FIITJEE

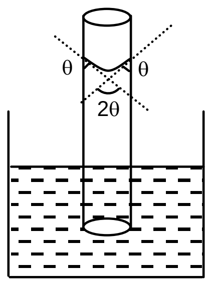
Solutions to JEE (Main)-2020

JEE–Main–2020 –Sept–2–Second–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. **C**

Sol. $2\theta = 60° \Rightarrow \theta = 30°$



$h = \frac{2T\cos\theta}{\rho r g}$

$= \frac{2(0.05)\cos 30^o}{(667)(0.15\times10^{-3})(10)}$

$= \frac{\sqrt{3}\times100}{667\times3} \simeq \frac{173.2}{2000}$ m. = 8.66 cm

2. **C**

Sol. $\Delta\ell = \ell \propto \Delta T \quad \Rightarrow \quad \frac{\Delta\ell}{\ell} = \alpha\Delta T = 0.02\%$

$\Delta\rho = -\rho\gamma\Delta T$

$\Rightarrow \quad \left|\frac{\Delta\rho}{\rho}\right| = \gamma\Delta T = 3\propto\Delta T = 3(0.02\%) = 0.06\%$

3. **A**

Sol. $\lambda = \frac{h}{p} = \frac{h}{mv}$

$\Rightarrow \quad \frac{\lambda_p}{\lambda_e} = \frac{m_e v_e}{m_p v_p} \Rightarrow 1.878\times10^{-4} = \left(\frac{9.1\times10^{-31}}{m_p}\right)\left(\frac{1}{5}\right)$

$\Rightarrow \quad m_p = \frac{9.1\times10^{-31}}{5\times1.878\times10^{-4}} = 0.97\times10^{-27}$ kg

4. **A**

Sol. ℓ = 10 × pitch

$= 10\times\frac{2\pi mv\cos\theta}{qB}$

$= \frac{20\times3.14\times1.67\times10^{-27}\times4\times10^5\times\frac{1}{2}}{1.6\times10^{-19}\times0.3}$

$= 4.36\times10^{-1}$ m. = 0.44 m.

5. **D**

Sol. On increasing the temperature, random velocity of molecules increases, therefore mean collision time between the molecules decreases. But the mean free path remains constant as it is product of velocity and time.

$\therefore$ B and C are correct option.

6. **A**

Sol. $\sigma 4\pi r^2 + \sigma 4\pi R^2 = Q$

$\Rightarrow \quad \sigma = \dfrac{Q}{4\pi(R^2 + r^2)}$

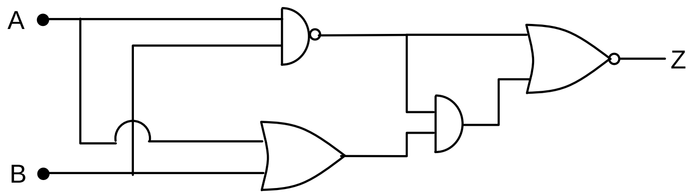
$\therefore \quad V_C = \dfrac{kq_1}{r} + \dfrac{kq_2}{R} = \dfrac{k(\sigma 4\pi r^2)}{r} + k\left(\dfrac{\sigma 4\pi R^2}{R}\right)$

$\Rightarrow \quad V_C = k\sigma 4\pi\ (r + R)$

$= K\left(\dfrac{Q}{R^2 + r^2}\right)(R + r)$

7. **D**

Sol.



A	B	Z
1	**0**	**0**
0	**0**	**0**
1	**1**	**1**
0	**1**	**0**

8. **A**

Sol. R = 100 Ω

$\tan\phi = \dfrac{X_C - X_L}{R} \Rightarrow \tan(-45^o) = \dfrac{-X_L}{R}$

$\Rightarrow \quad X_L = R = 100$

$\Rightarrow \quad Lw = 100$

$\Rightarrow \quad L = \dfrac{100}{2\pi f} = \dfrac{100}{2 \times 3.14 \times 1000}$

$\Rightarrow \quad L = 1.59 \times 10^{-2}$ H

Now of the option matches and the nearest option is (A).

9. **A**

Sol. $\vec{m} = Iab\,\hat{k} + Iab\,\hat{j}$

$\Rightarrow \quad |\vec{m}| = Iab\sqrt{2}$

Direction $\Rightarrow \dfrac{\hat{j} + \hat{k}}{\sqrt{2}}$

10. **C**

Sol. $\Delta E = h\nu = 13.6\ Z^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$eV

$\Rightarrow \quad h\nu = 13.6(1)^2\left[\dfrac{1}{n^2} - \dfrac{1}{(n+1)^2}\right]$eV

$\Rightarrow \quad \nu = \left(\frac{13.6\text{ eV}}{h}\right)\left[\frac{(n+1)^2 - n^2}{n^2(n+1)^2}\right]$

$\Rightarrow \quad \nu = \left(\frac{13.6\text{ eV}}{h}\right)\frac{2n+1}{n^2(n+1)^2}$

For n >> 1

$\Rightarrow \quad \nu = \left(\frac{13.6\text{ eV}}{h}\right)\frac{2n}{(n^2)\,n^2}$

$\Rightarrow \quad \nu \propto \frac{1}{n^3}$

11. **A**

Sol. $g_{h=h} = g_{d=h}$

$\Rightarrow \quad \frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(1-\frac{h}{R}\right)$

