FIITJEE

Solutions to JEE (Main)-2020

JEE–Main–2020 –Jan–9–Second–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

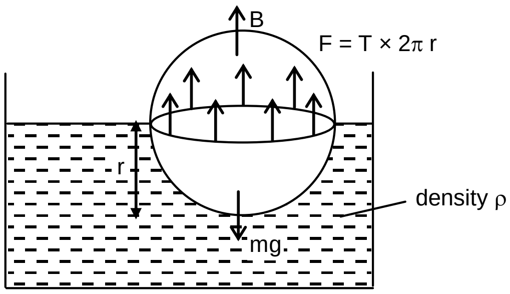
1. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet):

(A) $r=\sqrt{\frac{3T}{(2d-\rho)g}}$ (B) $r=\sqrt{\frac{T}{(d-\rho)g}}$

(C) $r=\sqrt{\frac{2T}{3(d+\rho)g}}$ (D) $r=\sqrt{\frac{T}{(d+\rho)g}}$

Ans. **A**

Sol.



$$dVg=\rho\left(\frac{V}{2}\right)g+T(2\pi r)$$

mg = B + F

$$\Rightarrow\ d\frac{4}{3}\pi r^3 g=\rho\cdot\frac{2}{3}\pi r^2 g+2\pi T$$

$$\Rightarrow\ \frac{2}{3}r^2 g(2d-\rho)=2T$$

$$\Rightarrow\ r=\sqrt{\frac{3T}{(2d-\rho)g}}$$

2. There is a small source of light at some depth below the surface of water (refractive index = $\frac{4}{3}$) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly):
[Use the fact that surface area of spherical cap of height h and radius of curvature r is $2\pi rh$]

(A) 21% (B) 17%

(C) 50% (D) 34%

Ans. **B**

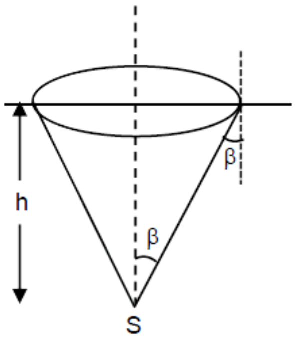
Sol.

$$\sin\beta=\frac{3}{4},\ \cos\beta=\frac{\sqrt{7}}{4}$$

Solid angle $d\Omega = 2\pi R^2(1-\cos\beta)$

Percentage of light $=\frac{2\pi R^2(1-\cos\beta)}{4\pi R^2}\times 100$

$$=\frac{1-\cos\beta}{2}\times 100=\left(\frac{4-\sqrt{7}}{8}\right)\times 100\approx 17\%$$



3. For the four sets of three measured physical quantifies as given below. Which of the following options is correct?
(i) $A_1 = 24.36$, $B_1 = 0.0724$, $C_1 = 256.2$
(ii) $A_2 = 24.44$, $B_2 = 16.082$, $C_2 = 240.2$
(iii) $A_3 = 25.2$, $B_3 = 19.2812$, $C_3 = 236.183$
(iv) $A_4 = 25$, $B_4 = 236.191$, $C_4 = 19.5$
(A) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3$
(B) $A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2 < A_4 + B_4 + C_4$
(C) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3 = A_4 + B_4 + C_4$
(D) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2$

Ans. **Bonus**

Sol. $A_1 + B_1 + C_1 = 24.36 + 0.0724 + 256.2 = 280.6324 = 280.6$
$A_2 + B_2 + C_2 = 24.44 + 16.082 + 240.2 = 280.722 = 280.7$
$A_3 + B_3 + C_3 = 25.2 + 19.2812 + 236.183 = 280.6642 = 280.7$
$A_4 + B_4 + C_4 = 25 + 236.191 + 19.5 = 280.691 = 281$
Answer should be $A_4 + B_4 + C_4 > A_3 + B_3 + C_3 = A_2 + B_2 + C_2 > A_1 + B_1 + C_1$

4. A plane electromagnetic wave is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$, with its polarization along the direction $\hat{k}$. The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant)

(A) $B_0 \frac{\hat{j}-\hat{i}}{\sqrt{2}} \cos\left(\omega t + k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$
(B) $B_0 \hat{k} \cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$
(C) $B_0 \frac{\hat{i}-\hat{j}}{\sqrt{2}} \cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$
(D) $B_0 \frac{\hat{i}+\hat{j}}{\sqrt{2}} \cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

Ans. **C**

Sol. EM wave is in direction $\rightarrow \frac{\hat{i}+\hat{j}}{\sqrt{2}}$
Electric field is in direction $\rightarrow \hat{k}$
$\vec{E}\times\vec{B} \rightarrow$ direction of propagation of EM wave.

5. A wire of length L and mass per unit length 6.0×10^{-3} kg m^{-1} is put under tension of 540 N. Two consecutive frequencies that it resonates at are: 420 Hz and 490 Hz. Then L in meters is
(A) 1.1 m
(B) 5.1 m
(C) 2.1 m
(D) 8.1 m

Ans. **C**

Sol. Fundamental frequency = 490 – 420 = 70 Hz.

$$70 = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow\ 70 = \frac{1}{2\ell}\sqrt{\frac{540}{6\times10^{-3}}}$$

$$\Rightarrow\ \ell = \frac{1}{2\times70}\sqrt{90\times10^{3}} = \frac{300}{140}$$

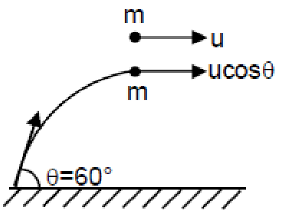
$$\Rightarrow\ \ell \approx 2.14 \text{ m}$$

6. A particle of mass m is projected with a speed u form the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is:

(A) $\frac{5}{8}\frac{u^2}{g}$ (B) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$ (C) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (D) $2\sqrt{2}\frac{u^2}{g}$

Ans. **C**

Sol.



$p_i = p_f$

mu + mu cos 60 = 2mv

$v = \frac{3u}{4}$

so horizontal range after collision

$$= v\sqrt{\frac{2H_{max}}{g}}$$

$$= \frac{3}{4}u\sqrt{\frac{2u^2\sin^2(60^o)}{2g^2}}$$

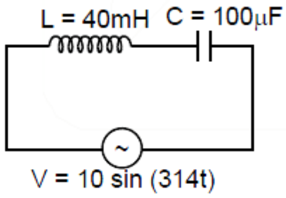
$$= \frac{3}{4}u^2\frac{\sqrt{\frac{3}{4}}}{g} = \frac{3\sqrt{3}u^2}{8g}$$

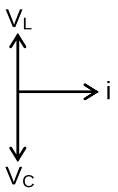
7. In LC circuit the inductance L = 40 mH and capacitance C = 100 μF. If a voltage V(t) = 10 sin (314 t) is applied to the circuit, the current in the circuit is given as

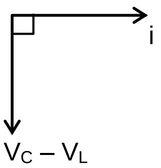
(A) 5.2 cos 314 t (B) 0.52 sin 314 t
(C) 0.52 cos 314 t (D) 10 cos 314 t

Ans. **C**

Sol.







$z = \sqrt{R^2 + (X_L - X_C)^2}$

R = 0

$Z = X_L - X_C$

$= \left|\omega L - \frac{1}{\omega C}\right|$

= 31.84 – 12.56

= 19.28

$X_L = X_C$

$V_L = V_C$

$i = \frac{V_0}{z}\sin\left(314t + \frac{\pi}{2}\right)$

$\therefore \quad i = \frac{V_0}{Z}\cos(314t)$

$\Rightarrow \quad i = \frac{10}{19.28}\cos(314t)$

$\Rightarrow$ i = 0.52 cos (314t)

8. Two gases argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to:

(A) 1.83 (B) 2.3
(C) 3.67 (D) 4.67

Ans. **A**

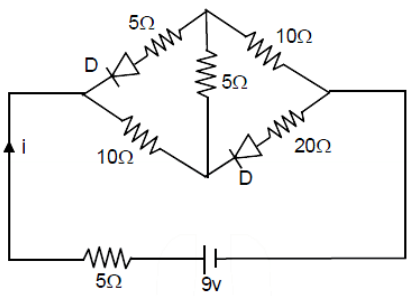
Sol. Mean free time = $\frac{1}{\sqrt{2}n\pi d^2 V_{rms}}$

$$\frac{t_{Ar}}{t_{Xe}} = \frac{d_{xe}^2}{d_{Ar}^2} \times \sqrt{\frac{m_1}{m_2}}$$

$$= \left(\frac{0.1}{0.07}\right)^2 \times \sqrt{\frac{40}{140}} = 1.07$$

Hence, the nearest answer is 1.

