FIITJEE

Solutions to JEE (Main)-2020

JEE–Main–2020 –Sept–6–Second–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. **C**

Sol. **Loop-I :**

$-20 + 2i_1 + 10(i_1 + i_2) + 5i_1 = 0$

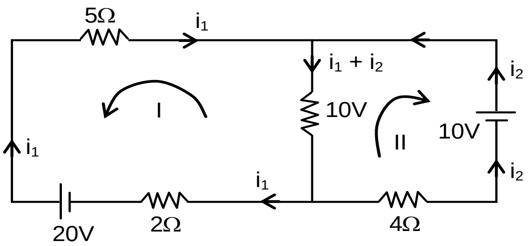
$17i_1 + 10i_2 = 20 \quad \ldots(1)$

Loop-II :

$-10 + 4i_2 + 10(i_1 + i_2) = 0$

$10i_1 + 14i_2 = 10 \quad \ldots(2)$

Equation (1) × 10 – Equation (2) × 17



$170i_1 + 100i_2 = 200$

$-170i_1 + 238i_2 = 170$

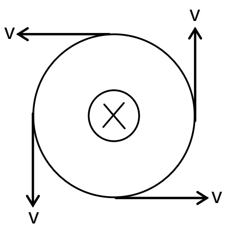
$-138i_2 = 30$

$$i_2 = -\frac{30}{138} = -0.217 \text{ A}$$

"–ve" sign indicates that current flows from "+ve" to "–ve" terminal in 10 V battery.

2. **D**

Sol. $\therefore \quad \frac{M}{L} = \frac{q}{2m}$



$\therefore \quad M = \frac{q}{2m}(mvr)$

$$M = \frac{qv}{2}\left(\frac{mv}{qB}\right)$$

$$M = \frac{mv^2}{2B}$$

As seen from figure Magnetic moment is opposite to field

So $\vec{M} = -\frac{mv^2\,\vec{B}}{2B^2}$

3. **A**

Sol. In steady state rate of flow of heat in all three rods are same.

$$\frac{dQ}{dt} = \frac{k_1A\,(100-70)}{\ell} = \frac{k_2A\,(70-20)}{\ell} = \frac{k_3A\,(20-0)}{\ell}$$

$30k_1 = 50k_2 = 20k_3$

$\therefore\; k_1 : k_3 = 2 : 3 \;\;\&\;\; k_2 : k_3 = 2 : 5$

4. **B**

Sol. By conservation of energy

$(K.E. + P.E.)_{initial} = (K.E. + P.E.)_{final}$

$$0 + \left(\frac{1}{4\pi\epsilon_0}\,\frac{2P}{a^3}\right) \times P = 2 \times \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{\frac{P^2}{2\pi\epsilon_o\ ma^3}} = \frac{P}{a}\sqrt{\frac{1}{2\pi\epsilon_o ma}}$$

5. **A**

Sol. Given wave is moving in "–ve" x-direction and the given magnetic field is along "+ve" z-direction. Since $C = \frac{E_0}{B_0}$

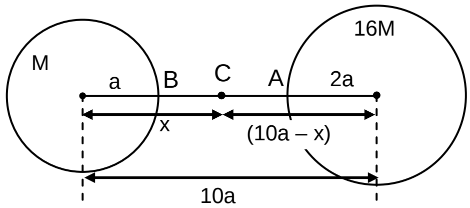
$E_0 = CB_0 = 1.2 \times 10^{-7} \times 3 \times 10^8 = 36$ N/C

Also $\because\ \vec{S} = \frac{1}{2\mu_o}\left(\vec{E} \times \vec{B}\right)$ So electric field is along "-ve" y-direction

$$\therefore\ \vec{E}(x, t) = \left[-36\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{j}\right]\frac{V}{m}$$

6. **D**

Sol. When a body fired from A, it should just cross the point 'C' where the gravitational field is zero.



$$\frac{GM}{x^2} = \frac{G(16M)}{(10a - x)^2} \Rightarrow \frac{1}{x} = \frac{4}{(10a - x)}$$

$\Rightarrow$ 10 a – x = 4x $\Rightarrow$ x = 2a

Now, potential at A, $V_A = -\frac{GM}{8a} - \frac{16\,GM}{2a} = -\frac{65\,GM}{8a}$

Potential at C, $V_C = -\frac{GM}{2a} - \frac{16\,GM}{8a} = -\frac{5\,GM}{2a}$

$$W = (V_C - V_A)m = \frac{1}{2}mv^2$$

$$v^2 = 2\,(V_C - V_A) = 2\left[\left(-\frac{5\,GM}{2a}\right) - \left(-\frac{65\,GM}{8a}\right)\right]$$

$$v^2 = \frac{45\,GM}{4a}$$

$$v = \frac{3}{2}\sqrt{\frac{5\,GM}{a}}$$

7. **A**

Sol. $\frac{mdv_x}{dt} = kv_y$ …(1) and $\frac{mdv_y}{dt} = kv_x$ …(2)

$$\frac{(2)}{(1)}\quad \frac{dv_y}{dv_x} = \frac{v_x}{v_y}$$

$V_y dv_y = v_x dv_x$

$v_y^2 = v_x^2 + C$

$v_y^2 - v_x^2 = C$ = Constant

Now, $\vec{v} \times \vec{a} = (v_x\hat{i} + v_y\hat{j}) \times \frac{k}{m}(v_y\hat{i} + v_x\hat{j})$

$= \frac{k}{m}\left[v_x^2\,\hat{k} - v_y^2\,\hat{k}\right] = \frac{k}{m}(v_x^2 - v_y^2)\,\hat{k}$ = Constant.