$\Rightarrow \quad 1 = \left(1+\frac{h}{R}\right)^2\left(1-\frac{h}{R}\right)$

$\Rightarrow \quad 1 = \left(1+\frac{2h}{R}+\frac{h^2}{R^2}\right)\left(1-\frac{h}{R}\right) \quad \Rightarrow \quad 1 = 1+\frac{h}{R}-\frac{h^2}{R^2}-\frac{h^3}{R^3}$

$\Rightarrow \quad \frac{h}{R}\left(\frac{h^2}{R^2}+\frac{h}{R}-1\right) = 0 \qquad \Rightarrow \quad \frac{h}{R} = \frac{-1\pm\sqrt{1+4}}{2}$

$\Rightarrow \quad h = \left(\frac{\sqrt{5}-1}{2}\right)R$

12. **A**

Sol. By conservation of angular momentum,

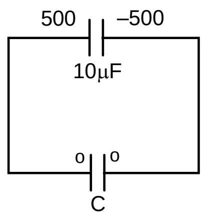
$I_1w_1 + I_2w_2 = (I_1 + I_2)\, w$

$\Rightarrow \quad w = \frac{I_1w_1 + I_2w_2}{I_1 + I_2} = \frac{(0.1)(10)+(0.2)(5)}{0.1+0.2} = \frac{2}{0.3} = \frac{20}{3}$

$K = \frac{1}{2}(I_1+I_2)w^2 = \frac{1}{2}(0.3)\left(\frac{20}{3}\right)\left(\frac{20}{3}\right) = \frac{20}{3}\text{ J}$

13. **A**

Sol.



By conservation of charge,

(50) (10) + 0 = (20) (10) + (20) C

$\Rightarrow \quad 500 = 20(10 + C)$

$\Rightarrow \quad 25 = 10 + C \quad \Rightarrow \quad C = 15\ \mu F$

14. **C**

Sol. $F = ma = m(-w^2 x) = 0 \quad \Rightarrow \quad A$

At t = T $\Rightarrow x \to \max \Rightarrow a = \max \Rightarrow B$

$v = \frac{dx}{dt}$ = slope of x – t curve $\Rightarrow$ C

$U = \frac{1}{2}mw^2x^2 \quad \& \quad K = \frac{1}{2}mw^2(A^2 - x^2)$

15. **D**

Sol. $\ell = n_1\beta_1 = n_2\beta_2 \Rightarrow n_1\frac{\lambda_1 D}{d} = n_2\frac{\lambda_2 D}{d}$

$\Rightarrow n_1\lambda = n_2\lambda_2$

$\Rightarrow (16)(700) = n_2(400) \Rightarrow n_2 = 28$

16. **D**

Sol. Efficiency $(\eta) = \frac{Q_{net}}{Q_+}$

$\Rightarrow \frac{50}{100} = \frac{1915 - 40 + 125 - Q}{1915 + 125} \Rightarrow \frac{1}{2} = \frac{2000 - Q}{2040}$

$\Rightarrow 1020 = 2000 - Q$

$\Rightarrow Q = 980$ J

17. **C**

Sol. $E = P^a A^b T^c$

$\Rightarrow ML^2T^{-2} = (MLT^{-1})^a (L^2)^b (T)^c$

$\Rightarrow a = 1$

$\Rightarrow a + 2b = 2 \Rightarrow b = \frac{1}{2}$

$-a + c = -2 \Rightarrow c = -1$

$\Rightarrow E = P A^{1/2} T^{-1}$

18. **A**

Sol. $X = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$; $y = \frac{1}{2}gt^2$

$\Rightarrow \frac{x}{y} = \frac{qE}{mg} \Rightarrow$ Straight line

19. **A**

Sol. In EM wave, wave velocity is in the direction perpendicular to both electric field & magnetic field.

$\Rightarrow \hat{v} = \hat{E} \times \hat{B} = \hat{k} \times \left(\frac{2\hat{i} - 2\hat{j}}{2\sqrt{2}}\right) = \frac{\hat{j} + \hat{i}}{\sqrt{2}}$

20. **C**

Sol. $V_{PQ} = \varepsilon_2 = \left(\frac{dv}{dx}\right)\ell$

$\Rightarrow 1.02 = \left(\frac{dv}{dx}\right)(51)$

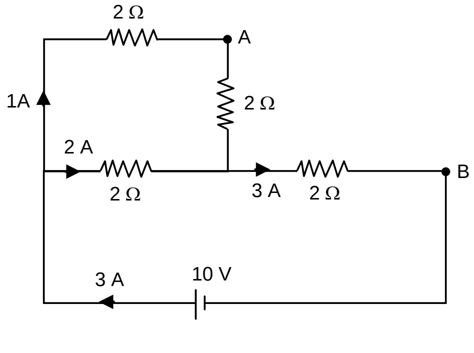
$\Rightarrow \frac{dv}{dx} = \frac{1.02}{51} = 0.02$ V/cm

21. **8**

Sol. When capacitor is fully charged, no current flows through it. Therefore, we can remove capacitor branch.

$V_{AB} = 2(1) + 2(3)$

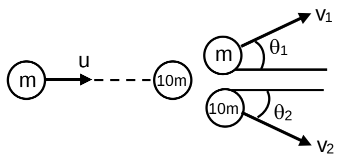
$= 2 + 6 = 8$ V



22. **10**

Sol. $\frac{1}{2}mv_1^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right) \Rightarrow v_1 = \frac{u}{\sqrt{2}}$

As the collision is perfectly elastic, energy remains conserved.