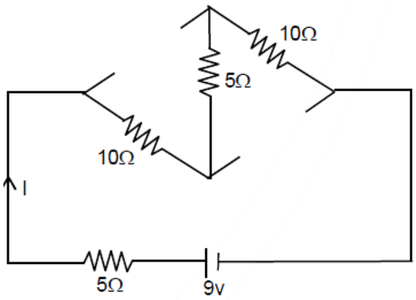
9. The current i in the network is:
(A) 0.6 A
(B) 0 A
(C) 0.2 A
(D) 0.3 A



Ans. **D**

Sol. Both diodes are in reverse biased.

$I = \frac{9}{30} = \frac{3}{10}$ A = 0.3 A



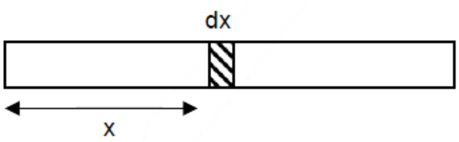
10. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at:

(A) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$ (B) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$

(C) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$ (D) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$

Ans. **C**

Sol.



$$X_{cm} = \frac{1}{M}\int_0^{\ell} X.dM$$

$$dM = \rho.dx = \left(a + b\left(\frac{x}{L}\right)^2\right)\cdot dx$$

$$x_{cm} = \frac{\int xdM}{\int dM} = \frac{\int x\rho dx}{\int \rho dx} = \frac{\int_0^L x\left(a + \frac{bx^2}{L^2}\right)dx}{\int_0^L \left(a + \frac{bx^2}{L^2}\right)dx}$$

$$= \frac{a\left(\frac{x^2}{2}\right)_0^L + \frac{b}{L^2}\left(\frac{x^4}{4}\right)_0^L}{a(x)_0^L + \frac{b}{L^2}\left(\frac{x^3}{3}\right)_0^L} = \frac{(2a+b)L}{(3a+b)4} \times 3$$

$$= \frac{3\ell}{4}\left(\frac{2a+b}{3a+b}\right)$$

11. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is

(A) $1:\sqrt{2}$ (B) 2 : 1 (C) $\sqrt{2}:1$ (D) 1 : 2

Ans. **C**

Sol. $\frac{du}{dv} = \frac{1}{2}\text{stress} \times \frac{\text{stress}}{y}$

$= \frac{1}{2}\frac{F^2}{A^2 y}$

$\frac{du}{dv} \propto \frac{1}{d^4}$; $\frac{\left(\frac{du}{dv}\right)_1}{\left(\frac{du}{dv}\right)_2} = \frac{d_2^4}{d_1^4} = \frac{1}{4}$

$\frac{d_1}{d_2} = (4)^{1/4}$; $\frac{d_1}{d_2} = \sqrt{2}:1$

12. Planet A has mass M and radius R. Plant B has half the mass and half the radius of Planet A. If the escape velocities from the planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$.

The value of 'n' is:

(A) 2 (B) 1 (C) 4 (D) 3

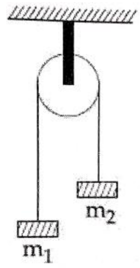
Ans. **C**

Sol. $v_e = \sqrt{\frac{2GM}{R}}$

$\therefore \quad \frac{v_1}{v_2} = \frac{\sqrt{\frac{2GM}{R}}}{\sqrt{\frac{2GM/2}{R/2}}} = 1 = \frac{n}{4}$

$\Rightarrow \quad n = 4.$

13. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released for rest. The angular speed of the wheel when m_1 descents by a distance h is:



(A) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+1}\right]^{\frac{1}{2}}$ (B) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+1}\right]^{\frac{1}{2}}$

(C) $\left[\frac{(m_1-m_2)}{(m_1+m_2)R^2+1}\right]^{\frac{1}{2}} gh$ (D) $\left[\frac{(m_1+m_2)}{(m_1+m_2)R^2+1}\right]^{\frac{1}{2}} gh$

Ans. **B**

Sol. $k_i + U_i = k_1 + k_2 + u_1 + u_2$

$$0 + 0 = \frac{1}{2}m_2 v^2 + \frac{1}{2}m_1 v^2 + \frac{1}{2}I\omega^2 - m_1 gh + m_2 gh$$

$$(m_1 - m_2)gh = \frac{1}{2}m_2(\omega R)^2 + \frac{1}{2}m_1(\omega R^2) + \frac{1}{2}I\omega^2$$

$$\sqrt{\frac{2(m_1 - m_2)gh}{\left(m_1 + m_2 + \frac{1}{R^2}\right)R^2}} = \omega$$

$$\omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$$

14. The energy required to ionize a hydrogen like ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground state?

(A) 11.4 nm (B) 35.8 nm
(C) 24.2 nm (D) 8.6 nm

Ans. **A**

Sol. $\frac{hc}{\lambda} = (13.6\ \text{eV})z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$n_1 = 1$; $n_2 = 3$

$\frac{hc}{\lambda} = (13.6\ \text{eV})\,(3^2)\left[\frac{1}{1^2} - \frac{1}{3^3}\right]$

$\Rightarrow \frac{hc}{\lambda} = (13.6\ \text{eV})\,(9)\left(\frac{8}{9}\right)$

Wavelength = $\frac{1240}{8 \times 13.6}$nm

λ = 11.39 nm

15. An electron of mass m and magnitude of charge $|e|$ initially at rest gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t ignoring relativistic effects is

(A) $\frac{|e|Et}{h}$ (B) $-\frac{h}{|e|Et}$ (C) $-\frac{h}{|e|E\sqrt{t}}$ (D) $-\frac{h}{|e|Et^2}$

Ans. **D**

Sol. $\lambda_D = \frac{h}{mv}$

$\therefore$ v = at

$v = \frac{eE}{m}t$ $\left(a = \frac{eE}{m}\right)$

$\lambda_D = \frac{h}{m\left(\frac{eE}{m}\right)t}$ $\lambda_0 = \frac{h}{eEt}$

$\frac{d\lambda_d}{dt} = -\frac{h}{|e|Et^2}$

16. A particle starts from the origin at t = 0 with an initial velocity of 3.0 $\hat{i}\,m/s$ and moves in the x-y plane with a constant acceleration $(6.0\,\hat{i} + 4.0\,\hat{j})\,m/s^2$. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is

(A) 40 (B) 60
(C) 32 (D) 50

Ans. **B**

Sol. $s_y = u_y t + \frac{1}{2}a_y t^2$

$32 = 0 + \frac{1}{2} \times 4t^2 \quad \Rightarrow \quad t = 4$ sec

$S_x = u_x t + \frac{1}{2}a_x t^2$

$= 3 \times 4 + \frac{1}{2} \times 6 \times 16 = 60$ m.

17. A small circular loop of conducting wire has radius a and carries current I. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T. If the mass of the loop is m then:

(A) $T = \sqrt{\frac{\pi m}{IB}}$ (B) $T = \sqrt{\frac{\pi m}{2IB}}$ (C) $T = \sqrt{\frac{2\pi m}{IB}}$ (D) $T = \sqrt{\frac{2m}{IB}}$

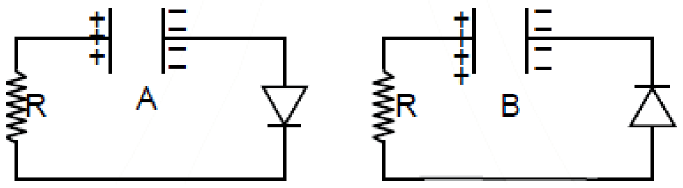
Ans. **C**

Sol. $\tau = MB \sin\theta = I\alpha$

$\pi R^2 I B\theta = \frac{mR^2}{2}\alpha$

$\omega = \sqrt{\frac{2\pi IB}{m}} = \frac{2\pi}{T} \quad ; \quad T = \sqrt{\frac{2\pi m}{IB}}$

18. Two identical capacitors A and B, charged to the same potential 5V are connected in two different circuits as shown below at time t = 0. If the charge on capacitors A and B at time t = CR is Q_A and Q_B respectively, then (Here e is the base of natural logarithm)

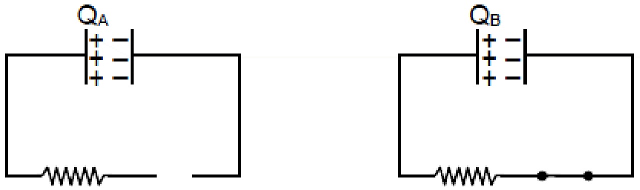


(A) $Q_A = VC, Q_B = CV$ (B) $Q_A = \frac{VC}{e}, Q_B = \frac{CV}{2}$

(C) $Q_A = VC, Q_B = \frac{VC}{e}$ (D) $Q_A = \frac{VC}{2}, Q_B = \frac{VC}{e}$

Ans. **C**

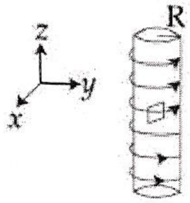
Sol. Maximum charge on capacitor = 5CV
(A) is reverse biased and (B) is forward biased
for case (A).



