FIITJEE

Solutions to JEE (Main)-2020

JEE–Main–2020 –Jan–9–First–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. Two particles of equal mass 'm' have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is:

(A) $\frac{1}{3}mu^2$ (B) $\frac{1}{8}mu^2$
(C) $\sqrt{\frac{2}{3}}mu^2$ (D) $\frac{3}{4}mu^2$

Ans. **B**

Sol. Conserving momentum

$$mv\,\hat{i} + m\left(\frac{v}{2}\hat{i} + \frac{v}{2}\hat{j}\right) = 2m\,(v_1\hat{i} + v_2\hat{j})$$

On solving

$$v_1 = \frac{3v}{4} \text{ and } v_2 = \frac{v}{4}$$

Change in K.E.

$$\left[\frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{v}{2}\sqrt{2}\right)^2\right] - \left[\frac{1}{2}(2M)\left(\frac{9v^2}{16} + \frac{v^2}{16}\right)\right]$$

$$= \frac{3mv^2}{4} - \frac{5mv^2}{8} = \frac{mv^2}{8}$$

2. Radiation, with wavelength 6561 Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to:

(A) 0.8 eV (B) 1.6 eV
(C) 1.8 eV (D) 1.1 eV

Ans. **A**

Sol. $KE_{max} = E - \phi$

$$= \frac{12400}{\lambda(\text{in Å})} - \phi \quad \text{(in eV)}$$

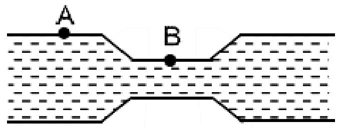
$$\therefore \quad r = \frac{\sqrt{2mKE}}{eB}$$

$$KE_{max} = \frac{r^2e^2B^2}{2m} \quad \text{(in J)}$$

$$= \frac{r^2eB^2}{2m} \quad \text{(in eV)}$$

$$\therefore \quad \phi = \frac{12400}{6561} - \frac{r^2eB^2}{2m} = 1.1 \text{ eV}$$

3. Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube. (density of water = 1000 kg m^{-3})



(Fig.)

(A) 1810 cm^3/s (B) 2420 cm^3/s
(C) 2720 cm^3/s (D) 3020 cm^3/s

Ans. **C**

Sol. Using equation of continuity

$$40\ V_A = 20\ V_B$$

$\Rightarrow$ $2V_A = V_B$

Using Bernoullies equation

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$\Rightarrow \quad P_A - P_B = \frac{1}{2}\rho\left(V_B^2 - V_A^2\right)$$

$$\Rightarrow \quad \Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right)$$

$$\Rightarrow \quad \Delta P = 500 \times \frac{3V_B^2}{4}$$

$$\Rightarrow \quad V_B = \sqrt{\frac{(\Delta P)\times 4}{1500}} = \sqrt{\frac{(700)\times 4}{1500}}\ m/s$$

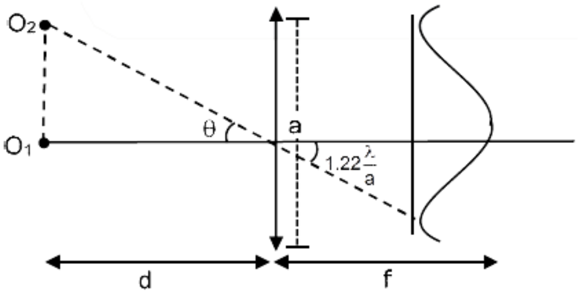
Volume flow rate = 20 × 100 × V_B = 2732 cm^3/s

4. The aperture diameter of a telescope is 5m. The separation between the moon and the earth is 4 × 10^5 km. With light of wavelength of 5500 Ä, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to:

(A) 200 m (B) 600 m
(C) 60 m (D) 20 m

Ans. **C**

Sol.



$$\theta = 1.22\frac{\lambda}{a}$$

Distance = $O_1O_2 = d\theta = 1.22\frac{\lambda}{a}d$

Distance = $O_1O_2 = \dfrac{1.22 \times 5893 \times 10^{-10} \times 4 \times 10^8}{5} \approx 57.5\ m$

$\therefore$ answer from options = 60 m (minimum distance)

5. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then, the combined body:

(A) starts moving in an elliptical orbit around the planet.
(B) continues to move in a circular orbit.
(C) Escapes from the Planet's Gravitational field.
(D) Falls vertically downwards towards the planet.

Ans. **A**

Sol. Conserving momentum

$$\frac{m}{2}\frac{v}{2}+mv=\left(m+\frac{m}{2}\right)V_f$$

$$V_f=\frac{5mV}{4\times\frac{3m}{2}}=\frac{5V}{6}$$

$v_f < v_{orb}$ (= v) thus the combined mass will go on to an elliptical path

$v_f < v_{orb}$ (= v)

6. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode and have mass $\frac{m}{4}$. The ratio of the specific heats $(C_V^A \text{ and } C_V^B)$ of gas A and B, respectively is

(A) 3 : 5
(B) 5 : 7
(C) 7 : 9
(D) 5 : 9

Ans. **B**

Sol. Molar heat capacity of A at constant volume = $\frac{5R}{2}$

Molar heat capacity of B at constant volume = $\frac{7R}{2}$

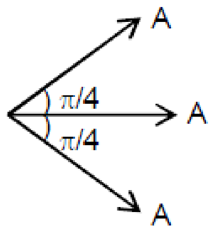
Dividing both, $\frac{(C_v)_A}{(c_v)_B}=\frac{5}{7}$

7. Three harmonic waves having equal frequency v and same intensity I_0, have phase angles $0, \frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to:

(A) $3I_0$
(B) I_0
(C) $0.2I_0$
(D) $5.8I_0$

Ans. **D**

Sol.



$$A_{res}=\left(\sqrt{2}+1\right)A$$

$$I_{res}=\left(\sqrt{2}+1\right)^2 I_0$$

$$=\left(3+2\sqrt{2}\right)I_0=5.8\,I_{0\,s}$$

8. A quantity f is given by $f=\sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of:

(A) energy
(B) momentum
(C) area
(D) volume

Ans. **A**

Sol. $[ML^2T^{-2}]$

$[hc] = [ML^3T^{-2}]$

$[c] = [LT^{-1}]$

$[G] = [M^{-1}L^3T^{-2}]$

9. If the screw on a screw-gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale the least count of the screw gauge is
(A) 0.001 cm (B) 0.001 mm
(C) 0.01 cm (D) 0.02 mm

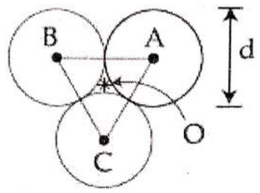
Ans. **A**

Sol. Pitch = $\frac{3}{6}$ = 0.5 mm

L.C. = $\frac{0.5\,\text{mm}}{50} = \frac{1}{100}\text{mm}$ = 0.01 mm

= 0.001 cm

10. Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is



(A) $\frac{15}{13}$ (B) $\frac{13}{15}$ (C) $\frac{23}{13}$ (D) $\frac{13}{23}$

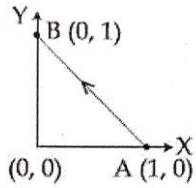
Ans. **D**

Sol. M. I. about P = $3\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}Md^2$

M. I. about B = $2\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M(d^2)\right] + \frac{2}{5}M\left(\frac{d}{2}\right)^2 = \frac{23}{10}Md^2$

Now ratio = $\frac{13}{23}$

11. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is: (all quantities are in SI units)



(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$

Ans. **C**

Sol. $W = \int \vec{F}\cdot d\vec{s}$

$= (-x\hat{i} + y\hat{j})\cdot(dx\hat{i} + dy\hat{j})$

$= \int_1^0 -x\,dx + \int_0^1 y\,dy$

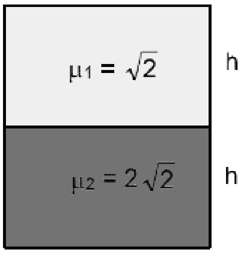
$= -\frac{x^2}{2}\Big|_1^0 + \frac{y^2}{2}\Big|_0^1 = \left(0 + \frac{1}{2}\right) + \left(\frac{1}{2}\right)$ = 1J

