PHYSICS, CHEMISTRY & MATHEMATICS

QP CODE: **RIT- 4**

Time Allotted: 3 Hours **Maximum Marks: 186**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. All the section can be filled in **PART-A & B** of OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **Part-A (01-04) –** Contains Six (04) multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **PART–A** (**05–12)** contains (8) Multiple Choice Questions which have **One or More Than One Correct** answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –1 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–1** marks, as a wrong option is also darkened.

(iii) **Part-B (01-06)** contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :_____

Batch :_____ **Date of Examination :**_____

Enrolment Number :_____

SECTION – I : PHYSICS

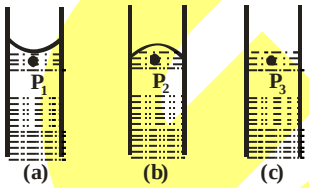
(PART – A)

SECTION – A

(Single Correct Answer Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. Compare the pressure at the point P in the three tubes shown in the figure :

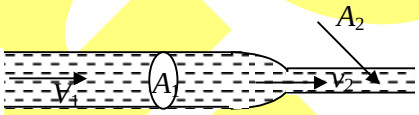


(A) $P_2 > P_1 > P_3$ (B) $P_3 > P_1 > P_2$

(C) $P_1 > P_3 > P_2$ (D) $P_2 > P_3 > P_1$

1. **D**

2. A liquid flows in a tube from left to right as shown in figure A_1 and A_2 are the cross-sections of the portions of the tube as shown. Then the ratio of speeds v_1/v_2 will be



(A) A_1/A_2 (B) A_2/A_1 (C) $\sqrt{A_2}/\sqrt{A_1}$ (D) $\sqrt{A_1}/\sqrt{A_2}$

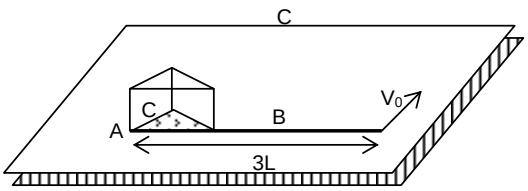
2. **B**

3. A uniform rod of length I is hinged at one extreme end and it is free to rotate in the vertical plane. If the rod is held vertical in the beginning and then released, the angular acceleration of the rod when it makes an angle of 45^0 with the horizontal is

(A) $\frac{3}{2\sqrt{2}}\frac{g}{\iota}$ (B) $\frac{6}{\sqrt{2}}\frac{g}{\iota}$ (C) $\sqrt{2}\frac{g}{\iota}$ (D) $\frac{2g}{\iota}$

3. **A**

4. A pyramid is placed on a horizontal surface whose base ABC is fixed on a horizontal surface. It is in the shape of an equilateral triangle of side L. One end of a string of length 3L is attached at A and other end attached to a small bead. The bead is given a velocity V_0 along the horizontal frictionless plane as shown. The time taken by the string to wound will be



(A) $\left[\frac{2L}{V_0}\right]\pi$ (B) $\left[\frac{3L}{V_0}\right]\pi$

(C) $\left[\frac{4L}{V_0}\right]\pi$ (D) none of these

4. **A**

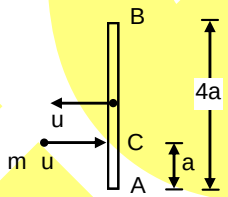
(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

5. A particle of mass m is projected with a velocity v making on angle θ with horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'H' is proportional to
(A) $V^{3/2}$ (B) V^3
(C) $H^{3/2}$ (D) H^3

5. **BC**

6. A rod AB of mass '3 m' and length '4 a' is moving translatory with velocity u on a frictionless horizontal surface. A small particle of mass 'm' is moving with equal and opposite velocity of rod as shown in figure. The particle collides perfectly elastically at the point C, which is at a distance 'a' from end A. Then choose the correct option(s).



(A) Angular momentum of the system about a point on the rod which is centre of the rod will be 2 mu a anticlockwise.
(B) Angular momentum of the system about a point on the rod which is centre of the rod will be mu a anticlockwise.
(C) Angular moment of the system about a point on the ground just below the centre of the rod will be 2 mu a anticlockwise.
(D) Angular moment of the system about a point on the ground just below the centre of the rod will be mu a anticlockwise.

6. **AD**

7. A bob is circulating in horizontal plane whose radius is constant with the help of ideal string so that it forms a conical pendulum. Then choose the correct option(s).
(A) Angular velocity of the string is constant.
(B) Magnitude of angular velocity of the string is constant.
(C) Direction of angular velocity of the string is varying.
(D) Both magnitude and direction of angular velocity are varying.

