FIITJEE

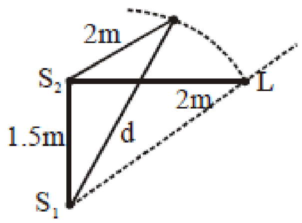
Solutions to JEE (Main)-2020

JEE–Main–2020 –Sept–5–Second–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. **B**

Sol.



Initially $S_2L = 2$ m

$$S_1L = \sqrt{2^2 + (3/2)^2}$$

$$S_1L = \frac{5}{2} = 2.5\text{ m}$$

$\Delta x = S_1L - S_2L = 0.5$ m

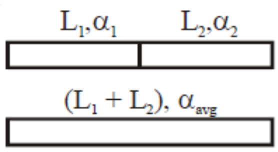
So since $\lambda = 1$ m. $\therefore$ $\Delta x = \frac{\lambda}{2}$

So white listener moves away from S_1. Then, Δx (= $S_1L - S_2L$) increases and hence, at $\Delta x = \lambda$ first maxima will appear. $\Delta x = \lambda = S_1L - S_2L$.

$1 = d - 2 \Rightarrow d = 3$ m.

2. **C**

Sol. At T°C $\quad L = L_1 + L_2$



At T + ΔT $\quad L_{eq} = L_1 + L_2$

where $L_1 = L_1(1 + \alpha_1\Delta T)$

$L_2 = L_2(1 + \alpha_2\Delta T)$

$L_{eq} = (L_1 + L_2)(1 + \alpha_{avg}\Delta T)$

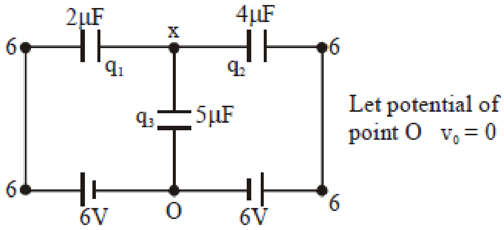
$\Rightarrow (L_1 + L_2)(1 + \alpha_{qev}\Delta T) = L_1 + L_2 + L_1\alpha_1\Delta T + L_2\Delta_2\Delta T$

$\Rightarrow (L_1 + L_2)\alpha_{avg} = L_1\alpha_1 + L_2\alpha_2$

$$\Rightarrow \alpha_{avg} = \frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$$

3. **A**

Sol.



Now, using junction analysis

We can say, $q_1 + q_2 + q_3 = 0$

$2(x - 6) + 4(x - 6) + 5(x) = 0$

$x = \frac{36}{11}$ $\quad q_3 = \frac{36(5)}{11} = \frac{180}{11}$

$q_3 = 16.36\ \mu C$

4. **A**

Sol. In adiabatic process

PV_γ = constant

$P\left(\frac{m}{\rho}\right)^\gamma$ = constant

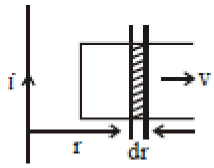
As mass is constant

$P \propto \rho\gamma$

$\frac{P_f}{P_i} = \left(\frac{\rho_f}{\rho_i}\right)^\gamma = (32)^{7/5} = 2^7 = 128$

5. **A**

Sol.



$B = \frac{\mu_0 i}{2\pi r}$

$\phi = \frac{\mu_0 i}{2\pi r}\ell dr$

$\Rightarrow \frac{d\phi}{2t} = \frac{\mu_0 i\ell}{2\pi r}\cdot\frac{dr}{dt}$

$\Rightarrow e = \frac{\mu_o}{2\pi}\cdot\frac{iv\ell}{r}$

$i = \frac{e}{R} = \frac{\mu_0}{2\pi}\cdot\frac{iv\ell}{Rr}$

6. **D**

Sol. $M = \mu_r NiA$

Here

μ_r = Relative permeability

N = Number of turns

i = Current

A = Area of cross section

$M = \mu_r NiA = \mu_r n\ell iA$

$M = \mu_r niV = 1000(1000)\ 0.5\ (10^{-3})$

$= 500 = 5 \times 10^2\ Am^2$

7. **A**

Sol. Energies of given Radiation can have
The following relation

$E_{\gamma\text{-Rays}} > E_{\text{X-Rays}} > E_{\text{microwave}} > E_{\text{AM Radiaowaves}}$

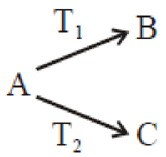
$\therefore \quad \lambda_{\gamma\text{-Rays}} < \lambda_{\text{X-Rays}} < \lambda_{\text{microwave}} < \lambda_{\text{AM Radiowaves}}$

According to tres.

(a) Microwave $\rightarrow 10^{-3}$ m (iv)
(b) Gamma Rays $\rightarrow 10^{-15}$ m (ii)
(c) AM Radio wve $\rightarrow 100$ m (i)
(d) X-Rays $\rightarrow 10^{-10}$ m (iii)

8. **B**

Sol.



$$\frac{1}{T_{eff}} = \frac{1}{T_1} + \frac{1}{T_2}$$

$$T_{eff} = \frac{T_1 T_2}{T_1 + T_2} = \frac{1000}{110} = \frac{100}{11} = 9.09$$

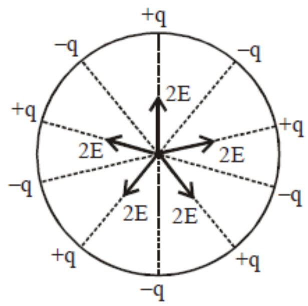
$$T_{eff} \cong 9$$

9. **C**

Sol. Potential of centre, = V =

$$V_C = \frac{K(\sum q)}{R}$$

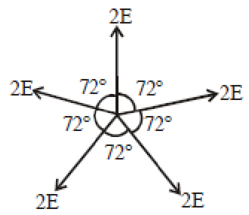
$$V_C = \frac{K(0)}{R} = 0$$



Electric field at centre $\vec{E}_B = \vec{E}_B = \sum \vec{E}$

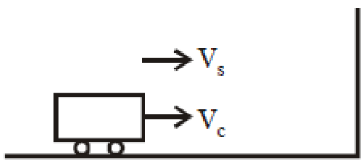
Let E be electric field produced by each charge at the centre, then resultant electric field will be

$E_C = 0$, since equal electric field vectors are acting at equal angle so their resultant is equal to zero.