12. A vessel of depth 2h is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}.$ The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be
(A) $\frac{h}{\sqrt{2}}$ (B) $\frac{h}{2(\sqrt{2}+1)}$ (C) $\frac{3}{4}h\sqrt{2}$ (D) $\frac{h}{3\sqrt{2}}$

Ans. **C**

Sol. $d = \frac{h}{\sqrt{2}} + \frac{h}{2\sqrt{2}}$

$\Rightarrow \quad d = \frac{h}{\sqrt{2}} \times \frac{3}{2} = \frac{3\sqrt{2}\,h}{4}$



13. A particle moving with kinetic energy E has de Broglie wavelength λ. If energy ΔE ;is added to its energy, the wavelength become $\lambda/2$. Value of ΔE, is:
(A) E (B) 3E (C) 2E (D) 4E

Ans. **B**

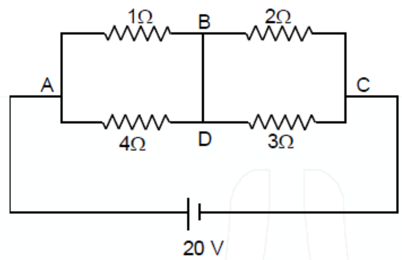
Sol. $\lambda = \frac{h}{\sqrt{2(KE)m}} \Rightarrow \lambda \propto \frac{1}{\sqrt{KE}}$

$\frac{\lambda}{\lambda/2} = \sqrt{\frac{KE_f}{KE_i}}$

$4KE_i = KE_f$

$\Rightarrow \quad \Delta E = 4KE_i - KE_i = 3KE = 3E$

14. In the given circuit diagram, a wire is joining points B and D. The current in this wire is:
(A) zero
(B) 2A
(C) 0.4 A
(D) 4A

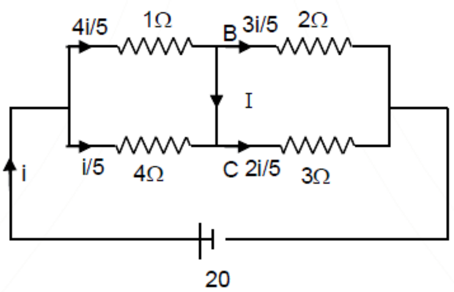


Ans. **B**

Sol. $R_{eff} = \frac{4}{5} + \frac{6}{5} = 2\,\Omega$

$i = \frac{20}{2} = 10A$

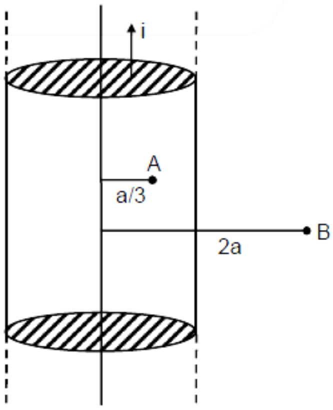
$I = \frac{4i}{5} - \frac{3i}{5} = +\frac{i}{5} = 2A$



15. A long, straight wire of radius a carries a current distributed uniformly over its cross-section. The ratio of the magnetic fields due to the wire at distance $\frac{a}{3}$ and 2a, respectively from the axis of the wire is:
(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) 2 (D) $\frac{3}{2}$

Ans. **A**

Sol.
$$B_A = \frac{\mu_0 ir}{2\pi a^2} = \frac{\mu_0 i \frac{a}{3}}{2\pi a^2} = \frac{\mu_0 i}{\pi a^2}\frac{a}{6} = \frac{\mu_0 i}{6\pi a}$$
$$B_B = \frac{\mu_0 i}{2\pi(2a)}$$
$$\frac{B_A}{B_B} = \frac{4}{6} = \frac{2}{3}$$



16. The electric fields of two plane electromagnetic plane waves in vacuum are given by
$\vec{E}_1 = E_0\hat{j}\cos(\omega t - kx)$ and
$\vec{E}_2 = E_0\hat{k}\cos(\omega t - ky)$
At t = 0, a particle of charge q is at origin with velocity $\vec{v} = 0.8\,c\hat{j}$ (c is the speed of light in vaccum). The instantaneous force experienced by the particle is:
(A) $E_0 q\left(0.8\hat{i} - \hat{j} + 0.4\hat{k}\right)$
(B) $E_0 q\left(0.4\hat{i} - 3\hat{j} + 0.8\hat{k}\right)$
(C) $E_0 q\left(0.8\hat{i} + \hat{j} + 0.2\hat{k}\right)$
(D) $E_0 q\left(-0.8\hat{i} + \hat{j} + \hat{k}\right)$

Ans. **C**

Sol. Magnetic field vectors associated with this electromagnetic wave are given by
$\vec{B}_1 = \frac{E_0}{c}\hat{k}\cos(kx - \omega t)$ & $\vec{B}_2 = \frac{E_0}{c}\hat{i}\cos(ky - \omega t)$
$\vec{F} = q\vec{E} + q(\vec{V}\times\vec{B})$
$= q(\vec{E}_1 + \vec{E}_2) + q(\vec{V}\times(\vec{B}_1 + \vec{B}_2))$
By putting the value of $\vec{E}_1, \vec{E}_2, \vec{B}_1$ & $\vec{B}_2$
The net Lorentz force on the charged particle is
$\vec{F} = qE_0\left[0.8\cos(kx - \omega t)\hat{i} + \cos(kx - \omega t)\hat{j} + 0.2\cos(ky - \omega t)\hat{k}\right]$
At t = 0 and at x = y = 0
$\vec{F} = qE_0[0.8\hat{i} + \hat{j} + 0.2\hat{k}]$

17. An electric dipole of moment $\vec{p} = (-\hat{i} - 3\hat{j} + 2\hat{k})\times 10^{-29}\text{cm}$ is at the origin (0, 0, 0). The electric field due to this dipole at $\vec{r} = +\hat{i} + 3\hat{j} + 5\hat{k}$ (note that $\vec{r}\cdot\vec{p} = 0$) is parallel to:
(A) $(-\hat{i} - 3\hat{j} + 2\hat{k})$
(B) $(+\hat{i} - 3\hat{j} - 2\hat{k})$
(C) $(-\hat{i} + 3\hat{j} - 2\hat{k})$
(D) $(+\hat{i} + 3\hat{j} - 2\hat{k})$

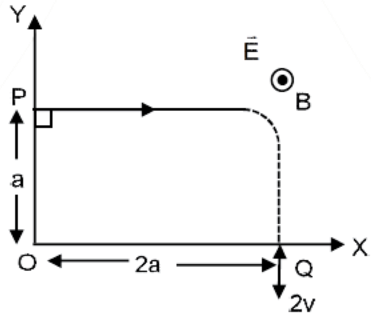
Ans. **D**

Sol. Since $\vec{p}\cdot\vec{r} = 0$
$\vec{E}$ must be antiparallel to $\vec{p}$
So, $\vec{E} = -\lambda(\vec{p})$
Where λ is a arbitrary positive constant
Now $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$
$\vec{A}\,||\,\vec{E}$

$\frac{a}{\lambda} = \frac{b}{3\lambda} = \frac{c}{-2\lambda} = k$

So, $\vec{A} = \lambda k(\hat{i} + 3\hat{j} - 2\hat{k})$

18. A charged particle of mass 'm' and charge 'q' moving under the influence of uniform electric field $E\vec{i}$ and a uniform magnetic field $B\vec{k}$ follows a trajectory from point P to Q as shown in figure. The velocities at P and Q are respectively, $v\vec{i}$ and $-2v\vec{j}$. Then which of the following statements (A, B, C, D) are the correct? (Trajectory shown is schematic and not to scale)



(a) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

(b) Rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$

(c) Rate of work done by both the fields at Q is zero.

(d) The difference between the magnitude of angular momentum of the particle at P and Q is 2 mav.