8. **D**

Sol. Using conservation of momentum

$m_1\overrightarrow{u_1} + m_2\overrightarrow{u_2} = m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2}$

$2m_2(\sqrt{3}\hat{i} + \hat{j}) + 0 = 2m_2\left(\hat{i} + \sqrt{3}\,\hat{j}\right) + m_2\vec{v}_2$

$\overrightarrow{V_2} = 2(\sqrt{3} - 1)(\hat{i} - \hat{j})$

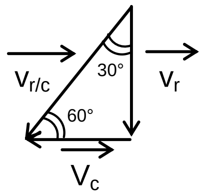
Angle between $\overrightarrow{v_1} \cdot \overrightarrow{v_2}$

$$\text{Cos}\,\theta = \frac{\overrightarrow{v_1} \cdot \overrightarrow{v_2}}{[v_1][v_2]} = \frac{2(\sqrt{3}-1)(1-\sqrt{3})}{2 \times 2\sqrt{2}(\sqrt{3}-1)} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$\Rightarrow \quad \theta = 105°$

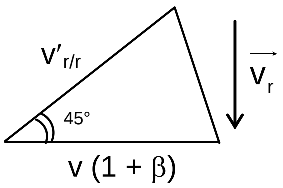
9. **D**

Sol.



$$\text{Tan } 60 = \frac{v_r}{v_c} = \frac{v_r}{v}$$

$v_r = v\sqrt{3} \qquad \ldots(1)$



$$\text{Tan } 45° = \frac{v_r}{v(1+\beta)}$$

$V_r = v(1 + \beta) \quad \ldots(2)$

By (1) to (2)

$v\sqrt{3} = v(1+\beta)$

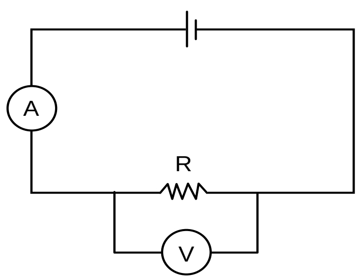
$\beta = \sqrt{3} - 1 = 0.73$

10. **C**

Sol. Total mass of reactant should be greater than that of product.
This condition is only fulfilled in case-3

11. **C**

Sol. In Ohm's law experiment, ammeter is used in series because in series same current will flow through it. But voltmeter is used in parallel to resistor to measure the potential difference across it.



12. **C**

Sol. (i) Inside the shell r < R

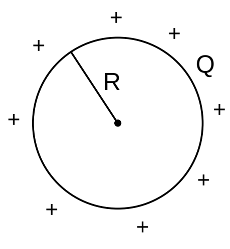
$E = 0 \Rightarrow F = 0$

(ii) On the surface r = R

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$$

$$\Rightarrow \quad F = \frac{1}{4\pi\epsilon_0}\frac{Qq}{R^2}$$

(iii) Outside the shall r > R



$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$\Rightarrow \quad F = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$$

13. **D**

Sol. Since significant figures show the degree of correctness of any measurement, so in any mathematical calculation we cannot increase the number of significant digits. Because the four reading has 3 significant digits so the answer should also have 3 significant digits only.

14. **D**

Sol. $P = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$

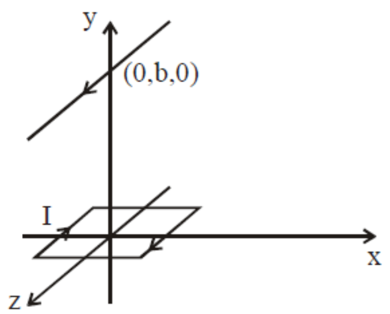
$P = \frac{2(\mu - 1)}{R} \qquad \ldots(1)$

$1.5\,P = \frac{(\mu - 1)}{R'}$; $\frac{1}{1.5} = \frac{2R'}{R}$

$R' = \frac{R}{3}$

15. **B**

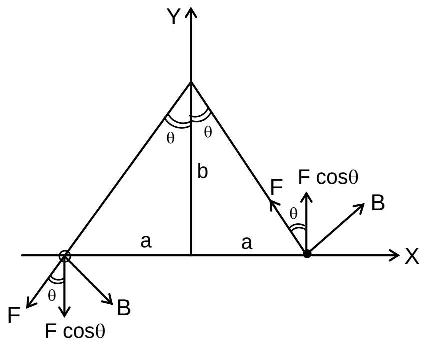
Sol. F = I B (2a)



$F = I\left(\frac{\mu_0 I}{2\pi\sqrt{a^2 + b^2}}\right)2a$

$F = \frac{\mu_0 I^2 a}{\pi\sqrt{a^2 + b^2}}$

Torque $\tau = 2F(\cos\theta) \times a$



$\tau = \frac{2 \times \mu_0 I^2 a^2}{\pi\sqrt{a^2 + b^2}} \times \frac{b}{\sqrt{a^2 + b^2}}$

$\tau = \frac{2\mu_0 I^2 a^2 b}{\pi(a^2 + b^2)}$

16. **D**

Sol. Using Bernoulli's equation

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

For horizontal tube $h_1 = h_2$

$P + \frac{1}{2}\rho v^2 = \frac{P}{2} + \frac{1}{2}\rho V^2$

$\frac{1}{2}\rho V^2 = \frac{P}{2} + \frac{1}{2}\rho v^2$

$V = \sqrt{\frac{P}{\rho} + v^2}$

17. **C**

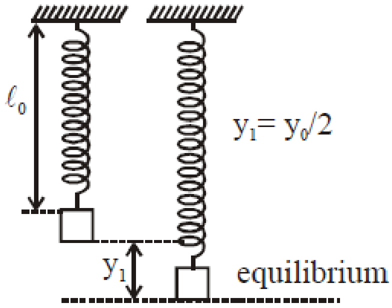
Sol. $y = y_0 \sin^2\omega t$

$y = \frac{y_0}{2}(1 - \cos 2\omega t)$

At t = 0, y = 0 extreme

At $\omega t = \frac{\pi}{2}$, $y = y_0$ extreme

At $\omega t = \frac{\pi}{4}$, $y = \frac{y_0}{2}$ mean



$\therefore \ y_1 = \frac{y_0}{2}$, at equilibrium mg = $ky_1 = \frac{ky_0}{2}$

$\frac{k}{m} = \frac{2g}{y_0}$

$2\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2g}{y_0}}$

$\omega = \frac{1}{2}\sqrt{\frac{2g}{y_0}} = \sqrt{\frac{g}{2y_0}}$

18. **B**

Sol. Mean relaxation time $T = \frac{\lambda}{V_{avg}} = \frac{\lambda}{\sqrt{\frac{8RT}{\pi m}}}$