10. **D**

Sol. f_1 = frequency heard by wall = $f_s = \left(\frac{v_s}{v_s - v_e}\right)$



f_2 = frequency heard y driver after reflection from wall

$$f_2 = \left(\frac{v_s + v_c}{v_s}\right) f_1 = \left(\frac{v_s + v_e}{v_s - v_e}\right) f_0$$

$$\frac{f_2}{f_0} = \frac{v_s - v_c}{v_s + v_c}$$

$$\frac{48}{44} = \frac{v_s - v_c}{v_s + v_c}$$

$12(v_s + v_c) = 11(v_s - v_c)$

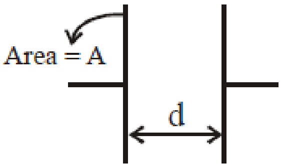
$23v_c = v_s$

$$v_c = \frac{v_s}{23} = \frac{345}{23} = 15 \text{ m/s}$$

$$= \frac{15 \times 18}{5} = 54 \text{ km/hr}$$

11. **D**

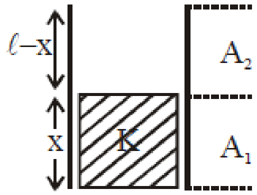
Sol.



Before inserting slab

$$C_i = \frac{\varepsilon_0 A}{d}$$

$$C_i = \frac{\varepsilon_0 \ell w}{d}$$



After inserting dielectric slab

$C_f = C_1 + C_2$

$$C_f = \frac{K\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d}$$

$$C_f = \frac{K\varepsilon_0 wx}{d} + \frac{\varepsilon_0 w(\ell - x)}{d}$$

$$C_f = 2C_i \Rightarrow \frac{K\varepsilon_0 wx}{d} + \frac{\varepsilon_0 w(\ell - x)}{d} = \frac{2\varepsilon_0 \ell w}{d}$$

$4x + \ell - x = 2\ell$

$$x = \frac{\ell}{3}$$

12. **B**

Sol. $$\frac{dm(t)}{dt} = bv^2$$

$$F_{thast} = v\frac{dm}{dt}$$

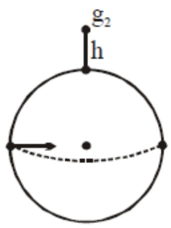
Force on satellite = $-\vec{v}\dfrac{dm(t)}{dt}$

$M(t)\, a = -v\, (bv^2)$

$$a = a\frac{bv^3}{M(t)}$$

13. **A**

Sol. $g_e = g - R\omega^2$



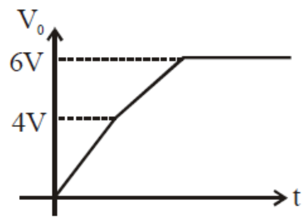
$$g_2 = g - \frac{2gh}{R}$$

Now $R\omega^2 = \frac{2gh}{R}$

$$h = \frac{R^2\omega^2}{2g}$$

14. **A**

Sol. Till input voltage reaches 4 V. No zener is in breakdown region. So $V_0 = V_i$ then now hen V_i changes between 4V to 6V one zener with 4V will breakdown are P.D. across this zener will become constant and remaining potential will drop acro resistance in series with 4V zener.

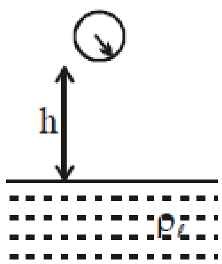


Now current in circuit increases Abruptly and source must have an internal resistance due to which. Some potential will get drop across the source also so correct graph between V_0 and t will be

We have to assume some resistance in series with source.

15. **B**

Sol.



After falling through h, the velocity be equal to terminal velocity.

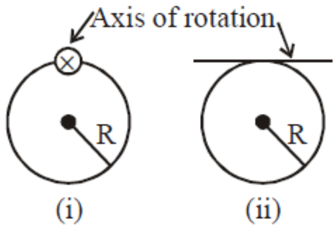
$$\sqrt{2gh} = \frac{2}{9}\frac{r^2 g}{\eta}(\rho_\ell - \rho)$$

$$\Rightarrow \quad h = \frac{2}{81}\frac{r^4 g(\rho_\ell - \rho)^2}{\eta^2}$$

$$\Rightarrow \quad h \propto r^4$$

16. **A**

Sol. Moment of inertia in case (i) is I_1



Moment of inertia in case (ii) is I_2

$I_1 = 2MR^2$

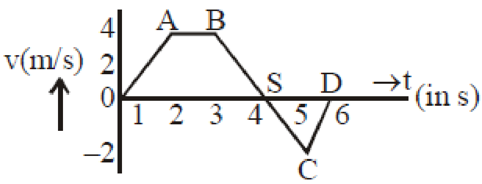
$I_2 = \frac{3}{2}MR^2$

$$T_1 = 2\pi\sqrt{\frac{I_1}{Mgd}} \quad ; \quad T_2 = 2\pi\sqrt{\frac{I_2}{Mgd}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{2MR^2}{\frac{3}{2}MR^2}} = \frac{2}{\sqrt{3}}$$

17. **B**

Sol. OS = $4 + \frac{1}{3} = \frac{13}{3}$



SD = $2 - \frac{1}{3} = \frac{5}{3}$

Area of OABS is A_1

Area of SCD is A_2

Distance = $|A_1| + |A_2|$

$$A_1 = \frac{1}{2}\left[\frac{13}{3} + 1\right]4 = \frac{32}{3}$$

$$A_2 = \frac{1}{2} \times \frac{5}{3} \times 2 = \frac{5}{3}$$

Distance = $|A_1| + |A_2|$

$$= \frac{32}{3} + \frac{5}{3} = \frac{37}{3}$$

18. **A**

Sol. $x = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \text{speed} \Rightarrow [x] = [L^1T^{-1}]$

$y = \frac{E}{B} = \text{speed} \Rightarrow [y] = [L^1T^{-1}]$

$z = \frac{\ell}{RC} = \frac{\ell}{\tau} \Rightarrow [z] = [L^1T^{-1}]$

So, x, y, z all have the same dimensions.

