FIITJEE

Solutions to JEE (Main)-2020

JEE–Main–2020 –Sept–3–First–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. **B**

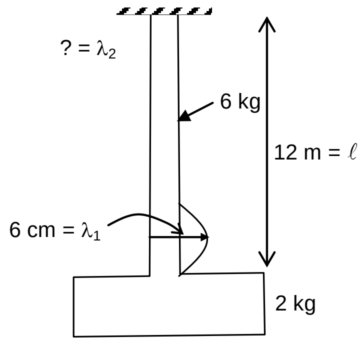
Sol. At lower end

$T_1 = 20 \Rightarrow v_1 = \sqrt{\frac{T_1}{\mu}} = \sqrt{\frac{20}{\mu}}$ ...(1)

At upper end

$T_2 = 80 \Rightarrow v_2 = \sqrt{\frac{T_2}{\mu}} = \sqrt{\frac{80}{\mu}}$

$= 2\mu$ ...(2)



$\therefore$ frequency remaining same

$f = f_2$

$\Rightarrow \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \Rightarrow \lambda_2 = \lambda_1 \frac{v_2}{v_1}$

$= 2\lambda = 12$ cm

2. **B**

Sol. If work function of metal be ϕ, then K.E. of emitted photo electron,

$k = h\nu\phi = \frac{hc}{\lambda} - \phi$ …(1)

$\therefore$ at $\lambda_1 = 500$ nm

$k_1 = \frac{hc}{\lambda_1} - \phi$ …(2)

At $\lambda_2 = 200$ nm

$k_2 = \frac{hc}{\lambda_2} - \phi = 3k_1$ (given)

$= 3\left(\frac{hc}{\lambda_1} - \phi\right)$

$\Rightarrow \frac{hc}{1}\left(\frac{3}{\lambda_1} - \frac{1}{\lambda_2}\right) = 2\phi$

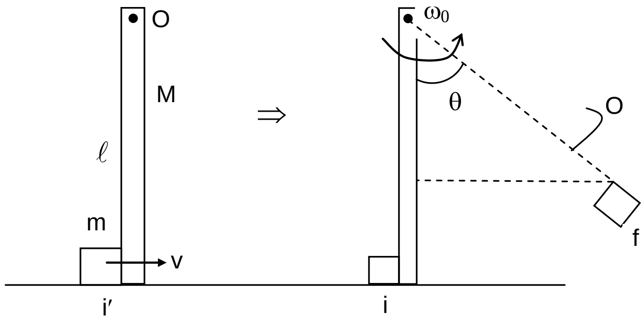
$\Rightarrow \phi = \frac{hc}{2}\left(\frac{3}{\lambda_1} - \frac{3}{\lambda_2}\right)$

$= \frac{hc}{2 \times 100\,\text{nm}}\left(\frac{3}{5} - \frac{1}{2}\right)$

$= \frac{1240}{2 \times 100}\frac{1}{10} = 0.62$ eV.

3. **D**

Sol.



COAM about O between (i) and (f)

$$mv\ell = \left(\frac{M\ell^2}{3} + m\ell^2\right)w_0$$

$$\frac{3mv}{(M\ell + 3m\ell)} = ab \qquad ...(1)$$

COTME between (i) and (f) positrons

$$\frac{1}{2}\left(m\ell^2 + \frac{M\ell^2}{3}\right)\left(\frac{3mv_o}{M\ell + 3m\ell}\right)^2 = mg(\ell - \ell\cos\theta) + Mg\left(\frac{\ell}{2} - \frac{\ell}{2}\cos\theta\right)$$

$$\Rightarrow \frac{\ell}{2} \times \frac{1}{3}\frac{9m^2v_0^2}{(M\ell + 3m\ell)} = \ell(1 - \cos\theta)\left(mg + \frac{Mg}{2}\right)$$

$$\Rightarrow \frac{3}{2} \times \frac{(1)^2 \times (1) \times 6^2}{(2 \times 1 + 3 \times 1 \times 1)} = (1)\,(1 - \cos\theta)\,(10 + 10)$$

$$\Rightarrow \frac{27 \times 2}{20 \times 5} = 0.54 = (1 - \cos\theta)$$

$\Rightarrow \cos\theta = 0.46$

$\Rightarrow \theta \approx 63°$

4. **D**

Sol. For rotating loop

$\varepsilon_0 = BA\omega = B(\pi ab)\omega$

$\therefore$ Average power loss

$$P_{avg} = \frac{E_0^2}{2R} = \frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$$

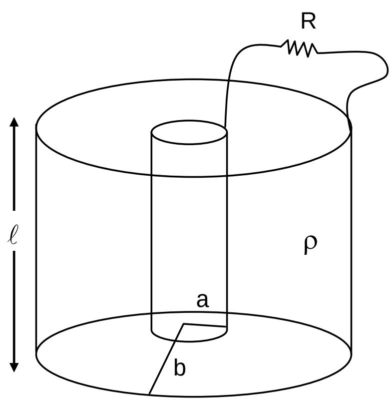
5. **D**

Sol. For maximum joule heating,

$R = R_{eq}$ of cylinder

$$= \int \frac{\rho dx}{(2\pi x\ell)}$$

$$= \frac{\rho}{2\pi\ell}\ell n\left(\frac{b}{a}\right)$$



6. **B**

Sol. $\frac{9}{16}N_o = N = N_o \quad e^{-\lambda t}$ ...(1)

