FIITJEE

Solutions to JEE (Main)-2021

JEE–Main–2021 –Feb–24–First–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

(PHYSICS)

Answers

Section-A

1.	D	**2.**	D	**3.**	C	**4.**	C
5.	D	**6.**	D	**7.**	A	**8.**	B
9.	A	**10.**	C	**11.**	B	**12.**	A
13.	D	**14.**	D	**15.**	B	**16.**	C
17.	C	**18.**	B	**19.**	B	**20.**	B

Section-B

1.	25	**2.**	25	**3.**	25600	**4.**	25
5.	2000	**6.**	1	**7.**	25	**8.**	75
9.	440	**10.**	15				

SECTION – A

Sol1. $\omega = \frac{2\pi}{T} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} = 8$

Sol2. $p = \frac{h}{\lambda}$

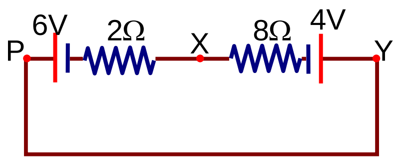
So, two photons having equal linear momenta have equal wavelength.
As wavelength decreases, momentum and energy of a photon increases.

Sol3.

$i = \frac{6-4}{2+8} = 0.2\,A$

$V_X + 4 + 0.2 \times 8 = V_Y$

$\Rightarrow V_Y - V_X = 5.6\,V$



Sol4. When connected in series, equivalent capacitance,

$C_1 = \frac{C \times C}{C + C} = \frac{C}{2}$

When connected in parallel, equivalent capacitance

$C_2 = C + C = 2C$

$\frac{C_1}{C_2} = \frac{C/2}{2C} = \frac{1}{4}$

Sol5. A : Series limit of Lyman series
B : Third line of Balmer series
C : Second line of Paschen series

Sol6. $\frac{dq}{dt} = i = 20t + 8t^2$

$\Rightarrow \int_0^q dq = \int_0^{15} \left(20t + 8t^2\right)dt.$

$\Rightarrow q = 20 \times \frac{15^2}{2} + 8.\frac{15^3}{3} = 11250\,C$

Sol7. Since, $\frac{x^2}{\alpha kT}$ should be dimensionless.

So, dimension of α, $[\alpha] = \frac{L^2}{ML^2T^{-2}} = M^{-1}T^2$

Dimension of $\alpha\beta^2$ should be that of W.

So, $[\alpha\beta^2] = ML^2T^{-2}$

$\Rightarrow [\beta^2] = \frac{ML^2T^{-2}}{M^{-1}T^2} = M^2L^2T^{-4} \Rightarrow [\beta] = MLT^{-2}$

Sol8. Velocity of block in equilibrium, in first case,

$v = A\omega = A.\sqrt{\frac{k}{M}}$

Velocity of block in equilibrium, is second case,

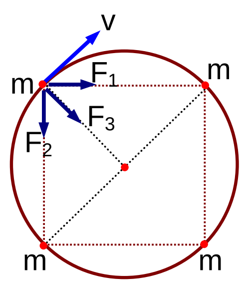
$v' = A'\omega' = A'\sqrt{\frac{k}{M+m}}$

From conservation of momentum,

$Mv = (M+m)\,v'$

$\Rightarrow MA\sqrt{\frac{k}{M}} = (M+m)\,A'\sqrt{\frac{k}{M+m}} \Rightarrow A' = A\sqrt{\frac{M}{M+m}}$

Sol9.



$F_1\cos45^\circ + F_2\cos45^\circ + F_3 = \frac{mv^2}{r}$

$\Rightarrow \frac{Gm^2}{\left(\sqrt{2}r\right)^2}.\frac{1}{\sqrt{2}} + \frac{Gm^2}{\left(\sqrt{2}r\right)^2}.\frac{1}{\sqrt{2}} + \frac{Gm^2}{(2r)^2} = \frac{mv^2}{r}$

$\Rightarrow v^2 = \frac{Gm}{r}\left(\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{1}{4}\right)$

$\Rightarrow v = \sqrt{\frac{Gm}{4r}\left(2\sqrt{2}+1\right)}$

Putting $m = 1\,kg$ and $r = 1\,m$,

$v = \frac{1}{2}\sqrt{G\,(1+2\sqrt{2})}$

Sol10. If charge (-Q) at origin is replaced by (+Q), then electric field at the centre of the cube is zero. Thus, electric field at the centre of the cube is as if only (-2Q) charge is present at the origin.

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{(-2Q)}{\left(\frac{\sqrt{3}}{2}a\right)^2}.\frac{1}{\sqrt{3}}\left(\hat{x} + \hat{y} + \hat{z}\right) = \frac{-2Q}{3\sqrt{3}\,\pi\varepsilon_0 a^2}\left(\hat{x} + \hat{y} + \hat{z}\right)$

Sol11. $\gamma = 3\alpha$

$\Delta V = \gamma V \Delta T = 3\alpha . a^3 . \Delta T$

Sol12. For part AM, slope of v – t graph is constant but negative. For part MB, slope of v – t graph is constant but positive.

Sol13. In isothermal process, temperature is constant.
In isochoric process, volume is constant.
In adiabatic process, there is no exchange of heat.
In isobaric process, pressure is constant.

Sol14. Focus of a spherical convex mirror is in the same side of centre of curvature. Thus, $f = +\frac{1}{2} r$.

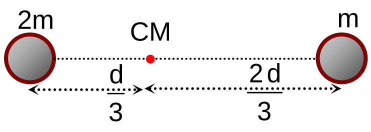
Sol15. $\alpha = \frac{\Delta I_C}{\Delta I_E} = \frac{3.5}{4} = \frac{7}{8}$

$\beta = \frac{\alpha}{1-\alpha} = \frac{7/8}{1-7/8} = 7$

Sol16.

$m\omega^2 . \frac{2d}{3} = \frac{G.m.2m}{d^2} \Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{3Gm}}$



Sol17. $I_1 = \frac{1}{2} mr^2$

$I_2 = \frac{1}{2} mr^2$

$I_3 = \frac{1}{2} mr^2$

$I_4 = \frac{2}{5} mr^2$.

Sol18. $W_{AB} = nRT \ln\frac{2V_1}{V_1} = nRT \ln 2$.

$W_{BC} = P_2(V_1 - 2V_1) = -P_2 V_1 = -\frac{1}{2} nRT$

$[\text{At B}, 2P_2 V_1 = nRT]$

$W_{CA} = 0$ [CA is isochoric process).