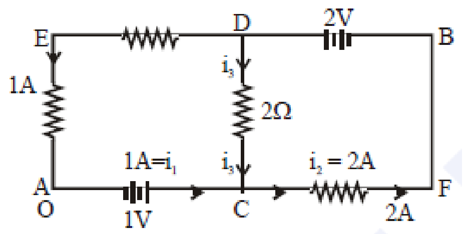
19. **D**

Sol. Figure of Merit = $C = \frac{i}{\theta}$

$= C = \frac{6 \times 10^{-3}}{2}$ = 3×10^{-3} Am^2

20. **A**

Sol. Let us assume the potential at A = V_A = 0.



Now at junction C, according to KCL

$i_1 + i_3 = i_2$

$1A + i_3 = 2A$

$i_3 = 2A$

Now analyse potential along ACDB

$v_A + 1 + i_3(2) - 2 = v_B$

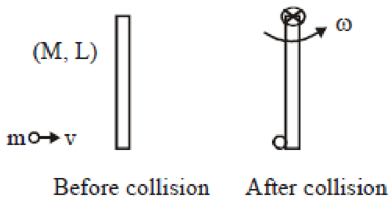
$0 + 1 + 2(1) - 2 = v_B$

$v_B = 3 - 2$

$v_B = 1$ amp.

21. **20.00**

Sol. $\vec{L}_i = \vec{L}_f$



$mvL = I\omega$

$mvL = \left(\frac{ML^3}{3} + mL^2\right)\omega$

$0.1 \times 80 \times 1 = \left(\frac{0.9 \times 1^2}{3} + 0.1 \times 1^2\right)\omega$

$8 = \left(\frac{3}{10} + \frac{1}{10}\right)\omega$; $8 = \frac{4}{10}\omega$; $\omega = 20\,\text{rad}\frac{\text{rad}}{\text{sec}}$

22. **2.00**

Sol. $E_1 = \phi + K_1$ …(1)

$E_2 = \phi + K_2$ …(2)

$E_1 - E_2 = K_1 - K_2$

Now $\frac{V_1}{V_2} = 2$

$\frac{K_1}{K_2} = 4$; $K_1 = 4K_2$

Now from equation (2)

$\Rightarrow \quad 4 - 2.5 = 4K_2 - K_2$

$1.5 = 3\,K_2$

$K_2 = 0.5$ eV

Now putting this

Value in equation (2)

$2.5 = \phi + 0.5$ eV

$\phi = 2$ eV

23. **5.00**

Sol. $\delta_{min} = (\mu - 1)\,A$

$= (1.5 - 1)1$

$= 0.5$

$$\delta_{min} = \frac{5}{10}$$

N = 5

24. **40.93**

Sol. $$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{N_2} = V_{H_2}$$

$$\sqrt{\frac{3RT_{N_2}}{M_{N_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$\frac{573}{28} = \frac{T_{H_2}}{2}$$

$\Rightarrow \quad T_{H_2} = 40.928$

25. **18.00**

Sol. P = constant

P = mav

$$m\frac{dv}{dt}v = P$$

$$\int_0^v vdv = \frac{P}{m}\int_0^t dt$$

$$\frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$\frac{dx}{dt} = \sqrt{\frac{2P}{m}}t^{1/2}$$

$$\int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$$

$$x = \sqrt{\frac{2P}{m}}\ \frac{t^{3/2}}{3/2} = \sqrt{\frac{2P}{m}} \times \frac{2}{3} t^{3/2}$$

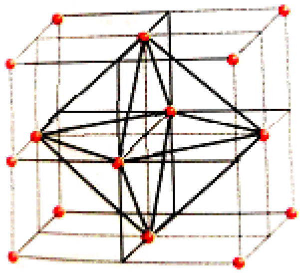
$$= \sqrt{\frac{2 \times 1}{2}} \times \frac{2}{3} \times 9^{3/2}$$

$$= \frac{2}{3} \times 27 = 18$$

PART –B (CHEMISTRY)

26. D

Sol. In FCC octahedral voids are present at the edge centers and body center

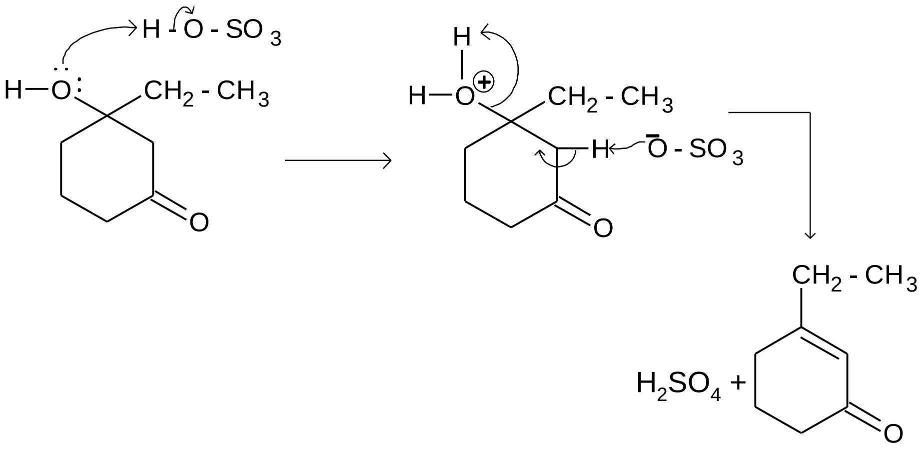


Consider a diagonal projected form edge centre passing through the body centre

Distance between octahedral voids = $\frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

27. A

Sol.



