FIITJEE

Solutions to JEE (Main)-2020

JEE–Main–2020 –Sept–2–First–Shift
PHYSICS, CHEMISTRY & MATHEMATICS

PART –A (PHYSICS)

1. **D**

Sol. Energy Density = $\frac{1}{2}\frac{B^2}{\mu_0}$

$B = \sqrt{2 \times \mu_0 \times \text{Energy density}}$

$B = \sqrt{2 \times 4\pi \times 10^{-7} \times 1.02 \times 10^{-8}}$ = 160 × 10^9 = 160 nT

2. **B**

Sol. Based on theory.

3. **B**

Sol. $f_x = \frac{1}{2\ell}\sqrt{\frac{T_x}{\mu}}$

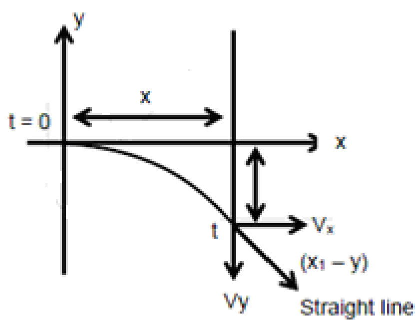
$f_y = \frac{1}{2\ell}\sqrt{\frac{T_y}{\mu}}$

$\frac{f_x}{f_y} = \frac{450}{300} = \sqrt{\frac{T_x}{T_y}}$

$\Rightarrow$ T_x/T_y = 9/4 = 2.25

4. **A**

Sol.



x > d path is straight line

$$\frac{-y = \frac{1}{2}at^2}{x - d} = \frac{at}{V_0}$$

$$\frac{-y - \frac{1}{2}a^2}{at} = \frac{x - d}{V_0}$$

$$\frac{-y}{at} - \frac{1}{2}\frac{d}{V_0} = \frac{x}{V_0} - \frac{d}{V_0}$$

$$-\frac{myV_0}{qEd} = \frac{x}{V_0} - \frac{d}{2V_0}$$

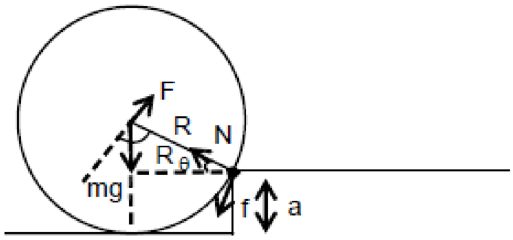
$$y = \frac{-qEd}{mV_0}\left(\frac{x}{V_0} - \frac{d}{2V_0}\right) \;;\; y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$$

5. **B**

Sol. Zero error = 0 + 7 × 0.1 = 0.070
Vernier reading = (3.1 + 4 × 0.01) –0.07 = 3.07

6. **B**

Sol.



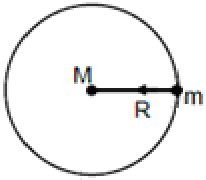
$FR > mg\cos\theta R$

$F > mg\cos\theta$

$F > mg\dfrac{\sqrt{R^2-(R-a)^2}}{R} \Rightarrow Mg\sqrt{1-\left(\dfrac{R-a}{R}\right)^2}$

7. **D**

Sol.



$M = \int \rho dV$

$M = \int_0^{r=R_0} \dfrac{k}{r} 4\pi r^2 dr$

$M = \dfrac{4\pi k R_0^2}{2} = 2\pi k R^2$

$F_G = \dfrac{GMm}{R_0^2} = 2\omega_0^2 R$

$\Rightarrow \quad \dfrac{G\dfrac{4\pi kR^2}{2}}{R^2} = \omega_0^2 R \qquad \Rightarrow \qquad \omega_0 = \sqrt{\dfrac{2\pi KG}{R}}$

$\because \quad T = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi\sqrt{R}}{\sqrt{2\pi KG}} = \sqrt{\dfrac{2\pi R}{KG}}$

$\Rightarrow \quad T^2 \propto R$

8. **D**

Sol. $\Delta P = d\sin\theta$

$= d\theta$

$= \dfrac{dy}{D} = \dfrac{10^{-3} \times 1.270\,\text{mm}}{1\,\text{m}}$ $1.27\ \mu\text{m}$

9. **B**

Sol. $\rho_m = 98 \times 10^{-8}$

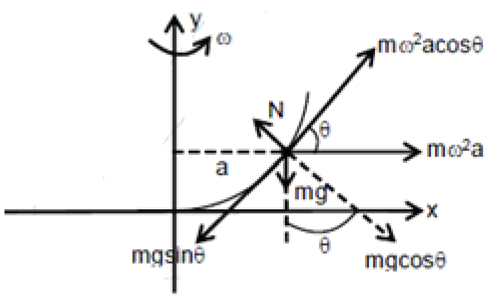
$\rho_A = 2.65 \times 10^{-8}$

$\rho_C = 1.724 \times 10^{-8}$

$\rho_T = 5.65 \times 10^{-8}$

10. **C**

Sol.