$\therefore \quad N' = N_o\, e^{-\frac{\lambda t}{2}}$

$= N_o\left(e^{-\lambda t}\right)^{1/2}$

$\Rightarrow \quad \frac{N'}{N_o} = \left(\frac{9}{16}\right)^{1/2} = \frac{3}{4}$

7. **A**

Sol. Coulomb's law

$\vec{F} = q\vec{V} \times \vec{B}$

$= (10^{-6})\ (2\hat{i} + 3\hat{j} + 4\hat{k}) \times (5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3}$

$= (10^{-9})\ \left[-30\hat{i} + 32\hat{j} - 9\hat{k}\right]$

8. **C**

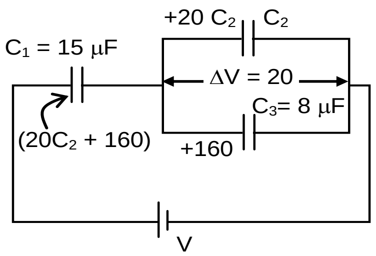
Sol. Angular width of a fringe is YDSE

$= \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{0.05 \times 10^{-3}}\,\text{rad}$

$= 10^{-2} \times \left(\frac{180}{3.14}\right) \approx 0.57^{o}$

9. **C**

Sol.



Total charge on all cap (left plates)

$\Delta V = 20$ V

$\Rightarrow \quad 750 = (20\ C_2 + 660) \times 1$

$\Rightarrow \quad 20\ C_2 = 590\ \mu C$

10. **B**

Sol. Power loss in AC

$P = \varepsilon_{rms}^2 R = \frac{\varepsilon_{rms}^2 R}{Z^2}$

$\therefore \quad \Delta Q = Pt$

$\Rightarrow \quad 2 \times 10 = \frac{(20)^2 \times 100 \times t}{(715600)}$

$\Rightarrow \quad t = 358$ sec.

11. **D**

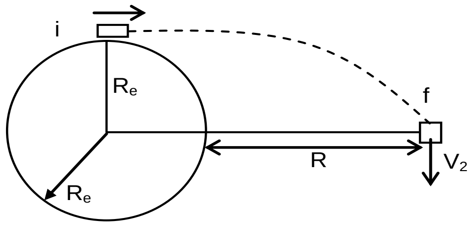
Sol. $\Delta P_1 = \frac{4T}{R_1} = 0.01$ ...(1)

& $\Delta P_2 = \frac{4T}{R_2} = 0.02$ ...(2)

$\therefore$ Ratio of volumes $\times \frac{R_1^3}{R_2^3} = \frac{1}{\left(\frac{1}{2}\right)^3} = 8:1$

12. **C**

Sol. $V = \sqrt{\frac{3}{2}}\sqrt{\frac{GMe}{R_e}} = \sqrt{\frac{36}{2R_e}}$



Between two positions

COAM

$$\sqrt[m]{\frac{36\,M_e}{2\,R_e}}\,R_e = m(R_e + R)V_2 \quad ...(1)$$

COTME

$$-\frac{GM_e m}{R_e} + \frac{1}{2}m\frac{3GM_e}{2R_e} = \frac{-GMm}{(R_e+R)} + \frac{1}{2}mv_2^2 \quad ...(2)$$

Solving:

$$\Rightarrow \quad -\frac{GM_e m}{4R_e} = -\frac{GM_e m}{(R_e+R)} + \frac{m}{2}\frac{\left(\frac{36\,M_e}{2\,R_e}\right)R_e^2}{(R_e+R)^2}$$

Let $R_e + R = x$

$$-\frac{1}{4R_e} = -\frac{1}{x} + \frac{3R_e}{4x^2}$$

$$\Rightarrow \quad -x^2 = -4R_e n + 3R_e^2 \quad ; \quad x^2 + 4\,R_e x + R_e^2 = 0$$

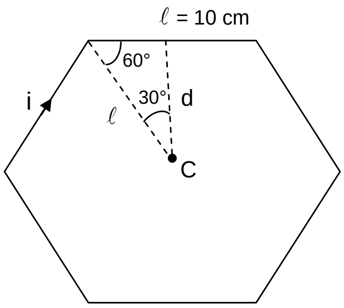
$$\Rightarrow \quad x = \frac{4\,R_e + \sqrt{16\,R_e^2 + 12\,R_e^2}}{2}$$

$= (2R_e + R_e)$

$= R = 2R_e$

13. **A**

Sol. $\vec{B}(\text{at C}) = 6 \times \frac{\mu_o i}{4\pi d}(\sin 30^o \times 2) \times 50$



$= \frac{\mu_o i}{\pi} \times \frac{3}{2 \times \left(0.1 \times \frac{\sqrt{3}}{2}\right)} \times 50$

$= \frac{\mu_o i}{\pi} \times (500\sqrt{3})$

14. **B**

Sol. $E_o = CB_o = 3 \times 10^8 \times 3 \times 10^{-8} = 9$ V/m

$\therefore \quad \vec{E} = -9\sin[200\,\pi(y + ct)]\,\hat{k}$

Direction of travel

$\frac{d}{dt}(y + ct) = 0$

$\Rightarrow \quad \frac{ds}{dt} = -C \rightarrow$ along $(-\hat{j})$

$\because \quad \vec{E} \times \vec{B}$ should be along $(-\hat{j})$

15. **C**

Sol. $LC = \frac{p}{N} = \frac{0.1\,cm}{50}$ = 0.02 mm = 0.002 cm

Hence, measurement should be a multiple of LC

16. **C**

Sol. $U = \frac{nfRT}{2}$

$= (1)(6)\frac{RT}{2} = 3RT$

17. **C**

Sol. Spheres are in parallel.