7. **AB**

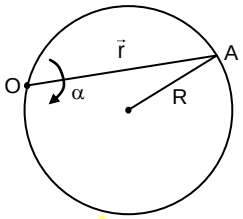
8. A person sitting firmly over a rotating stool has his arms folded with two identical balls. If he stretched his arms along with balls and then the work done by him
(A) zero (B) positive
(C) negative (D) any of these

8. **C**

9. A horizontal disc rotates freely about a vertical axis through its centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc in such a way that their axes coincide. After some time, both rotate with a common angular velocity
(A) some friction exists between the disc and the ring.
(B) the angular momentum of the disc plus ring is conserved.
(C) the final common angular velocity is $\frac{2}{3}$rd of the initial angular velocity of the disc.
(D) $\frac{2}{3}$ rd of the initial kinetic energy changes to heat.

9. **ABD**

10. A particle A starts circulating along a circle of radius R so that its position vector $\vec{r}$ relative to a point O rotates with the constant angular acceleration α as shown in figure then



(A) magnitude of velocity of the particle in the first t sec = 2α tR

(B) the angle subtended by path followed by the particle at the centre of the circle in the first t sec = αt^2.

(C) magnitude of its acceleration in the first t sec = 2α R.

(D) none of the above

10. **AB**

11. A solid body rotates about a fixed axis with an angular velocity $\omega = \sqrt{a - b\theta}$ where a, b are constant and θ is an angle of rotation from the initial position, then

(A) angular acceleration = $-\frac{b}{2}$

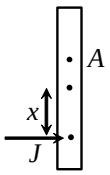
(B) angle of rotation in first t sec = $\left(\sqrt{a} - \frac{bt}{4}\right)t$

(C) angle of rotation in first t sec = $\left(\sqrt{a} + \frac{bt}{4}\right)t$

(D) none of the above

11. **AB**

12. A uniform rod of mass *m* and length *l* is placed in gravity free space and linear impulse *J* is given to the rod at a distance $x = l / 4$ from centre and perpendicular to the rod. Point *A* is at a distance $l / 3$ from centre as shown in the figure. Then



(A) Speed of centre of rod is $\frac{J}{m}$ (B) Speed of point *A* is zero

(C) Speed of upper end of rod is $\frac{J}{2m}$ (D) Speed of lower end of rod is $\frac{5}{2}\frac{J}{m}$

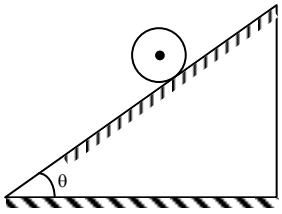
12. **ABCD**

(PART – B)

(Integer Type)

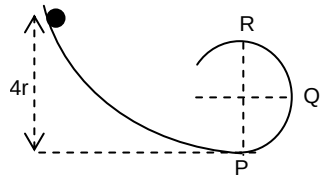
Part-C (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+4 marks** for correct answer and **there will be no negative marking.**

13. A uniform disc of mass m and radius R is rolling without slipping up a rough inclined plane which makes an angle 30^0 with the horizontal. If the coefficient of static: and kinetic friction are each equal to μ and the only force acting on the disc are gravitational and frictional force then if frictional force acting on it is $\frac{mg}{x}$ up the incline, find x



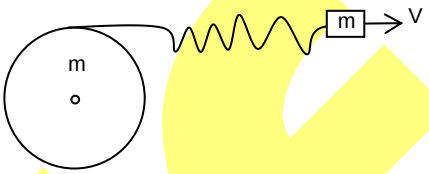
13. **6.00**

14. A solid sphere of mass m is released from the height 4r above the ground level on a smooth track shown in the figure. The track ends in the form of a circular arc of radius r. The speed of the sphere when it reaches the point Q is $\sqrt{X\frac{gr}{7}}$. Find X.



14. **30.00**

15. A block of mass m is attached to a pulley disc of equal mass m and radius r by means of a slack string as shown. The pulley is hinged about its centre on a horizontal table and the block is projected with an initial velocity of 5 m/s. Its velocity when the string becomes taut is $x\ m/s$. Find x.

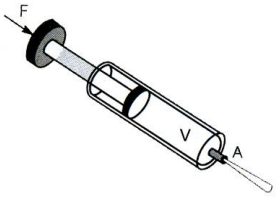


15. **3.33**
Range: 3.2 – 3.4

16. A large tank is filled with water to a height H. A small hole is made at the base of the tank. It takes T_1 time to decrease the height of water to $H/9$ and it takes T_2 time to take out the rest of water. Find T_1 / T_2.