$m\omega^2 a\cos\theta = mg\sin\theta$

$\omega = \sqrt{\dfrac{g\tan\theta}{a}}$

$y = 4cx^2$

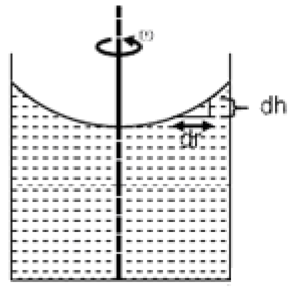
$\tan\theta = \dfrac{dy}{dx} = 8xC$

$(\tan\theta)_{a,b} = 8aC$

$\omega = \sqrt{\dfrac{g \times 8ac}{a}} = 2\sqrt{2gc}$

11. **A**

Sol. $\rho dr\omega^2 r = \rho g dh$



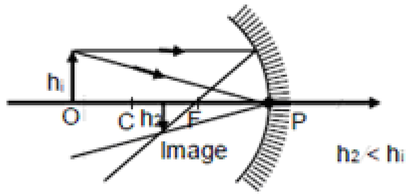
$$\omega^2\int_0^R r\,dr = g\int_0^h dh$$

$$\frac{\omega^2R^2}{2} = gh$$

$$h = \frac{\omega^2R^2}{2g} = \frac{25\omega^2}{2g}$$

12. **C**

Sol.



13. **A**

Sol. From momentum conservation

$$mu\hat{i} + 0 = mv\hat{j} + 3m\vec{v}'$$

$$\vec{v}' = \frac{u}{3}\hat{i} - \frac{v}{3}\hat{j}$$

From kinetic energy conservation $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3\,m)\left(\left(\frac{u}{3}\right)^2 + \left(\frac{v}{3}\right)^2\right)$

Solving, $v = \frac{u}{\sqrt{2}}$

14. **D**

Sol. $Y \propto F^aV^bA^c$ $\qquad Y = \left(\frac{F}{A}\right)$

$$\frac{MLT^{-2}}{L^2} \propto (M^1L^1T^{-2})^a(L^1T^{-1})^b(L^2)^c$$

$M^1L^{-1}T^{-2} \propto M^aL^{a+b+2c}\,T^{-2a-b}$

$a + b + 2c = -1$

$-2a + b = -2$

$a = 1, b = 0, c = -1$

$Y = F^1v^0A^{-1}$

15. **B**

Sol. From Given equation

$\mu = 0.6$

$A_m = \mu Ac$

$$\frac{A_{max} - A_{min}}{2} = A_c = 5 \qquad \ldots(1)$$

$$\frac{A_{max} - A_{min}}{2} = 3 \qquad \ldots(2)$$

From equation (1) + (2)

$A_{max} = 8$

From equation (1) – (2)

$A_{min} = 2$

16. **A**

Sol. $\frac{f_1n_1RT_1}{2}+\frac{r_2n_2RT_2}{2}=3\times\frac{5}{2}RT+\frac{5}{2}\times 3RT=15$

17. **B**

Sol. Pitch = (V cosθ)T

$= (V\cos\theta)\frac{2\pi m}{eB}$

$= (4\times 10^5\cos 60^o)\ \frac{2\pi}{0.3\times 10}\left(\frac{1.67\times 10^{-27}}{1.69\times 10^{19}}\right)$

= 4 cm

18. **A**

Sol. $P=\frac{E}{t}$

$= \frac{2}{235}\times\frac{6.023\times 10^{26}\times 200\times 1.6\times 10^{-19}}{30\times 24\times 60\times 60}$ = 60 W

19. **D**

Sol. V_A = 36 km/hr = 10 m/s

V_B = –72 km/hr = –20 m/s

V_{MA} = –1.8 km/hr = –0.5 m/s

$V_{man, B} = V_{man, A} + V_{A, B}$

$= V_{man, A} + V_A - V_B$

= –0.5 + 10 – (–20)

= –0.5 + 30 = 29.5 m/s

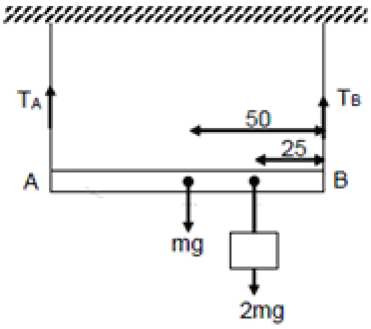
20. **B**

Sol. τ_{net} about B is zero at equilibrium

T_A 100 – mg × 50 – 2 mg × 25 = 0

T_A 100 = 100 mg

T_A = 1 mg



21. **15**

Sol. Flux as a function of time $\phi=\vec{B}\cdot\vec{A}=AB\cos(\omega t)$

emf induced,

$e=\frac{-d\phi}{dt}=AB\omega\sin(\omega t)$

Maximum value of emf = ABω

$= \pi R^2 B\omega$

$= 3.14\times 0.1\times 0.1\times 3\times 10^{-5}\times\frac{o}{0.2}=15$

22. **46**

Sol. $T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$

$T_2=T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$= T_1(10)^{\frac{7}{5}-1}$

$T_2 = T_1\,(10)^{2/5}$

$\Delta V = \frac{5}{2}nR\ ;\ \frac{5}{2}\times 5\times 3[10^{2/5}-1]\,(293)$

$\frac{625}{6}\times 1.5\times 293 = 461440 \approx 46\,\text{ks}$

23. **9**

Sol. $\frac{hc}{\lambda} = \phi + eV \qquad \ldots(1)$

$\frac{hc}{3\lambda} = \phi + \frac{eV}{4} \qquad \ldots(2)$

from (1) and (2)