$Q_A = CV$

so $q = q_{max}\,[e^{-t/RC}]$
$Q_B = CVe^{-1}$

19. An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has n turns/length and carries a current I. The electron gun shoots an electron along the radius of the solenoid with speed v. If the electron does not hit the surface of the solenoid, maximum possible value of v is (all symbols have their standard meaning):



(A) $\frac{2e\mu_0 nIR}{m}$ (B) $\frac{e\mu_0 nIR}{m}$ (C) $\frac{e\mu_0 nIR}{4m}$ (D) $\frac{e\mu_0 nIR}{2m}$

Ans. **D**

Sol. $R_{max} = \frac{R}{2} = \frac{mv_{max}}{qB} = \frac{mv_{max}}{e\mu_0 In}$

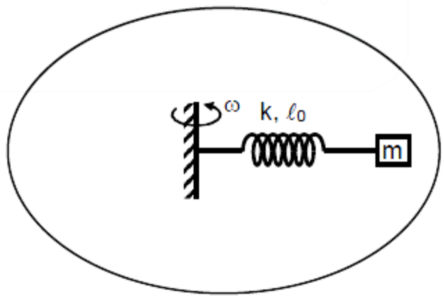
$V_{max} = \frac{Re\mu_0 In}{2m}$

20. A spring mass system (mass m, spring constant k and natural length ℓ) rests in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, ($k >> m\omega^2$) the relative change in the length of the spring is best given by the option:

(A) $\frac{m\omega^2}{3k}$ (B) $\frac{m\omega^2}{k}$ (C) $\frac{2m\omega^2}{3k}$ (D) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$

Ans. **B**

Sol. $m\omega^2 (\ell_0 + x) = kx$



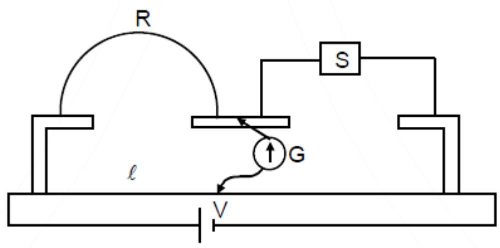
$\left(\frac{\ell_0}{x} + 1\right) = \frac{k}{m\omega^2}$

$x = \frac{\ell_0 m\omega^2}{k - m\omega^2}$

$k >> m\omega^2$

So, $\frac{x}{\ell_0}$ is equal to $\frac{m\omega^2}{k}$

21. In a meter bridge experiment S is a standard resistance. R is a resistance wire. It is found that balancing length is ℓ = 25 cm. If R is replaced by a wire of half length and half diameter that of R of same material, then the balancing distance ℓ' (in cm) will now be __________.



Ans. **40**

Sol. $\frac{x}{R} = \frac{75}{25} = 3$

$R = \frac{\rho\ell}{A} = \frac{4\rho\ell}{\pi d^2}$

$R' = \frac{4\rho\left(\frac{\ell}{2}\right)}{\pi\left(\frac{d}{2}\right)^2} = 2R$; then $\frac{X}{R'} = \left(\frac{100-\ell}{\ell}\right)$

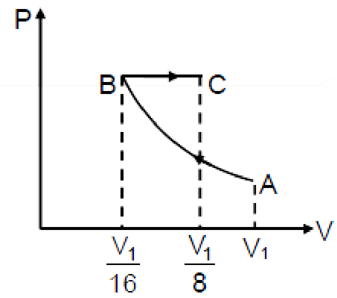
$\frac{100-\ell}{\ell} = \frac{X}{2R} = \frac{3}{2}$

ℓ = 40.00 cm

22. Starting at temperature 300 K, one mole of an ideal diatomic gas (γ = 1.4) is first compressed adiabatically form volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer) __________.

Ans. **1819**

Sol. PV^{γ} = constant



$TV^{\gamma-1}$ = constant

$$300\,(V_1)^{14-1} = T_B\left(\frac{V_1}{16}\right)^{2/5}$$

$T_B = 300 \times 2^{8/5}$

Now for BC process

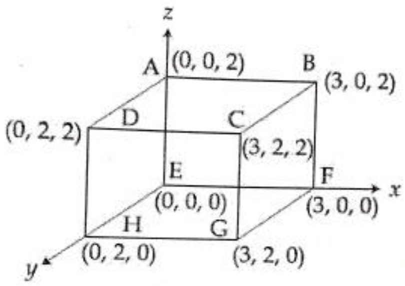
$$\frac{V_B}{T_B} = \frac{V_C}{T_C}$$

$$T_C = \frac{V_C T_B}{V_B} = 2 \times 300 \times 2^{8/5}$$

T_C = 1818.859

T_C = 1819 K

23. An electric field $\vec{E} = 4x\hat{i} - (y^2+1)\hat{j}\ N/C$ passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as ϕ_I and ϕ_{II} respectively. The difference between ($\phi_I - \phi_{II}$) is (in Nm^2/C) __________.



Ans. **48**

Sol. Flux via ABCD

$$\phi_1 = \int \vec{E}d\vec{A} = 0$$

Flux via BCEF

$$\phi_2 = \int \vec{E}d\vec{A}$$

$$\phi_2 = \int \vec{E}\vec{A} = \left[4x\hat{i} - (y^2+1)\hat{j}\right]\cdot 4\hat{i}$$

= 16x, x = 3

$$\phi_2 = 48\frac{N-m^2}{C}\ ;\ \phi_1 - \phi_2 = -48\frac{N-m^2}{C}$$

24. In a Young's double's slit experiment 15 fringes are observed on a small portion of the screen when light of wavelength 500 nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then the value of λ is (in nm) __________.

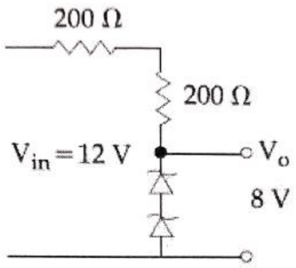
Ans. **750**

Sol. $$15 \times 500 \times \frac{D}{d} = 10 \times \lambda_2 \times \frac{D}{d}$$

λ_2 = 15 × 50 nm

λ_2 = 750 nm

25. The circuit as shown in figure is working as a 8 V dc regulated voltage source. When 12 V is used as input, the power dissipated (in mW) in each diode is (considering both zener diodes are identical) __________.



Ans. **40**

Sol. $i = \dfrac{(12-8)}{(200+200)} A = \dfrac{4}{400} = 10^{-2}$ A

$\because$ P = v × i

Power loss in each diode = (4) (10^{-2}) W = 40 mW.

PART –B (CHEMISTRY)

26. Amongst the following, the form of water with the lowest ionic conductance at 298 K is :
    (A) sea water
    (B) distilled water
    (C) saline water used for intravenous injection
    (D) water from a well

Ans. B

Sol. The form of H_2O with the lowest ionic conductance at 298 K is distilled water.