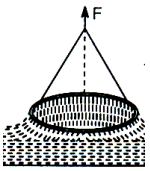
16. **2.00**

17. The cylinder initially contains a volume V of ideal fluid of density ρ. The small orifice at the end of the cylinder has cross-sectional area A. If you exert a constant force on the plunger. Then work must you do to empty the cylinder in time t is $W=\frac{\rho V^x}{2A^y t^z}$. Find x + y + z.



17. **7.00**

18. A ring is cut from a platinum tube of 8.5 cm internal and 9.5 cm external diameter. It is supported horizontally from a pan of a balance so that it comes in contact with the water in a glass vessel. The surface tension (in CGS unit) of water if an extra 9π weight is required to pull it away from water (g = 1000 cm/s^2) is K. Find the value of 'K'.



18. **500**

SECTION – II : CHEMISTRY

(PART – A)

SECTION – A
(Single Correct Answer Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. Which thermodynamic quantity does not change during working of an adiabatic system containing one mole of an ideal gas?
(A) Internal energy (B) Temperature
(C) Volume (D) Entropy

1. D

2. The correct statement between hexagonal BN and hexagonal graphite is
(A) BN is a better lubricant than graphite at room temperature
(B) BN is a high temperature lubricant and graphite is all temperature lubricant
(C) BN is a good conductor of electricity than graphite
(D) The inter-layer van der Waal's force in BN is weaker than that in graphite

2. B

3. $[Me_2SiO]_4$ is a cyclic silicone, containing eight membered ring. It can be polymerized in the presence of $Me_3Si – O – SiMe_3$. In the polymerizing process, $Me_3Si – O – SiMe_3$ acts as a
(A) dehydrating agent (B) chain propagating agent
(C) chain terminating agent (D) dehydrogenation agent

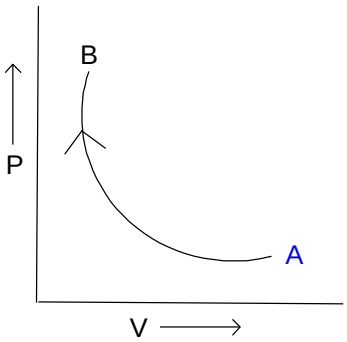
3. C

4. An isothermal and reversible compression of a system is carried out at 1000 K. If the volume of the system is compressed into $\frac{1}{10}^{th}$ of its original volume, how much work in kJ unit is done in the process? [$2.303nR = 18\ JK^{-1}$]
(A) 0.018 (B) 18
(C) 180 (D) 0.18

4. B

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

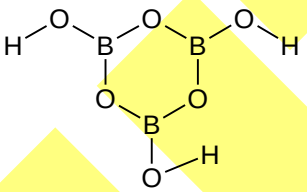
5. 

Choose correct statement(s) for the above thermodynamic process A → B containing an ideal gas in the system?
(A) It is a reversible process
(B) It is an isothermal process
(C) The work is done on the system by the surrounding
(D) Work done is accompanied with evolution of heat energy into the surrounding

5. ABC

6. Choose correct statement(s) regarding the following carbides
I. Al_4C_3 II. Mg_2C_3 III. Fe_3C IV. SiC
(A) I and II form same hydrocarbon upon hydrolysis
(B) The coordination number of carbon is greater than four in III
(C) IV. Si ≡ C is a molecular carbide
(D) Aqueous solution of II is basic in nature

6. BD

7. 

Choose correct statement(s) regarding the above compound
(A) It is a cyclic trimer of metaboric acid
(B) On strong heating it can be converted to B_2O_3
(C) In aqueous solution, it decomposes to H_2O_2 which acts as a bleach
(D) The oxidation numbers of oxygen atoms are different

7. AB

8. $2A(g) + B(g) \longrightarrow 4C(g)$

The following data are given for above thermochemical reaction.

Compounds	$\Delta_f H^o$ in kJ mol^{-1}	S_f^o in J K^{-1} mol^{-1}
A(g)	700	50
B(g)	80	460
C(g)	400	200

Choose correct statement(s) form the following
(A) if the reaction is carried out at 600 K, it becomes spontaneous
(B) this is an endothermic reaction
(C) the reaction will follow backward direction if the temperature is reduced to 520 K
(D) entropy factor favours forward reaction

8. ABD

9. Reaction of Rb and fullerence C_{60} forms a solid which is made up of $[Rb^+]_3\left[C_{60}^{3-}\right]$. The correct statement(s) regarding this compound is/are
(A) it is a crystalline solid
(B) it conducts electricity in solid state at very low temperature & high pressure
(C) the solid does not follow the law of electrical neutrality
(D) the anion contains five and six membered rings of carbon atoms