(A) (b), (c), (d)
(B) (a), (b), (c)
(C) (a), (c), (d)
(D) (a), (b), (c), (d)

Ans. **B**

Sol. (a) By work energy theorem

$W_{mag} + W_{ele} = \frac{1}{2}m(2v)^2 - \frac{1}{2}m(v)^2$

$0 + qE_0 2a = \frac{3}{2}mv^2$

$E_0 = \frac{3}{4}\frac{mv^2}{qa}$

(b) Rate of work done at A = power of electric force A = qE_0V

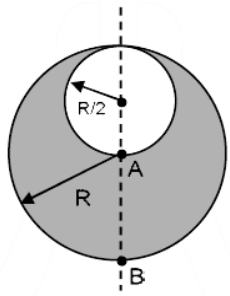
$= \frac{3}{4}\frac{mv^3}{a}$

(c) at Q, $\frac{dw}{dt} = 0$ for both forces

(d) $\Delta\vec{L} = (-m2v2a\hat{k}) - (-ma\hat{k})$

$|\Delta\vec{L}| = 3mva$

19. Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown the ratio $\frac{|\vec{E}_A|}{|\vec{E}_B|}$ of magnitude of electric field $\vec{E}_A$ and $\vec{E}_B$, respectively, at point A and B due to the remaining portion is:



(A) $\frac{18}{34}$
(B) $\frac{17}{54}$
(C) $\frac{18}{54}$
(D) $\frac{21}{34}$

Ans. **A**

Sol. For a solid sphere

$$E = \frac{\rho r}{3\varepsilon_0} \quad ; \quad E_A = \frac{-\rho R}{2(3\varepsilon_0)} \quad ; \quad |E_A| = \frac{-\rho R}{6\varepsilon_0}$$

Electric field at point B = $E_B = E_{1A} + E_{2A}$

E_{1A} = Electric field due to solid sphere of radius R at point B = $\frac{\rho R}{3\varepsilon_0}$

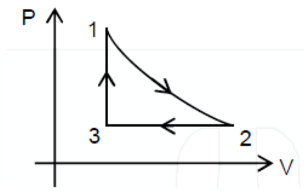
E_{2A} = Electric field due to solid sphere of radius R/2 (which having charge density $-\rho$)

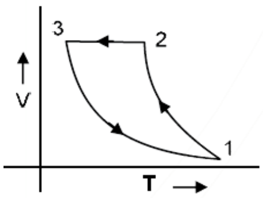
E_{2A} = R/2 = $-\frac{KQ' \times 4}{9R^2} = -\frac{\rho R}{54_{\varepsilon_0}}$

$$E_B = E_{1A} + E_{2A} = \frac{\rho R}{3\varepsilon_0} - \frac{\rho R}{54\varepsilon_0} = \frac{17\rho R}{54\varepsilon_0}$$

$$\left|\frac{E_A}{E_B}\right| = \frac{9}{17}$$

20. Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in the figure? Where, 1 $\rightarrow$ 2 is adiabatic. (Graphs are schematic and are not to scale)



(A)


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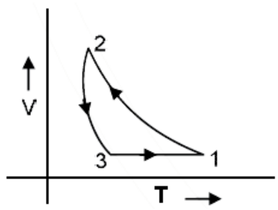
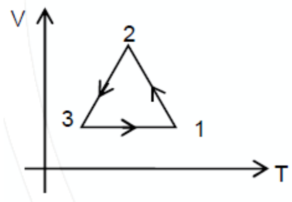
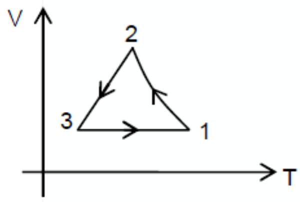


(C)


(D)


Ans. **D**

Sol. For process A – B

$PV = nRT$; as P increases

For process B – C

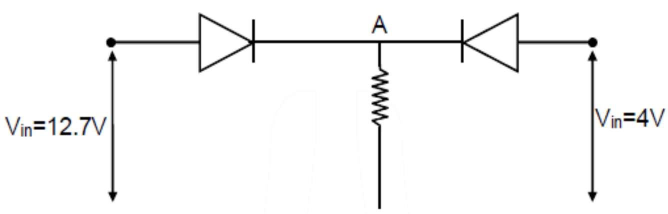
PV^γ = Constant

$\Rightarrow$ $TV^{\gamma-1}$ = Constant

For process C – A ; pressure is constant

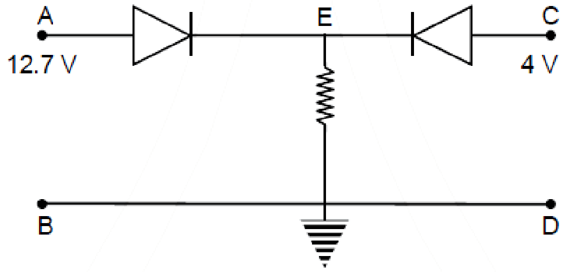
$V = kT$

21. Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased. Built in potential in each diode is 0.7 V. For the input voltages shown in the figure, the voltage (in Volts) at point A is __________.



Ans. **12**

Sol.

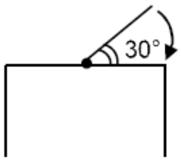


Let $V_B = 0$

Right diode is reversed biased and left diode is forward biased

$\therefore \quad V_E = 12.7 - 0.7 = 12$ Volt

22. One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\ s^{-1}$, where n is an integer. The value of n is __________.



Ans. **15**

Sol. $mg\frac{\ell}{2}\sin 30^{\circ} = \frac{1}{2}\frac{m\ell^2}{3}\omega^2$

Solving,

$\omega^2 = 15 \quad ; \quad \omega = \sqrt{15}$

23. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be __________.

Ans. **10**

Sol. $100 = \frac{L(0.25)}{0.025} \times 10^3$

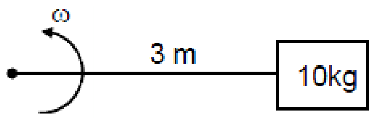
$\therefore \quad L = 100 \times 10^{-4}$ H

$= 10$ mH

24. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is (Breaking stress of wire = $4.8 \times 10^7\ Nm^{-2}$ and area of cross-section of the wire = $10^{-2}\ cm^2$) is

Ans. **4**

Sol.



$\frac{T}{A} = \sigma \quad \ldots(1)$

$T = m\omega^2\ell \quad \ldots(2)$

Solving,

$\omega = 4$ rad/s

25. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is __________.

Ans. **3**

Sol. $x^2 = at^2 + 2bt + c$

$2xv = 2at + 2b$

$xv = at + b$

$v^2 + ax = a$

$$ax = a - \left(\frac{at + b}{x}\right)^2$$

$$a = \frac{a(at^2 + 2bt + c) - (at + b)^2}{x^3}$$

$$a = \frac{ac - b^2}{x^3} \quad ; \quad a \propto x^{-3}$$

PART –B (CHEMISTRY)

26. If the magnetic moment of a dioxygen species is 1.73 B.M, it may be:
(A) O_2, O_2^- or O_2^+
(B) O_2 or O_2^-
(C) $O_2 + O_2^+$
(D) O_2^- or O_2^+

Ans. D

Sol. Magnetic moment = 1.73 BM
Unpaired electron = 1
$O_2 = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_2^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^1 = \pi^* 2p_y^1$
Hence O_2^-, O_2^+ have one unpaired electron.

27. The compound that cannot act both as oxidising and reducing agent is:
(A) H_2O_2
(B) H_2SO_3
(C) HNO_2
(D) H_3PO_4

Ans. D

Sol. $H_2O_2^{-1}$ $H_2SO_3^{+4}$
$H\overset{+3}{N}O_2$
$H_3\overset{+5}{P}O_4$
In H_3PO_4 phosphorus is in maximum oxidation state so cannot increase its oxidation number.