$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 + \frac{1}{2}(10\,m)v_2^2 \Rightarrow \frac{1}{4}mv^2 = 5mv_2^2$

$\Rightarrow v_2 = \frac{u}{2\sqrt{5}}$

By conservation of momentum along Y direction

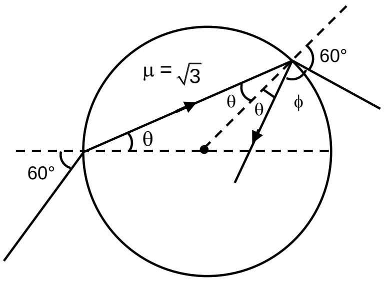
$0 + 0 = mv_1 \sin\theta, -10\,m\ v_2 \sin\theta_2$

$\Rightarrow m\frac{u}{\sqrt{2}}\sin\theta_1 = 10\,m\left(\frac{u}{2\sqrt{5}}\right)\sin\theta_2$

$\Rightarrow \sin\theta_1 = \sqrt{10}\sin\theta$

23. **90**

Sol.



$1 \sin 60° = \sqrt{3}\sin\theta$

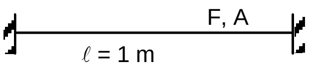
$\Rightarrow \theta = 30°$

$\Rightarrow \theta + \phi + 60° = 180$

$\Rightarrow \phi = 120° - 30° = 90°$

24. **35**

Sol.



$Y = \frac{\sigma}{\in} = \frac{F/A}{\in}$

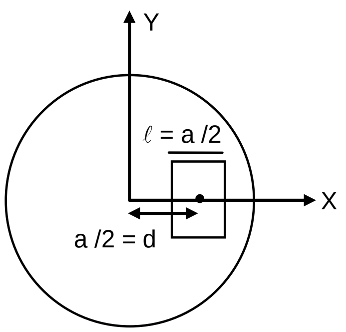
$\Rightarrow F = YA\in \quad \Rightarrow v = \sqrt{\frac{F\ell}{m}} = \sqrt{\frac{(YA\in)\ell}{m}} = \sqrt{\frac{Y\in}{\rho}}$

$\Rightarrow v = \sqrt{\frac{(9\times10^{10})(4.9\times10^{-4})}{9\times10^3}} = 70$ m/s

$f = \frac{v}{2l} = \frac{70}{2\times1} = 35$ Hz

25. **23**

Sol.



Assuming the density of the disc is σ.

$\Rightarrow M_{disc} = \sigma\,\pi a^2$

$\Rightarrow M_{hole} = \sigma\left(\frac{a}{2}\right)^2 = \frac{\sigma a^2}{4}$

$\Rightarrow X_{cm} = \frac{m_D X_D - m_H X_H}{m_D - m_H}$

$$\Rightarrow \qquad X_{CM} = \frac{\sigma\pi a^2(0) - \sigma\left(\frac{a}{2}\right)^2\left(\frac{a}{2}\right)}{\sigma\pi a^2 - \sigma\left(\frac{a}{2}\right)^2}$$

$$\Rightarrow \qquad X_{CM} = \frac{-\sigma\left(\frac{a}{4}\right)^2\left(\frac{a}{2}\right)}{\sigma a^2\left(\pi - \frac{1}{4}\right)} = -\frac{a}{2(4\pi - 1)} = -\frac{a}{x} \qquad \Rightarrow \quad x = 23.12$$

PART –B (CHEMISTRY)

26. D

Sol. First reaction is S_N1 in which rate does not depend on conc. of nucleophile. Second reaction is E_2 reaction in which rate depends on conc. of base.

27. A

Sol. For n= 4 possible values of ℓ = 0, 1, 2, 3 only ℓ = 2 & ℓ = 3 can have m = –2. So possible subshells are 2

28. A

Sol. (i) ion-ion interaction energy is inversely proportional to the distance between ions $\left(\frac{1}{r}\right)$

(ii) dipole-dipole interaction energy is inversely proportional to the third power of r $\left(\frac{1}{r^3}\right)$

(iii) The interaction energy of London force is inversely proportional to sixth power of distance between two interaction particles $\left(\frac{1}{r^6}\right)$

29. D

Sol. (i) XeF_5^- St. No. = (5 + 2) = 7
so hybridisation is = sp^3d^3
and structure is pentagonal planar.

(ii) XeO_3F_2 St. No. = 5
so hybridisation is = sp^3d
and structure is trigonal bipyramidal.

30. D

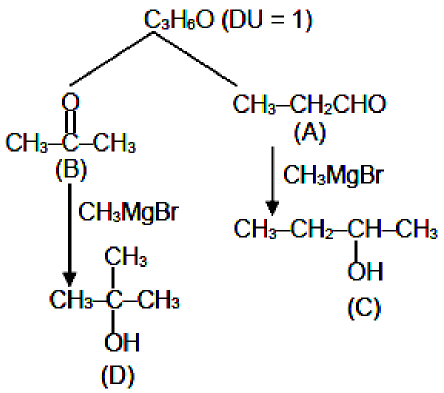
Sol. Li and Mg do not form solid bicarbonate. But react with N_2 to give nitrides.

$$6Li + N_2 \xrightarrow{\Delta} 2Li_3N$$

$$3Mg + N_2 \xrightarrow{\Delta} Mg_3N_2$$

31. B

Sol.