$\frac{hc}{\lambda}\left(1-\frac{1}{3}\right) = \frac{3}{4}eV \quad ; \quad \frac{hc}{\lambda}\frac{2}{3} = \frac{3}{4}eV$

$eV = \frac{8}{9}\frac{hc}{\lambda}\ ;\ \frac{hc}{\lambda} = \phi + \frac{8}{9}\frac{hc}{\lambda}$

$\phi = \frac{hc}{9\lambda} = \frac{hc}{\lambda_{th}}$

$\lambda_{th} = 9\lambda \quad ; \quad \therefore\ k = 9$

24. **4**

Sol. $c_1 = 5\mu F \qquad V_1 = 220$ Volt

$c_2 = 2.5\ \mu F \qquad V_2 = 0$

Heat loss; $\Delta H = U_i - U_f = \frac{1}{2}\frac{c_1c_2}{c_1+c_2}(v_1 - v_2)^2$

$= \frac{1}{2}\times\frac{5\times 2.5}{(5+2.5)}(220-0)^2\mu J$

$= \frac{5}{2\times 3}\times 22\times 22\times 100\times 10^{-6}\,J$

$= \frac{5\times 11\times 22}{3}\times 10^{-4}\,J = \frac{55\times 22}{3}\times 10^{-4}\,J$

$= \frac{1210}{3}\times 10^{-4}\,J = \frac{1210}{3}\times 10^{-3}\,J\times 4\times 10^{-2}$

According to questions

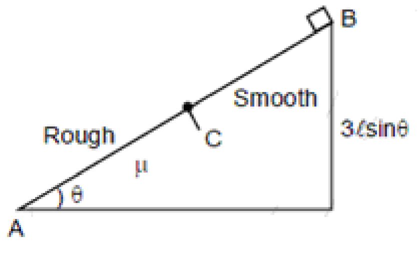
$\frac{x}{100} = 4\times 10^{-2}$

So, x = 4

Note: But given answer by JEE Main x = 36

25. **3**

Sol. Let AC = $\ell \qquad \therefore$ BC = $2\ell \qquad \therefore$ AB = 3ℓ



Apply work – Energy theorem

$W_f + W_{mg} = \Delta KE$

$Mg\,(3\ell)\sin\theta - \mu mg\cos\theta(\ell) = 0 + 0$

$\mu mg\cos\theta\ell = 3mg\ell\sin\theta$

$\mu = 3\tan\theta = k\tan\theta$

$\therefore \quad k = 3$

PART –B (CHEMISTRY)

26. B

Sol. For AB_4 compound possible geometry are

S. No.	Bond pair	Lone pair	Total	Hybridisation	Geometry	Polarity
1	4	0	4	SP^3	Tetrahedral	non polar
2	4	1	5	SP^3d	Sea-saw	Polar
3	4	2	6	sp^3d^2	Square Planar	non polar

Square pyramidal can be polar due to lone pair moment as the bond pair moments will get cancelled out.

27. C

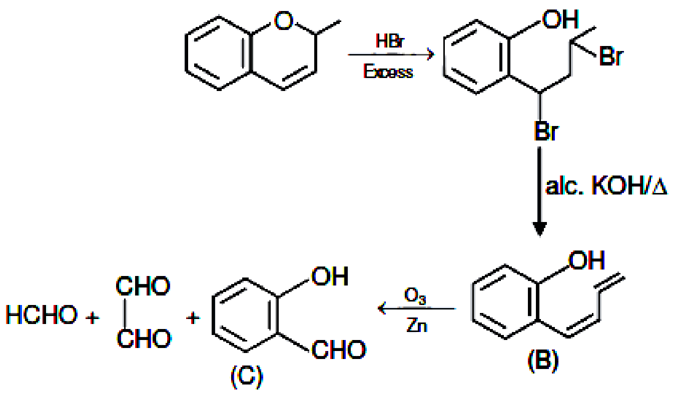
Sol. Rate of chemical reaction has nothing to do with value of equilibrium constant.

28. C

Sol. –I, – M effect of NO_2 increase reactivity towards nucleophilic addition reaction with HCN.

29. A

Sol.



30. C

Sol. (1) $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2$

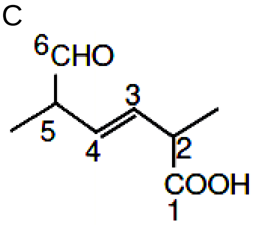
(2) $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

(3) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + H_2O$

(4) $NaN_3 \xrightarrow{300°C} 3N_2 + 2Na$

31. C

Sol.



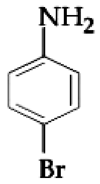
(2, 5-dimethyl-6-oxo-hex-3-enoic acid

32. D

Sol. Mole of Bromine = $\frac{0.08}{80} = 10^{-3}$ mole

Molar mass of compound = $\frac{0.172}{M} = 10^{-3}$

$M = \frac{0.172}{10^{-3}} = 172$ g

Molar mass of  = 80 + 72 + 6 + 14 = 172 g

33. A
Sol. Reaction path is S_N2 because OH^- is strong nucleophile, but hydroxyl ion will not attack on chiral centres and so there is retension of configuration.