$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = nRT\left(\ell n(2) - \frac{1}{2}\right)$

Sol19. If μ is Poisson's ratio,

Y = 3K (1- 2μ) …………(1)

and Y = 2η(1+μ)………. (2)

With the help of equations (1) and (2) , we can write

$\frac{3}{Y} = \frac{1}{\eta} + \frac{1}{3k} \Rightarrow K = \frac{\eta Y}{9\eta - 3Y}$

Sol20. Amplitude is proportional to the slit – width, thus,

$$\frac{A_1}{A_2} = 3$$

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\frac{A_1}{A_2} + 1\right)^2}{\left(\frac{A_1}{A_2} - 1\right)^2} = \frac{(3+1)^2}{(3-1)^2} = 4.$$

Section – B

Sol1. Potential difference across $2k\Omega$ is 5V, thus current through it,

$$i = \frac{5}{2\times 10^3} = 25\times 10^{-4}\,A.$$

Sol2. Percentage Modulation = $\frac{A_m}{A_c} \times 100 = \frac{20}{80} \times 100 = 25\%$

Sol3. Ratio of masses on two pistons of the hydraulic lift equals to that of their cross – section area.

$$\frac{A_1}{A_2} = \frac{100}{m}$$

Now, $\frac{4^2 A_1}{A_2 / 4^2} = \frac{M}{m} \Rightarrow \quad M = 256\frac{A_1}{A_2}.\, m = 25600\,kg.$

Sol4.

$f = W = 0.5\times 10 = 5\ N$

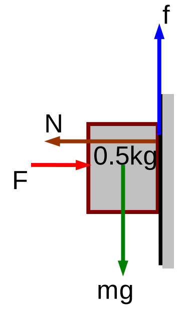
N = F

For block not to slide,

$f \le \mu N$

$\Rightarrow 5 \le 0.2F$

$\Rightarrow \ F \ge 25N$



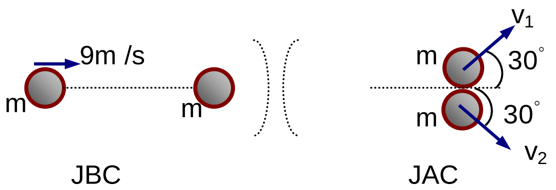
Sol5. Quality factor $= \frac{\omega L}{R} = \frac{2\times 3.14\times 10\times 10^6 \times 2\times 10^{-4}}{6.28} = 2000$

Sol6.

Using conservation of momentum in direction perpendicular to the original direction of motion,

$mv_1 \sin 30^\circ = mv_2 \sin 30^\circ$

$\Rightarrow \ \frac{v_1}{v_2} = 1$

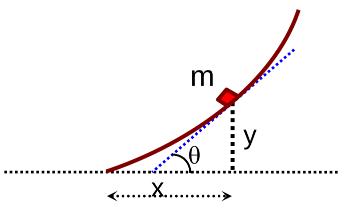


Sol7.

If the block does not slide,

$mg \sin\theta \le \mu\, mg \cos\theta$

$\Rightarrow \ \tan\theta \le \mu \Rightarrow \frac{dy}{dx} \le \mu \Rightarrow \ \frac{x}{2} \le 0.5 \Rightarrow \quad x \le 1\,m.$



Thus, $y \le \frac{1^2}{4} = 0.25\text{ m} = 25\text{cm}.$

Sol8. $I = I_0\cos^2\theta = 100 \times \cos^2\left(30^0\right) = 75\text{ Lumens}$

Note: As the incident light is unpolarized, intensity of emerging light does not depend on the polarization axis of polarizer.

Sol9. $\frac{N_p}{N_s} = \frac{V_p}{V_s} \Rightarrow N_p = \frac{V_p}{V_s} \times N_s = \frac{220}{12} \times 24 = 440$

Sol10. $v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{2\varepsilon_0 . 2\mu_0}} = \frac{1}{2\sqrt{\varepsilon_0\,\mu_0}} = \frac{c}{2} = 15 \times 10^7\text{ m/s}.$

PART –B (CHEMISTRY)

Answers

Section-A

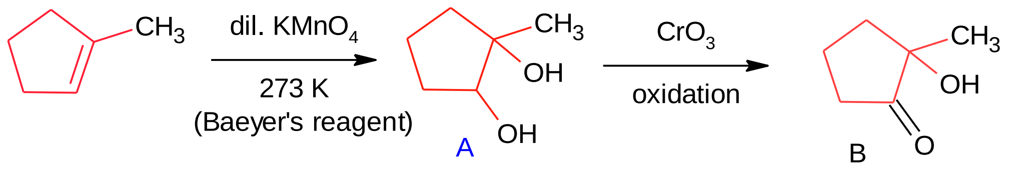
1.	A	**2.**	A	**3.**	B	**4.**	C
5.	C	**6.**	C	**7.**	B	**8.**	A
9.	A	**10.**	D	**11.**	D	**12.**	C
13.	B	**14.**	D	**15.**	A	**16.**	B
17.	A	**18.**	B	**19.**	A	**20.**	B

Section-B

1.	2	**2.**	1	**3.**	1380	**4.**	5
5.	36	**6.**	12	**7.**	2	**8.**	2
9.	8	**10.**	26				

SECTION – A

Sol1.

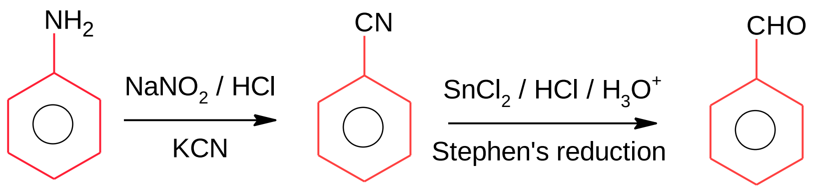


Sol2. Cu is the only element of 3d – series whose M^{2+} / M value is positive because of fact that low hydration enthalpy and high sublimation & ionization enthalpies.

Sol3. Al < Mg < Si < S < P
1st I. E increases along period with exception on moving from group 2 to group 13 and group 15 to group 16

Sol4. Blue cupric metaborate is reduced to cuprous metaborate in a luminous flame.
$2Cu(BO_2)_2 + 2NaBO_2 + C \xrightarrow{\text{Luminous}} 2CuBO_2 + Na_2B_4O_7 + CO$
Cupric metaborate is obtained by heating boric anhydride & copper sulphate in a non- luminous flame as
$CuSO_4 + B_2O_3 \xrightarrow{\text{Non-LuminiousFlame}} \underset{\text{Blue}}{Cu(BO_2)_2} + SO_3$.