$\therefore \quad T \propto \frac{1}{\sqrt{T}}$

19. **A**

Sol. de-Broglie wavelength

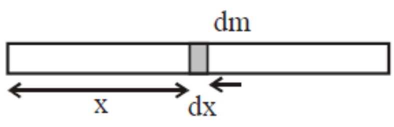
$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{3kT}{m}}} = \frac{h}{\sqrt{3mkT}}$

$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 4.64 \times 10^{-26} \times 1.38 \times 10^{-23} \times 400}}$

$\lambda = \frac{6.63}{2.77} \times 10^{-11}$ = 2.39 × 10^{-11} m ≈ 0.24 Å

20. **B**

Sol.



dm = λdx = $\lambda_0\left(1 + \frac{x}{L}\right)$dx

$M = \lambda_0 \int_0^L \left(1 + \frac{x}{L}\right)dx = \lambda_0 L + \lambda_0 \frac{L}{2} = \frac{3\lambda_0 L}{2}$ …(1)

$dI = dmx^2 = \lambda_0\left(1 + \frac{x}{L}\right)dx \times x^2$

$I = \lambda_0\left\{\int_0^L x^2 dx + \frac{1}{L}\int_0^L x^3 dx\right\} = \lambda_0\left\{\frac{L^3}{3} + \frac{L^3}{4}\right\}$

$I = \frac{7\lambda_0 L^3}{12} = \frac{7}{12}\left(\frac{2M}{3L}\right)L^3 = \frac{7}{18}ML^2$

21. **150**

Sol. As shown in figure.

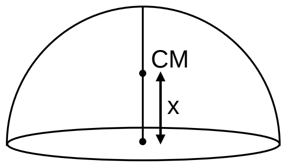
$\Delta I_C = (4.5 - 3)$ mA $= 1.5 \times 10^{-3}$ A

$\Delta I_B = (30 - 20)$ μA $= 10 \times 10^{-6}$ A

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{1.5 \times 10^{-3}}{10^{-5}} = 150$$

22. **3.00**

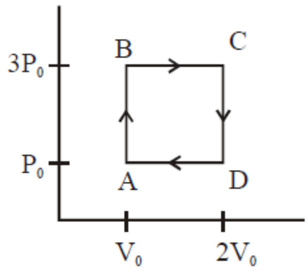
Sol. Centre of mass of solid sphere at



$$x = \frac{3R}{8}$$

$$x = \frac{3 \times 8}{8} = 3 \text{ cm}$$

23. **19.00**



Sol. $W_{Total} = (3P_o - P_o) \times (2V_o - V_o)$

$W_{Total} = 2P_oV_o$ …(1)

$Q_{in} = Q_{AB} + Q_{BC}$

$Q_{AB} = nC_V(T_B - T_A) = \frac{3}{2}nR(T_B - T_A)$

$Q_{AB} = \frac{3}{2}(3P_oV_o - P_oV_o) = 3P_oV_o$ …(2)

$Q_{BC} = nC_P(T_C - T_B) = \frac{5}{2}nR(T_C - T_B)$

$Q_{BC} = \frac{5}{2}\left[3P_o \times 2V_o - 3P_o \times V_o\right] = \frac{15}{2}P_oV_o$ …(3)

By (2) and (3) $Q_{in} = 3P_oV_o + \frac{15}{2}P_oV_o = \frac{21}{2}P_oV_o$

$$\eta = \frac{W_{Total}}{Q_{in}} \times 100 = \frac{2P_oV_o}{\frac{21}{2}P_oV_o} \times 100 = \frac{400}{21} \approx 19\%.$$

24. **9.00**

Sol. We know $I = 4I_o \cos^2\left(\frac{\phi}{2}\right)$ but $\phi = \frac{2\pi}{\lambda}x$

$$I = 4I_o \cos^2\left(\frac{\pi x}{\lambda}\right)$$

(i) when $x = \lambda$, $I = k$

i.e. $k = 4I_o \cos^2\pi$

$k = 4I_o$

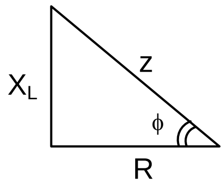
(ii) when $x = \frac{\lambda}{6}$

$$I' = k\cos^2\left(\frac{\pi}{6}\right) = k\left(\frac{3}{4}\right)$$

$$I' = \frac{9k}{12}$$

25. **400.00**

Sol.



$$P = \frac{V_{rms}^2}{z}\cos\phi$$

$$400 = \frac{(250)^2 \times 0.8}{z}$$

$\Rightarrow$ z = 125 Ω

$$\frac{R}{z} = \cos\phi \Rightarrow R = 125 \times 0.8 = 100\,\Omega$$

$$\frac{X_L}{z} = \sin\phi \Rightarrow X_L = 125 \times 0.6 = 75\,\Omega$$

$$\omega L = 75 \quad ; \quad L = \frac{75}{100\pi} = \frac{3}{4\pi}$$

For power factor unity, resonance should be there i.e. $X_L = X_C$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$(100\,\pi)^2 = \frac{4\pi}{3C}$$

$$C = \frac{4\pi}{3 \times 10^4 \times \pi^2} = \frac{4}{3\pi} \times 10^{-4} \quad ; \quad C = \frac{400}{3\pi}\,\mu F$$

PART –B (CHEMISTRY)

26. B

Sol. For oxide to be stable its ΔG value should be negative.