28. $[Pd(F)(Cl)(Br)(I)]^{2-}$ has n number of geometrical isomers. Then, the spin-only magnetic moment and crystal field stabilisation energy [CFSE] of $[Fe(CN)_6]^{n-6}$, respectively, are:
[Note: Ignore the pairing energy]
(A) 2.84 BM and –1.6 Δ_0
(B) 1.73 BM and –2.0 Δ_0
(C) 5.92 BM and 0
(D) 0 BM and –2.4 Δ_0

Ans. B

Sol. Number of geometrical isomers in square planar $[PdFClBr]^{2-}$ are 3
Hence, n = 3
$[Fe(CN)_6]^{3-}$
$Fe^{3+} = 3d^5$, according to CFT configuration planar is $t_{2g}^{221}e_g^{00}$
$\mu = \sqrt{n(n+2)} = 1.73$ B.M
CFSE = $-0.4\Delta_0 \times nt_{2g} + 0.6\Delta_0 \times n_{eq}$
$= -0.4\Delta_0 \times 5 = -2.0\Delta_0$

29. The electronic configurations of bivalent europium and trivalent cerium are: (atomic number: Xe = 54, Ce = 58, Eu = 63)
(A) [Xe] $4f^7 6s^2$ and [Xe] $4f^2 6s^2$
(B) [Xe] $4f^7$ and [Xe] $4f^1$
(C) [Xe] $4f^2$ and [Xe] $4f^7$
(D) [Xe] $4f^4$ and [Xe] $4f^9$

Ans. B

Sol. $Eu^{2+} \rightarrow [xe]4f^2$
Ce^{2+} : $[xe]4f^2$

30. For following reactions

$$A \xrightarrow{700\,K} \text{Product}$$

$$A \xrightarrow[\text{catalyst}]{500\,K} \text{Product}$$

it was found that the E_a is decreased by 30 kJ/mol in the presence of catalyst. If the rate remains unchanged, the activation energy for catalysed reaction is (Assume pre exponential factor is same):

(A) 135 kJ/mol (B) 105 kJ/mol
(C) 75 kJ/mol (D) 198 kJ/mol

Ans. B

Sol. $K_{catalyst} = K$

$$Ae^{-\frac{Ea_1}{RT_1}} = Ae^{-\frac{Ea_2}{RT_2}}$$

Ea_1 = energy of activation in presence of catalyst

$T_1 = 500$ K $T_2 = 700$ K

$$\frac{Ea_1}{T_1} = \frac{Ea_2}{T_2}$$

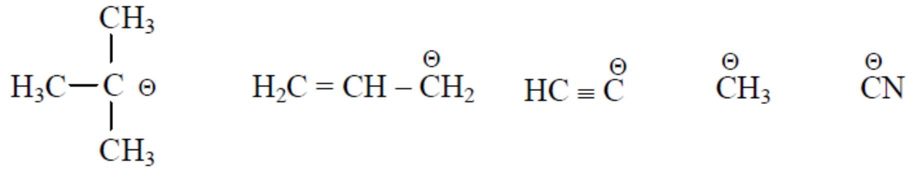
But $Ea_1 = Ea_2 - 30$

$$\frac{Ea_2 - 30}{500} - \frac{Ea_2}{700}$$

$5Ea_2 = 7Ea_2 - 210$

$$Ea_2 = \frac{210}{2} = 105 \text{ kJ/mole}$$

31. The increasing order of basicity for the following intermediates is (from weak to strong)

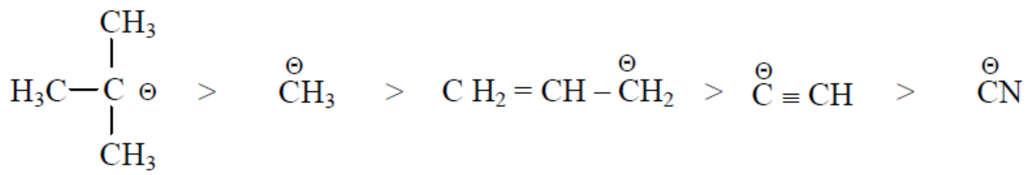


(i) (ii) (iii) (iv) (iv)

(A) (v) < (iii) < (ii) < (iv) < (i) (B) (iii) < (i) < (ii) < (iv) < (v)
(C) (v) < (i) < (iv) < (ii) < (iii) (D) (iii) < (iv) < (ii) < (i) < (v)

Ans. A

Sol. Basicity in inversely proportional to electronegativity.



(i) (iv) (ii) (iii) (iv)

i > iv > ii > iii > v

32. 'X' melts at low temperature and is a bad conductor of electricity in both liquid and solid state. X is:

(A) Silicon carbide (B) Mercury
(C) Zinc sulphide (D) Carbon tetrachloride

Ans. D

Sol. CCl_4 is a non conductor in solid and liquid phase.

33. The de Broglie wavelength of an electron in the 4^{th} Bohr orbit is:

(A) $6\pi a_0$ (B) $4\pi a_0$

(C) $2\pi a_0$ (D) $8\pi a_0$

Ans. D

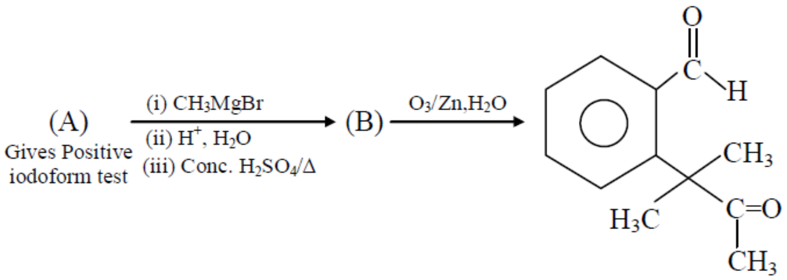
Sol. $2\pi r = n\lambda$

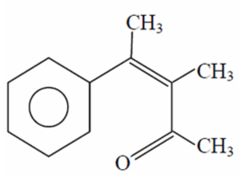
$$n = 4 \; \& \; r = a_0 \frac{n^2}{z} \Rightarrow 2\pi a_0 \frac{n^2}{z} = n\lambda$$

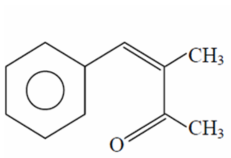
$$2\pi \frac{4^2 a_0}{1} = 4\lambda$$

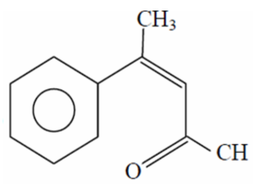
$$\lambda = 8\pi a_0$$

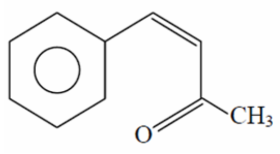
34. Identify (A) in the following reaction sequence:



(A) 

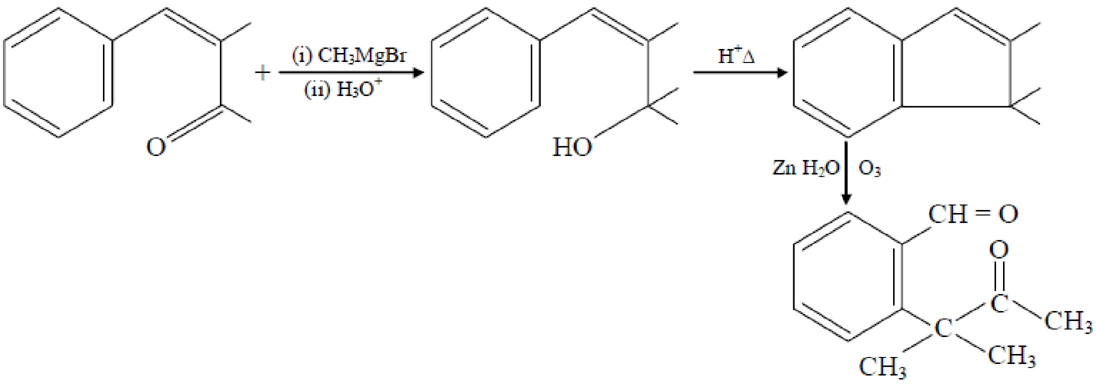
(B) 

(C) 

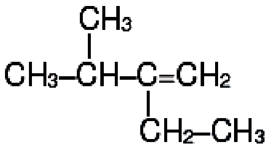
(D) 

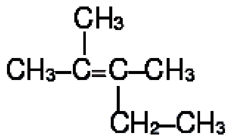
Ans. B

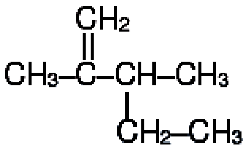
Sol.