So, $C_{aq} = C1 + C2 = 4\pi e_0(R_1 + R_2)$

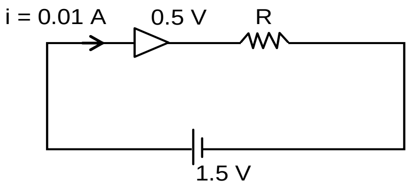
$\therefore$ Potential, $V = \left(\frac{q_1 + q_2}{c_1 + c_2}\right)$

$\therefore$ $q_1 = Gv = \frac{2}{3}R \times \frac{(q)}{(R)} = 6\ \mu C$

& $q_2 = 3\ \mu C$

18. **A**

Sol.



KVL

$1.5 - 0.5 - 0.01\ R = 0$

$\Rightarrow$ $R = 100\ \Omega$

19. **B**

Sol. Mass of material = CONSTANT

$\Rightarrow$ $(\pi R^2 L\rho) = M$ …(1)

$\therefore$ $I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right) = M\left(\frac{M}{4\pi\rho L} + \frac{L^2}{12}\right)$

$\therefore$ For I maximum / minimum

$0 = \frac{dI}{DL} = M\left(-\frac{M}{4\pi\rho L^2} + \frac{L}{6}\right)$

$\Rightarrow$ $\frac{M}{\pi\rho L} = \frac{4}{6}L^2$

$\Rightarrow$ $R^2 = \frac{2}{3}L^2$ $\Rightarrow$ $\frac{L}{R} = \sqrt{\frac{3}{2}}$

20. **D**

Sol. After burning, heat exchange occurs between helium and atmospheric. Hence, irreversible, isothermal process.

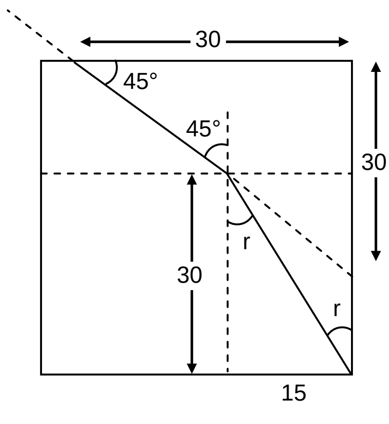
21. **158.00**

Sol. (1) sin (45°) = n sin r

$= \frac{n \times 15}{\sqrt{15^2 + 30^2}}$

$\Rightarrow$ $n = \frac{1}{\sqrt{2}} \times \sqrt{5} = \sqrt{\frac{5}{2}}$

$= 1.58 = \frac{158}{100} = \frac{N}{100}$



$\Rightarrow$ N = 158.00

22. **9.00**

Sol. Conservation of angular momentum

$$(5)\left(\frac{200\times R^2}{2}+80\,R^2\right)=\omega_f\left(\frac{200\times R^2}{2}\right)$$

$$\Rightarrow \quad \omega_p=\frac{5\times 180}{100} = 9.00$$

23. **101.25**

Sol. Height of capillary rise

$$h=\frac{2T}{r\rho g}$$

$$\Rightarrow \quad T=\frac{hrg\rho}{2}=\frac{0.15\times 0.015\times 10\times 900\times 10^{-2}}{2}$$

$= 10.125 \times 10^{-2}$ N/m

= 101.25 m N/m

24. **150.00**

Sol. W = ΔK = mgh

$\Rightarrow$ F × 0.2 = 0.15 × 10 × 20

$$\Rightarrow \quad F=\frac{30}{0.2} = 150 \text{ Newton}$$

25. **20.00**

Sol. Unfilled baker volume remains constant,

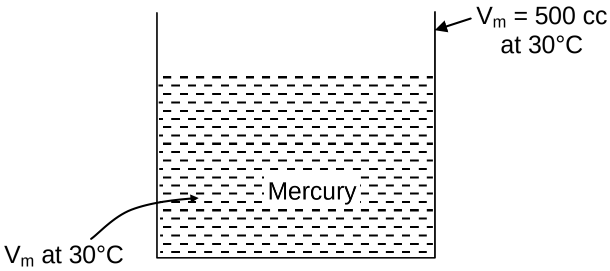
$\therefore \quad \Delta V_B = \Delta V_m$

$\Rightarrow \quad V_B\, Y_B\, \Delta T = V_M\, Y_M\, \Delta T$

$$\Rightarrow \quad V_M=\frac{V_B Y_B}{Y_M}=\frac{500\times 6\times 10^{-6}}{1.5\times 10^{-4}}$$

$= 500 \times 4 \times 10^{-2}$

= 20 cc



PART –B (CHEMISTRY)

26. A

Sol. Aqua regia is HNO_3 : HCl
1 : 3

$$HNO_3 + 3HCl \longrightarrow 2H_2O + NO + 3[Cl]$$

(i) $Au + 3[Cl] \longrightarrow AuCl_3 \xrightarrow{HCl} HAuCl_4$

(ii) $Pt + 4[Cl] \longrightarrow PtCl_4 \xrightarrow{HCl} H_2PtCl_6$

27. B

Sol. Mixture of weak acid and its salt with strong base acts as buffer solution.

28. B

Sol. Above reaction is S_N1 reaction as it proceed via formation of carbocation. Polar solvent is more suitable for S_N1 and racemisation takes place.

29. D

Sol.

	Species	Bond order
(A)	NO^+	3
(B)	NO^{2+}	2.5
(C)	NO^-	2
(D)	NO	2.5

Bond order strength is proportional to bond order.