27. B

Sol. ^{35}Cl ^{37}Cl

Molar ratio x 1 – x

M_{avg} $35 \times x + 37(1 - x) = 35.5$

$35x + 37 - 37x = 35.5$

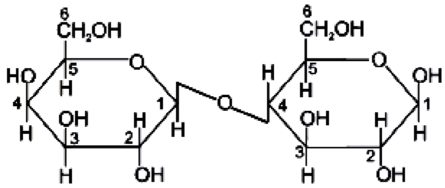
$2x = 1.5$

$X = \frac{3}{4}$

So, ratio of $^{35}Cl : ^{37}Cl = 3 : 1$

28. A

Sol.



The linkage is between C-1 of Galactose and C-4 of Glucose

$$\underset{(C_{12}H_{22}O_{11})}{\text{Lactose}}(\text{Milk sugar}) \xrightarrow{H_3O^+} \beta - \text{galactose} + \beta - \text{glucose}$$

It is hydrolysed by dilute acids or by the enzyme lactase, to an equimolecular mixture of D(+)-glucose and D(+)-galactose. Lactose is a reducing sugar.

29. D

Sol. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

$$K_C = \frac{[NH_3]^2}{[N_2][H_2]^3} = 64$$

For the reaction

$$NH_3(g) \rightleftharpoons \frac{1}{2}N_2(g) + \frac{3}{2}H_2(g)$$

$$K'_C = \frac{[N_2]^{1/2}[H_2]^{3/2}}{[NH_3]} = \frac{1}{\sqrt{K_C}} = \frac{1}{\sqrt{64}} = \frac{1}{8}$$

30. D

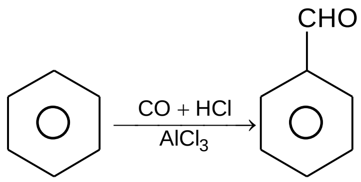
Sol. Dihydrogen of high degree of purity (>99.95%) is obtained by the electrolysis of warm aqueous barium hydroxide solution between nickel electrodes.

31. C

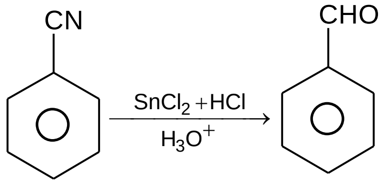
Sol. $2NO + N_2O_4 \longrightarrow 2N_2O_3$

32. C

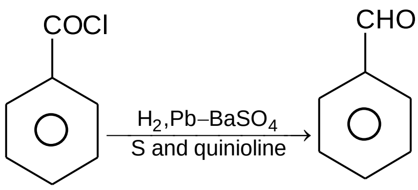
Sol.



Gatterman-Koach Reaction



Stephan Reduction



Rosenmund Reduction

33. A

Sol. In the ccp lattice of oxide ions effective number of O^{-2} ions = $8\times\frac{1}{8}+6\times\frac{1}{2}=4$

In the ccp lattice,
No. of octahedral voids = 4
No. of tetrahedral voids = 8
Given M_1 atoms occupies 50% of octahedral voids and M2 atoms occupies 12.5 of tetrahedral voids

No. of M_1 metal atoms = $4\times\frac{50}{100}=2$

No. of M_2 metal atoms = $8\times\frac{12.5}{100}=1$

$\therefore$ Formula of the compound = $(M_1)_2(M_2)O_4$

$\therefore$ Oxidation states of metals M_1 & M_2 respectively are +2 and +4.

34. B

Sol. Zn, Cd & Hg are purified by fractional distillation process.

35. A

Sol. For d^4 configuration if $\Delta_o < P$ the electronic configuration is $t_{2g}^3 e_g^1$

36. C

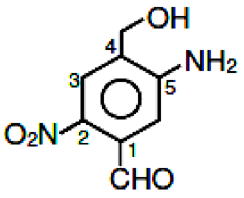
Sol. (I) Lucas reagent $\rightarrow$ Only ZnCl2/Conc.HCl
(II) Dumas method $\rightarrow$ CuO/Δ
(III) Kjeldahl's method $\rightarrow$ Conc. H_2SO_4/Δ
(IV) Heinsberg reagent $\rightarrow C_6H_5\ SO_2Cl$/aq.NaOH

37. B

Sol.
(i) $Ca(OH)_2$ is used in white wash.
(ii) Plaster of paris is used in making of molds for plaster statues.
(iii) NaCl is used in preparation of washing soda.
(iv) A suspension of $Mg(OH)_2$ in water is used in medicine as an antacid under name of milk of magnesia.

38. C

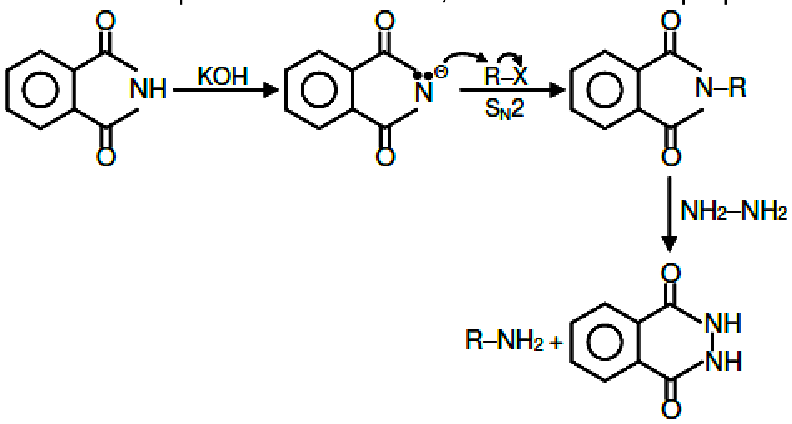
Sol.