Sol5.



Sol6. Major components of gun metal are Cu, Sn and Zn

Sol7. Sphalerite is ore of zinc consists of ZnS during its concentration group 1 cyanides are used as depressants like NaCN

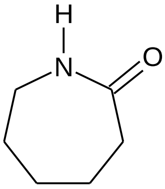
Sol8. Freundlich adsorption isotherm

$$\frac{x}{m} = kp^{1/n}$$

$$\log\frac{x}{m} = \frac{1}{n}\log p + \log k$$

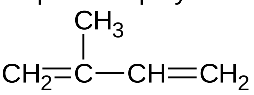
Comparing with
y = mx + C
so, slope of given graph is $\frac{1}{n}$ whose value varies from 0 to 1.

Sol9. Caprolactum is monomer of nylon-6



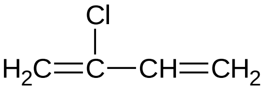
Caprolactum

Isoprene is polymer of natural rubber



Isoprene

2- chloro-1,3- butadiene or chloroprene is the polymer of neoprene which is a synthetic rubber

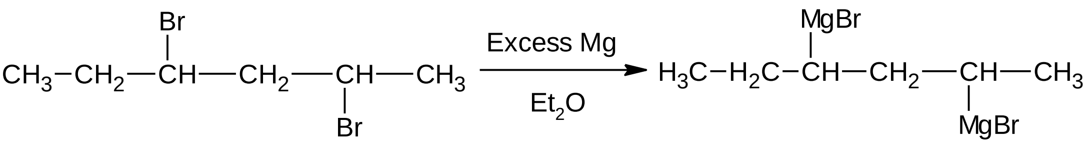


Chloroprene

Acrylonitrile is one of the monomer of Buna-N
$CH_2{=}CH{-}CN$ Acrylonitrile

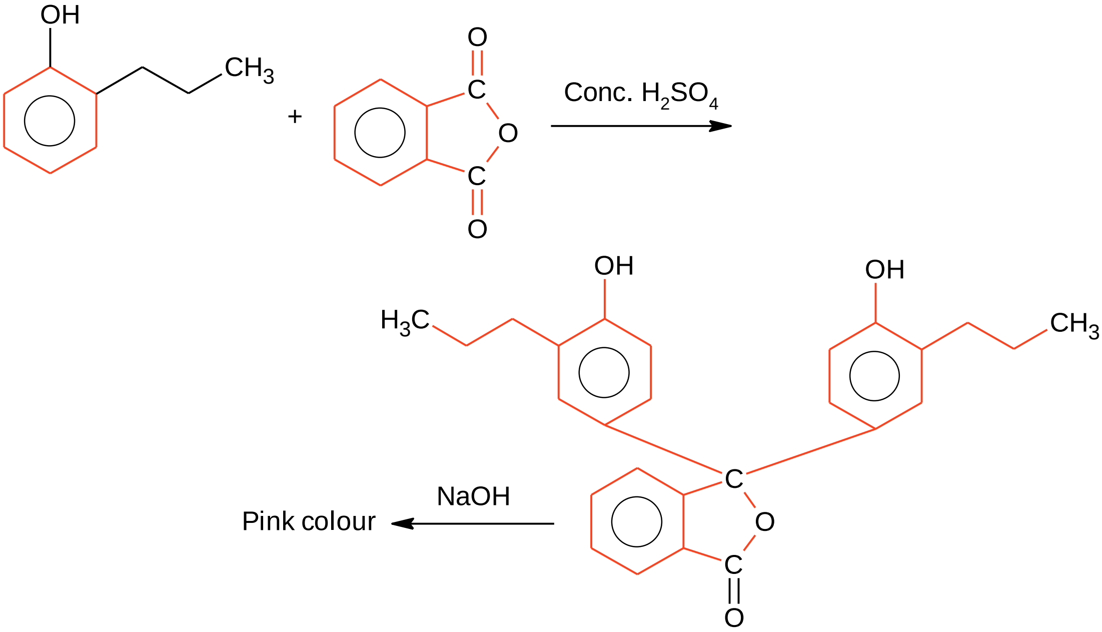
Sol10. In both reactions H_2O_2 act as reducing agent
In reaction (A) HOCl – O. A
H_2O_2 – R.A
In reaction (B) I_2 – O.A
H_2O_2 – R.A

Sol11.



Grignard reagent

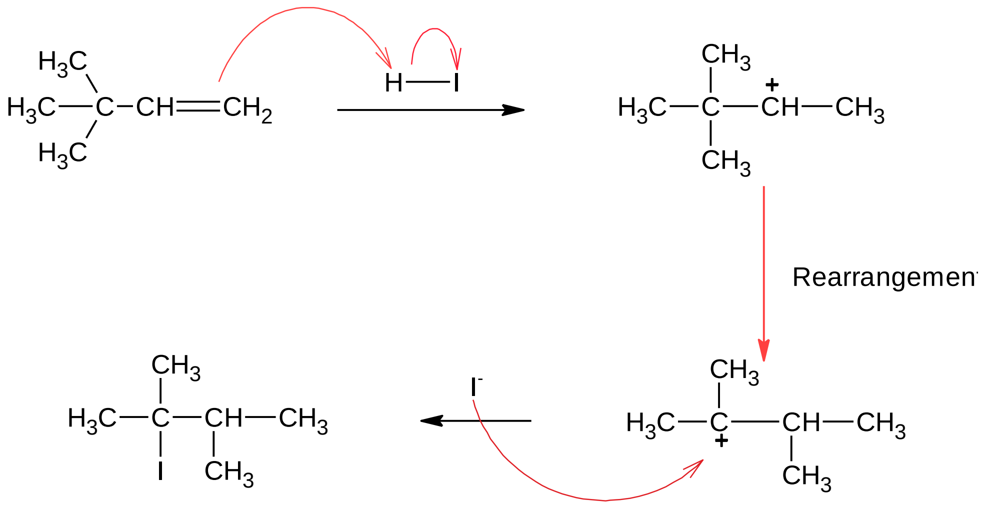
Sol12.



Phenolphthalein deriavtive

First reaction is EAS which will be given by that phenol derivative whose p- position is free for EAS reaction.

Sol13.