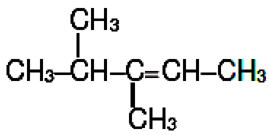


35. The major product (Y) in the following reactions is:

(A) 

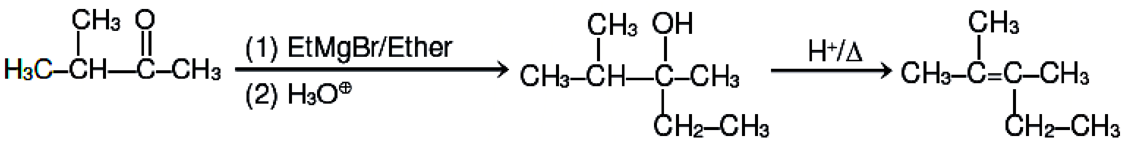
(B) 

(C) 

(D) 

Ans. B

Sol.



36. Complex X of composition $Cr(H_2O)_6Cl_n$ has a spin only magnetic moment of 3.83 BM. It reacts with $AgNO_3$ and shows geometrical isomerism. The IUPAC nomenclature of X is:

(A) Dichloridotetraaqua chromium (IV) chloride dihydrate

(B) Tetraaquadichlorido chromium (III) chloride dihydrate

(C) Tetraaquadichlorido chromium (IV) chloride dihydrate

(D) Hexaaqua chromium (III) chloride

Ans. B

Sol. $Cr(H_2O)_6Cl_n$

$(\mu_{complex})_{spin}$ = 3.8 B.M

n = 3

$Cr(H_2O)_6Cl_3$

Oxidation number of Cr should be +3

Compound so G.I so it will be $[Cr(H_2O)_4Cl_2]Cl.2H_2O$

IUPAC name = Tetraaquadichlorido chromium (III) chloride dihydrate

37. The acidic, basic and amphoteric oxides, respectively, are:

(A) Cl_2O, CaO, P_4O_{10} (B) MgO, Cl_2O, Al_2O_3

(C) Na_2O, SO_3, Al_2O_3 (D) N_2O_3, Li_2O, Al_2O_3

Ans. D

Sol. Non-metal oxides are acidic in nature

Alkali metal oxides are basic in nature

Al2O3 is amphoteric.

38. The K_{sp} for the following dissociation is 1.6×10^{-5}

$$PbCl_2(s) \rightleftharpoons Pb^{2+}(aq) + 2Cl^-(aq)$$

Which of the following choices is correct for a mixture of 300 mL 0.134 M $Pb(NO_3)_2$ and 100 mL 0.4 M NaCl?

(A) Not enough data provided
(B) $Q > K_{sp}$
(C) $Q < K_{sp}$
(D) $Q = K_{sp}$

Ans. B

Sol. $Q = [Pb^{2+}][Cl^-]^2$

$$= \left(\frac{300 \times 0.134}{400}\right)\left(\frac{100 \times 0.4}{400}\right)^2 = \frac{3 \times 0.134}{4}(0.1)^2$$

$Q = 1.005 \times 10^{-3}$

$Q > K_{sp}$

39. A chemist has 4 samples of artificial sweetener A, B, C and D. To identify these samples, he performed certain experiments and noted the following observations:

(i) A and D both form blue-violet colour with ninhydrin.
(ii) Lassaigne extract of C gives positive $AgNO_3$ test and negative $Fe_4[Fe(CN)_6]_3$ test.
(iii) Lassaigne extract of B and D gives positive sodium nitroprusside test.

Based on these observations which option is correct?

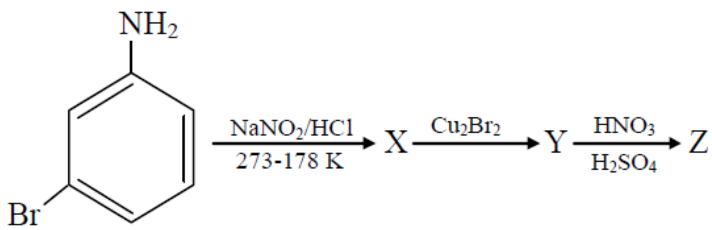
(A) A : Aspartame; B : Alitame; C : Saccharin; D : Sucralose
(B) A : Aspartame; B : Saccharin; C : Sucralose; D : Alitame
(C) A : Alitame; B : Saccharin; C : Aspartame; D : Sucralose
(D) A : Saccharin; B : Alitame; C : Sucralose; D : Aspartame

Ans. B

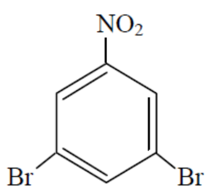
Sol. A – Aspartame
B – Saccharin
C – Sucralose
D – Alitame

(i) A & D give positive test with ninhydrin because both have free carboxylic and amine group.
(ii) C form precipitate with $AgNO_3$ because it has chlorine atoms.
(iii) B & D give positive test because both have S-atom.

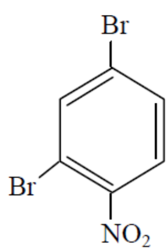
40. The major product Z obtained in the following reaction scheme is:

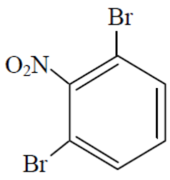


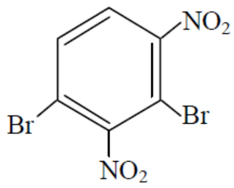
(A)



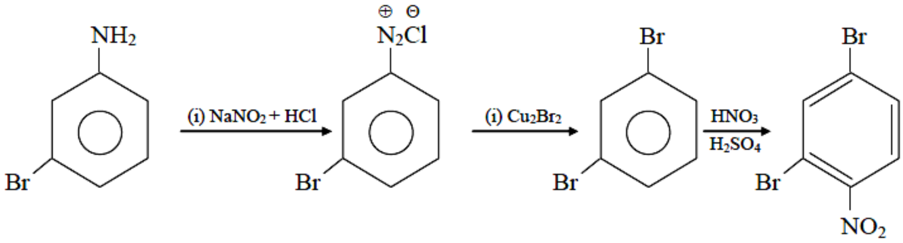
(B)



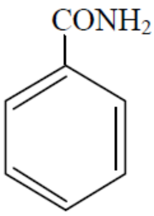
(C) 

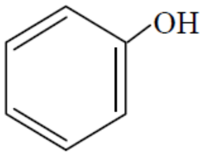
(D) 

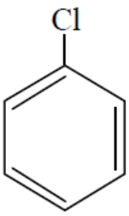
Ans. B

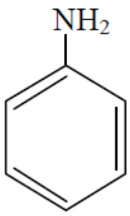
Sol. 

41. Which of these will produce the highest yield in Friedel Crafts reaction?

(A) 

(B) 

(C) 

(D) 

Ans. B

Sol. Aniline react lewis acid form anilinium complex. So phenol is most reactive among for nucleophellic substitution reaction.