5-Amino-4-(hydroxymethyl)-2-nitro benzene carbaldehyde.

39. A

Sol. From Gabriel phthalimide reaction, 1° Amine can be prepared.



40. C

Sol. Relative lowering in vapour pressure depends on no. of mole of solute greater the no. of mole of solute greater in RLVP and smaller will be vapour pressure. So order of vapour pressure is B > C > A.

41. D

Sol. For concentration cell $E^0_{cell} = 0$

Anode: $Cu(s) \longrightarrow Cu^{2+}(aq)_A$

Cathode: $Cu^{2+}(aq)_C \longrightarrow Cu(s)$

Overall: $Cu^{2+}(aq)_C \longrightarrow Cu^{2+}(aq)_A$

As $\Delta G = -nF E_{cell}$

If ΔG = -ve than E_{cell} is positive

$$E_{cell} = E^0_{cell} - \frac{0.059}{2}\log\frac{C_1}{C_2}$$

$$E_{cell} = \frac{-0.059}{2}\log\frac{C_1}{C_2}$$

$$E_{cell} > 0 \Rightarrow C_2 > C_1$$

42. C

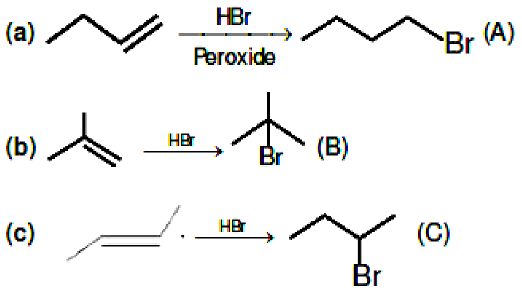
Sol. $\underset{X}{Na_2SO_3} + H_2SO_4 \longrightarrow \underset{Y}{SO_2} + Na_2SO_4 + H_2O$

$SO_2 + 2NaOH \longrightarrow \underset{X}{Na_2SO_4} + H_2O$

$Na_2SO_3 + SO_2 + H_2O \longrightarrow 2NaHSO_3$

43. A

Sol.



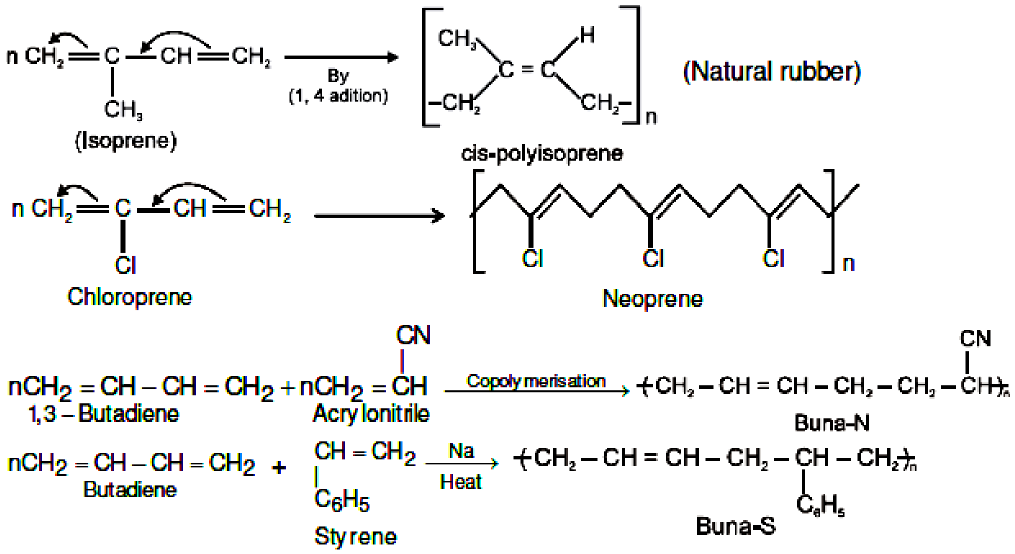
The boiling points of isomeric halo alkanes decrease with increase in branching.

44. A

Sol. Misch metal consists of Lanthanide metal (≈95%) and iron (≈5%) and traces of S, C, Ca and Al.

45. C

Sol.



46. 02.00

Sol. $AB_2 \rightleftharpoons \underset{s}{A^{2+}} + \underset{2s}{2B^-}$

$K_{sp} = 4s^3 = 3.20 \times 10^{-11}$

So, solubility = 2×10^{-4} mol L^{-1}

47. 48.00

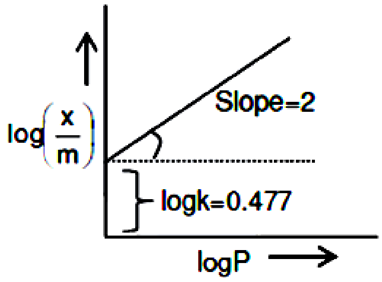
Sol.

$$\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$$

$$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log P$$

Slope $= \frac{1}{n} = 2$ So $n = \frac{1}{2}$.

