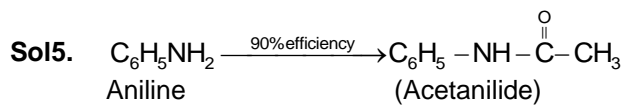
$$\text{Volume of } O_2 = 6 \times V_{C_x H_y}$$

$$V \left(x + \frac{y}{4} \right) = 6V$$

$$4 + \frac{y}{4} = 6$$

$$\frac{y}{4} = 2$$

$$y = 8$$



$$\frac{1.86}{93} = \frac{w}{135}$$

$$W = 2.70 \text{g}$$

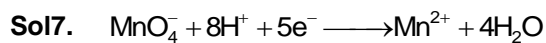
$$\text{Mass produced actual} = 2.70 \times \frac{90}{100} = 2.43 = 243 \times 10^{-2} \text{g}$$

Sol6. $W = 4.75 \text{g}$

$$n = \frac{4.75}{26} \quad T = 323 \text{K}, R = 0.0826$$

$$P = \frac{740}{760} \text{ atm}$$

$$V = \frac{nRT}{P} = \frac{4.75 \times 0.0826 \times 323}{26 \left(\frac{740}{760} \right)} = 5 \text{ litre}$$



$$E_1 = E^\circ - \frac{0.059}{5} \log \frac{[Mn^{2+}]}{[MnO_4^-]} \left[\frac{1}{[H^+]} \right]^8$$

$$[H^+] = 1 \text{M}$$

$$E_1 = E^\circ - \frac{0.059}{5} \log \frac{[Mn^{2+}]}{[MnO_4^-]}$$

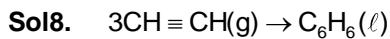
$$[H^+] = 10^{-4}$$

$$E_2 = E^\circ - \frac{0.059}{5} \log \frac{[Mn^{2+}]}{[MnO_4^-]} \times \left(\frac{1}{10^{-4}} \right)^8$$

$$= E^\circ - \frac{0.059}{5} \log \frac{[Mn^{2+}]}{[MnO_4^-]} + \frac{0.059}{5} \log 10^{-32}$$

So, difference in E_1 & E_2 is
 $= 0.3776 \text{ V}$

$$= 3776 \times 10^{-4} \text{ V}$$



$$\Delta G^\circ = -RT \ln K \dots\dots\dots(\text{i})$$

$$\Delta G^\circ = \Sigma \Delta G^\circ_{\text{P}} - \Sigma \Delta G^\circ_{\text{R}} \dots\dots\dots(\text{ii})$$

Equating (i) & (ii)

$$-2.303 RT \log k = 4.88 \times 10^5$$

$$\log K = \frac{-4.88 \times 10^5}{2.303 \times R \times T} = \frac{-488000}{5705.848} = -85.52 = -855 \times 10^{-1}$$

So; magnitude of $\log K = 855 \times 10^{-1}$.

Sol9. $\Delta T_f = k_f \cdot m$

$$T_f^\circ - T_f = 5.12 \times \frac{\left(\frac{10}{58}\right)}{\left(\frac{200}{1000}\right)}$$

$$5.5 - T_f = \frac{5.12 \times 5 \times 10}{58}$$

$$T_f = 1.086^\circ\text{C} = (1.086)^\circ\text{C} \cong 1^\circ\text{C}$$

Sol10. α - sulphur & β - sulphur – Diamagnetic, S_2 -form is paramagnetic due to presence of unpaired electron in π^* orbital like O_2 .

PART-C (MATHEMATICS)

Answers

Section-A

- | | | | |
|-------|-------|-------|-------|
| 1. C | 2. A | 3. D | 4. D |
| 5. D | 6. A | 7. C | 8. B |
| 9. C | 10. D | 11. B | 12. D |
| 13. C | 14. B | 15. A | 16. A |
| 17. C | 18. C | 19. C | 20. C |

Section-B

- | | | | |
|------|---------------------|----------------------|----------|
| 1. 1 | 2. Question Dropped | 3. 310 | |
| 4. 2 | 5. 56 | 6. 3 | 7. 31650 |
| 8. 2 | 9. 11 | 10. Question Dropped | |

SECTION-A

Sol1. $I = \int_0^2 f(x)dx = [xf(x)]_0^2 - \int_0^2 xf'(x)dx = 2e^2 - \int_0^2 xf'(x)dx \dots\dots\dots(A)$