27. The correct order of the spin-only magnetic moments of the following complexes is :
    (I) $[Cr(H_2O)_6]Br_2$ (II) $Na_4[Fe(CN)_6]$
    (III) $Na_3[Fe(C_2O_4)_3](D_0 > P)$ (IV) $(Et_4N)_2\,[CoCl_4]$
    (A) (I) > (IV) > (III) > (II) (B) (II) » (I) > (IV) > (III)
    (C) (III) > (I) > (IV) > (II) (D) (III) > (I) > (II) > (IV)

Ans. A

Sol. $[Cr(H_2O)_6]Br_2 : Cr^{2+} = [Ar]\, 4s^0\, 3d^4$

$$t_{2g}^3\, eg^1$$

$$n = 4,\ \mu = \sqrt{n(n+2)} = \sqrt{24}\ BM$$

$Na_4[Fe(CN)_6] : Fe^{2+} = [Ar]\, 4s^0\, 3d^6$

$$t_{2g}^6\ eg^0$$

$$n = 0,\ \mu = 0$$

$Na_3[Fe(C_2O_4)_3]: Fe^{3+} = [Ar]\, 3d^5$

$$t_{2g}^5\ eg^0$$

$$n = 1,\ \mu = \sqrt{3}\ BM$$

$(Et_4N)_2\,[CoCl_4] : Co^{2+} = [Ar]\, 3d^7$

$$e_g^4\ t_{2g}^3$$

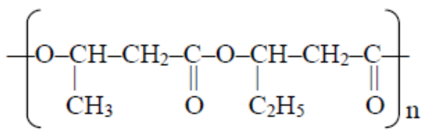
$$n = 3,\ \mu = \sqrt{15}\ BM$$

28. Which polymer has 'chiral' monomer(s)?
    (A) Nylon 6, 6 (B) Neoprene
    (C) PHBV (D) Buna-N

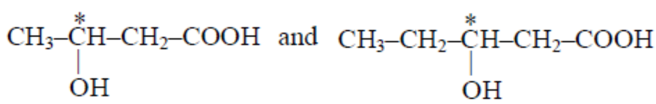
Ans. C

Sol.

PHBV :



Monomer :



3-Hydroxybutanoic acid and 3-Hydroxypentanoic acid

29. The first and second ionisation enthalpies of a metal are 496 and 4560 kJ mol^{-1}, respectively. How many moles of HCl and H_2SO_4, respectively, will be needed to react completely with 1 mole of the metal hydroxide ?

(A) 1 and 1 (B) 1 and 2
(C) 2 and 0.5 (D) 1 and 0.5

Ans. D

Sol. Metal: First ionization enthalpies = 496 kJ/mole
Second ionization enthalpies = 4560 kJ/mol
According to the given information, ionization enthalpies Metal belong to 1^{st} group i.e. Monovalent cation.

$$\underset{1\text{ mole}}{MOH} + \underset{1\text{ mole}}{HCl} \longrightarrow MCl + H_2O$$

$$\underset{1\text{ mole}}{2MOH} + \underset{1/2\text{ mole}}{H_2SO_4} \longrightarrow M_2SO_4 + H_2O$$

30. Among the statements (a) – (d), the correct ones are:
(a)Lithium has the highest hydration enthalpy among the alkali metals.
(b) Lithium chloride is insoluble in pyridine.
(c) Lithium cannot form ethynide upon its reaction with ethyne
(d) Both lithium and magnesium react slowly with H_2O

(A) (a), (b) and (d) only (B) (a) and (d) only
(C) (b) and (c) only (D) (a), (c) and (d) only

Ans. D

Sol. Lithium has the highest hydration enthalpy among the alkali metals due to small size.
Lithium chloride is covalent in nature so it's soluble in non-polar solvent.
Lithium and Magnesium react slowly with H_2O

31. The solubility product of $Cr(OH)_3$ at 298 K is 6.0×10^{-31}. The concentration of hydroxide ions in a saturated solution of $Cr(OH)_3$ will be :

(A) $(18 \times 10^{-31})^{1/4}$ (B) $(4.86 \times 10^{-29})^{1/4}$
(C) $(18 \times 10^{-31})^{1/2}$ (D) $(2.22 \times 10^{-31})^{1/4}$

Ans. A

Sol.
$$Cr(OH)_3 \rightleftharpoons \underset{s}{Cr^{3+}} + \underset{3s}{3OH^-}$$

$$K_{sp} = [Cr^{3+}][OH^-]^3$$
$$6 \times 10^{-31} = S \times (3S)^3$$
$$6 \times 10^{-31} = 27\ S^4$$
$$S = \left(\frac{6}{27} \times 10^{-31}\right)^{1/4}$$
$$[OH^-] = 3S$$
$$= 3\left(\frac{6}{27} \times 10^{-31}\right)^{1/4} = (18 \times 10^{-31})^{1/4}\ M$$

32. 5 g of zinc is treated separately with an excess of
(a) dilute hydrochloric acid and (b) aqueous sodium hydroxide.
The ratio of the volumes of H_2 evolved in these two reactions is : -

(A) 1 : 4 (B) 1 : 1
(C) 1 : 2 (D) 2 : 1

Ans. B

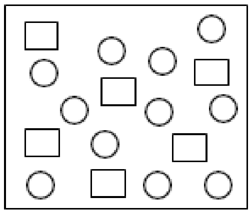
Sol. $Zn + 2\,dil.HCl \longrightarrow ZnCl_2 + H_2$

$Zn + 2NaOH \longrightarrow Na_2ZnO_2 + H_2$

Mole of Zn = $\dfrac{\text{Mole of dil. HCl}}{2} = \dfrac{\text{Mole of NaOH}}{2}$

$\dfrac{\text{volume of HCl}}{\text{volume of NaOH}} = \dfrac{1}{1}$

33. If the figure shown below reactant A (represented by square) is in equilibrium with product B (represented by circle). The equilibrium constant is:



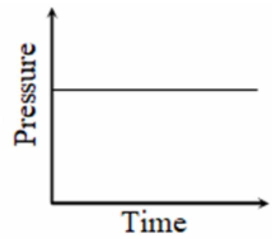
(A) 8
(B) 2
(C) 4
(D) 1

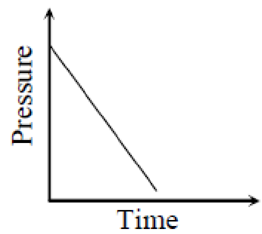
Ans. B

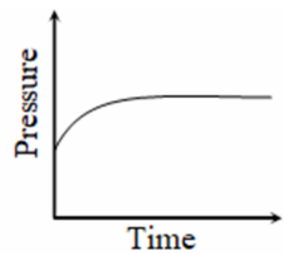
Sol. $A \rightarrow B$

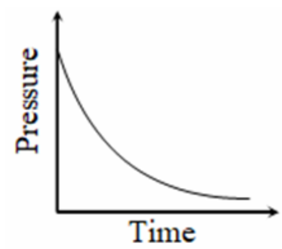
$K_{eq} = \dfrac{[B]}{[A]} = \dfrac{11}{6}$

34. A mixture of gases O_2, H_2 and CO are taken in a closed vessel containing charcoal. The graph that represents the correct behaviour of pressure with time is:

(A) 

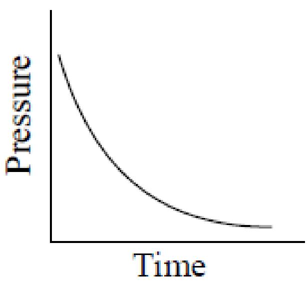
(B) 

(C) 

(D) 

Ans. D

Sol.

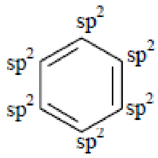


On increasing time, pressure will be decreases.

35. The number of sp^2 hybrid orbitals in a molecule of benzene is
(A) 18 (B) 6
(C) 12 (D) 24

Ans. A

Sol. Benzene : C_6H_6



In benzene, each carbon is sp^2 Hybridize
Total number of carbon = 6(sp^2 Hybri)
$\therefore$ Total hybrid orbital = $6 \times 3 = 18$

36. Biochemical Oxygen Demand (BOD) is the amount of oxygen required (in ppm):
(A) for the photochemical breakdown of waste present in 1 m^3 volume of a water body.
(B) for sustaining life in a water body.
(C) by bacteria to break-down organic waste in a certain volume of a water sample.
(D) by anaerobic bacteria to breakdown inorganic waste present in a water body.

Ans. C

Sol. Biochemical oxygen demand(BOD)
The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water.

37. The true statement amongst the following is :
(A) S is a function of temperature but DS is not a function of temperature.
(B) Both DS and S are functions of temperature.
(C) S is not a function of temperature but DS is a function of temperature.
(D) Both S and DS are not functions of temperature

Ans. B

Sol. $S = \int \frac{dq}{T}$

$\Delta S = nC\int_{T_1}^{T_2} dT$

Both ΔS and S are function of temperature.