34. C
Sol. In presence of sunlight CFC's molecule divides & release chlorine free radical, which react with ozone give chlorine monoxide radical ($ClO^\bullet$) and oxygen.

$$CF_2Cl_2(g) \xrightarrow{UV} \dot{C}l(g) + \dot{C}F_2Cl(g)$$

$$Cl^\bullet(g) + O_3(g) \longrightarrow ClO^\bullet(g) + O_2(g)$$

$$ClO^\bullet(g) + O(g) \longrightarrow Cl^\bullet(g) + O_2(g)$$

35. D
Sol. In this acid base Titrating there is no use of Bunsen burner and measuring cylinder other laboratory equipments will be required for getting the end point of titration.

36. C
Sol. For ideal gas
PM = dRT

$$d = \left[\frac{PM}{R}\right]\frac{1}{T}$$

So graph between d Vs T is not straight line.

37. B
Sol. Since spin only magnetic moment is 4.90 BM so number of unpaired electrons must be 4, so If the complex is octahedral, then it has to be high spin complex with configuration
$t_{2g}^{2,1,1}e_g^{1,1}$ in that case
CFSE = $4 \times (-0.4\Delta_0) + 2 \times 0.6\Delta_0 = -0.4\Delta_0$
If the complex is tetrahedral then its electronic configuration will be = $e_g^{2,1}t_{2g}^{1,1,1}$ and CFSE will be = $3 \times (-0.6\Delta_t) + 3 \times (0.4\Delta_t) = -0.6\Delta_t$

38. C
Sol. Bredig's Arc Method is used for preparation of colloidal sol's of less reactive metal like Au, Ag, Pt.

39. A & C
Sol. The vapour pressure of solution will be less than vapour pressure of pure solvent, so some vapour molecules will get condensed to maintain new equilibrium.

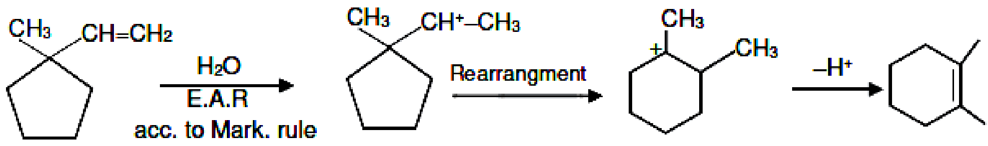
40. C
Sol. On moving Left to Right along a period.
Atomic Radius $\rightarrow$ decreases.
Electronegativity $\rightarrow$ Increases.
Electron gain enthalpy $\rightarrow$ Increases.
Ionisation Enthalpy $\rightarrow$ Increases.

41. B

Sol. With weak field ligands Mn(II) will be of high spin and with strong field ligands it will be of low spin. Ni(II) tetrahedral complexes will be generally of high spin due to sp^3 hybridisation. Mn(II) is of light pink color in aqueous solution.

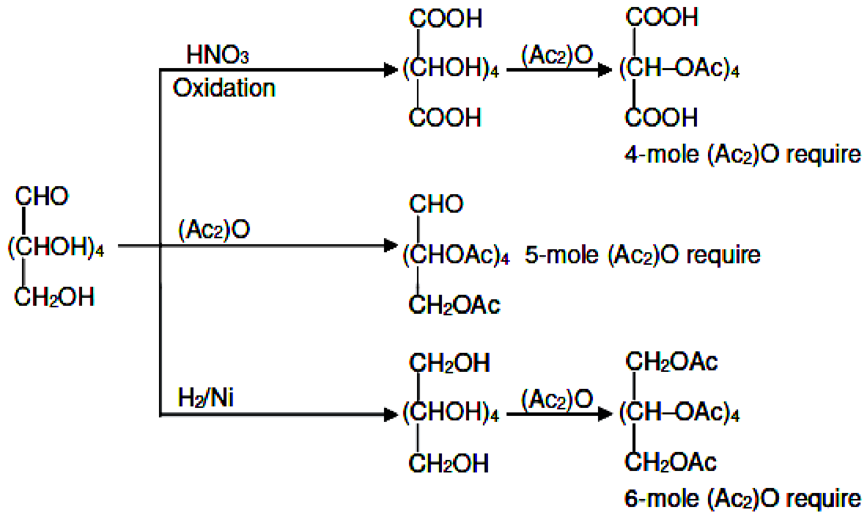
42. A

Sol.



43. A

Sol.



44. B

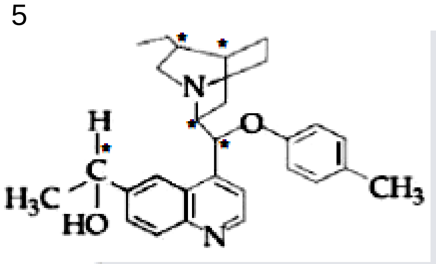
Sol. Cesium has lowest ionisation enthalpy and hence it can show photoelectric effect to the maximum extent hence it is used in photo electric cell.

45. C

Sol. 1, 2 and 3 are according to quantum theory but (4) is statement of kinetic theory of gases

46. 5

Sol.