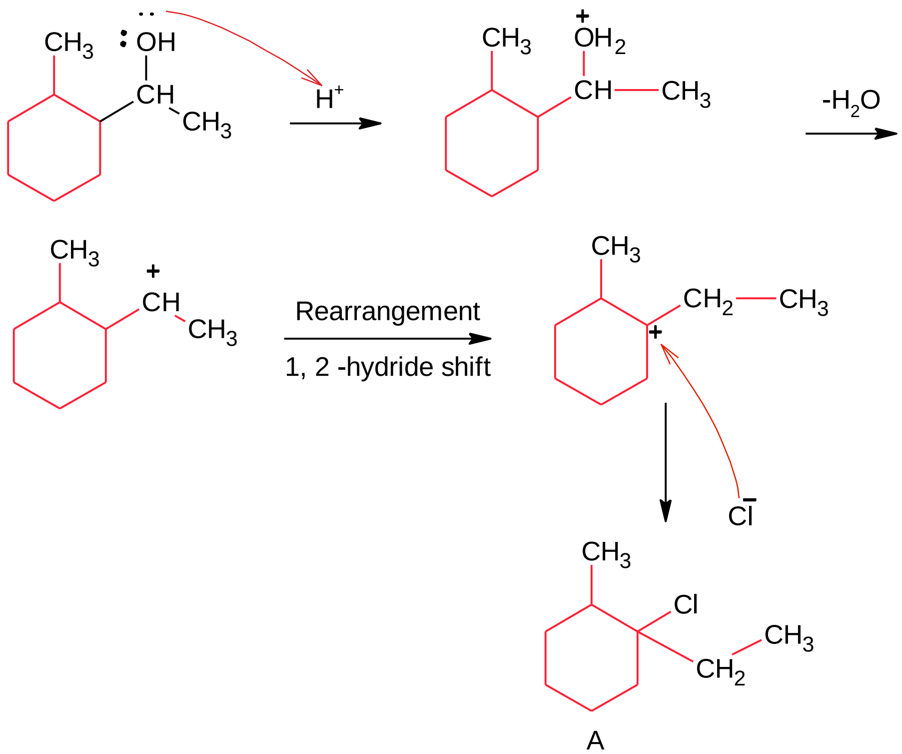


Sol14. Leaching of Al_2O_3 with alkali;

$Al_2O_3(s) + 2NaOH(aq) + 3H_2O(l) \rightarrow \underset{X}{2Na[Al(OH)_4](aq)}$

$\underset{X}{2Na[Al(OH)_4](aq)} + \underset{Y}{2CO_2(g)} \rightarrow \underset{Z}{Al_2O_3 \cdot xH_2O(s)} + 2NaHCO_3(aq)$

Sol15.

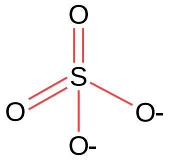


Sol16. Molybdenum oxide is used to convert alkane to aldehyde i.e propane to propanal

Sol17. During nitration of aniline, meta product is also formed, it is because of due to presence of anilinium ion. In anilinium $-\overset{+}{N}H_3$ group is meta directing group.

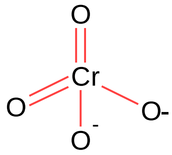
Sol18.

A. SO_4^{2-}



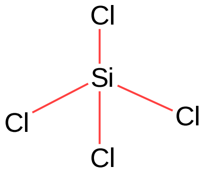
Tetrahedral

CrO_4^{2-}



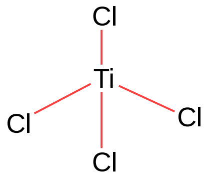
Tetrahedral

B $SiCl_4$



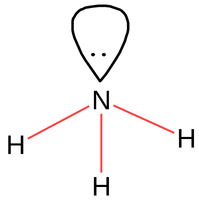
Tetrahedral

$TiCl_4$



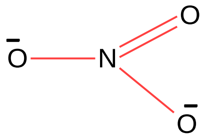
Tetrahedral

C. NH_3



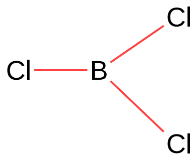
Pyramidal

NO_3^-



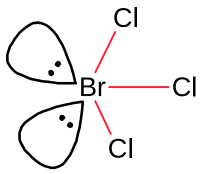
Trigonal planar

D. BCl_3



Trigonal planar

$BrCl_3$



Bent - T

Pair A & B are isostructural

Sol19. For stabilization of α - helix structure of protein, H- bonding is responsible which is in between $-\overset{\overset{\displaystyle H}{|}}{N}-$ and $-\overset{\overset{\displaystyle O}{\|}}{C}-$ group present in protein.

Sol20. Methane gas is produced during anaerobic degradation of vegetation that leads to global warming so considered as greenhouse gas like CO_2. Also it causes cancer.

SECTION-B

Sol1. BeO & $Be(OH)_2$ are amphoteric in nature, because they react with both acid and base

Sol2. Stability constant are:
$K_1 = 10^4$
$K_2 = 1.58 \times 10^3$

$K_3 = 5 \times 10^2$
$K_4 = 10^2$
Overall stability constant K will be
$K = K_1 \times K_2 \times K_3 \times K_4$
$= 7.9 \times 10^{11}$
Now, overall equilibrium constant for dissociation of $[Cu(NH_3)_4]^{2+}$ is

$$K' = \frac{1}{K} = \frac{1}{7.9 \times 10^{11}} = 0.126 \times 10^{-11}$$

$= 1.26 \times 10^{-12}$
So; x=1 (Rounded off to nearest integer)

Sol3. $A(g) \rightarrow B(g) \quad K_p = 100$
ΔG at 300 K and 1 atm
Using
$\Delta G = -RT \ln K_p$
$\Delta G = - R \times 300 \ln 100$
$= - R \times 300 \times 2 \times 2.3$
$\Delta G = -1380R$
So; x = 1380.

Sol4.

	$Cl_{2(g)}$	$\rightleftharpoons$	$2Cl_{(g)}$
Initially $\rightarrow$	1 mol		-
At eq.	1-x mol		2x mol

Here ; molecules of Cl_2 = atoms of Cl
i.e moles of Cl_2 = moles of Cl
So: 1-x=2x x= 1/3
Moles of Cl_2 at equilibrium = $\frac{2}{3}$
Moles of Cl at equilibrium = $\frac{2}{3}$
Total moles = $\frac{4}{3}$

Now: $P_{Cl_2} = \frac{2}{4} \times 1$ atm
$= \frac{1}{2}$ atm
$P_{Cl} = \frac{2}{4} \times 1\text{atm} = \frac{1}{2}$ atm

$$K_P = \frac{(P_{Cl})^2}{P_{Cl_2}} = \frac{\left(\frac{1}{2}\right)^2}{\frac{1}{2}} = \frac{1}{2}$$