38. A, B and C are three biomolecules. The results of the tests performed on them are given below:

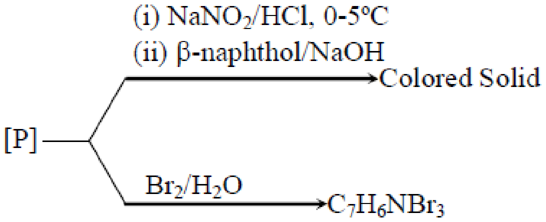
	Molisch's Test	**Barfoed Test**	**Biuret Test**
(A)	Positive	Negative	Negative
(B)	Positive	Positive	Negative
(C)	Negative	Negative	Positive

A, B and C are respectively:
(A) A = Lactose, B = Glucose, C = Alanine (B) A = Lactose, B = Glucose, C = Albumin
(C) A = Glucose, B = Fructose, C = Albumin (D) A = Lactose, B = Fructose, C = Alanine

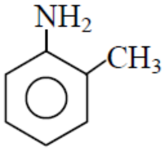
Ans. B

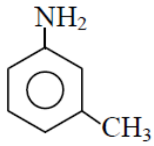
Sol. Lactose: Molisch's test
Glucose: Molisch's test and Barfoed test
Alumin: Biuret test

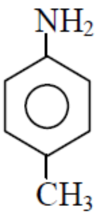
39. Consider the following reactions:

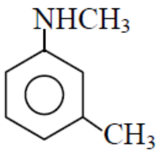


The compound[P] is

(A) 

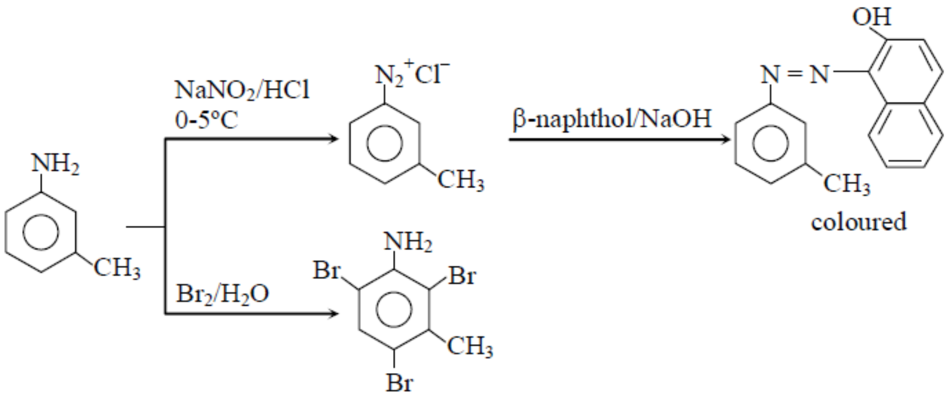
(B) 

(C) 

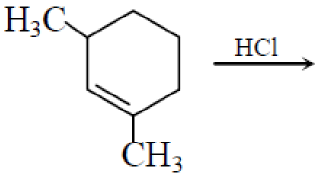
(D) 

Ans. B

Sol.



40. Which of the following reactions will not produce a racemic product?

(A) 

(B) $CH_3CH_2CH{=}CH_2 \xrightarrow{HBr}$

(C) $CH_3\text{–}\overset{\overset{\large O}{\|}}{C}CH_2CH_3 \xrightarrow{HCN}$

(D) $CH_3\text{–}\underset{H}{\overset{CH_3}{C}}\text{–}CH{=}CH_2 \xrightarrow{HCl}$

Ans. D

Sol. $CH_3\text{–}\underset{H}{\overset{CH_3}{C}}\text{–}CH{=}CH_2 \xrightarrow{HCl} CH_3\text{–}\underset{Cl}{\overset{CH_3}{C}}\text{–}CH_2\text{–}CH_3$

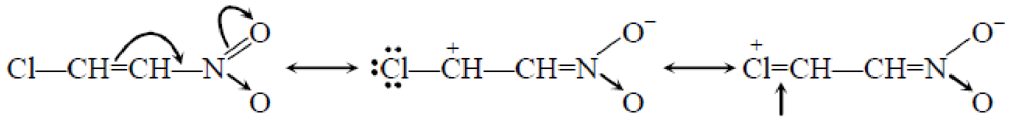
optical inactive

41. Which of the following has the shortest C–Cl bond?

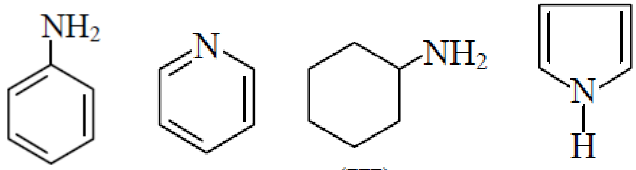
(A) $Cl - CH = CH - NO_2$
(B) $Cl - CH = CH - OCH_3$
(C) $Cl - CH = CH - CH_3$
(D) $Cl - CH = CH_2$

Ans. A

Sol.



42. The decreasing order of basicity of the following amines is

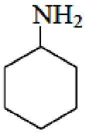


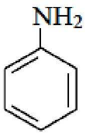
(I) (II) (III) (IV)

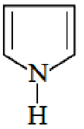
(A) (III) > (I) > (II) > (IV)
(B) (II) > (III) > (IV) > (I)
(C) (I) > (III) > (IV) > (II)
(D) (III) > (II) > (I) > (IV)

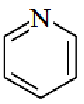
Ans. D

Sol.


: Nitrogen ℓ.p. not participate in resonance.


: Nitrogen ℓ.p. participate in resonance.


: Nitrogen ℓ.p. participate in resonance and increase the stability of the compound due to aromicity.


: Nitrogen ℓ.p. not participate in resonance due to increase the stability of the compound.

43. The reaction of $H_3N_3B_3Cl_3$(A) with $LiBH_4$ in tetrahydrofuran gives inorganic benzene (B). Further, the reaction of (A) with (C) leads to $H_3N_3B_3(Me)_3$. Compounds (B) and (C) respectively, are:

(A) Borazine and MeBr
(B) Boron nitride and MeBr
(C) Diborane and MeMgBr
(D) Borazine and MeMgBr

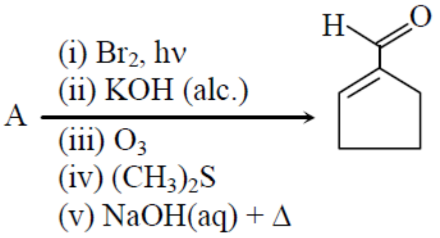
Ans. D

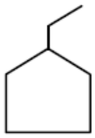
Sol.

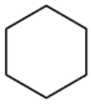
$$\underset{(A)}{B_3N_3H_3Cl_3} + LiBH_4 \longrightarrow \underset{\substack{(B)\\ \text{Inorganic} \\ \text{Benzene} \\ \text{or} \\ \text{Borazine}}}{B_3N_3H_6}$$

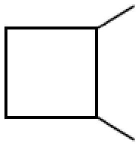
$$B_3N_3H_3Cl + \underset{(C)}{MeMgBr} \longrightarrow B_3N_3H_3(CH_3)_3 + 3MgBrCl$$

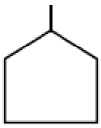
44. In the following reaction A is :



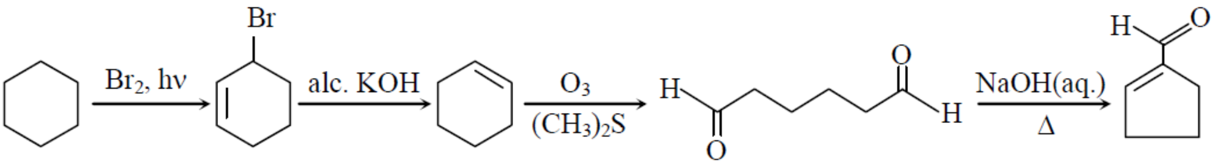
(A) 

(B) 

(C) 

(D) 

Ans. B

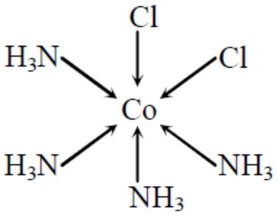
Sol. 