Put $I_1 = \int_0^2 xf'(x)dx \dots\dots\dots(i)$

Using properties $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I_1 = \int_0^2 (2-x)f'(2-x)dx = \int_0^2 (2-x)f'(x)dx \dots\dots\dots(ii)$

Adding (i) and (ii) we get

$2I_1 = 2 \int_0^2 f'(x)dx \Rightarrow I_1 = [f(x)]_0^2$

$f(2) - f(0) = e^2 - 1$

From (A) $I = 2e^2 - e^2 + 1 = e^2 + 1$

Sol2. $I = \int_1^3 [x^2 - 2x + 1 - 3]dx = \int_1^3 [(x-1)^2 - 3]dx = \int_1^3 [(x-1)^2]dx - 3 \int_1^3 dx \dots\dots\dots(A)$

$I_1 = \int_1^3 [(x-1)^2]dx$ Put $(x-1)^2 = t$

$I_1 = \frac{1}{2} \int_0^4 \frac{dt}{\sqrt{t}}$ $2(x-1)dx = dt$

$dx = \frac{dt}{2\sqrt{t}}$

$I_1 = \frac{1}{2} \left[0 \int_0^1 \frac{dt}{\sqrt{t}} + \int_1^2 \frac{dt}{\sqrt{t}} + 2 \int_2^3 \frac{dt}{\sqrt{t}} + 3 \int_3^4 \frac{dt}{\sqrt{t}} \right] = \frac{1}{2} \left\{ \left[\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_0^1 + 2 \left[\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_1^2 + 3 \left[\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_2^3 + \left[\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_3^4 \right\} = 5 - \sqrt{2} - \sqrt{3}$

Hence from (A)

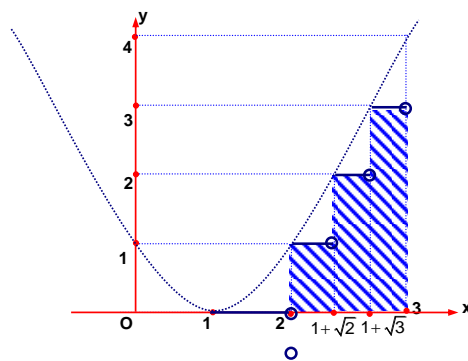
$= 5 - \sqrt{2} - \sqrt{3} - 6 = -1 - \sqrt{2} - \sqrt{3}$

2nd method

$$I = \int_1^3 \left(\left[(x-1)^2 \right] - 3 \right) dx = \int_1^3 \left[(x-1)^2 \right] dx - 3 \int_1^3 dx$$

$$\int_1^3 \left[(x-1)^2 \right] dx - 6 \dots \dots \dots (A)$$

Let $I_1 = \int_1^3 \left[(x-1)^2 \right] dx$



$$\therefore 0 \int_1^2 dx + 1 \int_2^{1+\sqrt{2}} dx + 2 \int_{1+\sqrt{2}}^{1+\sqrt{3}} dx + 3 \int_{1+\sqrt{3}}^3 dx$$

$$= (1 + \sqrt{2} - 2) + 2(1 + \sqrt{3} - 1 - \sqrt{2}) + 3(3 - 1 - \sqrt{3})$$

$$= -1 + \sqrt{2} + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3} = 5 - \sqrt{2} - \sqrt{3}$$

From (A), $I = 5 - \sqrt{2} - \sqrt{3} - 6 = -1 - \sqrt{2} - \sqrt{3}$

Sol3. $x - 2y = 1, x - y + kz = -2, ky + 4z = 6$

$$x - 2y + 0z - 1 = 0$$

$$x - y + kz + 2 = 0$$

$$0x + ky + 4z - 6 = 0$$

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 1(-4 - k^2) - 2(-4 + 0) = -4 - k^2 + 8 = -k^2 + 4$$

$$\therefore k^2 = 4 \Rightarrow k = \pm 2$$

For unique solution $\Delta \neq 0 \Rightarrow k \neq \pm 2$

$$\Delta_1 = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & k \\ 6 & k & 4 \end{vmatrix} = -(k+10)(k+2)$$