$K_P = 0.5 = 5 \times 10^{-1}$
Here : x =5

Sol5. $\Delta T_f = 0.5^oC$
$K_f = 1.86$
Using, density of water = 1g / mL
i = ?
$\Delta T_f = ik_f.\ m$

$$0.5 = i \times 1.86 \times \frac{9.45}{94.5 \times 500} \times 1000$$

i = 1.344
Now, using $\alpha = \frac{i-1}{n-1}$
n for $ClCH_2\ COOH$ = 2

$$\alpha = \frac{1.344 - 1}{2-1} = 0.344$$

Using

$K_a = \frac{C\alpha^2}{1-\alpha}$

$K_a = \frac{0.2\times(0.344)^2}{0.66} = 36\times10^{-3}$

So; x= 36

Sol6. The balanced equation is:

$S_8(s) + 12OH^-(aq) \longrightarrow 4S^{2-}(aq) + 2S_2O_3^{2-}(aq) + 6H_2O(\ell)$

Here; a = 12

Sol7. W = 4.5g

M.W = 90

V = 250ml

Using $M = \frac{W}{M.W\times V}\times1000$

$M = \frac{4.5}{90\times250}\times1000 = 0.2$

M = 2×10^{-1}M

So, x= 2

Sol8. using $\lambda = \frac{h}{\sqrt{2qVm}}$

$\lambda_{Li} = \frac{h}{\sqrt{2\times3e\times V\times8.3m}}$ m = mass of proton

$\lambda_p = \frac{h}{\sqrt{2\times e\times V\times m}}$

$\frac{\lambda_{Li}}{\lambda_p} = \sqrt{\frac{2eVm}{2\times24.9eVm}}$

$= 0.2 = 2\times10^{-1}$

So ; x = 2

Sol9. Coordination no. of an atom in BCC is 8.

Sol10. K = $3.3\times10^{-4}\ s^{-1}$

Time for 40 % completion ; t

Using $K = \frac{2.303}{t}\log_{10}\frac{[R]_0}{[R]}$

$3.3\times10^{-4} = \frac{2.303}{t}\log_{10}\frac{[R]_0}{0.6[R]_0}$

$t = \frac{2.303}{3.3}\times10^4\times0.22 \Rightarrow t=25.58$ mins

so; nearest integer is 26

PART–C (MATHEMATICS)

Answers

Section-A

1.	**B**	**2.**	**D**	**3.**	**A**	**4.**	**B**
5.	**C**	**6.**	**C**	**7.**	**B**	**8.**	**D**
9.	**D**	**10.**	**D**	**11.**	**C**	**12.**	**B**
13.	**D**	**14.**	**D**	**15.**	**C**	**16.**	**B**
17.	**A**	**18.**	**B**	**19.**	**C**	**20.**	**A**

Section-B

1.	**1**	**2.**	**10**	**3.**	**540**	**4.**	**9**
5.	**5**	**6.**	**3**	**7.**	**6**	**8.**	**75**
9.	**17**	**10.**	3				

SECTION – A

Sol1.

	I(6)	F(8)
Case I	2	4
Case II	3	6
Case III	4	8

Total $= {}^6C_2 \times {}^8C_4 + {}^6C_3 \times {}^8C_6 + {}^6C_4 \times {}^8C_8 = 15\times70+20\times28+15\times1=1625$

Sol2. Let total number of throws = n

Probability of getting 2 times = Probability of getting an even number 3 times.

$${}^nC_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2} = {}^nC_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{n-3}$$

[as probability of getting odd number = probability of getting even number = $\frac{1}{2}$]

${}^nC_2 = {}^nC_3 \Rightarrow n=5$

$\therefore$ Probability of getting an odd number for odd number of times =

$${}^5C_1\left(\frac{1}{2}\right)^5+{}^5C_3\left(\frac{1}{2}\right)^5+{}^5C_5\left(\frac{1}{2}\right)^5=\frac{2^4}{2^5}=\frac{1}{2}$$

Sol3.

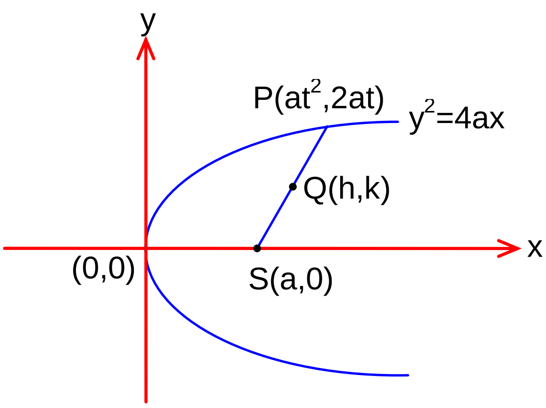
$h=\frac{a(1+t^2)}{2}$.........(i)

$k=at$............(ii)

From (i) & (ii) $\frac{2h}{a}=1+\frac{k^2}{a^2}$

$\therefore$ required locus of Q is $y^2=2a(x-a/2)$

Equation of directrix $x-a/2=-a/2\Rightarrow x=0$



Sol4. $f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$

$f'(x)=2x^2-x-2\cos x+2\cos x-(2x-1)\sin x$

$= (2x-1)(x-\sin x) \geq 0$ for $x \geq 0$

$f'(x) \geq 0 \ \forall x \geq 1/2$

$\therefore$ f(x) is increasing in $\left[\frac{1}{2}, \infty\right)$

Sol5. $\frac{dp}{dt} = 0.5p - 450$ and $P(0) = 850$

$\frac{dp}{dt} = 0.5(P-900)$

$\Rightarrow \frac{dp}{P-900} = 0.5dt$

$\int_{850}^{0} \frac{dp}{P-900} = \int_{0}^{T} 0.5dt$

$\Rightarrow \ln(P-900)\Big|_{850}^{0} = 0.5T$

$\frac{T}{2} = \ln\left|\frac{900}{50}\right| = \ln 18$

$T = 2\ln 18$

Sol6. $y = x^3$

$\frac{dy}{dx} = 3x^2 \Rightarrow \left.\frac{dy}{dx}\right|_{(t,t^3)} = 3t^2$

Equation of tangent $y - t^3 = 3t^2(x-t)$

Let again meet the curve at $Q(t_1, t_1^3)$

$\Rightarrow t_1^3 - t^3 = 3t^2(t_1 - t)$

$t_1^2 + tt_1 + t^2 = 3t^2$ $\quad [\because t_1 \neq t]$

$t_1^2 + tt_1 - 2t^2 = 0$

$\Rightarrow t_1 = -2t$

Required ordinate = $\frac{2t^3 + t_1^3}{3} = \frac{2t^3 - 8t^3}{3} = -2t^3$

Sol7. $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 3 & -2 & -k \\ 0 & -8 & 0 \\ 1 & 2 & -1 \end{vmatrix}, [R_2 - R_1 - 2R_3]$

$= -8(-3+k)$

For inconsistent $\Delta = 0 \Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & -2 & -k \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} = 32 - 40m \neq 0 \Rightarrow m \neq \frac{4}{5}$

Sol8.