Iodoform Test	–ve	+ve
Lucas Test	Immediate	after 5-10 Mint.
Ceric Ammonium nitrate	+ve	+ve

32. C

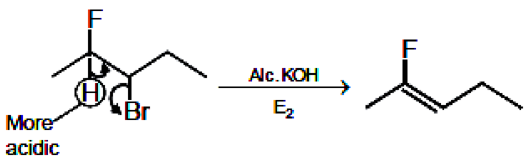
Sol. On moving left to right in a period.
Acidic character of oxides is increase.
3rd period element oxides.

$\underbrace{Na_2O \quad MgO}_{\text{Basic}} \quad \underbrace{Al_2O_3}_{\text{Amphoteric}} \quad \underbrace{SiO_2 \quad P_2O_5 \quad Cl_2O_7}_{\text{Acidic}}$

(i) Acidic character↑
(ii) Atomic No↑
So X have minimum Atomic No
& Z have maxima Atomic No
So correct order is X < Y < Z

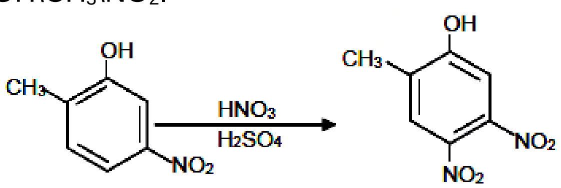
33. C
Sol. Seliwanoff reagent → [Resorcinol + Conc. HCl]
Use of Seliwanoff reagent is to distinguish aldoses and ketoses. Ketoses show red colour with Seliwanoff Reagent.

34. C
Sol. Stronger the ligand greater is splitting of d orbitals and smaller will be wave length of light absorbed.
The splitting power of ligands is $NH_3 > NC\bar{S} > F^-$
So order of wave length of light absorbed is $\lambda_{NH_3} < \lambda_{NC\bar{S}} < \lambda_{F^-}$

35. C
Sol.


36. B
Sol. Cast iron is made from pig iron which is used for production of wrought iron & steel.

37. A
Sol. Rate = $k[A]^a[B]^b$
from Exp (1) & (2) $\quad b = 2$
from Exp (1) & (3) $\quad a = 1$
from Exp (2) & (4) $\Rightarrow 3 = \left(\frac{x}{0.1}\right)^1 \quad$ so x = 0.3
from Exp (1) & (5) $\Rightarrow 48 = (3)^1\left(\frac{y}{0.1}\right)^2$
$(4)^2 = \left(\frac{y}{0.1}\right)^2 \quad$ so y = 0.4

38. B
Sol. This is electrophilic substitution reaction which is determine by electronic effect of $OH \backslash CH_3 \backslash NO_2$.


39. B
Sol. Acidic strength ∝ Stability of conjugate base
General order of acidic strength
$R - COOH > Ph - OH > R - C \equiv CH$
'c' is more acidic due to $-M$ effect of $-NO_2$.

40. D

Sol. In toilet cleaning liquid the main constituent is HCl, which can cause skin burn so it should be treated with $NaHCO_3$ which can easily consume the acid.

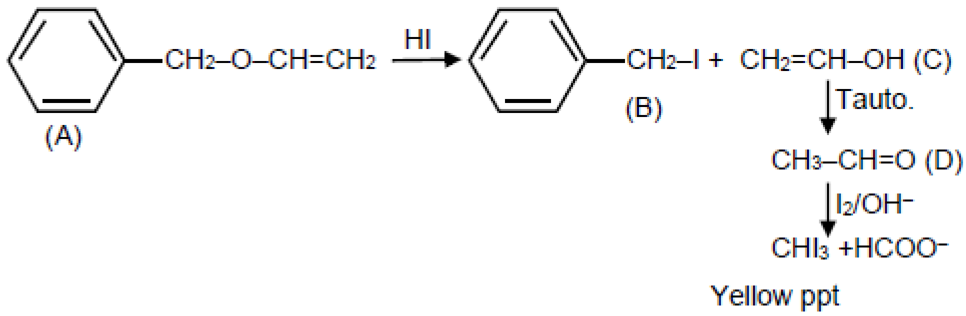
41. B

Sol. (a) When gas is adsorbed on metal surface. ΔH become less negative with progress of reaction.

(b) Gas with greater value of critical temperature (TC) absorbed more. As $TC(NH_3) > TC(N_2)$ So NH_3 absorbed more than N_2.

42. D

Sol. $C_9H_{10}O$



43. D

Sol. (1) $\overset{+2}{[Ni(NH_3)_2Cl_2]} \Rightarrow \quad Ni^{2+} \quad \Rightarrow \quad 3d^84s^0.$

$\Rightarrow$ sp^3 hybridisition.

$\Rightarrow$ tetrahedral.

so $[Ni(NH_3)_2Cl_2]$ do not show isomerism.

(2) $[Ni(NH_3)_4(H_2O)_2]^{2+}$, show geometrical isomerism.

(3) $[Ni(en)_3]^{2+}$, show optical isomerism.

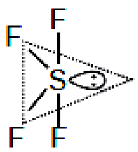
(4) $[Pt(NH_3)_2Cl_2]$, show geometrical isomerism.

44. B

Sol. When mango kept in concentrate salt solution then solvent (water) flow from mango to concentrate solution that's why mango shrinks this is called. "Osmosis"

45. B

Sol. $SF_4 \Rightarrow$ Steric No = 5 so hybridisation is sp^3d.