28. D

Sol. $Gas + Solid \rightleftharpoons GS \quad \Delta H = -ve$

Adsorbed gas

Adsorption of gas is an exothermic process. Increase in temperature reduces the extent of adsorption.

$\frac{x}{m} = K_P{}^{1/n}\,(n > 1)$

29. B

Sol. KNO_3, HCl and NaCl are strong electrolytes for these electrolytes of $\wedge_m$ with $\sqrt{c}$ will be liner which can be given as

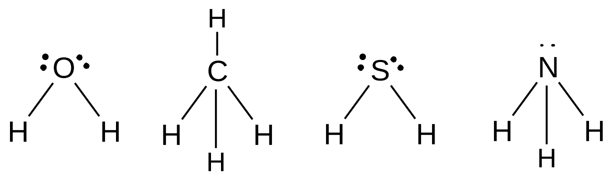
$\wedge_m = \wedge_m^0 - A\sqrt{c}$ for strong electrolyte

Since given variation is not linear it has to be a weak electrolyte

CH_3COOH is a weak electrolyte

30. B

Sol.



B.E ≈ 104° ≈ 109° ≈ 92° ≈ 107°

Using VSEPR, L.P – B.P repulsion we can safely say that CH_4 should have highest bond angle among the given

31. B

Sol. In isoelectronic species nuclear charge can be approximated as

Nuclear charge $\approx \frac{Z}{\text{no. of electrons}}$

	Al^{3+}	Mg^{2+}	Na^{+}	F^{-}	O^{2-}	N^{3-}
Nuclear Charge	$\frac{13}{10}$	$\frac{12}{10}$	$\frac{11}{10}$	$\frac{9}{10}$	$\frac{8}{10}$	$\frac{7}{10}$

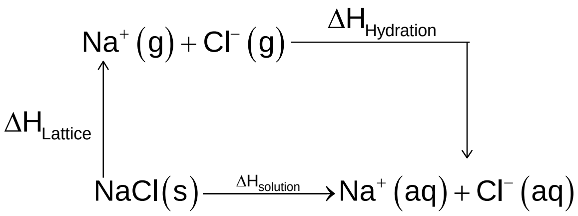
Minimum nuclear charge is in N^{3-} and maximum is in Al^{3+}

So order should be

$Al^{3+} < Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

32. B

Sol.



Hess's law

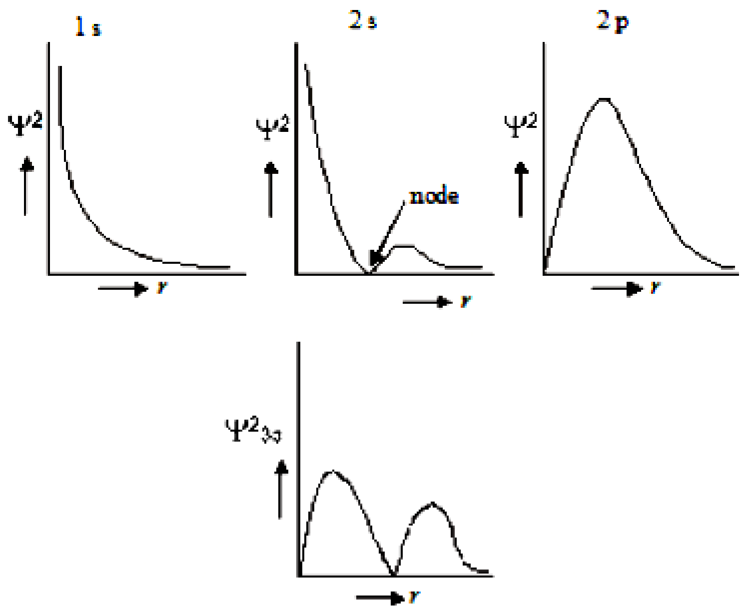
$\Delta H_{solution} = \Delta H_{lattice} + \Delta H_{hydration}$

$4 = 788 + \Delta H_{hydration}$

$\Delta H_{hydration} = -784\ kJ\ mol^{-1}$

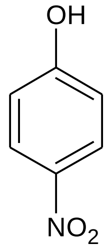
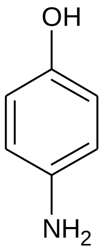
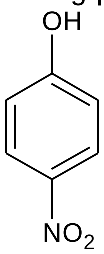
33. A

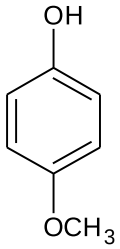
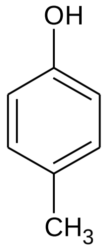
Sol. Probability density of plots



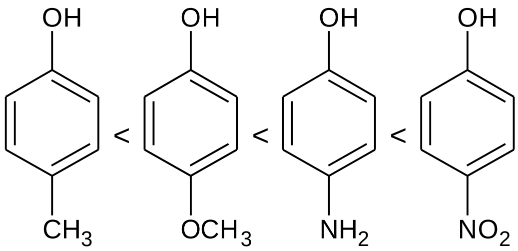
From the given graph answer is (1)

34. A

Sol. In  and  there is intermolecular hydrogen boding this hydrogen bonding will make there boiling point higher than other two. Now between these two hydrogen bonding is stronger in  (higher electronegativity of oxygen).