Intercept $\Rightarrow \log k = 0.477$ So $k = \text{Antilog}(0.477) = 3$

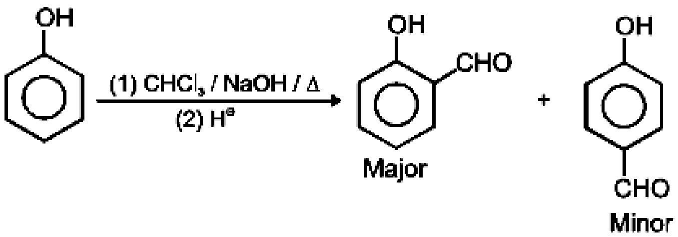


So $\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$

$= 3[0.04]^2 = 48 \times 10^{-4}$

48. 69.00

Sol. **Reimer-Tiemann formylation reaction :**



Molecular formula of product is $C_7H_6O_2$

Percentage weight of carbon $= \left(\frac{84}{122} \times 100\right) = 68.85\,\%$

49. 101.00

Sol. According to IUPAC convention for naming of elements with atomic number more than 100, different digits are written in order and at the end ium is added. For digits following naming is used.

0-nil
1-un
2-bi
3-tri
and so on…

50. 100.00

Sol.

$$\log\left(\frac{k_2}{k_1}\right) = \frac{Ea}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log(3.555) = \frac{Ea}{2.303R}\left[\frac{1}{303} - \frac{1}{313}\right]$$

$1.268 \times 8.314 \times 303 \times 313 = 10\ Ea$

So, Ea = 100 kJ

PART–C (MATHEMATICS)

51. C

Sol. Let $y = (ex)^x$

$\ell n y = [1 + \ell n x]$

$\frac{1}{y}\frac{dy}{dx} = (2 + \ell n\ x)$

$\Rightarrow dy = (ex)^x (2 + \ell n x) dx$

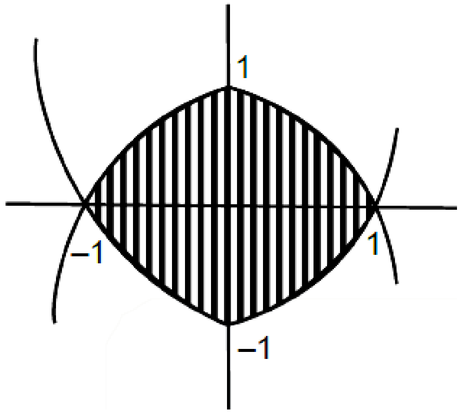
$\int_1^2 e^x . x^2 (2 + \log_e x) dx = (y)_1^2 = \left((ex)^x\right)_1^2 = 4e^2 - e$

52. B

Sol. Given curves are $y = x^2 - 1$ and $y = 1 - x^2$ so intersection point are $(\pm 1, 0)$ bounded area

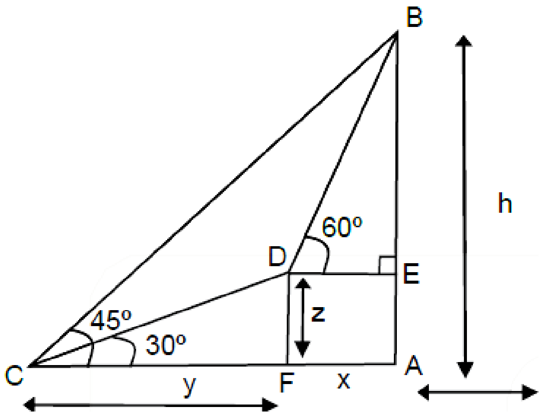
$= 4\int_0^1 (1 - x^2) dx = 4\left[x - \frac{x^3}{3}\right]_0^1$

$= 4\left(1 - \frac{1}{3}\right) = \frac{8}{3}$ sq. units



53. C

Sol.



In ΔCDF

$\sin 30^o = \frac{z}{1}$ [CD = 1 km (given)]

$z = \frac{1}{2}$ …………(1)

$\cos 30^o = \frac{y}{1} \Rightarrow = \frac{\sqrt{3}}{2}$

now in ΔABC

$\tan 45^o = \frac{h}{x+y}$

$\Rightarrow h = x + y$

$\Rightarrow x = h - \frac{\sqrt{3}}{2}$ ……………..(2)

Now

In ΔBDE,

$\tan 60^o = \frac{h-z}{x}$

$\sqrt{3}x = h - \frac{1}{2} \Rightarrow \sqrt{3}\left(h - \frac{\sqrt{3}}{2}\right) = h - \frac{1}{2} \Rightarrow \left(\sqrt{3}-1\right)h = 1$

$h = \frac{1}{\sqrt{3}-1}$km

54. D

Sol. $f(x) = \sin^2 x(\lambda + \sin x)$

$f'(x) = \sin x \cos x(2\lambda + 3\sin x)$

$\sin x = 0$ (one point)

$\sin x = -\frac{2\lambda}{3} \in (-1,1) \in \{0\}$ ………..(i)

$\lambda \in \left(\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

55. D

Sol. Given equation is $2x(2x+1) = 1 \Rightarrow 4x^2 + 2x - 1 = 0$ …………(1)

roots of equation (1) are α and β

$\therefore \alpha + \beta = -\frac{1}{2} \Rightarrow \beta = -\frac{1}{2} - \alpha$ ………..(2)

and

$4\alpha^2 + 2\alpha - 1 = 0 \Rightarrow \alpha^2 = \frac{1}{4} - \frac{\alpha}{2}$ ………..(3)

Now

$\Rightarrow \frac{\alpha}{2} = \frac{1}{4} - \alpha^2$

$\alpha = \frac{1}{2} - \frac{\alpha^2}{2}$

$\Rightarrow -\frac{1}{2} - \alpha = 2\alpha^2$

56. A

Sol. Applying Rolle's theorem in [0, 1] for function f(x)

$f'(c) = 0, c \in (0,1)$

again applying Rolle's theorem in [0, c] for function f'(x) is

$f''(c_1) = 0, c_1 \in (0, c)$

Option A is correct.