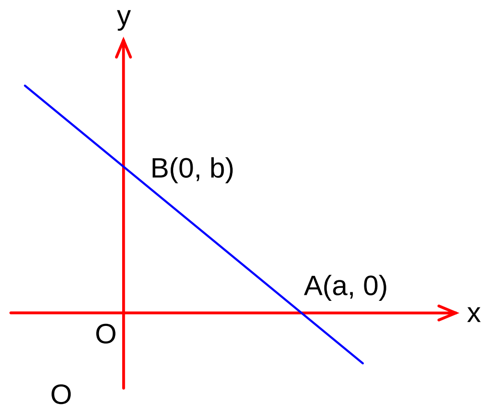
According to question,

$$\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{1}{4}$$

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{2}\text{.......(i)}$$

Equation of required line is $\frac{x}{a}+\frac{y}{b}=1$

Obviously B(2, 2) satisfying condition (i)



Sol9. $\int \frac{(\cos x-\sin x)dx}{\sqrt{8-\sin 2x}}=a\sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+c$

$$\int \frac{(\cos x-\sin x)dx}{\sqrt{9-(\sin x+\cos x)^2}}=$$

Put $(\sin x+\cos x)=t \Rightarrow (\cos x-\sin x)dx=dt$

$$=\int \frac{dt}{\sqrt{3^2-t^2}}=\sin^{-1}\frac{t}{3}+c=\sin^{-1}\left(\frac{\sin x+\cos x}{3}\right)+c$$

$=a=1, b=3$

$\therefore (a,b)\equiv(1,3)$

Sol10.

A	B	$A\vee B$	$A\wedge B$	$A\wedge(A\vee B)$	$A\vee(A\wedge B)$	$A\to B$	$A\wedge(A\to B)$	$A\wedge(A\to B)\to B$
T	T	T	T	T	T	T	T	T
T	F	T	F	T	T	F	F	T
F	T	T	F	F	F	T	F	T
F	F	F	F	F	F	T	F	T

$A\wedge(A\to B)\to B$ is a tautology

Sol11. $p+q=2$

$p^4+q^4=272$

$(p^2+q^2)^2-2p^2q^2=272$

$\left[(p+q)^2-2pq\right]^2-2p^2q^2=272$

Let pq = t $\Rightarrow (4-2t)^2-2t^2=272$

$2t^2-16t-256=0$

$\Rightarrow t^2-8t-128=0$

$\Rightarrow (t-16)(t+8)=0$

$\Rightarrow t=pq=16$

Required equation $x^2-2x+16=0$

Sol12. $\lim\limits_{x\to 0}\frac{\int_0^{x^2}\left(\sin\sqrt{t}\right)dt}{x^3}$ $\left(\frac{0}{0}\right)$ applying L'Hospital rule

$$= \lim_{x\to 0} \frac{\sin\left(\sqrt{x^2}\right).2x}{3x^2}$$

$$= \lim_{x\to 0} \frac{2\sin|x|}{3x}$$

$$= \frac{2}{3}, \text{ for } x > 0$$

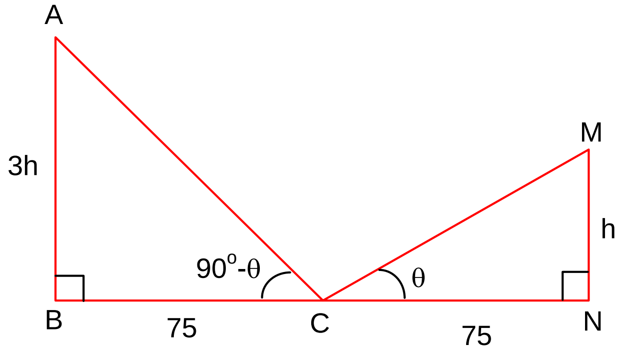
Sol13.

$BC = CN = x = 75$

$\cot\theta = \frac{3h}{75}$

$\tan\theta = \frac{h}{75}$

$\frac{3h^2}{75\times 75} = 1 \Rightarrow h = \frac{75}{\sqrt{3}} = 25\sqrt{3}$



Sol14. $f(x) = 2x-1 \quad g: R-\{1\} \to R$

$g(x) = \frac{x-1/2}{x-1}$

$f\{g(x)\} = 2\left(\frac{x-1/2}{x-1}\right) - 1 = \frac{2x-1-x+1}{x-1} = \frac{x}{x-1}, x \neq 1$

$y = \frac{x}{x-1} \Rightarrow x = \frac{y}{y-1} \quad \therefore y \neq 1$

one – one but not onto

Sol15. $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$

If $x = k, \quad k \in I$

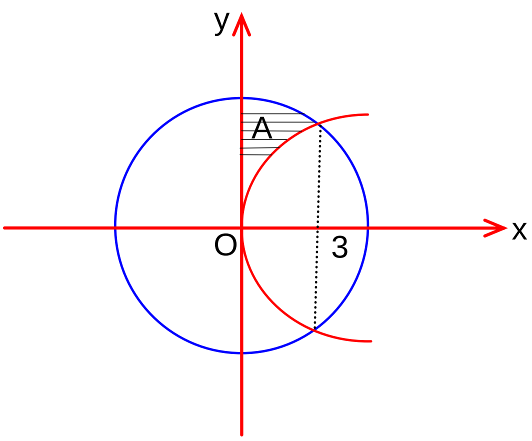
then $f(k) = 0$ as $\cos\left(\frac{2k-1}{2}\right)\pi = 0, \forall k \in I$

LHL $= \underset{h\to 0}{\text{Lt}}\,[k-h-1]\cos\left(\frac{2k-2h-1}{2}\right)\pi = \underset{h\to 0}{\text{Lt}}\,(k-2)\cos\left(\frac{2k-1}{2}\right)\pi \to 0$

RHL $= \underset{h\to 0}{\text{Lt}}\,[k+h-1]\cos\left(\frac{2k+2h-1}{2}\right)\pi = \underset{h\to 0}{\text{Lt}}\,(k-1)\cos\left(\frac{2k-1}{2}\right)\pi \to 0$

$\therefore f(x)$ is continuous $\forall x \in R$

Sol16.