42. B has a smaller first ionization enthalpy than Be. Consider the following statements:
(i) it is easier to remove 2p electron than 2s electron.
(ii) 2p electron of B is more shielded from the nucleus by the inner core of electrons than the 2s electrons of Be.
(iii) 2s electron has more penetration power than 2p electron
(iv) atomic radius of B is more than Be (atomic number B = 5, Be = 4)
The correct statements are:
(A) (i), (iii) and (iv)
(B) (i), (ii) and (iii)
(C) (i), (ii) and (iv)
(D) (ii), (iii) and (iv)

Ans. D
Sol. Radius of Boron is less than Radius of Be Hence IV statement is false. Rest are True. Statement

43. The correct order of heat of combustion for following alkadienes is:

(a) (b) (c)

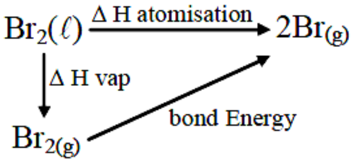
(A) (a) < (b) < (c)
(B) (a) < (c) < (b)
(C) (c) < (b) < (a)
(D) (b) < (c) < (a)

Ans. A

Sol. In isomers of Hydrocarbon heat of combustion depend upon their stability. Stability increase heat of combustion decrease.
Stability a > b > c
Heat of combustion c > b > a

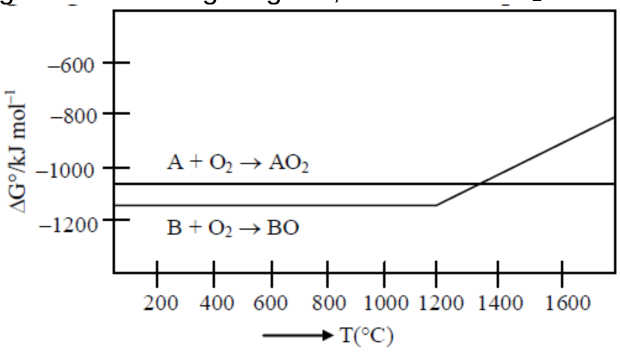
44. If enthalpy of atomisation for Br_2(A) is x kJ/mol and bond enthalpy for Br_2 is y kJ/mol, the relation between them:
(A) is x = y
(B) is x > y
(C) does not exist
(D) is x < y

Ans. B

Sol.


Δ H atomisation = Δ H Vap + Bond Energy
Hence.
x > y

45. According to the following diagram, A reduces BO_2 when the temperature is:



(A) > 1400 °C
(B) < 1400 °C
(C) < 1200 °C
(D) > 1200 °C but < 1400 °C

Ans. A

Sol. $A + BO_2 \longrightarrow B + AO_2$
ΔG = -ve
Only above 1400°C

46. The molarity of HNO_3 in a sample which has density 1.4 g/mL and mass percentage of 63% is (Molecular Weight of HNO_3 = 63)

Ans. 14

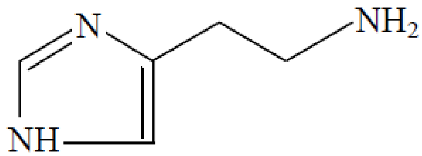
Sol. d = 1.4 g/mL
63/w/w

$$\text{Molarity} = \frac{63 \times 1.4}{63 \times 100} \times 1000 \text{ mole / litre} = 14 \text{ mole / L}$$

47. The mass percentage of nitrogen in histamine is___.

Ans. 37.84

Sol. Structure of Histamine is



Molecular formula = $C_5H_9N_3$

Molecular mass = 111

% of N = $\frac{42}{111}\times 100 = 37.84\%$

48. How much amount of NaCl should be added to 600 g of water (ρ = 1.00 g/mL) to decrease the freezing point of water to –0.2°C ?_____.

(The freezing point depression constant for water = 2 K kg mol^{-1})

Ans. 0.176

Sol. $\Delta T_f = 0.2$

$\Delta T_f = i\, K_f m$

i = 2 for NaCl

$0.2 = 2 \times 2 \times m$

$m = \frac{w}{58.5}\times\frac{1000}{600}$

$0.2 = 2 \times 2 \times \frac{w}{58.5}\times\frac{1000}{600}$

$w = \frac{0.2\times 58.5\times 600}{4\times 1000} = \frac{1.2\times 58.5}{40} = 0.176$

49. 108 g of silver (molar mass 108 g mol^{-1}) is deposited at cathode from $AgNO_3$ (aq) solution by a certain quantity of electricity. The volume (in L) of oxygen gas produced at 273 K and 1 bar pressure from water by the same quantity of electricity is__.

Ans. 0.568

Sol. $\left[n_{Ag}\right]_{deposit} = \frac{108}{108} = 1\,\text{mole}$

$Ag^{+} + 1e \longrightarrow Ag$

If charge is required for 1 mole of Ag

$H_2O \longrightarrow \frac{1}{2}O_2 + 2H^{+} + 2e^{-}$

2F charge deposit = ½ mole

1 F charge deposit = ¼ mole

PV = nRT

$V = \frac{1}{4}\times\frac{0.83\times 273}{1}$ = 5.675 L

50. The hardness of a water sample containing 10^{-3} M $MgSO_4$ expressed as $CaCO_3$ equivalents (in ppm) is________________.

(molar mass of $MgSO_4$ is 120.38 g/mol)

Ans. 100

Sol. Given: 10^{-3} M $MgSO_4$

i.e. 10^{-3} mole $MgSO_4$ present in 1 litre solution

$n_{MgSO_4} = n_{CaCO_3}$

Mass of $CaCO_3 = 10^{-3} \times 100$

ppm(in terms of $CaCO_3$) = $= \frac{10^{-3}\times 100}{1000}\times 10^{6} = 100$

PART–C (MATHEMATICS)

51. The product $2^{\frac{1}{4}}.4^{\frac{1}{16}}.8^{\frac{1}{48}}.16^{\frac{1}{128}}$.......... to ∞ is equal to

(A) $2^{\frac{1}{2}}$ (B) 2

(C) 1 (D) $2^{\frac{1}{4}}$

Ans. B

Sol. $2^{\frac{1}{4}+\frac{2}{16}+\frac{3}{48}+..........\infty}$

$= 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...........\infty} = \sqrt{2}$

52. Let the observations $x_i\left(1 \le i \le 10\right)$ satisfy the equation $\sum_{i=1}^{10}\left(x_i - 5\right) = 10$ and $\sum_{i=1}^{10}\left(x_i - 5\right)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, ..., x_{10} - 3$, then the ordered pair $\left(\mu, \lambda\right)$ is equal to:

(A) (6, 6) (B) (3, 3)

(C) (3, 6) (D) (6, 3)

Ans. B

Sol. Mean $\left(x_1 - 5\right) = \frac{\sum\left(x_i - 5\right)}{10} = 1$

$\therefore \lambda = \left\{\text{Mean}\left(x_i - 5\right)\right\} + 2 = 3$

$\mu = \text{var}\left(x_i - 5\right) = \frac{\sum\left(x_i - 5\right)^2}{10} - \frac{\sum\left(x_i - 5\right)}{10} = 3$

53. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and $\left(e_1, e_2\right)$ is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to:

(A) 16 (B) 14

(C) 17 (D) 15

Ans. C

Sol. $e_1 = \sqrt{1 - \frac{4}{18}} = \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$

$e_2 = \sqrt{1 + \frac{4}{9}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{9}$

$15e_1^2 + 3e_2^2 = k \Rightarrow k = 15\left(\frac{7}{9}\right) + 3\left(\frac{13}{9}\right)$

$\therefore \quad k = 16$

54. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}, B = \text{adj} A$ and $C = 3A$, then $\frac{|\text{adj}B|}{|C|}$ is equal to:

(A) 72 (B) 8
(C) 16 (D) 2

Ans. B

Sol. $|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = \left((9+4)-1(3-4)+2(-1-3)\right) = 13+1-8 = 6$

$|\text{adj}B| = |\text{adj}\,\text{adj}\,A| = |A|^{(n-1)^2} = |A|^4 = (36)^2$

$|C| = |BA| = 3^3 \times 6$

$\frac{|\text{adj}B|}{|C|} = \frac{36 \times 36}{3^3 \times 6} = 8$

55. If $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x} & ; x < 0 \\ b & ; x = 0 \\ \frac{\left(x+3x^2\right)^{1/3}}{x^{4/3}} & ; x > 0 \end{cases}$