Geometry is trigonal bipyramidal but shape is "See Saw".

46. 5

Sol. Mass ratio of C : H is 4 : 1 $\Rightarrow$ 12 : 3

& C : O is 3 : 4 $\Rightarrow$ 12 : 16

		mass	mole	mole ratio
so	C	12	1	1
	H	3	3	3
	O	16	1	1

Empirical formula $\Rightarrow CH_3O$

as compound is saturated acyclic so molecular formula is $C_2H_6O_2$.

$$C_2H_6O_2 + \frac{5}{2}O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(g)$$

mole 2 mole 5 mole

so required moles of O_2 is $\Rightarrow 5$

47. 144

Sol.
$$E^{\circ}_{cell} = E^{\circ}_{Cu^{+}/Cu} - E^{\circ}_{Cu^{2+}/Cu^{+1}}$$
$$= 0.52 - 0.16$$
$$= 0.36\ V$$
$$\boxed{E^{\circ}_{cell} = \frac{RT}{nF}\ln K_{eq}}$$
$$0.36 = \frac{0.025}{1}\ln k$$
$$\ln k = 14.4$$
$$= 144 \times 10^{-1}$$
Ans. 144

48. **–326400**

Sol.
$C_2H_5OH(\ell) + O_2(g) \longrightarrow 2O_2(g) + 3H_2O(\ell);\ \Delta H_C = -327\ Kcal$

$\Delta H_C = \Delta U_C + \Delta n_g RT$

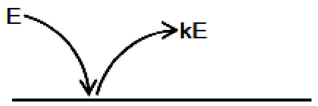
$-327 \times 10^3 = \Delta U_C + 1 \times 2 \times 300$

$\Delta U_C = -326400$ cal

So heat evolved as constant volume is –326400 cal

49. 222

Sol.



Metal (Work function = E_0)

$E = E_0 + (kE)_{max}$

$$\frac{hC}{\lambda} = 4.41 \times 10^{-19} + kE$$

$$\frac{6.63 \times 10^{-34} \times 3 \times 108}{300 \times 10^{-9}} = 4.41 \times 10^{-19} + kE$$

So, $(kE)_{max} = 6.63 \times 10^{-19} - 4.41 \times 10^{-19}$

$= 2.22 \times 10^{-19}$

$= 222 \times 10^{-21}$ J

50. 19

Sol.

	Compound	**Oxidation state of transition element.**
(i)	$K_2Cr_2O_7$	x = +6
(ii)	$KMnO_4$	y = +7
(iii)	K_2FeO_4	z = +6

so (x + y + z) = 19

PART–C (MATHEMATICS)

51. A

Sol. $A^T A = I$

$\Rightarrow a^2 + b^2 + c^2 = 1$ and $ab + bc + ca = 0$

Now, $(a+b+c)^2 = 1$

$\Rightarrow a+b+c = \pm 1$

So, $a^3 + b^3 + c^3 - 3abc$

$= (a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$

$= (\pm 1)(1-0) = \pm 1$

$\Rightarrow 3abc = 2 \pm 1 = 3,1$

$\Rightarrow abc = 1, \frac{1}{3}$

52. B

Sol. $\lambda = -\left(\sin^4\theta + \cos^4\theta\right)$

$\lambda = -\left(\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta\right)$

$\lambda = \frac{\sin^2 2\theta}{2} - 1$

$\frac{\sin^2 2\theta}{2} \in \left[0, \frac{1}{2}\right]$

$\lambda \in \left[-1, -\frac{1}{2}\right]$

53. B

Sol. $f(x) = a(x-3)(x-\alpha)$

$f(2) = a(\alpha - 2)$

$f(-1) = 4a(1+\alpha)$

$f(-1) + f(2) = 0 \Rightarrow a(\alpha - 2 + 4 + 4\alpha) = 0$

$a \neq 0 \Rightarrow 5\alpha = -2$

$\alpha = -\frac{2}{5} = -0.4$

$\alpha \in (-1, 0)$

54. D

Sol. Here normal is $\perp^r$ to both the lines. So normal vector to the plane is

$\vec{n} = \left(\hat{i} - 2\hat{j} + 2\hat{k}\right) \times \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$\vec{n} = -4\hat{i} + 5\hat{j} + 7\hat{k}$

Now equation of plane passing through (3, 1, 1) is

$\Rightarrow -4(x-3)+5(y-1)+7(z-1)=0$

$\Rightarrow -4x+12+5y-5+7z-7=0$

$\Rightarrow -4x+5y+7z=0$

The plane also passes through $(\alpha, -3, 5)$.

$-4\alpha+5\times(-3)+7\times(5)=0$

$\Rightarrow -4\alpha-15+35=0$

$\Rightarrow \alpha=5$

55. D

Sol. $a_1+a_2+a_3+...+a_{11}=0$

$\Rightarrow (a_1+a_{11})\times\frac{11}{2}=0$

$\Rightarrow a_1+a_{11}=0$

$\Rightarrow a_1+a_1+10d=0$, where d is the common difference

$\Rightarrow a_1=-5d$

$a_1+a_3+a_5+...+a_{23}=(a_1+a_{23})\times\frac{12}{2}=(a_1+a_1+22d)\times 6$

$=\left(2a_1+22\left(-\frac{a_1}{5}\right)\right)\times 6=-\frac{72}{5}a_1 \Rightarrow K=-\frac{72}{5}$