47. 96500

Sol. $E^0_{cell} = E^0_{Sn^{2+}|Sn} - E^0_{Cu^{2+}|Cu}$

= -0.16 – 0.34 = -0.50 V

$\Delta G^0 = -nFE^0_{cell}$

= -2 × 96500 × (-0.5) = 96500 J

= 96.5 kJ = 96500 J

48. 6

Sol. The oxidation states of iron in these compounds will be

A = +2

B = +4

C = 0

The sum of oxidation states will be = 6.

49. 189000 to 190000

Sol. $\Delta H = \Delta U + \Delta n_g RT$

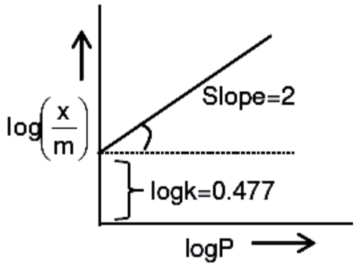
$41000 \times 5 = \Delta U + 5 \times 8.314 \times 373$

$205000 = \Delta U + 15505.61$

$\Delta U = 189494.39 \text{ J} = 189494 \text{ J}$

50. 48

Sol. $\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$



$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log P$

Slope $= \frac{1}{n} = 2$ So $n = \frac{1}{2}$.

Intercept $\Rightarrow \log k = 0.477$ So $k = \text{Antilog } (0.477) = 3$

So $\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$

$= 3[4]^2 = 48$

PART–C (MATHEMATICS)

51. A

Sol. $|x| < 1, |y| < 1, x \neq y$

$(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+....$

By multiplying and dividing x – y:

$$\frac{(x^2-y^2)+(x^3-y^3)+(x^4-y^4)+....}{x-y}$$

$$=\frac{(x^2+x^3+x^4+...)-(y^2+y^3+y^4+.....)}{x-y}$$

$$=\frac{\frac{x^2}{1-x}-\frac{y^2}{1-y}}{x-y}$$

$$=\frac{(x^2-y^2)-xy(x-y)}{(1-x)(1-y)(x-y)}$$

$$=\frac{x+y-xy}{(1-x)(1-y)}$$

52. D

Sol. Let t_{r+1} denotes

$r+1^{th}$ term of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$

$t_{r+1}={}^{10}C_r\alpha^{10-r}(x)^{\frac{10-r}{9}}.\beta^r x^{-\frac{r}{6}}$

$={}^{10}C_r\,\alpha^{10-r}\beta^r(x)^{\frac{10-r}{9}-\frac{r}{6}}$

If t_{r+1} is independent of x

$\frac{10-r}{9}-\frac{r}{6}=0 \quad \Rightarrow \quad r=4$

maximum value of t_5 is 10 K (given)

$\Rightarrow \quad {}^{10}C_4\,\alpha^6\beta^4$ is maximum

By AM $\geq$ GM (for positive numbers)

$$\frac{\frac{\alpha^3}{2}+\frac{\alpha^3}{2}+\frac{\beta^2}{2}+\frac{\beta^2}{2}}{4}\geq\left(\frac{\alpha^6\beta^4}{16}\right)^{\frac{1}{4}}$$

$\Rightarrow \alpha^6\beta^4 \leq 16$

So, $10K={}^{10}C_4 16$

$\Rightarrow$ K = 336

53. C

Sol. Let L be the common normal to parabola $y = x^2 + 7x + 2$ and line $y = 3x - 3$

$\Rightarrow$ slope of tangent of $y = x^2 + 7x + 2$ at P = 3

$\Rightarrow \left.\frac{dy}{dx}\right]_{\text{For P}} = 3$

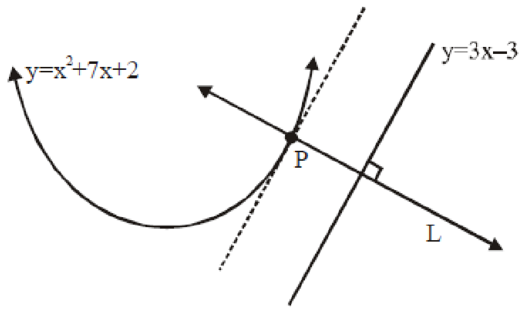
$\Rightarrow 2x + 7 = 3 \Rightarrow x = -2 \Rightarrow y = -8$

So P (–2, –8)

Normal at P : x + 3y + C = 0

$\Rightarrow$ C = 26 (P satisfies the line)

Normal : x + 3y + 26 = 0



54. B

Sol. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{8}-i\cos\frac{2\pi}{9}}\right)^3$

$$= \left(\frac{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)+i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)-i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}\right)^3$$

$$= \left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$$

$$= \left(\frac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-2i\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}\right)^3$$

$$= \left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3$$

$$= \left(\frac{e^{i5\pi/36}}{e^{-i5\pi/36}}\right)^3 = \left(e^{i5\pi/18}\right)^3$$

$$= \cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$$

$$= -\frac{\sqrt{3}}{2}+\frac{i}{2}$$

55. B

Sol. Two points on the line (L say) $\frac{x}{3}=\frac{y}{2}, z=1$ are (0, 0, 1) and (3, 2, 1)

So dr's of the line is < 3, 2, 0 >

Line passing through (1, 2, 1), parallel to L and coplanar with given plane is

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + t(3\hat{i} + 2\hat{j}),\ t \in R(-2, 0, 1)$ satisfies the line (for t = –1)

$\Rightarrow (-2, 0, 1)$ lies on given plane.