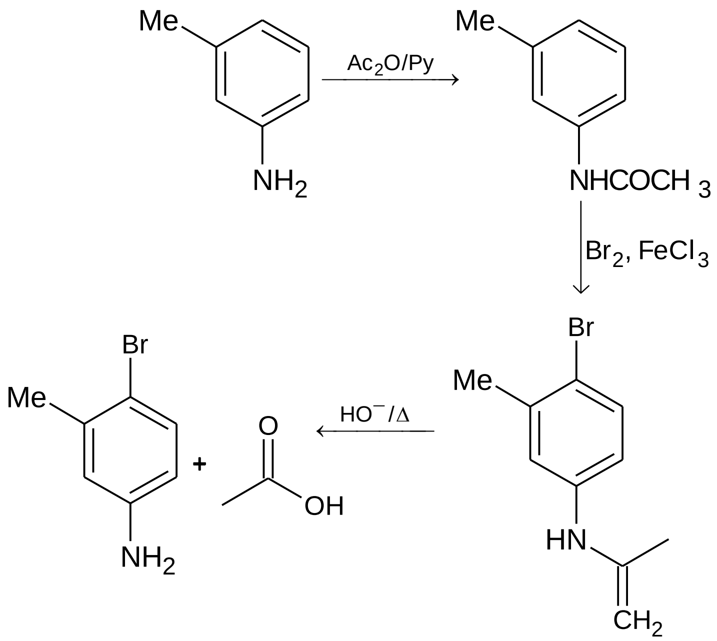
Boiling point of  will be higher than  due to higher molar mass (and dipole-dipole interaction)

Now order is



35. D

Sol.



36. A

Sol. $\ell nK = -\frac{E_a}{RT} + I$

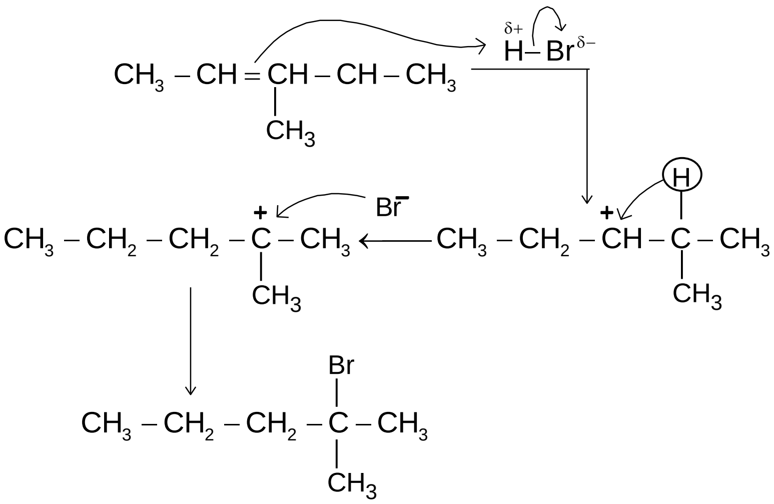
$-\frac{E_a}{R} = \text{slope}$ slope is negative

$\Rightarrow -\frac{E_a}{R} = -\frac{10-0}{5-0}$

$E_a = 2R$

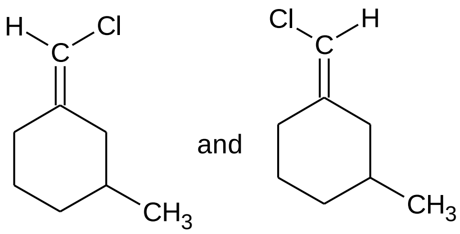
37. B

Sol.



38. A

Sol.



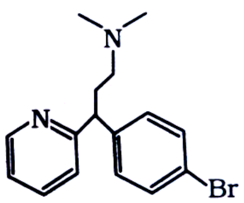
Geometrical isomers

39. A

Sol. Zone refining is used to obtain high purity elements which are used in the manufacture of semiconductors. Boron and silicon both are used in semiconductors.

40. C

Sol.



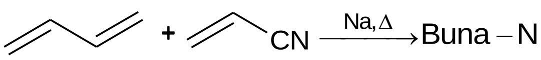
Brompheniramine
(Dimetapp, Dimetane)

Anti Histamine(Given in NCERT)

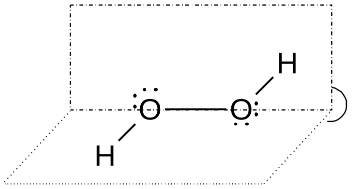
41. A

Sol. Nylon 6, Nylon 6, 6 & Bakelite are condensation polymers.

Buna – N- Addition polymerization



42. A

Sol. 

90^o.2 angle

Hydrogen peroxide has open book type structure. It is coloureless in free state

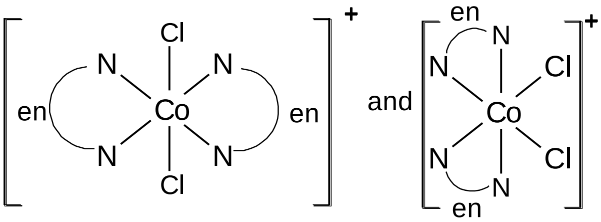
43. A

Sol. $NH_3 + \underset{(excess)}{3Cl_2} \longrightarrow NCl_3 + 3HCl$

44. B

Sol. Boiling and clark's method ($Ca(OH)_2$) are used for removing temporary hardness. Whereas, calgon, sodium carbonate ion exchange method are used for removing permanent hardness.

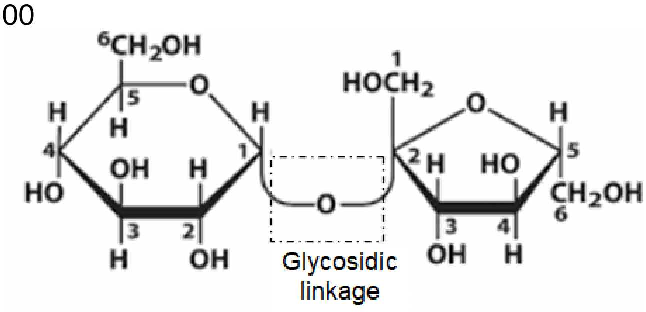
45. B

Sol. 

(A) Trans
Optically inactive due to presence of plane of symmetry

(B) Cis
Optically active no plane of symmetry

46. 9.00

Sol. 