30. B

Sol. Acidic strength $\propto$–I, –M effect due to strong –I, –M effect of 3 – $COOCH_3$, it has most acidic Hydrogen.

31. B

Sol. $[Ti(H_2O)_6]^{3+}$ = $Ti^{3+} = 3d^1\ 4s^0$

$t_{2g}^{1,0,0,}, e_g^{0,0}$

CFSE = $\left[-0.4n_{t_{2g}} + 0.6n_{eg}\right]\Delta_0 + n(p)$

= [-0.4 $\times$ 1]20300 = -8120 cm^{-1}

$$= \frac{-8120}{83.7} kJ/mole = -97 kJ/mole$$

32. A

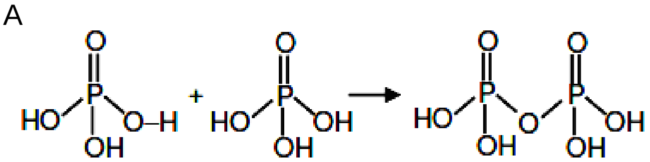
Sol. Only cis-$[CrCl_2(ox)_2]^{3-}$ show optical isomerism while its trans form do not show optical isomerism due to presence of plane of symmetry.

33. B

Sol. Burning of fossil fuels (which contain sulphur and nitrogenous matter) such as coal and oil in power stations and furnaces produce sulphur dioxide and nitrogen oxides which causes acid rain.

34. A

Sol.



Pyrophosphoric acid

No. of P =O bond = 2.
P–OH bond = 4.
P–O–P bond = 1.

35. D

Sol. un = 1
nil = 0
enn = 9
So atomic number = 109

36. C

Sol. Glycerol can be separated from spent-lye in soap industry by using reduce pressure distillation technique.

37. D

Sol. (i) Ionic mobilities decrease with increase in temperature due to increase in random motion and hence decrease in relaxation time so decrease in drift speed.
(ii) NaCl is completely soluble salt while $BaSO_4$ is sparingly soluble salt so $C_1 >> C_2$.
(iii) On increase in temperature conductance increase.

38. C

Sol. (i) Though solubility of gas will decrease with increase in temperature but this conclusion can not be drawn from the given table.
(ii) For γ ; $(P)_\gamma = (K_H)_\gamma.(X)_\gamma$

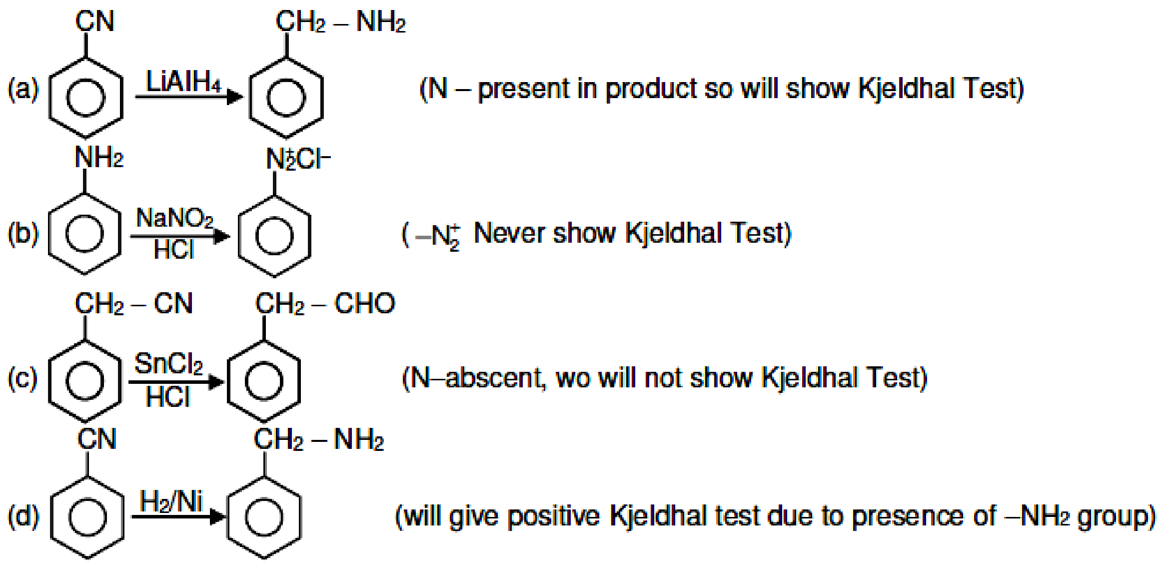
$$= 2\times10^{-2}\left[\frac{55.5}{55.5+\frac{1000}{18}}\right] = 10^{-2}\text{ bar}$$

(iii) For $\delta \Rightarrow P_\delta = (k_H)_\delta.(X)_\delta = 0.5 \times 10^3 \times\frac{1}{2} = 250$ bar

(iv) From Henry's law
$P = k_H(X)$
Higher the value of kH smaller will be solubility so γ is more soluble.

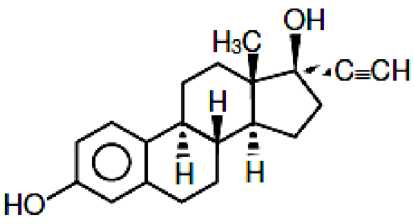
39. A

Sol.



40. D

Sol.



Novestrol (Anti Fertility Drugs)

Novestrol has phenolic functional group, alcoholic functional group and Terminal alkyne.

41. B

Sol. At room temperature water is liquid and has boiling point 373 K due to hydrogen bonding. Where as H_2S is gas and it has no hydrogen bonding. Hence boiling point of H_2S is less than 300 K [Boiling point of H_2S is -60^oC].