For no solution

$$\Delta = 0, \Delta_1 \neq 0$$

$$k=2$$

Sol4. $\frac{a+20}{2} = 3 + 7r$

$$a + 20 = 6 + 14r \dots\dots\dots(i)$$

$$\frac{b + 6}{2} = 2 + 5r$$

$$b + 6 = 4 + 10r$$

$$b = -2 + 10r \dots\dots\dots(ii)$$

$$\frac{9 - a - 9}{2} = 1 - 9r$$

$$-a = 2 - 18r$$

$$a = 18r - 2 \dots\dots\dots(iii)$$

Solving (i) and (iii) we get

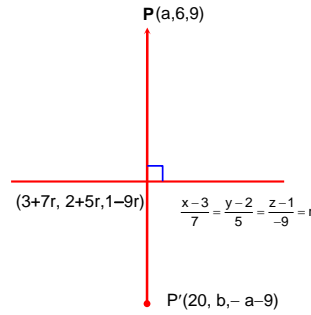
$$20 + 18r - 2 = 6 + 14r$$

$$4r = 8 - 20 = -12$$

$$r = -3$$

$$\therefore a = 14 + 14(-3) = -56 \text{ and } b = -2 - 30 = -32$$

$$|a + b| = |-56 - 32| = 88$$



Sol5.

$$\frac{a + 2 + a}{3} = \frac{10}{3}$$

$$2a + 2 = 10$$

$$2a = 8 \Rightarrow a = 4 \dots\dots\dots(i)$$

$$\text{and } \frac{c + b + b}{3} = \frac{7}{3}$$

$$2b + c = 7 \dots\dots\dots(ii)$$

Since a, b, c are in A.P

$$2b = a + c$$

$$\text{From (i) } \therefore 2b = 4 + c \dots\dots\dots(iii)$$

Solving (ii) and (iii)

$$4 + c + c = 7$$

$$2c = 3$$

$$c = \frac{3}{2}$$

$$\therefore 2b = 4 + \frac{3}{2} = \frac{11}{2}$$

$$\therefore b = \frac{11}{4}$$

As per question

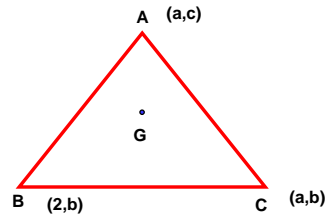
$$\alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{1}{a}$$

$$= \frac{-11}{4} = -\frac{11}{4}, \quad \alpha\beta = \frac{1}{4}$$

$$\alpha^2 + \beta^2 - \alpha\beta = (\alpha + \beta)^2 - 3\alpha\beta$$

$$= \frac{121}{16} - \frac{3}{4}$$

$$= \frac{121 - 192}{16} = \frac{-71}{16}$$

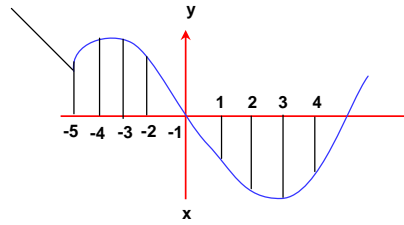


Sol6. let $f: \mathbb{R} \rightarrow \mathbb{R}$

$$f'(x) = \begin{cases} -55, & x < -5 \\ 6x^2 - 6x - 120, & -5 < x < 4 \\ 6x^2 - 6x - 36, & x > 4 \end{cases}$$

$$= \begin{cases} -55, & x < -5 \\ 6(x-5)(x+4), & -5 < x < 4 \\ 6(x-3)(x+2), & x > 4 \end{cases}$$

$f(x)$ increasing in $x \in (-5, -4) \cup (4, \infty)$



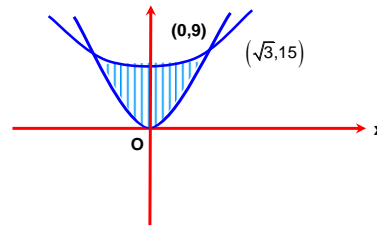
Sol7. Total possibilities = $2^5 \times 2^5$

Farounable case = ${}^5C_2 \times 3^3 = 10 \times 3^3$

$$\therefore \text{required probability} = \frac{10 \times 3^3}{2^5 \times 2^5} = \frac{5 \times 27}{2^9} = \frac{135}{2^9}$$