45. The isomer (s) of $[Co(NH_3)_4Cl_2]$ that has/have a Cl – Co – Cl angle of 90º, is/are :

(A) meridional and trans
(B) cis only
(C) trans only
(D) cis and trans

Ans. B

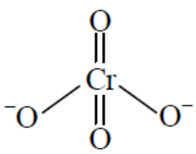
Sol. $[Co(NH_3)_4Cl_2]$

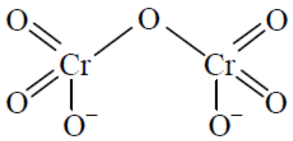


46. The sum of the total number of bonds between chromium and oxygen atoms in chromate and dichromate ions is ____________________

Ans. 12

Sol.

Chromate = CrO_4^{2-} 

dichromate = $Cr_2O_7^{2-}$ 

Total number of Cr–O bond = 12

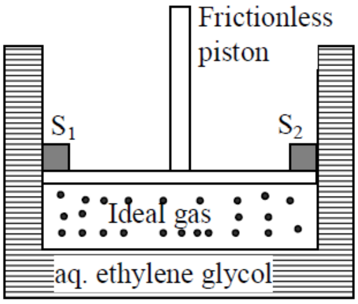
47. A sample of milk splits after 60 min. at 300 K and after 40 min. at 400 K when the population of *lactobacillus acidophilus* in it doubles. The activation energy (in kJ/mol) for this process is closest to ____________

$$\text{(Given, } R = 8.3 \text{ J mol}^{-1}\text{K}^{-1},\ \ell n\left(\frac{2}{3}\right) = 0.4,\ e^{-3} = 4.0)$$

Ans. 3.98

Sol.
$$\ell n\frac{K_2}{K_1} = \frac{Ea}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$
$$\ell n\frac{60}{40} = \frac{Ea}{8.3}\left[\frac{1}{300} - \frac{1}{400}\right]$$
$$Ea = 3.98 \text{ kJ/mole.}$$

48. A Cylinder containing an ideal gas (0.1 mol of 1.0 dm^3) is in thermal equilibrium with a large volume of 0.5 molal aqueous solution of ethylene glycol at its freezing point. If the stoppers S_1 and S_2 (as shown in the figure) are suddenly withdrawn, the volume of the gas in litres after equilibrium is achieved will be____.
(Given, K_f(water) = 2.0 K kg mol^{-1}, R = 0.08 dm^3 atm K^{-1} mo^{-1})



Ans. 2.18

Sol. $\Delta T_f = K_f \times i \times m$
$\Delta T_f = 2.0 \times 1 \times 0.5$
$\Delta T_f = 1$
$273 - T_1 = 1$
$T_1 = 272$ K
$$P = \frac{nRT}{V}$$
$$P = \frac{0.1 \times 0.08 \times 272}{1}$$
P = 2.176 atm
Apply Boyle's law
$P_1V_1 = P_2V_2$
$2.176 \times 1 = 1 \times V_2$
$V_2 = 2.17$

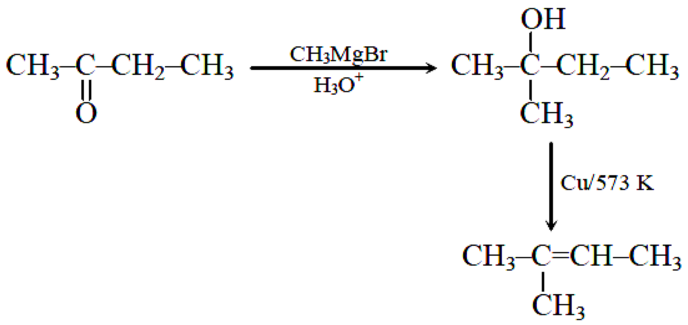
49. Consider the following reactions

$$A \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CH_3MgBr} B \xrightarrow[573\text{ K}]{Cu} \text{2-methyl-2-butene}$$

The mass percentage of carbon in A is____.

Ans. 66.67

Sol.



$$\% \text{ carbon} = \frac{\text{Atomic mass} \times \text{Atomicity}}{\text{Molar mass}} \times 100$$

$$= \frac{12 \times 4}{72} \times 100 = 66.66\%$$

50. 10.30 mg of O_2 is dissolved into a liter of sea water of density 1.03 g/mL. The concentration of O_2 in ppm is_______________.

Ans. 10

Sol. $\text{PPM} = \frac{\text{Mass of } O_2}{\text{Mass of water}} \times 10^6 = \frac{10.30 \times 10^{-3}}{1030} \times 10^6 = 10$

PART–C (MATHEMATICS)

51. If $x=\sum_{n=0}^{\infty}(-1)^n\tan^{2n}\theta$ and $y=\sum_{n=0}^{\infty}\cos^{2n}\theta$, for $0<\theta<\frac{\pi}{4}$, then:

(A) $y(1-x)=1$ (B) $y(1+x)=1$

(C) $x(1+y)=1$ (D) $x(1-y)=1$

Ans. A

Sol. Use $1+r+r^2+........\infty=\frac{1}{1-r}, |r|<1$

$x=\frac{1}{1+\tan^2\theta}=\cos^2\theta$

$y=\frac{1}{1+\tan^2\theta}=\cos^2\theta$

$y=\frac{1}{1-\cos^2\theta}=\frac{1}{\sin^2\theta}$

$x+\frac{1}{y}=1$

52. If $p\rightarrow(p\wedge\sim q)$ is false, then the truth values of p and q are respectively:

(A) F, T (B) F, F

(C) T, T (D) T, F

Ans. C

Sol.

P	Q	$\sim Q$	$P\wedge\sim Q$	$P\rightarrow(P\wedge\sim Q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

53. Let $a,b\in R, a\neq 0$ be such that the equation $ax^2-2bx+5=0$ has a repeated root α, which is also a root of the equation, $x^2-2bx-10=0$. If β is the other root of this equation, then $\alpha^2+\beta^2$ is equal to:

(A) 28 (B) 24

(C) 26 (D) 25

Ans. D

Sol. $2\alpha=\frac{2b}{a}\Rightarrow\alpha=\frac{b}{a}$ and $\alpha^2=\frac{5}{a}$

$\Rightarrow\frac{b^2}{a^2}=\frac{5}{a}$

$\Rightarrow b^2=5a$ …….(i) $(a\neq 0)$

$\alpha+\beta=2b$ ……..(ii)

and $\alpha\beta=-10$ …….(iii)

$\alpha = \frac{b}{a}$ is also root of $x^2 - 2bx - 10 = 0$

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

by (i) $\Rightarrow 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow 20a^2 = 5a$

$\Rightarrow a = \frac{1}{4}$ and $b^2 = \frac{5}{4}$

$\alpha^2 = 20$ and $\beta^2 = 5$

Now $\alpha^2 + \beta^2$

$= 5 + 20 = 25$

54. Let $[t]$ denote the greatest integer $\le t$ and $\lim\limits_{x\to 0} x\left[\frac{4}{x}\right] = A$. Then the function, $f(x) = [x^2]\sin(\pi x)$ is discontinuous, when x is equal to:

(A) $\sqrt{A+21}$ (B) $\sqrt{A+5}$

(C) $\sqrt{A}$ (D) $\sqrt{A+1}$

Ans. D

Sol. $\lim\limits_{x\to 0}\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right) = A \Rightarrow \lim\limits_{x\to 0} 4 - x\left\{\frac{4}{x}\right\} = A$

$\Rightarrow 4 - 0 = A$

check when

(A) $x = \sqrt{A+21} \Rightarrow x = 5 \Rightarrow$ continuous

(B) $x = \sqrt{A+5} \Rightarrow x = 3 \Rightarrow$ continuous

(C) $x = \sqrt{A} \Rightarrow x = 2 \Rightarrow$ continuous

(D) $x = \sqrt{A+1} \Rightarrow x = \sqrt{5} \Rightarrow$ discontinuous

55. Let a function $f:[0,5] \to R$ be continuous, $f(1) = 3$ and F be defined as:

$F(x) = \int_1^x t^2 g(t)dt$, where $g(t) = \int_1^t f(u)du$. Then for the function F, the point x = 1 is:

(A) a point of local minima (B) a point of inflection.