56. C

Sol. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}=e^{\lim_{x\to 0}\frac{1}{x}\left(\tan\left(\frac{\pi}{4}+x\right)-1\right)}$

$=e^{\lim_{x\to 0}\left(\frac{1+\tan x-1+\tan x}{x(1-\tan x)}\right)}=e^{\lim_{x\to 0}\frac{2\tan x}{x(1-\tan x)}}=e^2$

57. A

Sol. $x^2-y^2\sec^2\theta=10$

$\Rightarrow \frac{x^2}{10}-\frac{y^2}{10\cos^2\theta}=1$

Hence, eccentricity of hyperbola $(e_H)=\sqrt{1+\frac{10\cos^2\theta}{10}}$ …………(1)

$\left\{e=\sqrt{1+\frac{b^2}{a^2}}\right\}$

Now, equation of ellipse is $x^2\sec^2\theta+y^2=5$.

$\Rightarrow \frac{x^2}{5\cos^2\theta}+\frac{y^2}{5}=1$

Hence, eccentricity of ellipse $(e_E)=\sqrt{1-\frac{5\cos^2\theta}{5}}$ $\left\{e=\sqrt{1-\frac{a^2}{b^2}}\right\}$

$= \sqrt{1-\cos^2\theta} = |\sin\theta| = \sin\theta$ ....(2)

$\left\{\because \theta \in \left(0, \frac{\pi}{2}\right)\right\}$

Given $e_H = \sqrt{5} e_E$

Hence, $1+\cos^2\theta = 5\sin^2\theta$

$1+\cos^2\theta = 5\left(1-\cos^2\theta\right)$

$\cos^2\theta = \frac{2}{3}$ ....(3)

Now, length of latus rectum of ellipse $= \frac{2a^2}{b} = \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$

58. D

Sol. $y \ge x^2 \Rightarrow$ upper region of $y = x^2$

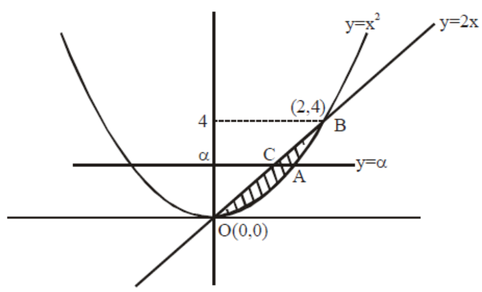
$y \le 2x \Rightarrow$ lower region of $y = 2x$

According to question, area of OABCO = 2 (area of OACO)

$\Rightarrow \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right) dy = 2\int_0^\alpha \left(\sqrt{y} - \frac{y}{2}\right) dy$

$\Rightarrow \frac{4}{3} = 2\left(\frac{2}{3}\alpha^{3/2} - \frac{1}{4}\alpha^2\right)$

$\Rightarrow 3\alpha^2 - 8\alpha^{3/2} + 8 = 0$



59. A

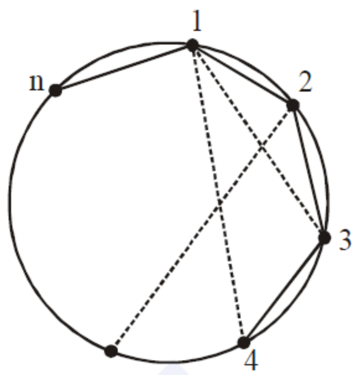
Sol. Number of blue lines = number of sides = n

Number of red lines = number of diagonals

$= {}^nC_2 - n$

${}^nC_2 - n = 99n \Rightarrow \frac{n(n-1)}{2} = 100n$

$\Rightarrow n = 201$



60. A

Sol. $3 + 2\sqrt{-54} = 3 + 6\sqrt{6}\,i = \left(3 + \sqrt{6}\,i\right)^2$

$3 - 2\sqrt{54} = \left(3 - \sqrt{6}\,i\right)^2$

$\left(3 + 2\sqrt{-54}\right)^{1/2} - \left(3 - 2\sqrt{-54}\right)^{1/2} = \pm\left(3 + \sqrt{6}\,i\right) \pm \left(3 - \sqrt{6}\,i\right)$

$= 6, -6, 2\sqrt{6}\,i, -2\sqrt{6}\,i$

61. C

Sol. $f'(x) = \dfrac{\dfrac{x}{1+x} - \ell n(1+x)}{x^2}$

$= \dfrac{x-(1+x)\ell n(1+x)}{x^2(1+x)}$

Suppose $h(x) = x-(1+x)\ell n(1+x)$

$\Rightarrow h'(x) = 1-\ell n(1+x)-1 = -\ell n(1+x)$

$h'(x) > 0 \,\forall\, x \in (-1, 0)$

$h'(x) < 0 \,\forall\, x \in (0, \infty)$

$h(0) = 0 \Rightarrow h'(x) \le 0 \,\forall\, x \in (-1, \infty)$

$\Rightarrow f'(x) \le 0 \;\forall\; x \in (-1, \infty)$

$\Rightarrow f(x)$ is a decreasing function for all $x \in (-1, \infty)$.

62. C

Sol. $2x^2 dy = (2xy + y^2)dx$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2xy+y^2}{2x^2}$ (Homogeneous D.E.)