α - D – Glucose

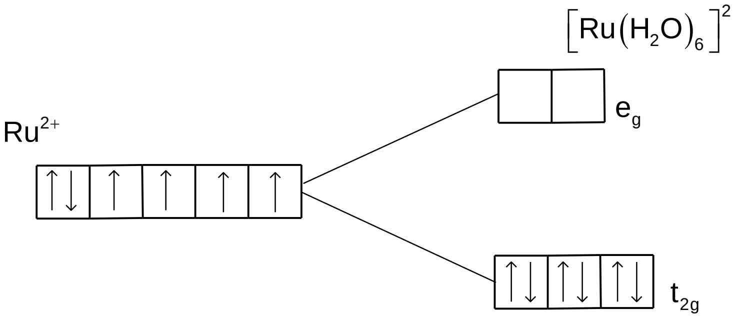
β- D - Fructose

Sucrose

47. 0.00

Sol. $\Delta_0 > P$

Pairing of electron will take place



Number of unpaired electron = 0

$\mu = 0.00$

48. –13.49

Sol. $2\ A(g) \rightarrow A_2(g)$

$\Delta n_g = 1 – 2$

$\Delta n_g = 1$

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta H = -20 \times 10^3 + (-1)8.31 \times 298$

$\Delta H = -22477.572\ J\ mol^{–1}$

$\Delta G = \Delta H - T\Delta S$

$\Delta G = -22477.572 – (-30)298 = -22477.572 + 8940$

$\Delta G = -13537.57\ J\ mol^{–1}$

49. 50.00

Sol. $\underset{n_f=6}{K_2Cr_2O_7} + \underset{n_f=3}{FeC_2O_4} \longrightarrow Fe^{3+} + \underset{+H_2O}{CO_2} + Cr^{3+} + K^{+}$

Now apply equivalent concept

$$\underbrace{0.02 \times 6}_{\text{Normality}} \times V = \frac{0.288}{\frac{144}{3}} \times 10^3$$

$$V = \frac{0.288 \times 10^3}{48 \times 6 \times 0.02} = 50.00\ mL$$

50. 16.00

Sol. $X + Y \rightleftharpoons 2Z$

	X	Y	2Z
Initial	1.0	1.5	0.5
At equ^m	1 – x	1.5 – x	0.5 + 2x
equ^m conc	0.75	1.25	1.0

0.5 + 2x = 1

x = 0.25

$$K = \frac{1}{0.75 \times 1.25}$$

$$K = \frac{16}{15}$$

X = 16.00

PART–C (MATHEMATICS)

51. D

Sol. $\left(\frac{-1+\sqrt{3}i}{1-i}\right)^{30} = \left(\frac{2\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)}{\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)}\right)^{30}$

$= \frac{2^{30}\left(\cos 20\pi + i\sin 20\pi\right)}{2^{15}\left(\cos\frac{15\pi}{2}-i\sin\frac{15\pi}{2}\right)}$

$= \frac{2^{15}\left(1+0i\right)}{\left(0+i\right)} = -2^{15}i$

52. C

Sol. So $D = 0 \to \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0 \quad \Rightarrow \quad k^2 = 9$

$x + y + 3z = 0$ …………….(1)

$x + 3y + 9z = 0$ …………….(2)

$3x + y + 3z = 0$ …………….(3)

(1) – (3)

$x = 0 \Rightarrow y + 3z = 0$

$\frac{y}{z} = -3$

So $x + \left(\frac{y}{z}\right) = -3$

53. B

Sol. $\lim_{x\to 0} \frac{\left(e^{\frac{\sqrt{1+x^2+x^4}-1}{x}}-1\right)}{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)}$

put $\frac{\sqrt{1+x^2+x^4}-1}{x} = t$

clearly $x \to 0 \Rightarrow t \to 0$

$\therefore$ given limit $= \lim_{t\to 0}\frac{e^t-1}{t} = 1$

54. A

Sol. Line are coplanar

so $\begin{vmatrix} \alpha & 5-\alpha & 1 \\ 2 & -1 & 1 \\ +1 & +3 & 2 \end{vmatrix} =$

$-5\alpha + (\alpha - 5)3 + 7 = 0$

$-2\alpha = 8 \Rightarrow \alpha = -4$

$\Rightarrow L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

Now by cross checking option (A) is correct.

55. D

Sol. $\frac{dy}{dx} + 2\tan x . y = 2\sin x$

I.F. $= e^{\int 2\tan x\, dx} = \sec^2 x$

Solution is $y.\sec^2 x = \int 2\sin x.\sec^2 x\, dx + C$

$y\sec^2 x = 2\sec x + C$

$0 = 2.2 + c \Rightarrow c = -4$

$y\sec^2 x = 2\sec x - 4$

$y\left(\frac{\pi}{4}\right) = \sqrt{2} - 2$

56. C

Sol. $L = \sin\left(\frac{\pi}{16} + \frac{\pi}{8}\right)\sin\left(\frac{\pi}{16} - \frac{\pi}{8}\right)$

$\sin\frac{3\pi}{16}.\sin\left(-\frac{\pi}{16}\right)$

$= \frac{1}{2}\left(\cos\left(\frac{3\pi}{16} + \frac{\pi}{16}\right) - \cos\left(\frac{3\pi}{16} - \frac{\pi}{16}\right)\right) =$

$= \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \cos\frac{\pi}{8}\right)$

$M = \cos\left(\frac{\pi}{16} + \frac{\pi}{8}\right)\cos\left(\frac{\pi}{16} - \frac{\pi}{8}\right)$

$\cos\frac{3\pi}{16}.\cos\left(-\frac{\pi}{16}\right)$

$= \frac{1}{2}\left(\cos\left(\frac{3\pi}{16} + \frac{\pi}{16}\right) + \cos\left(\frac{3\pi}{16} - \frac{\pi}{16}\right)\right)$

$= \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \cos\frac{\pi}{8}\right)$

57. C

Sol. $e^y y' x^4 + 4x^3 e^y + 2y' \frac{1}{2\sqrt{y+1}} = 0$ at (1, 0)

$y' + 4 + y' = 0 \quad \Rightarrow \quad y' = -2$

equation of tangent at $(1, 0)$ is $2x + y - 2 = 0$

So option (C) is correct.