$x^2+y^2=36$...........(i)

$y^2=9x$...........(ii)

Solving (i) & (ii),

$x^2+9x-36=0$

$(x+12)(x-3)=0$

$x=3$

Let $A=\int_0^3\left(\sqrt{36-x^2}-3\sqrt{x}\right)dx$

$=\left[\frac{x\sqrt{36-x^2}}{2}+18\sin^{-1}\frac{x}{6}\right]_0^3-3.\left[\frac{x^{3/2}}{3/2}\right]_0^3$

$=\frac{3\times3\sqrt{3}}{2}+18\times\frac{\pi}{6}-2\times3\sqrt{3}=3\pi-\frac{3\sqrt{3}}{2}$

$\therefore$ Required area =

$=\frac{1}{2}\pi(6)^2+2\left(3\pi-\frac{3\sqrt{3}}{2}\right)=\left(24\pi-3\sqrt{3}\right)$ sq. unit

Sol17. Any point on line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=r$ is $P(r+3,2r+4,2r+5)$ lies on $x+y+z=17$, $5r+12=17$

r = 1

$P(4,6,7)$ $A(1,1,9)$ dist $AP=\sqrt{9+25+4}=\sqrt{38}$

Sol18. $e^{\left(\cos^2x+\cos^4x+\cos^6x+......\infty\right)\log_e 2}$

$=e^{\frac{\cos^2x}{1-\cos^2x}\log_e 2}=e^{\cot^2x\log_e 2}=2^{\cot^2x}$

$t^2-9t+8=0$

$(t-8)(t-1)=0$

$t=2^{\cot^2x}=8=2^3$

$\Rightarrow\cot^2x=3=\cot^2\frac{\pi}{6}$

$\Rightarrow x=n\pi\pm\frac{\pi}{6},\ 0<x<\frac{\pi}{2}\Rightarrow x=\frac{\pi}{6}$

$\frac{2\sin x}{\sin x+\sqrt{3}\cos x}=\frac{2\times\frac{1}{2}}{\frac{1}{2}+\sqrt{3}\times\frac{\sqrt{3}}{2}}=\frac{1}{2}$

Sol19. Let direction ratio of the normal to the required plane are l,m,n

$\therefore 3l+m-2n=0$

$\therefore 2l-5m-n=0$

$\therefore\frac{l}{-11}=\frac{m}{-1}=\frac{n}{-17}$

Equation of required plane

$11(x-1)+1(y-2)+17(z+3)=0$

$\Rightarrow 11x+y+17z+38=0$

Sol20. $\left(-{}^{15}C_1+2\ {}^{15}C_2-3\ {}^{15}C_3+.....-15\ {}^{15}C_{15}\right)+\left({}^{14}C_1+{}^{14}C_3+.....+{}^{14}C_{11}\right)$

$=A+B(\text{say})$

$(1+x)^{15} = \sum {}^{15}C_r x^r$

differentiating $15(1+x)^{14} = \Sigma\, r.\, {}^{15}C_r x^{r-1}$

put $= x = -1$

$\Rightarrow 0 = \Sigma(-1)^{r-1} r. {}^{15}C_r$

$0 = {}^{15}C_1 - 2\,{}^{15}C_2 + ...$

$\Rightarrow 0 = -{}^{15}C_1 + 2.{}^{15}C_2 - .... = A$

${}^{14}C_1 + {}^{14}C_3 + ... + {}^{14}C_{11} + {}^{14}C_{13} = 2^{13}$

$B + {}^{14}C_{13} = 2^{13}$

$B = 2^{13} - {}^{14}C_{13} = 2^{13} - 14$

$\therefore A + B = 2^{13} - 14$

Section B

Sol1. $\sum_{r=1}^{n} \tan^{-1}\frac{1}{1+r+r^2} = \sum_{r=1}^{n} \tan^{-1}\frac{(r+1)-r}{1+r(r+1)}$

$= \sum_{r=1}^{n}\left[\tan^{-1}(r+1) - \tan^{-1} r\right]$

$= \tan^{-1}2 - \tan^{-1} + ... + \tan^{-1}(n+1) - \tan^{-1} n$

$= \tan^{-1}(n+1) - \tan^{-1}(1)$

For $n \to \infty$ value $= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$\therefore \tan\left(\frac{\pi}{4}\right) = 1$

Sol2. $z + \alpha|z-1| + 2i = 0$

$z + 2i = -\alpha|z-1|$

Let $z = x + iy$

$\Rightarrow x + i(y+2) = -\alpha\sqrt{(x-1)^2 + y^2}$

for $\alpha \in R$ $y + 2 = 0 \Rightarrow y = -2$

$x^2 = \alpha^2\left[(x-1)^2 + 4\right]$

$x^2 = \alpha^2\left(x^2 - 2x + 5\right)$

$\frac{x^2}{\alpha^2} = x^2 - 2x + 5$

$x^2\left(\frac{1}{\alpha^2} - 1\right) + 2x - 5 = 0$

$x \in R \Rightarrow D = 4 + 20\left(\frac{1}{\alpha^2} - 1\right) \geq 0$

$1 + 5\left(\frac{1}{\alpha^2} - 1\right) \geq 0$

$\frac{1}{\alpha^2} \geq \frac{4}{5} \Rightarrow \alpha^2 \leq \frac{5}{4} \Rightarrow \frac{-\sqrt{5}}{2} \leq \alpha \leq \frac{\sqrt{5}}{2}$

$4\left(p^2 + q^2\right) = 4\left(\frac{5}{4} + \frac{5}{4}\right) = 10$

Sol3. $M = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & c_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$M^T M = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$\text{Tr}\left(M^T M\right) = a_1^2 + b_1^2 + c_1^2 + a_2^2 + b_2^2 + c_2^2 + a_3^2 + b_3^2 + c_3^2 = 7$

all $a_i, b_i, c_i \in \{0,1,2\}$ for $i = 1,2,3$

Case 1 7 one's and two zeros which can occur in $^9C_7 = {}^9C_2 = 36$ ways

Case 2 One 2 three 1's five zeros $= {}^9C_1 \times {}^8C_3 \times {}^5C_5 = 9 \times \frac{8 \times 7 \times 6}{6} = 9 \times 56 = 504$

$\therefore$ total such matrices = 504 + 36 = 540

Sol4. $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = a \forall x \in \left(0, \frac{\pi}{2}\right)$