$\left\{\begin{array}{c} \text{Let } y = xt \\ \Rightarrow \dfrac{dy}{dx} = t + x\dfrac{dt}{dx} \end{array}\right\}$

$\Rightarrow t + x\dfrac{dt}{dx} = \dfrac{2x^2t + x^2t^2}{2x^2}$

$\Rightarrow t + x\dfrac{dt}{dx} = t + \dfrac{t^2}{2}$

$\Rightarrow 2\int \dfrac{dt}{t^2} = \int \dfrac{dx}{x}$

$\Rightarrow 2\left(-\dfrac{1}{t}\right) = \ell n x + C$

$\Rightarrow -\dfrac{2x}{y} = \ell n x + C$

Put x = 1 and y = 2 to get C = –1.

$\Rightarrow \dfrac{-2x}{y} = \ell n x - 1$

$\Rightarrow y = \dfrac{2x}{1-\ell n x}$

$\Rightarrow f(x) = \dfrac{2x}{1-\log_e x}$

So, $f\left(\dfrac{1}{2}\right) = \dfrac{1}{1+\log_e 2}$

63. D

Sol. Given $P=\begin{bmatrix}1 & 2 & 1\\ -2 & 3 & -4\\ 1 & 9 & -1\end{bmatrix}$.

Here $|P|=0$ and also given $PX=0$

$$\Rightarrow\begin{bmatrix}1 & 2 & 1\\ -2 & 3 & -4\\ 1 & 9 & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=0$$

$$\left.\begin{aligned}x+2y+z=0\\ \Rightarrow -2x+3y-4z=0\\ x+9y-z=0\end{aligned}\right\}$$

$D=0$, so the system has infinitely many solutions.

By solving these equations, we get $x=-\frac{11\lambda}{2},\ y=\lambda,\ z=\frac{7\lambda}{2}$.

Also, given $x^2+y^2+z^2=1$.

$$\Rightarrow\left(-\frac{11\lambda}{2}\right)^2+(\lambda)^2+\left(\frac{7\lambda}{2}\right)^2=1$$

$$\Rightarrow\lambda=\pm\frac{1}{\sqrt{\frac{121}{4}+1+\frac{49}{4}}}$$

There are 2 values of λ.
So, there are 2 solution sets for (x, y, z).

64. B

Sol. $y=(1+x)^{2y}+\cos^2\left(\sin^{-1}x\right)$

Put $x=0$.

$y=(1+0)^{2y}+\cos^2\left(\sin^{-1}0\right)=2$

So, we have to find the normal at (0, 2).

Now, $y=e^{2y\ln(1+x)}+\cos^2\left(\cos^{-1}\sqrt{1-x^2}\right)$

$y=e^{2y\ln(1+x)}+\left(\sqrt{1-x^2}\right)^2$

$y=e^{2y\ln(1+x)}+\left(1-x^2\right)$ ....(1)

Now differentiate w.r.t. x.

$y'=e^{2y\ln(1+x)}\left[\frac{2y}{1+x}+\ln(1+x).2y'\right]-2x$

Put $x=0$ and $y=2$.

$y'=e^{4\ln 1}\left[4+\ln 1.2y'\right]-0$

$y'=4=$ slope of tangent to the curve

Hence, equation of normal at (0, 2) is $y-2=-\frac{1}{4}(x-0)$

$\Rightarrow x+4y=8$

65. A

Sol. $\sim p \wedge (p \vee q) \to q$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q) \to q$

$\equiv C \vee (\sim p \wedge q) \to q$

$\equiv (\sim p \wedge q) \to q$

$\equiv \sim (\sim p \wedge q) \vee q$

$\equiv (p \vee \sim q) \vee q$

$\equiv (p \vee q) \vee (\sim q \vee q)$

$\equiv (p \vee q) \vee T$

So, $\sim p \wedge (p \vee q) \to q$ is a tautology.

66. D

Sol. $S = \left(x + x^2 + x^3 + x^4 + \ldots 9\,\text{terms}\right) + \left(ka + ka + ka + ka + \ldots 9\,\text{terms}\right)$

$+\left(0 + 2a + 4a + 6a + \ldots 9\text{ terms}\right)$

$\Rightarrow S = x\left(\dfrac{x^9 - 1}{x - 1}\right) + 9ka + 72a$

$\Rightarrow S = \dfrac{\left(x^{10} - x\right) + (9k + 72)a(x - 1)}{x - 1}$

Comparing with the given sum, we get $9k + 72 = 45$.

$\Rightarrow k = -3$

67. C

Sol. Given E_1, E_2, E_3 are pairwise independent events.

So $P(E_1 \cap E_2) = P(E_1).P(E_2)$

and $P(E_2 \cap E_3) = P(E_2).P(E_3)$

and $P(E_3 \cap E_1) = P(E_3).P(E_1)$

and $P(E_1 \cap E_2 \cap E_3) = 0$.

Now $P\left(\dfrac{\bar{E}_2 \cap \bar{E}_3}{E_1}\right) = \dfrac{P\left(E_1 \cap \left(\bar{E}_2 \cap \bar{E}_3\right)\right)}{P(E_1)}$

$= \dfrac{P(E_1) - \left[P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)\right]}{P(E_1)}$