Answer of the question is (B)

We can check other options by finding equation of plane

Equation plane : $\begin{vmatrix} x-1 & y-2 & z-1 \\ 1+2 & 2-0 & 1-1 \\ 2+2 & 1-0 & 2-1 \end{vmatrix} = 0$

$\Rightarrow 2(x-1) - 3(y-2) - 5(z-1) = 0$

$\Rightarrow 2x - 3y - 5z + 9 = 0$

56. D

Sol. $|A| \neq 0$

For $(P): A \neq I_2$

So, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$|A|$ can be –1 or 1

So (P) is false.

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow tr(A) = 2$

$\Rightarrow$ Q is true

57. C

Sol. $\frac{|x|}{2} + \frac{|y|}{3} = 1$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

Area of Ellipse $= \pi ab = 6\pi$

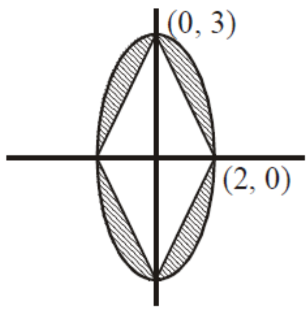
Required area,

$= \pi \times 2 \times 3$ – (Area of quadrilateral)

$= 6\pi - \frac{1}{2} 6 \times 4$

$= 6\pi - 12$

$= 6(\pi - 2)$



58. D

Sol. $\sigma^2 =$ variance

$\mu =$ mean

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$$

$\mu = 17$

$$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b)}{17} = 17$$

$\Rightarrow 9a + b = 17$ ……….(1)

$\sigma^2 = 216$

$$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b-17)^2}{17} = 216$$

$$\Rightarrow \frac{\sum_{x=1}^{17}a^2(x-9)^2}{17} = 216$$

$\Rightarrow a^2 81 - 18 \times 9a^2 + a^2 3 \times (35) = 216$

$\Rightarrow a^2 = \frac{216}{24} = 9 \quad \Rightarrow \quad a = 3(a > 0)$

$\Rightarrow$ From (1), b = – 10

So, $a + b = -7$

59. C

Sol. Slope of tangent to the curve $y = x + \sin y$

at (a, b) is $\frac{2-\frac{3}{2}}{\frac{1}{2}-0} = 1$

$\Rightarrow \left.\frac{dy}{dx}\right]_{x=a} = 1$

$\frac{dy}{dx} = 1 + \cos y . \frac{dy}{dx}$ (from equation of curve)

$\Rightarrow \left.\frac{dy}{dx}\right]_{x=a \text{ and } y=b} = 1 + \cos b = 1$

$\Rightarrow \quad \cos b = 0$

$\Rightarrow \quad \sin b = \pm 1$

Now, from curve $y = x + \sin y$

$b = a + \sin b$

$\Rightarrow |b - a| = |\sin b| = 1$

60. D

Sol. $\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x, y > 0$

$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$

By integrating both sides:

$\ell n|y+1| = -\ell n|2+\sin x| + \ell nK$

$\Rightarrow y + 1 = \frac{K}{2+\sin x} \qquad (y+1>0)$

$\Rightarrow y(x) = \frac{K}{2+\sin x} - 1$

Given $y(0) = 1 \Rightarrow K = 4$

So, $y(x) = \frac{4}{2+\sin x} - 1$

$a = y(\pi) = 1$

$b = \left.\frac{dy}{dx}\right]_{x=\pi} = \left.\frac{-\cos x}{2+\sin x}\left(y(x)+1\right)\right]_{x=\pi} = 1$

So, $(a,b) = (1,1)$

61. D

Sol. Let p denotes statement
p : I reach the station in time.
q : I will catch the train.
Contrapositive of $p \to q$ is $\sim q \to \sim p$
$\sim q \to \sim p$: I will not catch the train, then I do not reach the station in time.

62. D

Sol. $f(x) = \sin\left(\frac{|x|+5}{x^2+1}\right)$

For domain:

$-1 \le \frac{|x|+5}{x^2+1} \le 1$

Since $|x|+5$ and x^2+1 is always positive

So $\frac{|x|+5}{x^2+1} \ge 0 \; \forall x \in R$

So for domain:

$\frac{|x|+5}{x^2+1} \le 1$

$\Rightarrow |x| + 5 \le x^2 + 1$

$\Rightarrow 0 \le x^2 - |x| - 4$

$\Rightarrow 0 \le \left(|x| - \frac{1+\sqrt{17}}{2}\right)\left(|x| - \frac{1-\sqrt{17}}{2}\right)$

$\Rightarrow |x| \ge \frac{1+\sqrt{17}}{2}$ or $|x| \le \frac{1-\sqrt{17}}{2}$ (Rejected)

$\Rightarrow x \in \left(-\infty, -\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$