Sol8. Area of shaded region

$$\begin{aligned} & 2 \int_0^{\sqrt{3}} (2x^2 + 9 - 5x^2) dx \\ &= 2 \int_0^{\sqrt{3}} (9 - 3x^2) dx \\ &= \int_0^{\sqrt{3}} (3 - x^2) dx = 6 \left[3x - \frac{x^3}{3} \right]_0^{\sqrt{3}} = 6 \left[\frac{9\sqrt{3} - 3\sqrt{3}}{3} \right] = 12\sqrt{3} \end{aligned}$$



Sol9. $A^T = A$ and $B^T = -B$

$$C = A^2 B^2 - B^2 A^2$$

$$C^T = (A^2 B^2)^T - (B^2 A^2)^T = B^2 A^2 - A^2 B^2$$

$C^T = -C$. Hence C is skew symmetric matrix

$$\therefore \det(C) = 0$$

Hence system have infinite solutions

Sol10. $x + \sqrt{3}y = 2\sqrt{3}$, and $m = -\frac{1}{\sqrt{3}}, C = 2$

$$(1) \frac{x^2}{\frac{9}{2}} - \frac{y^2}{\frac{1}{2}} = 1 \Rightarrow c = \sqrt{a^2 m^2 - b^2} = \sqrt{\frac{9}{2} \times \frac{1}{3} - \frac{1}{2}} = 1, \text{ not possible}$$

$$(2) y^2 = \frac{1}{6\sqrt{3}}x, c = \frac{a}{m} = \frac{1}{24\sqrt{3}} \times \left(\frac{-1}{\sqrt{3}} \right) = -\frac{1}{72}, \text{ not possible}$$

$$(3) c = a\sqrt{1+m^2} = \sqrt{7} \times 2 = 2\sqrt{7}, \text{ not possible}$$

$$(4) \frac{x^2}{9} + \frac{y^2}{1} = 1, C = \sqrt{9m^2 + 1} = \sqrt{9 \times \frac{1}{3} + 1} = 2$$

Sol11. ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2)$

$${}^{n+1}C_2 + 2 \sum_{r=1}^{n-1} {}^{r+1}C_2$$

$$= {}^{n+1}C_2 + 2 \sum_{r=1}^{n-1} \frac{(r+1)!}{(r-1)!2!} = {}^{n+1}C_2 + \sum_{r=1}^{n-1} (r+1)r$$

$$= \frac{(n+1)!}{2!(n+1)!} + \frac{(n-1)n(2(n-1)+1)}{6} + \frac{(n-1)n}{2} = \frac{n(n+1)}{2} + \frac{n(n-1)(2n-1)}{6} + \frac{n(n-1)}{2}$$

$$= \frac{n}{2} \left[n+1+n-1 + \frac{(n-1)(2n-1)}{3} \right] = \frac{n}{2} \left[2n + \frac{2n^2 - 3n + 1}{3} \right]$$

$$= \frac{n(6n + 2n^2 - 3n + 1)}{6} = \frac{(2n^2 + 3n + 1)}{6} = \frac{n(2n+1)(n+1)}{6}$$

Sol12. $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 + \lambda (\vec{r} \cdot (\hat{i} - 2\hat{j}) + 2) = 0$

$$\vec{r} \cdot ((1+\lambda)\hat{i} + (1-2\lambda)\hat{j}) - 1 + 2\lambda = 0$$

$$\therefore \text{point } (1,0,2) = \hat{i} + 2\hat{k}$$

$$\therefore 1 + \lambda + 2 - 1 + 2\lambda = 0 \Rightarrow \lambda = -\frac{2}{3}$$

$$\therefore \vec{r} \cdot \left(\frac{\hat{i}}{3} + \frac{7\hat{j}}{3} + \hat{k} \right) - \frac{7}{3} = 0$$

$$\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$$

Sol13. $\sim(\sim p \wedge (p \vee q)) = p \vee (\sim p \wedge \sim q)$
 $= (p \vee \sim p) \wedge (p \vee \sim q)$
 $= p \vee \sim q$