58. B

Sol. Let $x = \tan\theta$

$$y_1 = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$

$$x = \sin\phi, y_2 = \tan^{-1}\left(\frac{2\sin\phi\cos\phi}{\cos 2\phi}\right)$$

$$= \tan^{-1}(\tan 2\phi) = 2\phi = 2\sin^{-1}x$$

$$\frac{dy_1}{dy_2} = \frac{dy_1/dx}{dy_2/dx} = \frac{\frac{1}{(1+x^2)}.\frac{1}{2}}{2.\frac{1}{\sqrt{1-x^2}}}$$

$$= \frac{\sqrt{1-x^2}}{4(1+x^2)} = \frac{\sqrt{1-\frac{1}{4}}}{4\left(1+\frac{1}{4}\right)} = \frac{\sqrt{3}}{10}$$

59. A

Sol. Let a, ar, ar^2........G.P.

$T_2 + T_3 + T_4 = 3 \quad \Rightarrow \quad ar(1+r+r^2) = 3$ ....(i)

$T_6 + T_7 + T_8 = 243 \quad \Rightarrow \quad ar^5(1+r+r^2) = 243$ ....(ii)

by (i) and (ii)

$r^4 = 81 \quad \Rightarrow \quad r = 3$

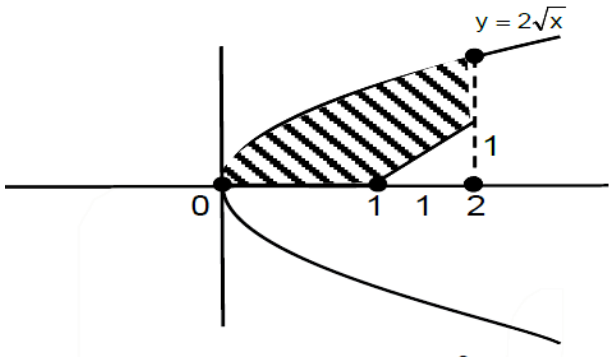
$\therefore \quad a = \frac{1}{13}$

$$S_{50} = \frac{a(r^{50}-1)}{r-1} = \frac{3^{50}-1}{26}$$

60. B

Sol. $y = [x](x-1)$

$$= \begin{cases} 0 & 0 \le x < 1 \\ x-1 & 1 \le x < 2 \end{cases}$$



$$\text{Area } = \int_0^2 2\sqrt{x}.dx - \frac{1}{2}(1)(1)$$

$$= \left(\frac{4x^{3/2}}{3}\right)_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2}$$

61. B

Sol. $I = \int \frac{\cos\theta}{2\sin^2\theta + 7\sin\theta + 3} d\theta$

$\sin\theta = t \quad \Rightarrow \quad \cos\theta \, d\theta = dt$

$= \frac{1}{2}\int \frac{1}{t^2 + \frac{7}{2}t + \frac{3}{2}} dt$

$= \frac{1}{2}\int \frac{1}{\left(t + \frac{7}{4}\right)^2 - \left(\frac{5}{4}\right)^2} dt$

$= \frac{1}{2}\ln\left|\frac{2t+1}{t+3}\right| + c$

$= \frac{1}{5}\ln\left|\frac{2\sin\theta+1}{\sin\theta+3}\right| + c$

So $A = \frac{1}{5}$

$B(\theta) = \frac{5(2\sin\theta + 1)}{\sin\theta + 3}$

62. B

Sol. $\alpha + \beta = \frac{3}{7}. \alpha\beta = -\frac{2}{7}$

$\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2} = \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{(1-\alpha^2)(1-\beta^2)}$

$= \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{1 + (\alpha\beta)^2 - (\alpha^2+\beta^2)}$

$\Rightarrow \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{1 + (\alpha\beta)^2 - (\alpha+\beta)^2 + 2\alpha\beta}$

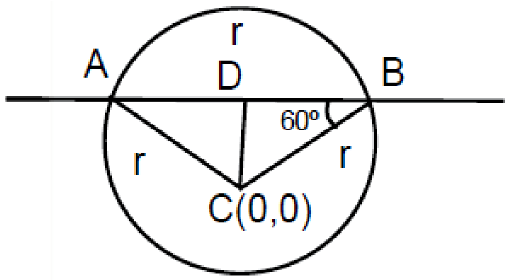
$= \frac{\frac{3}{2} + \frac{2}{7}\left(\frac{3}{7}\right)}{1 + \left(\frac{2}{7}\right)^2 - \left(\frac{3}{7}\right)^2 - 2\left(\frac{2}{7}\right)} = \frac{27}{16}$

63. B

Sol. $AB = r,\ AD = \frac{r}{2}$

$CD = r\sin 60^o = \frac{\sqrt{3}r}{2}$

$\Rightarrow \frac{|0+0-3|}{\sqrt{1^2+2^2}} = \frac{\sqrt{3}r}{2} \Rightarrow r = 2\sqrt{\frac{3}{5}} \Rightarrow r^2 = \frac{12}{5}$



64. A

Sol.

p	q	$q \to p$	$p \vee q$	$r: p \to (q \to p)$	$s: p \to (p \vee q)$	$r \to s$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T

65. C

Sol. $A \to 5Q$ $\quad B \to 5Q$ $\quad C \to 5QA$

A_1, A_2, A_3, A_4, A_5 $\quad B_1, B_2, B_3, B_4, B_5$ $\quad C_1, C_2, C_3, C_4, C_5$

$A_1A_2A_3\,B_1C_1 \Rightarrow {}^3C_1 \times {}^5C_3 \times {}^5C_1 \times {}^5C_1 = 750$