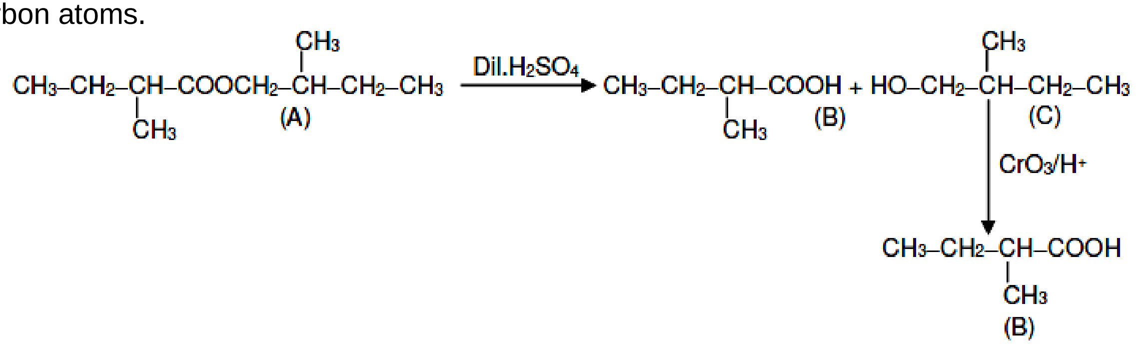
42. A

Sol. The diameter of the dispersed particles is not much smaller than the wavelength of the light used. The intensity of scattered light depends on the difference between the refractive indice of the D.P and D.M., In lyophobic colloids, this difference is appreciable and therefore the tyndal effect is quite well defined but in lyophilic sols the difference is very small and the tyndal effect is very weak. So, to show Tyndall effect the refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

43. BD

Sol. (B) contains eight carbon atoms whereas the molecular formula $C_{10}H_{20}O_2$ contains ten carbon atoms.

(D)

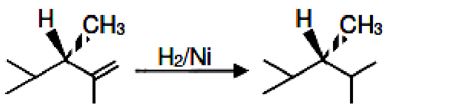


44. D

Sol. Zero order reaction is always multi step reaction.

45. B

Sol.



Optically inactive

46. 100

Sol. Molarity of H_2O_2 solution = $\left(\dfrac{\text{Volume strength}}{11.2}\right)$

Volume strength = $8.9 \times 11.2 = 99.68$ V

47. 142

Sol. Sodium metal :

$E = E_0 + (KE)_{max}$; $E^0_{cell} = 0.22$ V

Cell reaction

Cathode : $AgCl(s) + e^- \longrightarrow Ag(s) + Cl^-(aq)$

Anode : $\frac{1}{2}H_2(g) \longrightarrow H^+(aq) + e^-$

Overall : $AgCl(s) + \frac{1}{2}H_2(g) \longrightarrow Ag(s) + H^+(aq) + Cl^-(aq)$

$$E_{cell} = E^0_{cell} - \frac{0.06}{1}\log [H^+][Cl^-]$$

$$E_{cell} = 0.22 - \frac{0.06}{1}\log [10^{-1}][10^{-1}] = 0.22 + 0.12 = 0.34 \text{ V}$$

$(KE)_{max} = E_{cell} = 0.34$ eV

So $E = 2.3 + 0.34 = 2.64$ eV = Energy of photon incident

For potassium metal :

$E = E_0 + (KE)_{max}$

$2.64 = 2.25 + (KE)_{max}$

$(KE)_{max} = 0.39 = E_{cell}$

Cell reaction

Cathode : $AgCl(s) + e^- \longrightarrow Ag(s) + Cl^-(aq)$

Anode : $\frac{1}{2}H_2(g) \longrightarrow H^+(aq) + e^-$

Overall : $AgCl(s) + \frac{1}{2}H_2(g) \longrightarrow Ag(s) + H^+(aq) + Cl^-(aq)$

$$E_{cell} = E^0_{cell} - \frac{0.06}{1}\log [H^+][Cl^-]$$

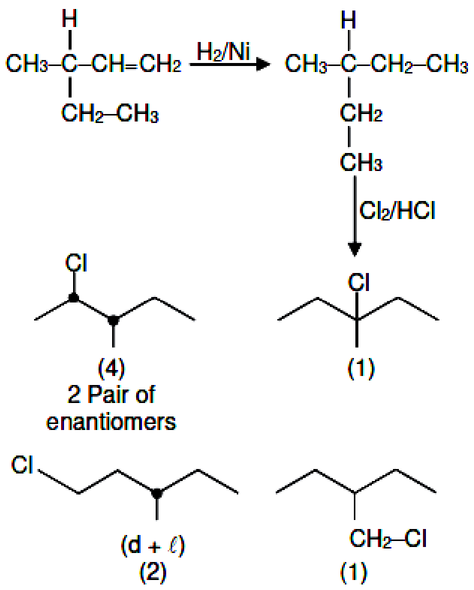
$0.39 = 0.22 - 0.12 \log [H^+]$

$0.17 = 0.12 \times pH$

$pH = 17/12 = 1.4166 = 1.42$

48. 8

Sol.