57. A

Sol. $y=\left(\frac{2}{\pi}x-1\right)\operatorname{cosec}x$

$\Rightarrow \frac{dy}{dx}=\frac{2}{\pi}\operatorname{cosec}x-\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x\cot x$

$\Rightarrow \frac{dy}{dx}+\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x\cot x=\frac{2}{\pi}\operatorname{cosec}x$

$\Rightarrow \frac{dy}{dx}+y\cot x=\frac{2}{\pi}\operatorname{cosec}x\,k$

$\Rightarrow P(x)=\cot x$

58. A

Sol. Equation of line is

$\frac{x}{3}+\frac{y}{1}=1$

$\Rightarrow x+3y-3=0$

If image is (x_1,y_1) then $\frac{x_1+1}{1}=\frac{y_1+4}{3}=-2\frac{-1-12-3}{10}$

$x_1+1=\frac{y_1+4}{3}=\frac{16}{5}$

$\Rightarrow x_1=\frac{11}{5},y_1+1=\frac{28}{5}$

59. C

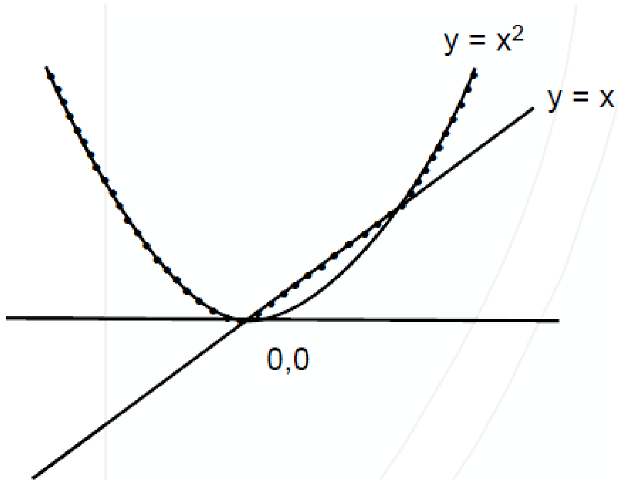
Sol. $f'(c)=1+\ell nc=\frac{e}{e-1}$

$\ell nc=\frac{1}{e-1}$

$c=e^{\frac{1}{e-1}}$

60. A

Sol.



61. D

Sol. $A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$\Rightarrow A^4 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

$B = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 4\theta + \cos\theta & \sin 4\theta + \sin\theta \\ -(\sin 4\theta + \sin\theta) & \cos 4\theta + \cos\theta \end{bmatrix}$

$B = (\cos 4\theta + \cos\theta)^2 + (\sin 4\theta + \sin\theta)^2$

$= 2 + 2(\cos 4\theta.\cos\theta + \sin 4\theta.\sin\theta)$

$= 2 + 2\cos(4\theta - \theta)$

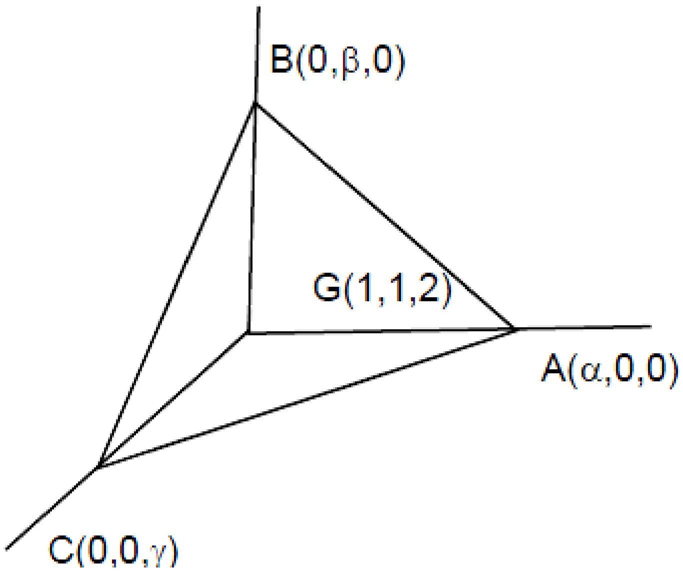
$= 2 + 2.\cos 3\theta$

$|B| = 2 + 2\cos\frac{3\pi}{5}$

$= 2 - \left(\frac{\sqrt{5}-1}{2}\right) = \frac{5-\sqrt{5}}{2} \in (1, 2)$

62. D

Sol. Let $A(\alpha,0,0), B(0,\beta,0), C(0,0,\gamma)$ then $G\left(\frac{\alpha}{3},\frac{\beta}{3},\frac{\gamma}{3}\right) = (1,1,2)$



$\alpha = 3, \beta = 3, \gamma = 6$

$\therefore$ Equation of plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

$\Rightarrow \frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

$\Rightarrow 2x + 2y + z = 6$

$\therefore$ Required line $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

63. C

Sol. Let $a_1 a_1+d, a_1+2d$......... first A.P.