(C) not a critical point (D) a point of local maxima

Ans. A

Sol. $F'(x) = x^2 g(x)$

$\Rightarrow F'(1) = 1.g(1) = 0$ ……..(1) $(\because g(1) = 0)$

Now $F''(x) = 2xg(x) + x^2 g'(x)$

$\Rightarrow F''(x) = 2xg(x) + x^2 f(x)$ $(\because g'(x) = f(x))$

$\Rightarrow F''(1) = 0 + 1\times 3$

$\Rightarrow F''(1) = 3$ ……(2)

From (1) and (2) F (x) has local minimum at x = 1

56. A random variable X has the following probability distribution:

X	:	1	2	3	4	5
P(X)	:	K^2	2K	K	2K	$5K^2$

Then $P(X>2)$ is equal to:

(A) $\frac{1}{36}$ (B) $\frac{7}{12}$

(C) $\frac{23}{36}$ (D) $\frac{1}{6}$

Ans. C

Sol. $\sum p_i = 1 \Rightarrow 6k^2 + 5k = 1$

$6k^2 + 5k - 1 = 0$

$6k^2 + 6k - k - 1 = 0$

$(6k-1)(k+1) = 0 \Rightarrow k = -1$ (rejected); $k = \frac{1}{6}$

$P(x > 2) = k + 2k + 5k^2$

$= \frac{1}{6} + \frac{2}{6} + \frac{5}{36} = \frac{6+12+5}{36} = \frac{23}{36}$

57. If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} = \lambda \tan\theta + 2\log_e|f(\theta)| + C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to:

(A) $(-1, 1+\tan\theta)$ (B) $(1, 1-\tan\theta)$

(C) $(1, 1+\tan\theta)$ (D) $(-1, 1-\tan\theta)$

Ans. A

Sol. $\int \frac{\frac{\sec^2\theta}{1+\tan^2\theta}}{1-\tan^2\theta} + \frac{2\tan\theta}{1-\tan^2\theta} d\theta$

$= \int \frac{\sec^2\theta(1+\tan^2\theta)}{(1+\tan\theta)^2} d\theta$

$= \int \frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta} d\theta$

$\tan\theta = t \Rightarrow \sec^2\theta \, d\theta = dt$

$= \int \left(\frac{1-t}{1+t}\right) dt = \int \left(-1 + \frac{2}{1+t}\right) dt$

$= -t + 2\log(1+t) + C$

$= -\tan\theta + 2\log(1+\tan\theta) + C$

$\Rightarrow \quad \lambda = -1$ and $f(\theta) = 1 + \tan\theta$

58. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}; y(1)=1;$ then a value of x satisfying $y(x)=e$ is:

(A) $\sqrt{2}e$
(B) $\frac{1}{2}\sqrt{3}e$
(C) $\sqrt{3}e$
(D) $\frac{e}{\sqrt{2}}$

Ans. C

Sol. Put $y=vx$

$$\Rightarrow V+x\frac{dy}{dx}=\frac{vx^2}{x^2+v^2x^2}$$

$$\Rightarrow\left(\frac{1+V^2}{V^3}\right)dv=-\frac{dx}{x}$$

$$\Rightarrow\frac{-1}{2V^2}+\ell nV=-\ell nx+c$$

$$\Rightarrow-\frac{x^2}{2y^2}+\ell ny=c$$

When $x=1, y=1\Rightarrow c=\frac{-1}{2}$

When $y=e\Rightarrow\frac{-x^2}{2e^2}+1=\frac{-1}{2}$

$$\Rightarrow x^2=3e^2$$

59. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100}a_{2n+1}=200$ and $\sum_{n=1}^{100}a_{2n}=100$, then $\sum_{n=1}^{200}a_n$ is equal to:

(A) 225
(B) 300
(C) 150
(D) 175

Ans. C

Sol. $a_3+a_5+a_7+.......+a_{204}=\frac{ar^2\left(1-r^{200}\right)}{1-r^2}\quad\rightarrow(1)$

$a_2+a_4+a_6+........+a_{200}=\frac{ar\left(1-r^{200}\right)}{1-r^2}\quad\rightarrow(2)$

(1) + (2) $\Rightarrow a_2+a_3+a_4+........+a_{201}=300$

$r\left(a_1+a_2+a_3+......+a_{201}\right)=300$

$\sum_{n=1}^{100}a_n=\frac{300}{r}$

Also $(1)\div(2)\Rightarrow r=2$

60. If z be a complex number satisfying $\left|Re(z)\right|+\left|Im(z)\right|=4$, then $|z|$ cannot be:

(A) $\sqrt{\frac{17}{2}}$
(B) $\sqrt{10}$

(C) $\sqrt{8}$ (D) $\sqrt{7}$

Ans. D

Sol. $z = x + iy$

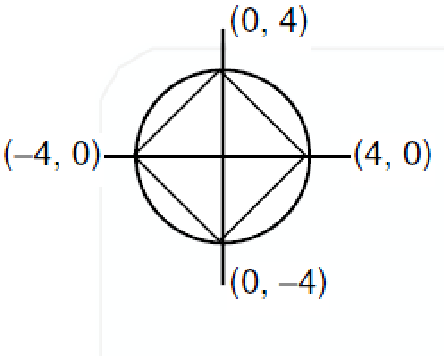
$|x| + |y| = 4$

Minimum value of $|z| = 2\sqrt{2}$

Maximum value of $|z| = 4$

$|z| \in \left[\sqrt{8}, \sqrt{16}\right]$

So $|z|$ can't be $\sqrt{7}$



61. Given : $f(x) = \begin{cases} x, & 0 \le x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x \le 1 \end{cases}$ and $g(x) = \left(x - \frac{1}{2}\right)^2, x \in R$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines $2x = 1$ and $2x = \sqrt{3}$, is:

(A) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (B) $\frac{1}{2} + \frac{\sqrt{3}}{4}$

(C) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ (D) $\frac{1}{2} - \frac{\sqrt{3}}{4}$

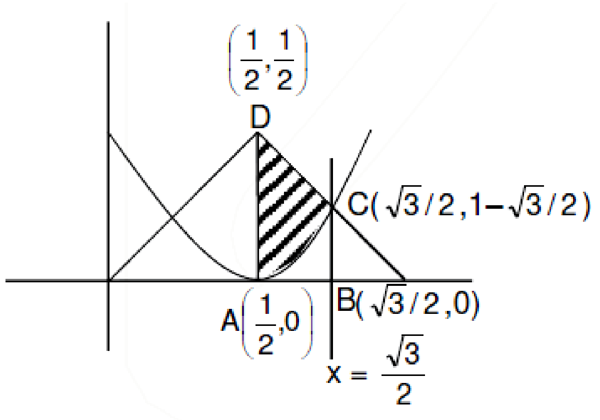
Ans. C

Sol. Required area

$= \text{Area of trapezium ABCD} - \int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx$

$= \frac{1}{2}\left(\frac{\sqrt{3} - 1}{2}\right)\left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \frac{1}{3}\left(\left(x - \frac{1}{2}\right)^3\right)_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$

$= \frac{\sqrt{3}}{4} - \frac{1}{3}$



62. In the expansion of $\left(\frac{x}{\cos\theta} + \frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{16} \le \theta \le \frac{\pi}{8}$, then the ratio $l_2 : l_1$ is equal to:

(A) 1 : 16 (B) 1 : 8

(C) 8 : 1 (D) 16 : 1

Ans. D

Sol. $T_{r+1} = {}^{16}C_r \left(\frac{x}{\cos\theta}\right)^{16-r} \left(\frac{1}{x\sin\theta}\right)^r$

for $r = 8$ term is free from 'x

$$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta\cos^8\theta}$$

$$T_9 = {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$$

In $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$, $L_1 = {}^{18}C_8\, 2^8$ $\because$ (Min value of L_1 at $\theta = \frac{\pi}{4}$)

In $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$, $L_1 = {}^{16}C_8 \dfrac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 . 2^8 . 2^4$ ($\because$ min value of L_2 at $\theta = \frac{\pi}{8}$)

$$\frac{L_2}{L_1} = \frac{{}^{16}C_8 . 2^8\, 2^4}{{}^{16}C_8 . 2^8} = 16$$

63. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is:

(A) $\frac{945}{2^{11}}$ (B) $\frac{945}{2^{10}}$

(C) $\frac{965}{2^{11}}$ (D) $\frac{965}{2^{10}}$

Ans. Bonus

Sol. Case – I : Exactly two box contain 2, 3 balls and other two box does not contains 2, 3 balls equals to ${}^4C_2$

$${}^4C_2 \times 2 \times {}^{10}C_2 \times {}^8C_3 \times \left(2^5 - \frac{5!}{2!3!} \times 2!\right)$$

$$= 2^4 \times 3^2 \times 45 \times 8 \times 7 = 2^7 \times 3^4 \times 5 \times 7$$