49. 143

Sol. $d = \dfrac{Z \times M}{Na \times Volume}$

$2.7 = \dfrac{Z \times 27}{6.02 \times 10^{23} \times \left[4.05 \times 10^{-3}\right]^3}$

Z = 4 $\Rightarrow$ fcc unit cell

For fcc unit cell $4r = \sqrt{2}a$

$r = \dfrac{1.414 \times 405}{4}$ = 143.1675 pm = 143.17 pm

50. 47

Sol. Let total mole of solution = 1

So mole of glucose = 0.1

Mole of H_2O = 0.9

%(w/w) of H_2O = $\left[\dfrac{0.9 \times 18}{0.9 \times 18 + 0.1 \times 180}\right] \times 100 = 47.368 = 47.37$

PART–C (MATHEMATICS)

51. D

Sol. α, β are roots of $x^2 + px + 2 = 0$

$\Rightarrow \alpha^2 + p\alpha + 2 = 0$ and $\beta^2 + p\beta + 2 = 0$

$\Rightarrow \frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + px + 1 = 0$

But $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + 2qx + 1 = 0$

$\Rightarrow p = 2q$

Also $\alpha + \beta = -p \qquad \alpha\beta = 2$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \left(\frac{\alpha^2 - 1}{\alpha}\right)\left(\frac{\beta^2 - 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right)$$

$$= \frac{(-p\alpha - 3)(-p\beta - 3)(\alpha\beta + 1)^2}{(\alpha\beta)^2} =$$

$$= \frac{9}{4}\left(p^2\alpha\beta + 3p(\alpha + \beta) + 9\right)$$

$$= \frac{9}{4}\left(9 - p^2\right) = \frac{9}{4}\left(9 - 4q^2\right)$$

52. B

Sol. Ellipse : $\frac{x^2}{4} + \frac{y^2}{3} = 1$

eccentricity $= \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$\therefore$ foci $= (\pm 1, 0)$

For hyperbola, given $2a = \sqrt{2} \Rightarrow a = \frac{1}{\sqrt{2}}$

$\therefore$ hyperbola will be $\frac{x^2}{1/2} - \frac{y^2}{b^2} = 1$

eccentricity $= \sqrt{1 + 2b^2}$

$\therefore$ foci $= \left(\pm\sqrt{\frac{1 + 2b^2}{2}}, 0\right)$

$\because$ Ellipse and hyperbola have same foci

$\Rightarrow \sqrt{\frac{1 + 2b^2}{2}} = 1$

$\Rightarrow b^2 = \frac{1}{2}$

$\therefore$ Equation of hyperbola: $\frac{x^2}{1/2} - \frac{y^2}{1/2} = 1$

$\Rightarrow x^2 - y^2 = \frac{1}{2}$

Clearly, $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$ does not lie on it.

53. C

Sol. $A : D \geq 0$

$\Rightarrow (m+1)^2 - 4(m+4) \geq 0$
$\Rightarrow m^2 + 2m + 1 - 4m - 16 \geq 0$
$\Rightarrow m^2 - 2m - 15 \geq 0$
$\Rightarrow (m-5)(m+3) \geq 0$
$\Rightarrow m \in (-\infty, -3] \cup [5, \infty)$
$\therefore A = (-\infty, -3] \cup [5, \infty)$
$B = [-3, 5)$
$A - B = (-\infty, -3) \cup [5, \infty)$
$A \cap B = \{-3\}$
$B - A = (-3, 5)$
$A \cup B = R$

54. B

Sol. $y^2 + \ln(\cos^2 x) = y \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

for $x = 0$ $\quad y = 0$ or 1

Differentiating wrt x
$\Rightarrow 2yy' - 2\tan x = y'$
At $(0,0)\, y' = 0$
At $(0,1)\, y' = 0$
Differentiating wrt x
$2yy'' + 2(y')^2 - 2\sec^2 x = y''$
At (0, 0) y” = –2
At (0, 1) y” = 2
$\therefore |y''(0)| = 2$

55. B

Sol. Let $P=\left(3t^2,6t\right); N=\left(3t^2,0\right)$

$M=\left(3t^2,3t\right)$

Equation of MQ : y = 3t

$\therefore \quad Q=\left(\frac{3}{4}t^2,3t\right)$

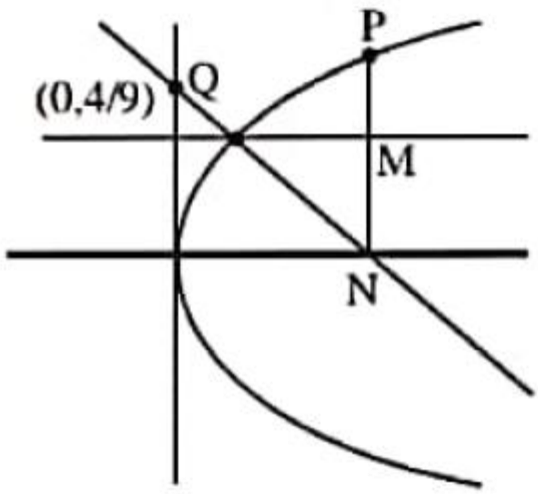
Equation of NQ

$y=\frac{3t}{\left(\frac{3}{4}t^2-3t^2\right)}\left(x-3t^2\right)$

y – intercept of NQ $=4t=\frac{4}{3}\Rightarrow t=\frac{1}{3}$

$\therefore \quad MQ=\frac{9}{4}t^2=\frac{1}{4}$

$PN=6t=2$



56. C

Sol. $\vec{r}=\hat{i}(1+12\ell)+\hat{j}(-1)+\hat{k}(\ell)$

$\vec{r}=\hat{i}(2+m)+\hat{j}(m-1)+\hat{k}(-m)$

For intersection

$1+2\ell=2+m$ …………(i)

$-1=m-1$ …………(ii)

$\ell=-m$ …………(iii)

from (ii) m = 0

from (iii) $\ell=0$

These values of m and ℓ do not satisfy equation (1).