Sol14. $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$

$$f(x)f''(x) = f'(x)f'(x)$$

$$\frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}, \text{ Integrating on both sides}$$

$$\ln |f'(x)| = \ln |f(x)| + \ln |c|$$

$$f'(x) = cf(x), f'(0) = cf(0) \Rightarrow c = 2$$

$$\frac{f'(x)}{f(x)} = 2, \text{ again integrating on both side}$$

$$\ln f(x) = 2^x + k$$

$$f(x) = e^{2^x+k}$$

$$f(0) = e^k = e^k = 1 \Rightarrow k = 0$$

$$\therefore f(x) = e^{2^x} \quad [\because e = 2.718]$$

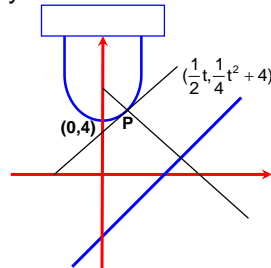
$$\therefore e^2 \in (6,9)$$

Sol15.

$$y = x^2 + 4$$

$$x^2 = y - 4$$

$$y = 4x - 1$$



$$PQ = \frac{|4 \times \frac{1}{2}t - \frac{1}{4}t^2 - 4 - 1|}{\sqrt{17}}$$

$$p = \frac{|t^2 - 8t - 20|}{4\sqrt{17}}$$

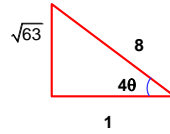
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$$\frac{dp}{dt} = \frac{1}{4\sqrt{17}}(2t-8) \text{ for maximum / minimum, } \frac{dp}{dt} = 0 \Rightarrow t = 4$$

also $\frac{d^2p}{dt^2} > 0$ at $t = 4$

hence closest point becomes at $t = 4$ is (2,8)

Sol16. $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$



Put $\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8} = \theta$

$$\sin 4\theta = \frac{\sqrt{63}}{8} \therefore \cos 4\theta = \frac{1}{8}$$

$$2\cos^2 2\theta = \frac{1}{8} + 1 \Rightarrow \cos 2\theta = \frac{3}{4}$$

$$\Rightarrow \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{3}{4}$$

(adding 1 both side)

$$\frac{2}{\sec^2 \theta} = \frac{7}{4}$$

$$\cos^2 \theta = \frac{7}{8} \Rightarrow \sec^2 \theta = \frac{8}{7}$$

$$\tan^2 \theta = \frac{8}{7} - 1 = \frac{1}{7}$$

$$\tan \theta = \frac{1}{\sqrt{7}}$$

Sol17. (A)

P	q	~p	~q	p→q	~q∧(p→q)	(~q∧(p→q))→~p
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

tautology

(B)

p∨q	(p∨q)~p	((p∨q)∧~p)→q
T	F	T
T	F	T
T	T	T
F	F	T

tautology

Sol18. $y = ax^2 + bx + c$

$$a + b + c = 2$$

$$\therefore \frac{dy}{dx} = 2ax + b \Rightarrow \frac{dy}{dx}\bigg|_{(0,0)} = b = 1$$

It passes through (0,0),

$$0 = 0 + 0 + c \Rightarrow c = 0$$

$$a = 1$$

$$\therefore a = 1, b = 1, c = 0$$

Sol19. $S = 432 \text{ km/hr}$

$$\tan 60^\circ = \frac{h}{y}$$

$$h = \sqrt{3}y$$

$$\tan 30^\circ = \frac{h}{x+y}$$

$$x+y = \sqrt{3}h$$

$$x+y = \sqrt{3} \cdot \sqrt{3}y$$

$$x+y = 3y$$

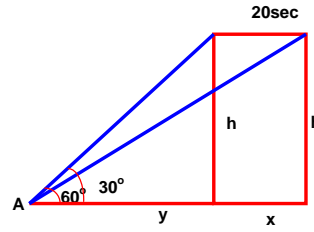
$$x = 2y$$

$$\therefore x = 432 \times \frac{1000}{3600} \times 20$$

$$x = 2400 \text{ m}$$

$$y = 1200 \text{ m}$$

$$\therefore h = 1200\sqrt{3} \text{ m}$$



Sol20. $x \frac{dy}{dx} + y = bx^4$

$$\frac{dy}{dx} + \frac{y}{x} = bx^3$$

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$xy = b \int x^4 dx = \frac{bx^5}{5} + c$$

$$\text{at } (1,2), 1 \cdot 2 = \frac{b}{5} + c \Rightarrow c = 2 - \frac{b}{5}$$

$$\therefore \int_1^2 f(x) dx = \int_1^2 \left(\frac{bx^4}{5} + \frac{10-b}{5x} \right) dx = \left[\frac{bx^5}{25} + \left(\frac{10-b}{5} \right) \ln x \right]_1^2$$