Case – II : Two box contains 2 balls each and two box contain 3 balls each equals to

$$\frac{10!}{2!3!2!3!2!2!} \times 4!$$

$$= 2^5 \times 3^3 \times 5^2 \times 7$$

$$\Rightarrow \text{probability} = \frac{2^5 \times 3^3 \times 5 \times 7(12+5)}{4^{10}} = \frac{3^3 \times 5 \times 7 \times 17}{2^{15}}$$

64. If $A = \{x \in R : |x| < 2\}$ and $B = \{x \in R : |x-2| \ge 3\}$; then:

(A) $B - A = R - (-2, 5)$ (B) $A \cap B = (-2, -1)$

(C) $A - B = [-1, 2)$ (D) $A \cup B = R - (2, 5)$

Ans. A

Sol. $A = \{x : x \in (-2, 2)\}$

$B = \{x : x \in (-\infty, -1] \cup [5, \infty)\}$

$A \cap B = \{x : x \in (-2, -1]\}$

$A \cup B = \{x : x \in (-\infty, 2) \cup [5, \infty)\}$

$A - B = \{x : x \in (-1, 2)\}$

$B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$

65. Let $a-2b+c=1$. If $f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then:

(A) $f(-50)=501$
(B) $f(50)=1$
(C) $f(50)=-501$
(D) $f(-50)=-1$

Ans. B

Sol. Use $R_1 \to R_1+R_3-2R_2$

$$f(x)=\begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

= 1

66. Let f and g be differentiable functions on R such that fog is the identity function. If for some $a,b \in R, g'(a)=5$ and $g(a)=b$, then $f'(b)$ is equal to:

(A) 1
(B) 5
(C) $\frac{1}{5}$
(D) $\frac{2}{5}$

Ans. C

Sol. Given that $f(g(x))=x$

$\Rightarrow f'(g(x)).g'(x)=1$

Put $x=a$

$\Rightarrow f'(g(a))g'(a)=1$

$\Rightarrow f'(b)\times 5=1$

$\Rightarrow f'(b)=\frac{1}{5}$

67. If one end of a focal chord AB of the parabola $y^2=8x$ is at $A\left(\frac{1}{2},-2\right)$, then the equation of the tangent to it at B is:

(A) $2x-y-24=0$
(B) $x-2y+8=0$
(C) $2x+y-24=0$
(D) $x+2y+8=0$

Ans. B

Sol. Given $2at_1=-2 \Rightarrow t_1=\frac{-1}{2}$

Also $t_1t_2=-1 \Rightarrow t_2=2$

Equation of tangent is $t2y=x+at_2^2$

$\Rightarrow 2y=x+8$

68. If $x=2\sin\theta-\sin 2\theta$ and $y=2\cos\theta-\cos 2\theta$, $\theta\in[0,2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta=\pi$ is:

(A) $-\frac{3}{8}$
(B) $\frac{3}{2}$
(C) $\frac{3}{4}$
(D) $-\frac{3}{4}$

Ans. Bonus mark

Sol. $\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$

$\frac{dy}{dx} = \frac{\sin 2\theta - \sin\theta}{\cos\theta - \cos 2\theta} = \cot\frac{3\theta}{2}$

$\frac{d^2y}{dx^2} = \frac{-3}{2}\operatorname{cosec}^2\frac{3\theta}{2}.\left(\frac{d\theta}{dx}\right)$

$= \frac{-3}{2}\operatorname{cosec}^2\left(\frac{3\theta}{2}\right).\frac{1}{(2\cos\theta - 2\cos 2\theta)}$

$\left.\frac{d^2y}{dx^2}\right|\theta = \pi = \left(\frac{-3}{2}\right)\frac{1}{4\sin\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$

$= +\frac{3}{8}$

69. The length of the minor axis (along y – axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is:

(A) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (B) $\frac{1}{2}\sqrt{\frac{5}{3}}$

(C) $\frac{1}{2}\sqrt{\frac{11}{3}}$ (D) $\sqrt{\frac{5}{6}}$

Ans. C

Sol. $y = mx + c$, compare with $x + 6y = 8$

$\Rightarrow m = \frac{-1}{6}$

$c^2 = a^2m^2 + b^2$

$\Rightarrow \frac{16}{9} = \frac{a^2}{36} + \frac{4}{3}$

$\Rightarrow a^2 = 16$

$e = \sqrt{1 - \left(\frac{b}{a}\right)^2} = \sqrt{\frac{11}{12}}$

70. The following system of linear equation

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$ has

$x - 2y - 6z = 0$,

(A) infinitely many solutions, (x, y, z) satisfying $x = 2z$.

(B) no solution

(C) only the trivial solution.

(D) infinitely many solutions, (x, y, z) satisfying $y = 2z$.

Ans. A

Sol. $\begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$

$= 7(-20) - 6(-20) - 2(-10)$

$= -140 + 120 + 20 = 0$

So infinite non – trivial solution exist

71. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0, (k > 0)$ touch each other at a point, then the largest value of k is ____________.

Ans. 36

Sol. Two circles touches each other if $C_1C_2 = |r_1 \pm r_2|$

Distance between $C_2(3,0)$ and $C_1(0,4)$ is either $\sqrt{k} + 1 = 5$ or $|\sqrt{k} - 1| = 5$

$\Rightarrow k = 16$ or $k = 36$

$\Rightarrow$ maximum value of k is 36

72. If $C_r = {}^{25}C_r$ and $C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k,$ then k is equal to __________

Ans. 51

Sol. $\sum_{r=0}^{25}(4r+1)\,{}^{25}C_r = 4\sum_{r=0}^{25} r\,{}^{25}C_r + \sum_{r=0}^{25}{}^{25}C_r$

$= 4\sum_{r=1}^{25} r \times \frac{25}{r}\,{}^{24}C_{r-1} + 2^{25} = 100\sum_{r=1}^{25}{}^{24}C_{r-1} + 2^{25}$

$= 100.2^{24} + 2^{25} = 2^{25}(50+1) = 51.2^{25}$

So $k = 51$

73. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}, |\vec{b}| = 5, \vec{b}.\vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b} \times \vec{c}$, then $|\vec{a} \times (\vec{b} \times \vec{c})|$ is equal to____.

Ans. 30

Sol. $\vec{b}.\vec{c} = 10 \Rightarrow |\vec{b}||\vec{c}|\cos\left(\frac{\pi}{3}\right) = 10 \Rightarrow$

$5.|\vec{c}|.\frac{1}{2} = 10 \Rightarrow |\vec{c}| = 4$

Also, $\vec{a}.(\vec{b} \times \vec{c}) = 0$

$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}||\vec{b} \times \vec{c}|\sin\left(\frac{\pi}{2}\right)$

$\sqrt{3} \times |\vec{b}||\vec{c}|\sin\frac{\pi}{3} \times 1 = \sqrt{3} \times 5 \times 4 \times \frac{\sqrt{3}}{2} = 30$

74. If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}(\lambda \in R)$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ______

Ans. 3

Sol. Lines must be intersecting

$\Rightarrow (2s-1, 4s+3, 3s-1) = (2t-3, 6t-2, \lambda t+1)$

$2s-1 = 2t-3,\ 4s+3 = 6t-2,\ 3s-1 = \lambda t+1$

$\Rightarrow t = \frac{1}{2},\ s = -\frac{1}{2}, \lambda = -7$

distance of plane contains given lines from given plane is same as distance between point (–3, –2, 1) from given plane.

Required distance equal to $\frac{|-69+20-2+48|}{\sqrt{529+100+4}} = \frac{3}{\sqrt{633}} = \frac{k}{\sqrt{633}} \Rightarrow k = 3$

75. The number of terms common to the two A.P.'s 3, 7, 11,…………..407 and 2, 9, 16, …..709 is ______________

Ans. 14

Sol. First common term = 23

common difference $= 7 \times 4 = 28$

Last term ≤ 407

$\Rightarrow \quad 23 + (n-1) \times 28 \le 407$

$\Rightarrow \quad (n-1) \times 28 \le 384$

$\Rightarrow \quad n \le 13.71 + 1$

$n \le 14.71$

So $\quad$ n = 14