9. ABD

10. $6NH_2CONH_2 \xrightarrow[300^oC]{\text{High pressure}} 6NH_3 + 3CO_2 + (P)$
Correct statement(s) for product(P) is/are
(A) it is a cyclic compound
(B) it is a trimer of cyanamide
(C) it is used for making polymers and plastics
(D) it contains no oxygen atoms

10. ABCD

11. The order of the following properties of $SnCl_2$ and $SnCl_4$ is/are
(A) reducing agent: $SnCl_2 > SnCl_4$ (B) Lewis acid strength: $SnCl_4 > SnCl_2$
(C) melting point: $SnCl_2 > SnCl_4$ (D) boiling point: $SnCl_4 > SnCl_2$

11. ABCD

12. Which of the following characteristics of a thermodynamics system does not change in isothermal irreversible process?
(A) Internal energy (B) Enthalpy
(C) Entropy (D) Gibbs energy

12. AB

(PART – B)

(Integer Type)

Part-C (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+4 marks** for correct answer and **there will be no negative marking.**

13. In the dimer $Al_2(CH_3)_2(t\text{-}C_4H_9)_4$, what is the molar mass or atomic mass of the species in g mol^{-1} unit, that forms bridges between two aluminum atoms?

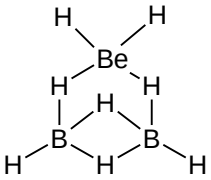
13. 15

14. How much heat is absorbed in kJ mol^{-1} unit, by one mole of an ideal monoatomic gas at constant pressure, by heating it from 1000 K to 3000 K?
$\left[Cv = \frac{3}{2}R\right]$ [R = 8.314 J K^{-1} mol^{-1}]

14. 41.57

15. The standard entropy(S^o) of a substance in liquid state is 205 J K^{-1} mol^{-1} and that in vapour state is 330 J K^{-1} mol^{-1}. What will be it's boiling point in 'K' unit if the heat of vapourisation of the liquid is 40 kJ mol^{-1}?

15. 320

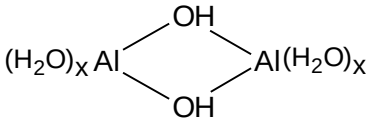
16. 

The structure of beryllium borohydride is given above. How many 3c – 2e bond(s) is/are present in above molecule?

16. 4

17. $\left[Al(H_2O)_6\right]^{3+} + CN^- \longrightarrow \left[Al(H_2O)_5 OH\right]^{2+} + HCN$

If the aqueous solution of above reaction contains the ion,



What is the value of 'x'?

17. 5

18. $2A(g) + 3B(g) \longrightarrow 4C(g)$

What is the enthalpy change of above reaction in kJ unit at 500 K if he internal energy change of the reaction is 8.397 kJ?

18. 4.24

Space For Rough Work

SECTION – III : MATHEMATICS

(PART – A)

SECTION – A
(Single Correct Answer Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. We have 21 identical balls, which needs to be distributed among 3 Boys A, B, C. Such that A always gets even number of balls. Number of possible ways of doing this is
(A) 142 (B) 120
(C) 126 (D) 132

1. D

2. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7+\cdots}}}$ having n radical signs then by method of induction which of the following is true?
(A) $a_n > 7\ \forall\ n \geq 1$ (B) $a_n > 3\ \forall\ n \geq 1$
(C) $a_n < 4\ \forall\ n \geq 1$ (D) $a_n < 3\ \forall\ n \geq 1$

2. **C**

3. Sum of the series $\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+.....$ upto n terms is equal to
(A) $1-\frac{1}{n^2}$ (B) $\frac{n(n+2)}{(n+1)^2}$
(C) $\frac{n^2+2n+2}{(n+1)^2}$ (D) $1+\frac{1}{n^2}$

3. B

4. The total number of six digit numbers $\overline{abcdef}$ having the property that $a<b\leq c<d<e\leq f$ is equal to
(A) ${}^{10}C_6$ (B) ${}^{12}C_6$
(C) ${}^{11}C_6$ (D) None of these

4. C

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

5. If $a_1, a_2, ...a_n$ are positive real numbers whose product is a fixed number c
(A) minimum value of $a_1+a_2+.....+a_{n-1}+2a_n$ is $n(2c)^{1/n}$
(B) maximum value of $a_1+a_2+.....+a_{n-1}+2a_n$ cannot be calculated
(C) minimum value of $a_1+a_2+.....+a_{n-1}+2a_n$ is $(2c)^{1/n}$
(D) None of these