(A) –2 (B) –1
(C) 0 (D) 1

Ans. C

Sol. LHL = a + 3

f (0) = b

RHL $= \lim_{h \to 0} \left(\frac{(1+3h)^{\frac{1}{3}} - 1}{h} \right) = 1$

$\therefore \quad a = -2$

$\quad b = 1$

$\therefore \quad a + 2b = 0$

56. If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is 336 k, then k is equal to:

(A) 6 (B) 7
(C) 4 (D) 8

Ans. D

Sol. Number of numbers

$= 8 \times 8 \times 7 \times 6 = 2688 = 336k \Rightarrow k = 8$

			2	

57. In a box, there are 20 cards, out of which 10 are labeled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A – card is obtained. The probability that the second A – card appears before the third B – card is:

(A) $\frac{9}{16}$ (B) $\frac{15}{16}$

(C) $\frac{13}{16}$ (D) $\frac{11}{16}$

Ans. D

Sol. AA + ABA + BAA + ABBA + BBAA + BABA

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$$

58. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^{x} + 1 = 0$ is:

(A) 2 (B) 4

(C) 3 (D) 1

Ans. D

Sol. Let $e^{x} = t \in (0, \infty)$

Given equation $t^4 + t^3 - 4t^2 + t + 1 = 0$

$$t^2 + t - 4 + \frac{1}{t} + \frac{1}{t^2} = 0$$

$$\left(t^2 + \frac{1}{t^2}\right) + \left(t + \frac{1}{t}\right) - 4 = 0$$

Let $t + \frac{1}{t} = \alpha$

$$\left(\alpha^2 - 2\right) + \alpha - 4 = 0$$

$$\alpha^2 + \alpha - 6 = 0$$

$$\alpha = -3, 2$$

$$\Rightarrow \alpha = 2$$

$$\Rightarrow e^{x} + e^{-x} = 2$$

$x = 0$ only solution

59. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/ min. When the thickness of ice is 5 cm, then the rate (in cm/min) at which of the thickness of ice decreases, is:

(A) $\frac{5}{6\pi}$ (B) $\frac{1}{36\pi}$

(C) $\frac{1}{18\pi}$ (D) $\frac{1}{54\pi}$

Ans. C

Sol. Let thickness = x cm

Total volume $v = \frac{4}{3}\pi(10 + x)^3$

$$\frac{dv}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt} \quad \text{……….(i)}$$

Given $\frac{dv}{dt} = 50\ cm^3 / min$

At $x = 5\,cm$

$$50 = 4\pi(10+5)^2 \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{18\pi} cm/min$$

60. The value of $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x} dx$ is equal to:

(A) 2π (B) 4π

(C) π^2 (D) $2\pi^2$

Ans. C

Sol. $\int_0^{\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x} + \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x} dx = \int_0^{\pi} \frac{2\pi\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$= 2\pi \int_0^{\pi/2} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} + \frac{\cos^8 x}{\sin^8 x + \cos^8 x} dx$

$= 2\pi \int_0^{\pi/2} 1\, dx = 2\pi \times \frac{\pi}{2} = \pi^2$

61. Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z+3i|$ is:

(A) $\frac{15}{4}$ (B) $2\sqrt{3}$

(C) $\sqrt{10}$ (D) $\frac{7}{2}$

Ans. D

Sol. $x^2 + (y-1)^2 = x^2 + (y+2)^2$

$-2y + 1 = 4y + 4$

$6y = -3 \Rightarrow y = -\frac{1}{2}$

$x^2 + y^2 = \frac{25}{4} \Rightarrow x^2 = \frac{24}{4} = 6$

$\Rightarrow \quad z = \pm\sqrt{6} - \frac{i}{2}$

$|z+3i| = \sqrt{6 + \frac{25}{4}} = \sqrt{\frac{49}{4}}$

$|z+3i| = \frac{7}{2}$

62. Negation of the statement:

$\sqrt{5}$ is an integer or 5 is irrational' is:

(A) $\sqrt{5}$ is an integer and 5 is irrational

(B) $\sqrt{5}$ is not an integer or 5 is not irrational

(C) $\sqrt{5}$ is not an integer and 5 is not irrational

(D) $\sqrt{5}$ is irrational or 5 is an integer

Ans. C

Sol. $\sqrt{5}$ is not an integer and 5 is not an irrational number $\sim(p \vee q) = \sim p \wedge \sim q$

63. Let C be the centroid of the triangle with vertices (3, –1), (1, 3) AND (2, 4). Let P be the point of intersection of the lines $x+3y-1=0$ and $3x-y+1=0$. Then the line passing through the points C and P also passes through the point:

(A) (–9, –7) (B) (–9, –6)
(C) (7, 6) (D) (9, 7)

Ans. B

Sol. D (2, 2)

Point of intersection $P\left(-\frac{1}{5}, \frac{2}{5}\right)$ equation of line DP $8x-11y+6=0$

64. If for some α and β in R, the intersection of the following three planes

$x+4y-2z=1$

$x+7y-5z=\beta$

$x+5y+\alpha z=5$

is a line in R^3, then $\alpha+\beta$ is equal to:

(A) 0 (B) 2
(C) 10 (D) –10

Ans.

Sol. $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$

$(7\alpha+25)-(4\alpha+10)+(-20+14)=0$

$3\alpha+9=0 \Rightarrow \alpha=-3$

Also $D_z = 0 \Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0$

$1(35-5\beta)-(15)+1(4\beta-7)=0$

$\beta=13$

65. Let f be any function continuous on $[a,b]$ and twice differentiable on (a, b). If for all $x \in (a,b), f'(x)>0$ and $f''(x)<0$, then for any $c \in (a,b) \frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than:

(A) $\frac{c-a}{b-c}$ (B) $\frac{b+a}{b-a}$
(C) $\frac{b-c}{c-a}$ (D) 1

Ans.

Sol. Let use LMVT for $x \in [a,c]$

$\frac{f(c)-f(a)}{c-a} = f'(\alpha), \alpha \in (a,c)$

also use LMVT for $x \in [c,b]$

$\frac{f(b)-f(c)}{b-c} = f'(\beta), \beta \in (c,b)$

$\because f''(x) < 0 \Rightarrow f'(x)$ is decreasing

$f'(\alpha) > f'(\beta)$

$\frac{f(c)-f(a)}{c-a} > \frac{f(b)-f(c)}{b-c}$

$\frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c}$ ($\because$ f (x) is increasing)

66. The value of $\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right)$ is:

(A) $\frac{1}{\sqrt{2}}$
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) $\frac{1}{2\sqrt{2}}$

Ans.

Sol. $\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8}-3\cos\frac{\pi}{8}\right]+\sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8}-4\sin^3\frac{\pi}{8}\right]$

$=4\cos^6\frac{\pi}{8}-4\sin^6\frac{\pi}{8}-3\cos^4\frac{\pi}{8}+3\sin^4\frac{\pi}{8}$

$=4\left[\left(\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}\right)\right]\left[\left(\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}+\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)\right]-3\left[\left(\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}\right)\right]$

$=\cos\frac{\pi}{4}\left[4\left(1-\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)-3\right]=\frac{1}{\sqrt{2}}\left[1-\frac{1}{2}\right]=\frac{1}{2\sqrt{2}}$

67. A circle touches the y – axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle?

(A) $3x-4y-24=0$
(B) $3x+4y-6=0$
(C) $4x-3y+17=0$
(D) $4x+3y-8=0$

Ans. D

Sol. Equation of family of circle

$(x-0)^2+(y-4)^2+\lambda x=0$

$\Rightarrow$ passes (2, 0)

$4+16+2\lambda=0 \Rightarrow \lambda=-10$

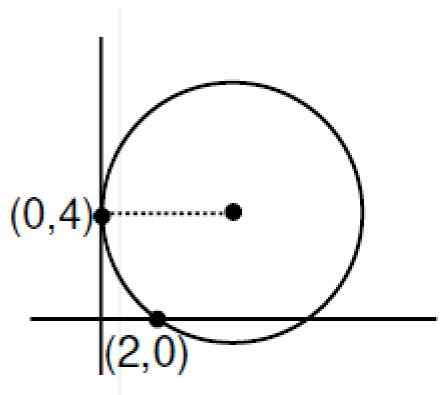
$x^2+y^2-10x-8y+16=0$

centre (5, 4). $R=\sqrt{25+16-16}=5$

Check the options.