As per question

$$\frac{32b}{25} + (10-b) \ln 2 - \frac{b}{32} = \frac{62}{5}$$

$$\frac{31}{25}b + (10-b) \ln 2 = \frac{62}{5}$$

$$(10-b) \ln 2 = \frac{62}{5} - \frac{31}{25}b = \frac{31}{5} \left(2 - \frac{b}{5} \right)$$

$$\therefore b = 10$$

SECTION - B

Sol1. $L_1 = \frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

$$L_2 = \frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$$

$$SD = \frac{|(\vec{a}_1 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 2\hat{i} - 3\hat{j} + \hat{k}$$

$$\frac{\vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|} = \frac{2\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{14}}$$

$$\vec{b}_2 \times \vec{b}_1 = (0 - \lambda)\hat{i} + \left(-2\lambda - \frac{1}{2}\right)\hat{j} + (\lambda - 0)\hat{k}$$

$$\therefore \text{SD} = \frac{\left|-2\lambda + 3\left(2\lambda + \frac{1}{2}\right) + \lambda\right|}{\sqrt{14}} = \frac{\left|5\lambda + \frac{3}{2}\right|}{\sqrt{14}}$$

$$\therefore \frac{\left|5\lambda + \frac{3}{2}\right|}{\sqrt{14}} = \frac{\sqrt{7}}{2\sqrt{2}} \Rightarrow \left|5\lambda + \frac{3}{2}\right| = \frac{7}{2}$$

$$\therefore 5\lambda + \frac{3}{2} = \frac{7}{2} \Rightarrow \therefore 5\lambda = 2 \Rightarrow \lambda = \frac{2}{5}$$

$$\text{or } 5\lambda + \frac{3}{2} = -\frac{7}{2} \Rightarrow 5\lambda = -5 \Rightarrow \lambda = -1$$

$$\therefore |\lambda| = 1$$

Sol2.

$$\sum_{i=0}^k \binom{10}{i} \binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i} \binom{13}{k+1-i}$$

$$\sum_{i=0}^k {}^{10}C_i {}^{15}C_{k-i} + \sum_{i=0}^{k+1} {}^{12}C_i {}^{13}C_{k+1-i}$$

$${}^{10}C_0 {}^{15}C_k + {}^{10}C_1 {}^{15}C_{k-1} + {}^{10}C_2 {}^{15}C_{k-2} + \dots + {}^{10}C_k {}^{15}C_0$$

Equating the coefficient of x^k in $(1+x)^{10}(x+1)^{15}$

= Equating the coefficient of x^k in $(1+x)^{25} = {}^{25}C_k \dots \dots \dots$ (i)

$$\sum_{i=0}^{k+1} {}^{12}C_i {}^{13}C_{k+1-i}$$

$$= {}^{12}C_0 {}^{13}C_{k+1} + {}^{12}C_1 {}^{13}C_k + {}^{12}C_2 {}^{13}C_{k-1} + \dots + {}^{12}C_{k+1} {}^{13}C_0$$

= Equating the coefficient of x^{k+1} in $(1+x)^{12}(x+1)^{13}$

= Equating the coefficient of x^{k+1} in $(1+x)^{25} = {}^{25}C_{k+1} \dots \dots \dots$ (ii)

From (i) and (ii) ${}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$

Hence $26 \geq k+1 \Rightarrow k \leq 25$

But maximum value of k is not defined bonus

Sol3.

$$\frac{2^{21} \left(\frac{-1+i\sqrt{3}}{2}\right)^{21}}{(\sqrt{2})^{24} \left(\frac{1-i}{\sqrt{2}}\right)^{24}} + \frac{2^{21} \left(\frac{1+\sqrt{3}i}{2}\right)^{21}}{(\sqrt{2})^{24} \left(\frac{1+i}{\sqrt{2}}\right)^{24}} = k$$

$$2^9 \frac{e^{14\pi i}}{e^{-i6\pi}} + 2^9 \frac{e^{i7\pi}}{e^{i6\pi}} = k$$

$$2^9 \left[\frac{1}{1} + \frac{-1}{1} \right] = k$$

$$0 = k$$

$$\therefore n = [|k|] = 0$$

$$\begin{aligned} & \sum_{j=0}^5 (j+5)^2 - \sum_{j=0}^5 (j+5) \\ &= \sum_{j=0}^5 (j+5)(j+5-1) = \sum_{j=0}^5 (j+5)(j+4) \\ &= \sum_{j=0}^5 (j^2 + 9j + 20) = \frac{5 \times 6 \times 11}{6} + \frac{9 \times 5 \times 6}{2} + 20 \times 6 \\ & 55 + 135 + 120 = 310 \end{aligned}$$