Hence the two lines do not intersect for any values of ℓ and m.

57. B

Sol. Equation of $AB=\vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(3\hat{j}-3\hat{k}\right)$

Let coordinates of M $=\left(1,(1+3\lambda),-3\lambda\right)$.

$\overrightarrow{PM}=-3\hat{i}+(3\lambda-1)\hat{j}-3(\lambda+1)\hat{k}$

$\overrightarrow{AB}=3\hat{j}-3\hat{k}$

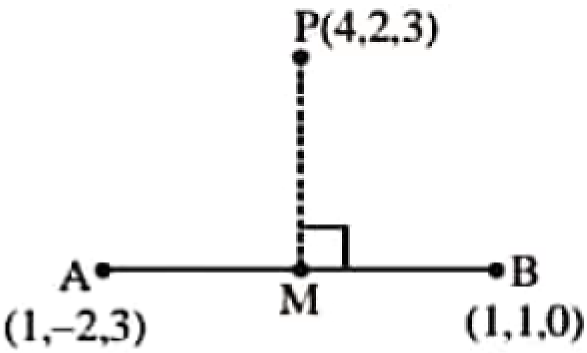
$\because \overrightarrow{PM}\perp\overrightarrow{AB}\Rightarrow\overrightarrow{PM}.\overrightarrow{AB}=0$

$\Rightarrow 3(3\lambda-1)+9(\lambda+1)=0$

$\Rightarrow \lambda=-\frac{1}{3}$

$\therefore\ M=(1,0,1)$

Clearly M lies on $2x+y-z=1$.



58. D

Sol. $\left(1+e^{-x}\right)\left(1+y^2\right)\frac{dy}{dx}=y^2$

$\Rightarrow\left(1+y^{-2}\right)dy=\left(\frac{e^x}{1+e^x}\right)dx$

$\Rightarrow \left(y - \frac{1}{y}\right) = \ell n\left(1 + e^x\right) + c$

$\therefore$ It passes through (0, 1) $\Rightarrow c = -\ell n 2$

$\Rightarrow y^2 = 1 + y\ell n\left(\frac{1+e^x}{2}\right)$

59. C

Sol. $\therefore \sigma^2 \le \frac{1}{4}(M - m)^2$

Where M and m are upper and lower bounds of values of any random variable.

$\therefore \quad \sigma^2 < \frac{1}{4}(10-0)^2$

$\Rightarrow \quad 0 < \sigma < 5$

$\therefore \quad \sigma \ne 6$

60. A

Sol. $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}$

$= Ax^3 + Bx^2 + Cx + D.$

$R_2 \to R_2 - R_1 \qquad R_3 \to R_3 - R_2$

$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-1 & x-1 & x-1 \\ x-2 & 2(x-2) & 6(x-2) \end{vmatrix}$

$= (x-1)(x-2)\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix}$

$= -3(x-1)^2(x-2) = -3x^3 + 12x^2 - 15x + 6$

$\therefore B + C = 12 - 15 = -3$

61. C

Sol. $p \to \sim(p \wedge \sim q)$

$= \sim p \vee \sim(p \wedge \sim q)$

$= \sim p \vee \sim p \vee q$

$= \sim p \vee q$

62. A

Sol. $\int_{-\pi}^{\pi} \left|\pi - |x|\right| dx = 2\int_0^{\pi} |\pi - x| dx$

$= 2\int_0^{\pi} (\pi - x) dx$

$= 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \pi^2$

63. A

Sol. Sum obtained is a multiple of 4.

$A = \{(1,3),(2,2),(3,1),(2,6),(3,5),(4,4),(5,3)(6,2),(6,6)\}$

B : Score of 4 has appeared at least once.

$B = \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4),(4,1),(4,2),(4,3),(4,5),(4,6)\}$

Required probability $= P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$$= \frac{\frac{1}{36}}{\frac{9}{36}} = \frac{1}{9}$$

64. C

Sol. Sum of 1st 25 terms = sum of its next 15 terms

$\Rightarrow (T_1 + ... + T_{25}) = (T_{26} + ... + T_{40})$

$\Rightarrow (T_1 + ... + T_{40}) = 2(T_1 + ... + T_{25})$

$\Rightarrow \frac{40}{2}\left[2 \times 3 + (39d)\right] = 2 \times \frac{25}{2}\left[2 \times 2 + 24d\right]$

$\Rightarrow d = \frac{1}{6}$

65. C

Sol. $T_{r+1} = {}^nC_r\,(3)^{\frac{n-r}{2}}\,(5)^{\frac{r}{8}} \qquad (n \geq r)$

Clearly r should be a multiple of 8.

$\because$ there are exactly 33 integral terms

Possible values of r can be

0, 8, 16, ………,32×8

$\therefore$ least value of n = 256

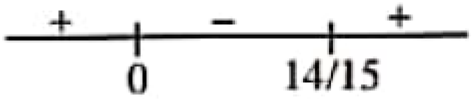
66. D

Sol. $f(x) = (3x - 7)x^{2/3}$

$\Rightarrow f(x) = 3x^{5/3} - 7x^{2/3}$

$\Rightarrow f'(x) = 5x^{2/3} - \frac{14}{3x^{1/3}}$

$= \frac{15x - 14}{3x^{1/3}} > 0$



$\therefore\ f'(x) > 0\ \forall\, x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

67. A

Sol. $2\pi - \left(\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right)$

$$= 2\pi - \left(\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right)$$
$$= 2\pi - \left(\tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right)$$
$$= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