$A_1A_2B_1B_2C_1 \Rightarrow {}^3C_2 \times {}^5C_2 \times {}^5C_2 \times {}^5C_1 = 1500$

$\therefore$ Total = 2250

66. A

Sol. Given $x + a = y + b + 1 = z + c$

Now $\begin{vmatrix} x & a+y & a+x \\ y & b+y & b+y \\ z & c+y & c+z \end{vmatrix} = \begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix} (C_3 \to C_3 - C_1)$

$= \begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} (C_2 \to C_2 - C_3)$

$= y\begin{vmatrix} x & 1 & b \\ y & 1 & b \\ z & 1 & c \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$y.\begin{vmatrix} x & 1 & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix} = y\begin{vmatrix} x & 1 & a \\ a-b & 0 & -(a-b) \\ z-x & 0 & c-a \end{vmatrix}$

$= y(a-b)\begin{vmatrix} x & 1 & a \\ 1 & 0 & -1 \\ z-x & 0 & c-a \end{vmatrix}$

$= -y(a-b)(c-a+z-x) = y(a-b)$

67. A

Sol. Given $\log_{7^{\frac{1}{2}}} x + \log_{7^{\frac{1}{3}}} x + \log_{7^{\frac{1}{4}}} x + ...$ 20 times = 460

$\Rightarrow \quad (2+3+4.....+21)\log_7 x = 460$

$\Rightarrow \quad \frac{20}{2}(2+21)\log_7 x = 460$

$\Rightarrow \quad \log_7 x = 2$

$\Rightarrow \quad x = 49$

68. A

Sol. $c^2 = 36(1+m^2)$ ....(1)

$c^2 = 100m^2 - 64$ ....(2)

$100m^2 - 64 = 36 + 36m^2$

69. A

Sol. $5+3+7+a+b = 25 \Rightarrow a+b = 10$

S.D. $= \sqrt{\frac{5^2+3^2+7^2+a^2+b^2}{2} - 5^2} = 2$

$= \frac{a^2+b^2+83}{5} - 25 = 4 \Rightarrow a^2+b^2 = 62$

$\Rightarrow (a+b)^2 - 2ab = 62 \Rightarrow ab = 19$

So equation whose roots are a and b is $x^2 - 10x + 19 = 0$

70. D

Sol. $f(x) = (3x^2 + ax - 2 - a)e^x$

$f'(x) = (3x^2 + ax - 2 - a)e^x + e^x(6x + a)$

$= e^x(3x^2 + (a+6)x - 2)$

$\because$ x = 1 is a critical point

$\therefore \quad f'(1) = 0$

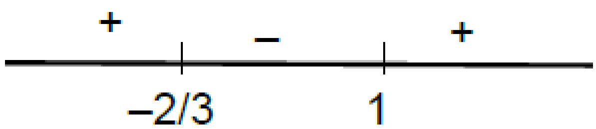
$\therefore \; 3 + a + 6 - 2 = 0$

$a = -7$

$\therefore f'(x) = e^x(3x^2 - x - 2)$

$= e^x(3x^2 - 3x + 2x - 2)$

$= e^x(3x+2)(x-1)$

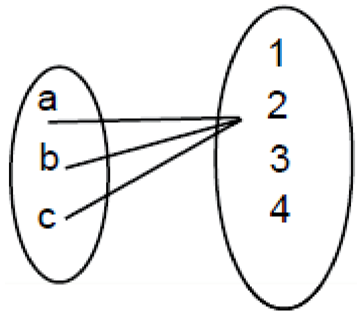


$\therefore$ maxima at $x = \frac{-2}{3}$

$\therefore$ minima at x = 1

71. 19.00

Sol. Only '2' in range $\rightarrow$ 1 function
one element out of 1, 3, 4 is in range with '2'
number of ways $= {}^{3}C_{1}.\frac{3!}{2!.1!}.2! = 18$
(Select one from 1, 3, 4 and distribute among a, b, c)
Total function $= 1 + 18 = 19$



72. 120.00

Sol. $\left(1+x+x^2+x^4\right)^6 = (1+x)^6.\left(1+x^2\right)^6$

Coefficient of $x^4 = {}^6C_4.\,{}^6C_0 + {}^6C_2.\,{}^6C_1 + {}^6C_0.\,{}^6C_2$
$= 15 + 90 + 15$
$= 120$

73. 0.50

Sol. $y = x^2 - 3x + 2,$ $\quad$ $x + y = a, x - y = b$

$2x_1 - 0 = 31$	$2x_2 - 3 = -1$
$x_1 = 2$	$x_2 = 1$
$x_1 = 4 - 6 + 2 = 0$	$x_2 = 0$
$(2,0)$	$(1, 0)$
$b = 2$	$a = 1$

$\therefore \frac{a}{b} = \frac{1}{2} = 0.5$

74. 6.00

Sol. $\vec{b}.\vec{a} = \vec{c}.\vec{a}$

$\left|\vec{a}+\vec{b}-\vec{c}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\left(\vec{a}.\vec{b} - \vec{b}.\vec{c} - \vec{a}.\vec{c}\right)$

$= 4 + 16 + 16 + 2\left(\vec{a}.b - 0 - \vec{a}.\vec{b}\right) = 36$

$\Rightarrow \left|\vec{a}+\vec{b}-\vec{c}\right| = 6$

75. 11.00

Sol. Let probability of hitting the target $= p \Rightarrow p = \frac{1}{2}$

Let n be the minimum number of bombs
According to given condition

$1 - \left({}^nC_0P^0(1-P)^n + {}^nC_1P^1(1-P)^{n-1}\right) \geq \frac{99}{100}$

$\Rightarrow \quad 2^n \geq (n+1)100$

$n = 10 \quad \Rightarrow \quad 2^{10} \geq 1100$ Reject
$n = 11 \quad \Rightarrow \quad 2^{11} \geq 1200$ Select