Let $\sin x = t, t \in (0, 1)$

$g(t) = \frac{4}{t} + \frac{1}{1-t}$

$g'(t) = -\frac{4}{t^2} + \frac{1}{(1-t)^2}$

$g''(t) = \frac{8}{t^3} + \frac{1}{(1-t)^3}$

$g'(t) = 0 \Rightarrow t = \frac{2}{3}$

$g''\left(\frac{2}{3}\right) > 0$

$\therefore g(t)_{\text{minimum}} = \frac{4}{2/3} + \frac{1}{1 - 2/3} = 9$

Minimum value of α for which solution exist = 9

Sol5. Sum of elements $A \cap (B \cup C) = 274 \times 400$

In set B numbers of the form 9k + 2 are $\{101, 109, ...992\}$

$\therefore \text{sum} = \frac{100}{2}(101 + 992) = \frac{100 \times 1093}{2}$..........(i)

Another possible number is 9k + 5 forms are $\{104, ...995\}$

$\therefore \text{sum} = \frac{100}{2}(104 + 995) = \frac{100}{2} \times 1099$........(ii)

$\therefore \text{Total} = \frac{100}{2} \times [1093 + 1099] = 100 \times 1096 = 274 \times 4 \times 100 = 274 \times 400$

$\therefore$ possible value of $\ell = 5$

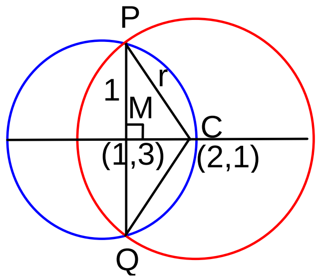
Sol6.

$x^2 + y^2 - 2x - 6y + 6 = 0$ centre (1, 3)

$r = \sqrt{1+9-6} = 2$

$CM = \sqrt{1+4} = \sqrt{5}$

$r = \sqrt{5+4} = 3$



Sol7. Let $P(B_1) = a \qquad P(B_2) = b \qquad P(B_3) = c$

Given $a(1-b)(1-c) = \alpha$..........(i)

$b(1-a)(1-c) = \beta$..........(ii)

$c(1-b)(1-a) = \gamma$..........(iii)

$(1-a)(1-b)(1-c) = p$..........(iv)

$(\alpha - 2\beta)p = \alpha\beta$

$\Rightarrow \left[a(1-b)(1-c) - 2b(1-a)(1-c)\right]p = ab(1-a)(1-b)(1-c)^2$

$\Rightarrow (1-c)^2(1-a)(1-b)\left[a(1-b) - 2b(1-a)\right] = ab(1-a)(1-b)(1-c)^2$

$\Rightarrow a - ab - 2b + 2ab = ab \Rightarrow a = 2b$........(v)

Again $(\beta - 3\gamma)p = 2\beta\gamma$

$\Rightarrow (1-a)^2(1-b)(1-c)\left[b(1-c) - 3c(1-b)\right] = 2bc(1-a)^2(1-b)(1-c)$

$\Rightarrow b - bc - 3c + 3bc = 2bc \Rightarrow b = 3c$.......(vi)

$$\Rightarrow \frac{P(B_1)}{P(B_3)} = \frac{a}{c} = \frac{2b}{b/3} = 6$$

Sol8. $\vec{c} = \alpha\vec{a} + \beta\vec{b}$......(i)

$\vec{a}\,.\,\vec{c} = 7 \qquad \vec{b}\,.\,\vec{c} = 0$

$\vec{a} = -\hat{i} + \hat{j} + \hat{k} \Rightarrow |\vec{a}| = \sqrt{3}$

$\vec{b} = 2\hat{i} + \hat{k} \Rightarrow |\vec{b}| = \sqrt{5} \qquad \vec{a}\,.\,\vec{b} = -2 + 1 = -1$

From (i) $\vec{a}\,.\,\vec{c} = \alpha|\vec{a}|^2 - \beta$

$3\alpha - \beta = 7$.........(ii)

$\bar{b}\,.\,\bar{c} = \alpha\bar{b}.\bar{a} + \beta|\vec{b}|^2 \Rightarrow -\alpha + 5\beta = 0$.........(iii)

Solving $\alpha = \frac{5}{2}$ and $\beta = \frac{1}{2}$

$\Rightarrow \vec{c} = \frac{5}{2}\left(-\hat{i} + \hat{j} + \hat{k}\right) + \frac{1}{2}\left(2\hat{i} + \hat{k}\right) = \frac{-3}{2}\hat{i} + \frac{5}{2}\hat{j} + 3\hat{k}$

$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\left(\vec{a}\,.\,\vec{b} + \vec{b}\,.\,\vec{c} + \vec{c}\,.\,\vec{a}\right) = \frac{75}{2}$

$\Rightarrow 2|\vec{a} + \vec{b} + \vec{c}|^2 = 75$

Sol9. $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$ and $Q = \left[q_{ij}\right] \Rightarrow PQ = kI_3$

$q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$

$$PQ = kI_3 \Rightarrow P^{-1} = \frac{Q}{k} = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{pmatrix}^{-1}$$

$$\therefore q_{23} = \frac{P_{32}}{|P|} \qquad |P| = 12\alpha + 20$$

$$\frac{1}{k}.\left(-\frac{k}{8}\right) = -\frac{3\alpha + 4}{12\alpha + 20} \Rightarrow \alpha = -1 \qquad \therefore |P| = -12 + 20 = 8$$

$$P \text{ is} = kI_3$$

$$|P|\,|Q| = (kI_3) \Rightarrow 8.\frac{k^2}{2} = k^3$$

$$k \neq 0 \Rightarrow k = 4$$

$$\therefore \alpha^2 + k^2 = 1 + 16 = 17.$$

Sol10. $\int_{-a}^{a} \left(|x| + |x-2|\right)dx = 22, a > 2$

$$\int_{-a}^{0} (-2x+2)dx + \int_{0}^{2} (x - x + 2)dx + \int_{2}^{a} (2x-2)dx = 22$$

$$\Rightarrow -2\left.\frac{(x-1)^2}{2}\right|_{-a}^{0} + 2x\Big|_0^2 + 2\left.\frac{(x-1)^2}{2}\right|_2^a = 22$$

$$\Rightarrow 2a^2 + 2 = 20 \Rightarrow a^2 = 9 \Rightarrow a = 3$$

$$\therefore \int_{3}^{-3} \left(x + [x]\right)dx = -\int_{-3}^{3} \left(2x - \{x\}\right)dx = -\int_{-3}^{3} 2xdx + 6\int_{0}^{1} xdx = 6.\left.\frac{x^2}{2}\right|_0^1 = 3$$