68. D

Sol. LHL : $\lim\limits_{x\to 0^-}\left|\frac{1-x-x}{\lambda-x-1}\right| = \left|\frac{1}{\lambda-1}\right|$

RHL : $\lim\limits_{x\to 0^+}\left|\frac{1-x+x}{\lambda-x+0}\right| = \left|\frac{1}{\lambda}\right|$

For existence of limit

LHL = RHL

$\Rightarrow \frac{1}{|\lambda-1|} = \frac{1}{|\lambda|} \Rightarrow \lambda = \frac{1}{2}$

$\therefore L = \frac{1}{|\lambda|} = 2$

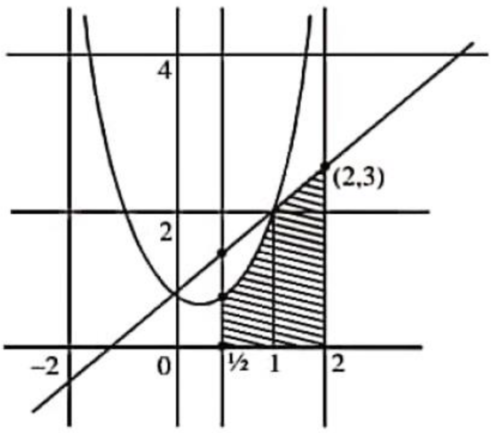
69. C

Sol. $0 \le y \le x^2+1,\ 0 \le y \le x+1, \frac{1}{2} \le x \le 2$

Required area

$= \int_{1/2}^{1}\left(x^2+1\right)dx + \frac{1}{2}(2+3)\times 1$

$= \frac{19}{24} + \frac{5}{2} = \frac{79}{24}$



70. C

Sol. $S = \left(2.\ ^1P_0 - 3.\ ^2P_1 + 4.\ ^3P_2 ....\text{upto 51 terms}\right) + \left(1! - 2! + 3! - ....\text{upto 51 terms}\right)$

$\left[\because\ ^nP_{n-1} = n!\right]$

$\therefore \quad S = \left(2\times 1! - 3\times 2! + 4\times 3! .... + 52.51!\right) + \left(1! - 2! + 3! ....(51)!\right)$

$= \left(2! - 3! + 4! .... + 52!\right) + \left(1! - 2! + 3! - 4! + ... + (51)!\right)$

$= 1! + 52!$

71. 10

Sol. $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (x^2+1)^2 + x^2 & x(x^2+1)+x \\ x(x^2+1)+x & x^2+1 \end{bmatrix}$$

$$a_{11} = (x^2+1)^2 + x^2 = 109$$
$$\Rightarrow x = \pm 3$$
$$a_{22} = x^2 + 1 = 10$$

72. 3

Sol. $\because$ center lies on $x + y = 2$ and in 1st quadrant

center $= (\alpha, 2-\alpha)$ where $\alpha > 0$ and $2 - \alpha > 0 \Rightarrow 0 < \alpha < 2$

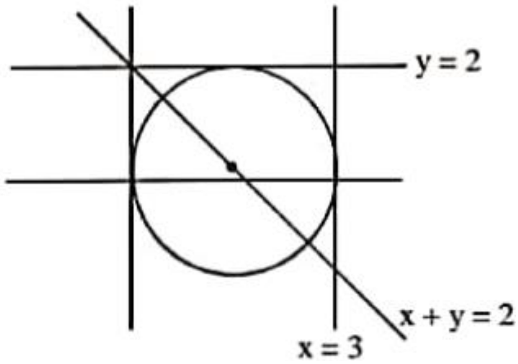
$\because$ circle touches x = 3 and y = 2

$\Rightarrow |3-\alpha| = |2-(2-\alpha)|$ = radius

$\Rightarrow |3-\alpha| = |\alpha| \Rightarrow \alpha = \dfrac{3}{2}$

$\therefore$ radius = α

$\Rightarrow$ Diameter $= 2\alpha = 3$.



73. 8

Sol. $\lim\limits_{x\to 0}\left\{\dfrac{1}{x^8}\left(1-\cos\dfrac{x^2}{2}-\cos\dfrac{x^2}{4}+\cos\dfrac{x^2}{2}\cos\dfrac{x^2}{4}\right)\right\} = 2^{-k}$

$$\Rightarrow \lim_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)}{4\left(\frac{x^2}{2}\right)^2}\frac{\left(1-\cos\frac{x^2}{4}\right)}{16\left(\frac{x^2}{4}\right)^2} = \frac{1}{8}\times\frac{1}{32} = 2^{-k}$$

$$\Rightarrow 2^{-8} = 2^{-k} \Rightarrow k = 8$$

74. 4

Sol. $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\text{.......to}\infty\right)}$

$$= \left(\frac{4}{24}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}$$

$$= \left(\frac{1}{2}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{4}{25}\right)} = \left(\frac{1}{2}\right)^{-2} = 4$$

75. 4

Sol. $\left(\dfrac{1+i}{1-i}\right)^{m/2} = \left(\dfrac{1+i}{i-1}\right)^{n/3} = 1$

$\Rightarrow \left(\frac{(1+i)^2}{2}\right)^{m/2} = \left(\frac{(1+i)^2}{-2}\right)^{n/3} = 1$

$\Rightarrow (i)^{m/2} = (-i)^{n/3} = 1$

$\Rightarrow \frac{m}{2} = 4k_1$ and $\frac{n}{3} = 4k_2$

$\Rightarrow m = 8k_1$ and $n = 12k_2$

Least value of $m = 8$ and $n = 12$

$\therefore$ GCD = 4