Option (4)

$\left|\frac{4\times5+3\times4-8}{5}\right|=\frac{24}{5}\neq5$



68. If for all real triplets (a, b, c), $f(x)=a+bx+cx^2$; then $\int_0^1 f(x)dx$ is equal to:

(A) $\frac{1}{2}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$
(B) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$
(C) $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$
(D) $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$

Ans. B

Sol. $\int_0^1 \left(a+bx+cx^2\right)dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}\Big|_0^1 = a+\frac{b}{2}+\frac{c}{3}$

$f(1)=a+b+c$

$f(0)=a$

$f\left(\frac{1}{2}\right)=a+\frac{b}{2}+\frac{c}{4}$

Now $\frac{1}{6}\left(f(1)+f(0)+4f\left(\frac{1}{2}\right)\right)=\frac{1}{6}\left(a+b+c+a+4\left(a+\frac{b}{2}+\frac{c}{4}\right)\right)$

$=\frac{1}{6}(6a+3b+2c)=a+\frac{b}{2}+\frac{c}{3}$

69. If $f'(x)=\tan^{-1}(\sec x+\tan x), -\frac{\pi}{2}<x<\frac{\pi}{2},$ and $f(0)=0$, then $f(1)$ is equal to:

(A) $\frac{\pi-1}{4}$ (B) $\frac{\pi+1}{4}$

(C) $\frac{\pi+2}{4}$ (D) $\frac{1}{4}$

Ans. B

Sol. $f'(x)=\tan^{-1}(\sec x+\tan x)=\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

$=\tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}\right)=\tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right)$

$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)=\frac{\pi}{4}+\frac{x}{2}$

$(f'(x))dx=\frac{\pi}{4}+\frac{x}{2}dx$

$f(x)=\frac{\pi}{4}x+\frac{x^2}{4}+c$

$f(0)=c=0 \Rightarrow f(x)=\frac{\pi}{4}x+\frac{x^2}{4}$

So $f(1)=\frac{\pi+1}{4}$

70. The integral $\int \frac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ is equal to:

(where C is a constant of integration)

(A) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-13/7}+C$ (B) $-\left(\frac{x-3}{x+4}\right)^{-1/7}+C$

(C) $\left(\frac{x-3}{x+4}\right)^{1/7}+C$ (D) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{3/7}+C$

Ans.

Sol. $\int\left(\frac{x-3}{x+4}\right)^{\frac{-6}{7}}\frac{1}{(x+4)^2}dx$

Let $\frac{x-3}{x+4}=t^7$,

$\frac{7}{(x+4)^2}dx=7t^6dt$

$\int t^{-6}t^6\,dt=t+c$

71. The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is ___________

Ans. 8

Sol. Let $A(1,-1,3), B(2,-4,11)$

$\overrightarrow{AB}=\hat{i}-3\hat{j}+8\hat{k}$

C (–1, 2, 3), D (3, –2, 10)

$\overrightarrow{CD}=4\hat{i}-4\hat{j}+7\hat{k}$

projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}=\frac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{CD}|}$

$=\left(\frac{4+12+56}{\sqrt{16+16+49}}\right)=\frac{72}{9}=8$

72. If the vectors, $\vec{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k},\ \vec{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and $\vec{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k}\,(a\in R)$ are coplanar and $3(\vec{p}.\vec{q})^2-\lambda|\vec{r}\times\vec{q}|^2=0$, then the value of λ is _________.

Ans. 1

Sol. $\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix}=0\Rightarrow a+1+a+a=0$

$\Rightarrow a=-\frac{1}{3}$

$\vec{P}=\frac{2}{3}\hat{i}-\frac{1}{3}\hat{j}-\frac{1}{3}\hat{k}$

$\vec{Q}=\frac{1}{3}\left(-\hat{i}+2\hat{j}-\hat{k}\right)$

$\vec{R}=\frac{1}{3}\left(-\hat{i}-\hat{j}+2\hat{k}\right)$

$\vec{P}.\vec{Q}=\frac{1}{9}\begin{vmatrix} i & j & k \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}=\frac{1}{9}\left(i(4-1)-j(-2-1)+k(1+2)\right)$

$=\frac{1}{9}(3i+3j+3k)=\frac{i+j+k}{3}$

$|\vec{R}\times\vec{Q}|=\frac{1}{3}\sqrt{3}\Rightarrow|\vec{R}\times\vec{Q}|^2=\frac{1}{3}$

$$3\left(\vec{P}\times\vec{Q}\right)^2-\lambda\left|\vec{R}\times\vec{Q}\right|^2=0$$

$$3.\frac{1}{9}-\lambda.\frac{1}{3}=0\Rightarrow\lambda=1$$

73. If for $x\geq 0$, $y=y(x)$ is the solution of the differential equation, $(x+1)dy=\left((x+1)^2+y-3\right)dx, y(2)=0$, then $y(3)$ is equal to _______________

Ans. 3

Sol. $\frac{dy}{dx}=(1+x)+\left(\frac{y-3}{1+x}\right)$

$$\frac{dy}{dx}-\frac{1}{(1+x)}y=(1+x)-\frac{3}{(1+x)}$$

$$\text{I.F.}=e^{-\int\frac{1}{(1+x)}dx}=\frac{1}{(1+x)}$$

$$\therefore\quad \frac{d}{dx}\left(\frac{y}{1+x}\right)=1-\frac{3}{(1+x)^2}$$

$$\frac{y}{1+x}=x+3(1+x)^{-1}+c$$

$$y=(1+x)\left[x+\frac{3}{(1+x)}+c\right]$$

$\therefore$ at $x=2, y=0$ $\therefore$ $0=3(2+1+c)\Rightarrow c=-3$

$\therefore$ at x = 3, y = 3

74. The coefficient of x^4 in the expansion of $\left(1+x+x^2\right)^{10}$ is ______________.

Ans. 615

Sol. General term $\frac{10!}{\alpha!\beta!\gamma!}x^{\beta+2\gamma}$

for coefficient of $x^4\Rightarrow\beta+2\gamma=4$

$\gamma=0,\beta=4,\alpha=6 \quad\Rightarrow\quad \frac{10!}{6!4!0!}=210$

$\gamma=1,\beta=2,\alpha=7 \quad\Rightarrow\quad \frac{10!}{7!2!1!}=360$

$\gamma=2,\beta=0,\alpha=8 \quad\Rightarrow\quad \frac{10!}{8!0!2!}=45$

Total = 615

75. The number of distinct solutions of the equation, $\log_{\frac{1}{2}}|\sin x|=2-\log_{\frac{1}{2}}|\cos x|$ in the interval $[0,2\pi]$, is _________.

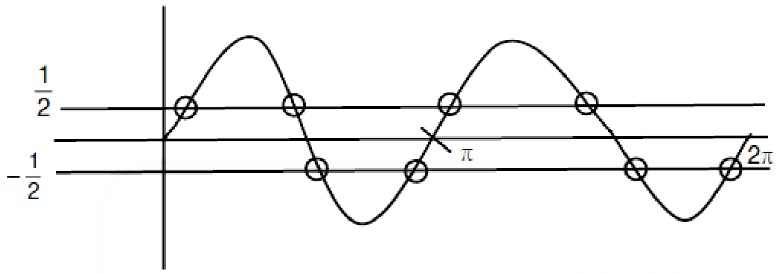
Ans. 8

Sol. $\log_{1/2}|\sin x|=2-\log_{1/2}|\cos x|$

$$\log_{1/2}|\sin x\cos x|=2$$

$$|\sin x \cos x| = \frac{1}{4}$$

$$\sin 2x = \pm\frac{1}{2}$$



Number of solution = 8.