$= \dfrac{P(E_1) - P(E_1).P(E_2) - P(E_1)P(E_3) + 0}{P(E_1)}$

$= 1 - P(E_2) - P(E_3)$

$= \left[1 - P(E_3)\right] - P(E_2)$

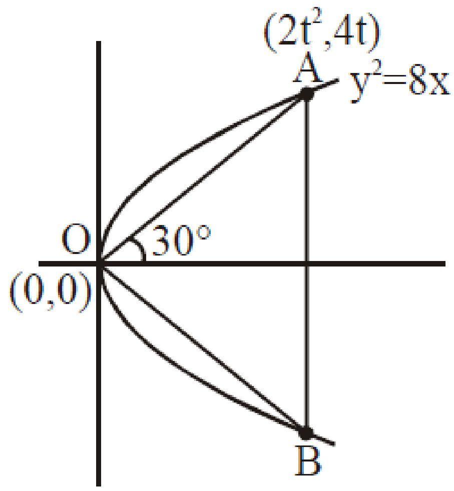
$= P\left(E_3^C\right) - P(E_2)$

68. A

Sol. $\tan 30^o = \frac{4t}{2t^2} = \frac{2}{t} \Rightarrow t = 2\sqrt{3}$

$AB = 8t = 16\sqrt{3}$

Area $= \frac{\sqrt{3}}{4}\left(16\sqrt{3}\right)^2 = 192\sqrt{3}$



69. C

Sol. $f(x+y) = f(x) + f(y)$

$\Rightarrow f(n) = nf(1) = 2n$

$g(n) = \sum_{k=1}^{n-1}(2k) = 2\left(\frac{(n-1)n}{2}\right) = n(n-1)$

$g(n) = 20 \Rightarrow n(n-1) = 20$

$n = 5$

70. C

Sol. Given the points (1, 2) and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y - 1 = 0$.

$\Rightarrow (1+2-1)(\sin\theta + \cos\theta - 1) > 0$

$\Rightarrow \sin\theta + \cos\theta > 1$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{3\pi}{4}$

$\Rightarrow 0 < \theta < \frac{\pi}{2}$

71. 91

Sol. Put $\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5},\ 0 < \alpha < \frac{\pi}{2}$

Now $\frac{3}{5}\cos kx - \frac{4}{5}\sin kx = \cos\alpha.\cos kx - \sin\alpha.\sin kx = \cos(\alpha + kx)$

As we have to find derivate at x = 0, we have $\cos^{-1}(\cos(\alpha + kx)) = \alpha + kx$

$\Rightarrow y = \sum_{k=1}^{6} k(\alpha + kx) = \sum_{k=1}^{6}\left(k\alpha + k^2x\right)$

$\Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = \frac{(6)(7)(13)}{6} = 91$

72. 3.00

Sol. Let a be the first term and d be the common difference of the given A.P., where d > 0.

$$\overline{X} = a + \frac{0 + d + 2d + ... + 10d}{11} = a + 5d$$

$$\text{Variance } = \frac{\sum\left(\overline{X} - x_i\right)^2}{11}$$

$$\Rightarrow 90 \times 11 = 2\left(25d^2 + 16d^2 + 9d^2 + 4d^2 + d^2\right) = 110d^2$$

$$\Rightarrow d = \pm 3 \Rightarrow d = 3$$

73. 1.0

Sol. $3 < 3x < 6$

Take cases when $3 < 3x < 4,\ 4 < 3x < 5,\ 5 < 3x < 6$.

$$\text{Now } \int_1^2 \left|2x - [3x]\right| dx$$

$$= \int_1^{4/3} (3 - 2x)dx + \int_{4/3}^{5/3} (4 - 2x)dx + \int_{5/3}^{2} (5 - 2x)dx$$

$$= \frac{2}{9} + \frac{3}{9} + \frac{4}{9} = 1$$

74. 118

Sol. $^nC_{r-1} : {^nC_r} : {^nC_{r+1}} = 2 : 5 : 12$

$$\text{Now } \frac{^nC_{r-1}}{^nC_r} = \frac{2}{5}$$

$\Rightarrow 7r = 2n + 2$ ………..(1)

$$\frac{^nC_r}{^nC_{r+1}} = \frac{5}{12}$$

$\Rightarrow 17r = 5n - 12$ …………..(2)

On solving (1) and (2), we get $n = 118$.

75. 0.8

Sol. Using section formula, we get

$$\overline{OP} = \frac{2\lambda + 1}{\lambda + 1}\hat{i} + \frac{\lambda + 1}{\lambda + 1}\hat{j} + \frac{3\lambda + 1}{\lambda + 1}\hat{k}$$

$$\text{Now } \overline{OB}.\overline{OP} = \frac{4\lambda + 2 + \lambda + 1 + 9\lambda + 3}{\lambda + 1}$$

$$= \frac{14\lambda + 6}{\lambda + 1}$$

$$\overline{OA} \times \overline{OP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \frac{2\lambda + 1}{\lambda + 1} & 1 & \frac{3\lambda + 1}{\lambda + 1} \end{vmatrix} = \frac{2\lambda}{\lambda + 1}\hat{i} + \frac{-\lambda}{\lambda + 1}\hat{j} + \frac{-\lambda}{\lambda + 1}\hat{k}$$

$$\left|\overrightarrow{OA} \times \overrightarrow{OP}\right|^2 = \frac{4\lambda^2 + \lambda^2 + \lambda^2}{(\lambda + 1)^2} = \frac{6\lambda^2}{(\lambda + 1)^2}$$

$$\Rightarrow \frac{14\lambda + 6}{\lambda + 1} - 3 \times \frac{6\lambda^2}{(\lambda + 1)^2} = 6$$

$$\Rightarrow 10\lambda^2 - 8\lambda = 0$$

$$\Rightarrow \lambda = 0, \frac{8}{10}$$

$$\Rightarrow \lambda = 0.8$$