Sol4. $(x+1)^2 + |x-5| = \frac{27}{4}$

Case I $x < 5$

$$x^2 + 2x + 1 - x + 5 = \frac{27}{4}$$

$$x^2 + x + 6 = \frac{27}{4} \Rightarrow 4x^2 + 4x - 3 = 0$$

$$(2x+3)(2x-1) = 0 \Rightarrow x = -\frac{3}{2}, \frac{1}{2}$$

Case II $x \geq 5$

$$x^2 + 2x + 1 + x - 5 = \frac{27}{4}$$

$$x^2 + 3x - 4 = \frac{27}{4}$$

$$4x^2 + 12x - 43 = 0$$

$$x = \frac{-12 \pm \sqrt{144 + 43 \times 16}}{2 \times 4} = \frac{-12 \pm \sqrt{832}}{8} \text{ (rejected)}$$

Because $x > 5$

Sol5. Let $P(h,k)$

$$\sqrt{(h-5)^2 + k^2} = 3\sqrt{(h+5)^2 + k^2}$$

$$h^2 - 10h + 25 + k^2 = 9h^2 + 90h + 225 + 9k^2$$

$$8h^2 + 8k^2 + 100h + 200 = 0$$

$$x^2 + y^2 + \frac{25}{2}x + 25 = 0$$

$$r^2 = \frac{625}{16} - 25 = \frac{625 - 400}{16} = \frac{225}{16}$$

$$4r^2 = \frac{225}{4} = 56.25$$

Sol6. Let a, ar, ar^2, ar^3 are in G.P

$$a + ar + ar^2 + ar^3 = \frac{65}{12}$$

$$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$$

$$\frac{a(1-r^4)}{1-r} = \frac{65}{12} \times \frac{18}{65}$$

$$\frac{1 \left(1 - \frac{1}{r^4} \right)}{1 - \frac{1}{r}}$$

$$(ar)^2 r = \frac{3}{2} \Rightarrow a = \frac{2}{3}, r = \frac{3}{2}$$

$$\therefore \text{third term} = ar^2 = \frac{2}{3} \times \frac{9}{4} = \frac{3}{2}$$

$$\therefore \alpha = \frac{3}{2} \Rightarrow 2\alpha = 3$$

Sol7.

A	B	C
-	-	1
-	-	2
-	-	3

Number of groups = ${}^{10}C_1(2^9 - 2) = 5100$

Number of groups = ${}^{10}C_2(2^8 - 2) = 11430$

Number of groups = ${}^{10}C_3(2^7 - 2) = 15120$

Total number of groups = 31650

Sol8. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x} \dots\dots\dots(i)$

Replace x by $\frac{1}{x}$

$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x \dots\dots\dots(ii)$

$a\left(f(x) + f\left(\frac{1}{x}\right)\right) + \alpha\left(f(x) + f\left(\frac{1}{x}\right)\right) = b\left(x + \frac{1}{x}\right) + \beta\left(x + \frac{1}{x}\right)$

$(a + \alpha)\left(f(x) + f\left(\frac{1}{x}\right)\right) = (b + \beta)\left(x + \frac{1}{x}\right)$

$\therefore \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{b + \beta}{a + \alpha} = \frac{2}{1} = 2$

Sol9. $\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{9+k^2}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

$\Rightarrow k < \frac{10\sqrt{10}}{3} + 1 \Rightarrow k \leq 11$

\therefore maximum value of k is 11

Sol10. Equation of normal is $4x - 3y + 1 = 0$

Equation of tangent is $3x + 4y - 43 = 0$

Area of triangle = $\frac{1}{2} \left(\frac{43}{3} + \frac{1}{4} \right) \times 7$

= $\frac{1}{2} \times \left(\frac{172+3}{12} \right) \times 7 = \frac{1225}{24}$

$\therefore 24A = 1225$