So, $a = \frac{1+\sqrt{17}}{2}$

63. C

Sol. α and β are roots of $5x^2 + 6x - 2 = 0$

$\Rightarrow 5\alpha^2 + 6\alpha - 2 = 0$

$\Rightarrow 5\alpha^{n+2} + 6\alpha^{n+1} - 2\alpha^{n} = 0$ ……….(1)

(By multiplying α^n)

Similarly $5\beta^{n+2} + 6\beta^{n+1} - 2\beta^{n} = 0$ ………..(2)

By adding (1) and (2)

$5S_{n+2} + 6S_{n+1} - 2S_n = 0$

For n = 4

$5S_6 + 6S_5 = 2S_4$

64. C

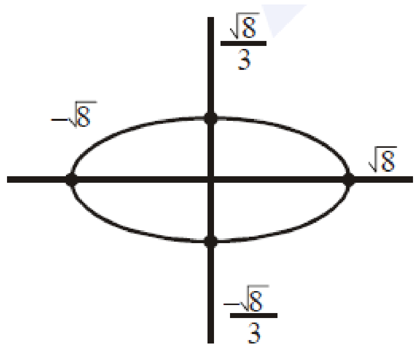
Sol. $R\{(x,y): x, y \in z, x^2 + 3y^2 \le 8\}$

For domain of R^{-1}

Collection of all integral of y's

For $x = 0, 3y^2 \le 8$

$\Rightarrow y \in \{-1, 0, 1\}$



65. D

Sol. Since $p(x)$ has relative extreme at x = 1 and 2

so $p'(x) = 0$ at x = 1 and 2

$\Rightarrow p'(x) = A(x-1)(x-2)$

$\Rightarrow p(x) = \int A(x^2 - 3x + 2)dx$

$p(x) = A\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right) + C$ …………..(1)

$P(1) = 8$

From (1)

$8 = A\left(\frac{1}{3} - \frac{3}{2} + 2\right) + C$

$\Rightarrow 8 = \frac{5A}{6} + C \Rightarrow 48 = 5A + 6C$ ………..(3)

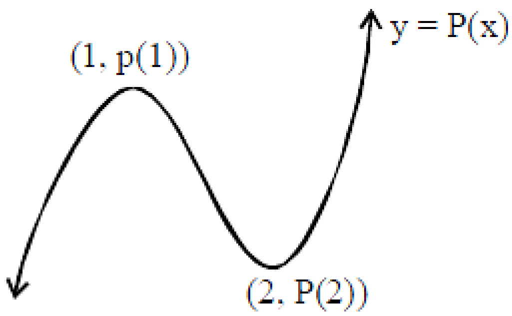
$P(2) = 4$

$\Rightarrow 4 = A\left(\frac{8}{3} - 6 + 4\right) + C$

$\Rightarrow 4 = \frac{2A}{3} + C \Rightarrow 12 = 2A + 3C$ …………(4)

From 3 and 4, C = –12

So P (0) = C = – 12



66. B

Sol. $f(x) = \begin{cases} ae^x + be^{-x}, & -1 \le x < 1 \\ cx^2, & 1 \le x \le 3 \\ ax^2 + 2cx, & 3 < x \le 4 \end{cases}$

For continuity at x = 1

$\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x)$

$\Rightarrow \quad ae + be^{-1} = c \quad \Rightarrow \quad b = ce - ae^2$ ……….(1)

For continuity at x = 3

$\lim_{x\to3^-} f(x) = \lim_{x\to3^+} f(x)$

$\Rightarrow 9c = 9a + 6c$

$\Rightarrow c = 3a$ ………(2)

$f'(0) + f'(2) = e$

$\left(ae^x - be^x\right)_{x=0} + \left(2cx\right)_{x=2} = e$

$\Rightarrow \ a - b + 4c = e$ ………..(3)

From (1), (2) and (3)

$a - 3ae + ae^2 + 12a = e$

$\Rightarrow \ a\left(e^2 + 13 - 3e\right) = e$

$\Rightarrow a = \dfrac{e}{e^2 - 3e + 13}$

67. B

Sol. Let three terms of G.P. are $\frac{a}{r}, a, ar$ product = 27

$\Rightarrow a^3 = 27 \Rightarrow a = 3$

$S = \frac{3}{r} + 3r + 3$

For $r > 0$

$\dfrac{\frac{3}{r} + 3r}{2} \ge \sqrt{3^2}$ (By AM $\ge$ GM)

$\Rightarrow \frac{3}{r} + 3r \ge 6$ ……….(1)

For $r < 0$, $\frac{3}{r} + 3r \le -6$ …………(2)

From (1) and (2)

$S \in (-\infty, -3] \cup [9, \infty)$

68. D

Sol. $2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

For no solution:

$$D = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$\Rightarrow 2\left(-2 - \lambda^2\right) + 1(1 - \lambda) + 2(\lambda + 2) = 0$

$\Rightarrow -2\lambda^2 + \lambda + 1 = 0$

$\Rightarrow \lambda = 1, -\frac{1}{2}$

$D_x = \begin{vmatrix} 2 & -1 & 2 \\ -4 & 2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -1 & 2 \\ -2 & -2 & \lambda \\ \lambda & \lambda & 1 \end{vmatrix}$