$a_{40}=a_1+39d=-159$ ....(1)

$a_{100}=a_1+99d=-399$ ....(2)

From equation (1) and (2)

$d=-4,\ a_1=-3$

Now

$b_{100}=a_{70}$

$\Rightarrow b_1+99d=a_1+69d$

$b_1+99x-2=-3+69\times-4$ (According to question D = d + 2)

$\Rightarrow b_1=-81$

64. D

Sol. Equation of normal at $\left(ae, \frac{b^2}{a}\right)$

$\frac{a^2x}{ae}-\frac{b^2y}{b^2/a}=a^2-b^2$

It passes through (0, –b)

$ab=a^2e^2$

$a^2b^2=a^4e^4$ $\left(b^2=a^2\left(1-e^2\right)\right)$

$1-e^2=e^4$

65. D

Sol. $fof\left(x\right)=\frac{a-f\left(x\right)}{a+f\left(x\right)}=x$

$\Rightarrow \frac{a-ax}{1+x}=f\left(x\right)$

$\Rightarrow \frac{a\left(1-x\right)}{1+x}=\frac{a-x}{a+x}$

$\Rightarrow a=1$

So $f\left(x\right)=\frac{1-x}{1+x}$

$f\left(-\frac{1}{2}\right)=3$

66. C

Sol. $T_{r+1}={}^{10}C_r.\left(\frac{-K}{x^2}\right)^r\left(\sqrt{x}\right)^{10-r}$

$={}^{10}C_r.\left(-K\right)^r.x^{5-\frac{5r}{2}}$ for constant term $\Rightarrow 5-\frac{5r}{2}=0\Rightarrow r=2$

$\Rightarrow T_3={}^{10}C_2.K^2=405$

$\Rightarrow \frac{10(9)}{2}K^2 = 405$

$\Rightarrow K^2 = 9 \Rightarrow |K| = 3$

67. D

Sol. $y = x^2, (2, 4)$

tangent at (2, 4) is $\frac{1}{2}(y+4) = 2x$

$y + 4 = 4x \Rightarrow 4x - y - 4 = 0$

Equation of circle $(x-2)^2 + (y-4)^2 + \lambda(4x - y - 4) = 0$

It passes through (0, 1)

$\therefore 4 + 9 + \lambda(0 - 1 - 4) = 0$

$13 = 5\lambda \Rightarrow \lambda = \frac{13}{5}$

$\therefore$ circle is $x^2 - 4x + 4 + y^2 - 8y + 16 + \frac{13}{5}(4x - y - 4) = 0$

$\Rightarrow x^2 + y^2 + \left(\frac{52}{5} - 4\right)x - \left(8 + \frac{13}{5}\right)y + 20 - \frac{52}{5} = 0$

$\Rightarrow x^2 + y^2 + \frac{32}{5}x - \frac{53}{5}y + \frac{48}{5} = 0$

$\because$ centre is $\left(-\frac{16}{5}, \frac{53}{10}\right)$

68. C

Sol. $(x + iy)^1 = i(x^2 + y^2)$

$\Rightarrow x^2 - y^2 + 2ixy = i(x^2 + y^2)$

compare real and imaginary parts

$\Rightarrow x = y$

69. A

Sol. $P{:}n^3 - 1$ is even, q : n is odd

contrapositive of $p \to q = \sim q \to \sim p$

$\Rightarrow$"If n is not odd then $n^3 - 1$ is not even"

$\Rightarrow$For an integer n, if n is even, then $n^3 - 1$ is odd.

70. B

Sol. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$\Rightarrow \alpha = 1.4 - P(A \cap B) - \beta \Rightarrow \alpha + \beta = 1.4 - P(A \cap B)$ ………..(1)

again

$P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A \cap B) = .2$ ………..(2)

by (1) and (2) $\alpha = 1.2 - \beta$

now $0.85 \le \alpha \le 0.95$

$\Rightarrow 0.85 \le 1.2 - \beta \le 0.95 \Rightarrow \beta \in [0.25, 0.35]$

71. 5

Sol. $f(x) = a^x$

$\Rightarrow f(x) = a^x$

$\Rightarrow f(1) = a = 3$

So $f(x) = 3^x$

$$\sum_{i=1}^{n} f(i) = 363$$

$\Rightarrow 3 + 3^2 + ...3^n = 363$

$$\frac{3(3^n - 1)}{2} = 363$$

$3^n = 243 \Rightarrow n = 5$

72. 3

Sol. $$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ 0 & \lambda - 3 & -\lambda + 3 \\ 3 - \lambda & 0 & \lambda - 3 \end{vmatrix} = 0$$

$C_1 \rightarrow C_1 + C_3$

$$\begin{vmatrix} 3\lambda - 1 & 3\lambda + 1 & 2\lambda \\ 3 - \lambda & \lambda - 3 & 3 - \lambda \\ 0 & 0 & \lambda - 3 \end{vmatrix} = 0$$

$\Rightarrow (\lambda - 3)^2 [6\lambda] = 0 \Rightarrow \lambda = 0, 3$

Sum of values of $\lambda = 3$

73. 1

Sol. $|\vec{x} + \vec{y}|^2 = |\vec{x}|^2$

$I \Rightarrow |\vec{y}|^2 + 2\vec{x}.\vec{y} = 0$ ....(1)

and $(2\vec{x} + \lambda\vec{y})\vec{y} = 0$ s

$\Rightarrow \lambda|\vec{y}|^2 + 2\vec{x}.\vec{y} = 0$ ....(2)

by (1) and (2) $\lambda = 1$

74. 6

Sol.

x_i (observation)	0	2	2^2		2^n
f_i (frequency)	${}^nC_0$	${}^nC_1$	${}^nC_2$		${}^nC_n$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\frac{0 \times {}^nC_0 + 2 \times {}^nC_1 + 2^2 \times {}^nC_2 \ldots\ldots 2^n \times {}^nC_n}{{}^nC_0 + {}^nC_1 + {}^nC_1 \ldots\ldots {}^nC_n} = \frac{3^n - 1}{2^n} = \frac{728}{2^n}$$

$\Rightarrow \quad 3^n = 3^6$

$\Rightarrow \quad n = 6$

75. 120

Sol. Consonants are L, T, T, R

Vowels are E, E

Total number of words (with or without meaning) from letters of word 'LETTER'

$= \frac{6!}{2!2!} = 180$

Total number of words (with or without meaning) from letters of word 'LETTER' if vowels are together $= \frac{5!}{2!} = 60$

$\therefore$ Required $= 180 - 60 = 120$