$= 2(1+\lambda)$ which is not equal to zero for $\lambda = 1, -\frac{1}{2}$

69. A

Sol. Slope of tangent is 2, Tangent of hyperbola

$\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1) is $\frac{xx_1}{4} - \frac{yy_1}{2} = 1$ (T = 0)

Slope : $\frac{1}{2}\frac{x_1}{y_1} = 2 \Rightarrow x_1 = 4y_1$ ……….(1)

(x_1, y_1) lies on hyperbola

$\Rightarrow \frac{x_1^2}{4} - \frac{y_1^2}{2} = 1$ ……….(2)

From (1) and (2)

$\frac{(4y_1)^2}{4} - \frac{y_1^2}{2} = 1 \Rightarrow 4y_1^2 - \frac{y_1^2}{2} = 1$

$\Rightarrow 7y_1^2 = 2 \Rightarrow y_1^2 = \frac{2}{7}$

Now $x_1^2 + 5y_1^2 = (4y_1)^2 + 5y_1^2$

$= (21)y_1^2 = 21 \times \frac{2}{7} = 6$

70. A

Sol. Let B_1 be the event where Box – I is selected and $B_2 \rightarrow$ where box – II selected

$P(B_1) = P(B_2) = \frac{1}{2}$

Let E be the event where selected card is non prime.

For B_1: Prime numbers:

$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

For B_2 : Prime numbers:

$\{31, 37, 41, 43, 47\}$

$P(E) = P(B_1) \times P\left(\frac{E}{B_1}\right) + P(B_2)P\left(\frac{E}{B_2}\right)$

$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}$

Required probability:

$P\left(\frac{B_1}{E}\right) = \frac{\frac{1}{2} \times \frac{20}{30}}{\frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{3}{4}} = \frac{8}{17}$

71. 309.00

Sol. MOTHER

$1 \to E$

$2 \to H$

$3 \to M$

$4 \to O$

$5 \to R$

$6 \to T$

So position of word MOTHER in dictionary

$2\times 5! + 2\times 4! + 3\times 3! + 2! + 1$

$= 240 + 48 + 18 + 2 + 1$

= 309

72. 2.00

Sol. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$|\vec{a}-\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 8$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} + |\vec{a}|^2 + |\vec{c}|^2 - 2\vec{a}.\vec{c} = 8$

$\Rightarrow 4 - 2\left(\vec{a}.\vec{b} + \vec{a}.\vec{c}\right) = 8$

$\Rightarrow \vec{a}.\vec{b} + \vec{a}.\vec{c} = -2$

$|\vec{a}+2\vec{b}|^2 + |\vec{a}+2\vec{c}|^2$

$= |\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a}.\vec{b} + |\vec{a}|^2 + 4|\vec{c}|^2 + 4\vec{a}.\vec{c}$

$= 10 + 4\left(\vec{a}.\vec{b} + \vec{a}.\vec{c}\right)$

= 10 – 8

= 2

73. 40.00

Sol. $\lim\limits_{x\to 1} \dfrac{x + x^2 + x^3 + ... + x^n - n}{x-1} = 820$

$\Rightarrow \lim\limits_{x\to 1}\left(\dfrac{x-1}{x-1} + \dfrac{x^2-1}{x-1} + ...\dfrac{x^n-1}{x-1}\right) = 820$

$\Rightarrow 1 + 2 + ... + n = 820$

$\Rightarrow n(n+1) = 2\times 820$

$\Rightarrow n(n+1) = 40\times 41$

Since $n \in N$, so $n = 40$

74. 9.00

Sol. Circle $x^2 + y^2 - 2x - 4y + 4 = 0$

$\Rightarrow (x-1)^2 + (y-2)^2 = 1$

Centre : (1, 2) radius = 1

line $3x + 4y - k = 0$ intersects the circle at two distinct points.

$\Rightarrow$ distance of centre from the line < radius

$$\Rightarrow \left|\frac{3\times1+4\times2-k}{\sqrt{3^2+4^2}}\right|<1$$

$\Rightarrow |11-k|<5$

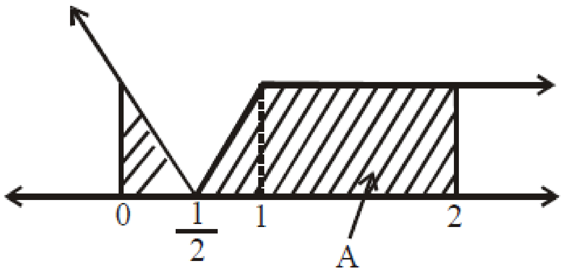
$\Rightarrow 6<k<16$

$\Rightarrow k\in\{7,8,9,.......15\}$ since $k\in I$

Number of K is 9.

75. 1.50

Sol. $\int_0^2 \left||x-1|-x\right| dx$



Let $f(x)=\left||x-1|-x\right|$

$$=\begin{cases} 1, & x\ge 1 \\ |1-2x|, & x\le 1 \end{cases}$$

$$A=\frac{1}{2}+1=\frac{3}{2}$$

Or

$$\int_0^{1/2}(1-2x)dx+\int_{1/2}^{1}(2x-1)+\int_0^2 1dx$$

$$=\left[x-x^2\right]_0^{\frac{1}{2}}+[x]_1^2$$

$$=\frac{3}{2}$$