5. AB

6. If first three terms of sequence $\frac{1}{16}, a, b, \frac{1}{6}$ are in geometric series and the last three terms are in harmonic series, then the values of *a* and *b* will be

(A) $a=-\frac{1}{4}, b=1$ (B) $a=\frac{1}{12}, b=\frac{1}{9}$

(C) $a=4, b=1$ (D) none of these

6. AB

7. A man wants to distribute 101 coins of a rupee each, among his 3 sons with the condition that no one receives more money than the combined total of other two. The number of ways of doing this is

(A) ${}^{103}C_2 - 3.\ {}^{52}C_2$ (B) $\frac{{}^{103}C_2}{3}$

(C) 1275 (D) $\frac{{}^{103}C_2}{6}$

7. AC

8. a, b, c are in 3 distinct numbers in H.P., a, b, c >0, then

(A) $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P. (B) $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

(C) $a^5 + c^5 \geq 2b^5$ (D) $\frac{a-b}{b-c} = \frac{a}{c}$

8. ABCD

9. Let $a_1, a_2, a_3, \ldots$ and $b_1, b_2, b_3, \ldots$ Be arithmetic progression such that $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$, then

(A) The common difference in progression 'a_1' is equal but opposite in sign to the common difference in progression 'b_j'

(B) $a_n + b_n = 100$ for any n

(C) $(a_1+b_1), (a_2+b_2), (a_3+b_3), \ldots$ are in A.P.

(D) $\sum_{r=1}^{100}(a_r + b_r) = 10^4$

9. ABCD

10. If n objects are arranged in a row, then the number of ways of selecting three of these objects so that no two of them are next to each other is

(A) $\frac{1}{6}(n-2)(n-3)(n-4)$ (B) ${}^{n-2}C_3$

(C) ${}^{n-3}C_3 + {}^{n-3}C_2$ (D) none of these

10. ABC

11. The number of ways in which 200 things can be divided into groups of 100 pairs is

(A) $\frac{200!}{2^{100}}$ (B) $\left(\frac{101}{2}\right)\left(\frac{102}{2}\right)\left(\frac{103}{2}\right)\ldots\left(\frac{200}{2}\right)$

(C) $\frac{200!}{2^{100}(100!)}$ (D) (1. 3. 5…..199)

11. BCD

12. A contest consists of ranking 10 songs of which 6 are Indian classics and 4 are western songs. Number of ways of ranking so that
(a) There are exactly 3 Indian classics in top 5 is $(5!)^3$
(b) Top rank goes to an Indian classic is 6(9!)
(c) The ranks of all western songs are consecutive, is 4!7!
(d) The six Indian classics are in a specified order is $^{10}P_4$

12. ABCD

(PART – B)

(Integer Type)

Part-C (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+4 marks** for correct answer and **there will be no negative marking.**

1. The number of 7 digit numbers whose sum of the digits equals 10 and which is formed by using the digits 1, 2 and 3 only is

1. 77

2. Find the number of integral solutions of $x+y+z\le 29$ such that $x>0, y>1, z>2$.

2. 2600

3. In a geometric progression with common ratio q, the sum of the first 109 terms exceeds the sum of the first 100 terms by 12. If sum of the first nine terms of the progression is $\frac{\lambda}{q^{100}}$ then find the value of λ.

3. 12.00

4. The maximum number of points of intersection of five lines and four circle is

4. 62

5. Sum of three numbers in G.P. is 21 and the sum of their squares is 189. If the common ratio of the G.P. is ‘a’ or ‘b’ then a + b equals to

5. 2.50

6. The value of expression $\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}.....\frac{2021}{2019!+2020!+2021!}=\frac{1}{k_1}-\frac{1}{k_2!}$, then k_2-k_1 is

6. 02019.00

Space For Rough Work

FIITJEE INTERNAL TEST

BATCHES:

PAPER-1

JEE ADVANCED LEVEL

ANSWER KEY

ANSWER KEYS

Physics

Chemistry

Mathematics

Mathematics

Section – A (37 to 54)

MCQ Single Correct Type (37 – 40) (+3, –1)

37. D

Sol. Let number of balls A gets be 2x

$2x + y + z = 21$ where $x \in [0, 20]$

$y + z = 21 - 2x$ $y, z \geq 0$

for non negative integer solution

$^{21-2x+2-1}C_{2-1} = {}^{22-2x}C_1$

$\sum_{x=0}^{10}(22-2x) = 132$

38. **C**

Sol. $a_1 = \sqrt{7} < 4$

$a_2 = \sqrt{7+\sqrt{7}} < 4$ and so on

39. B

Sol. Given series is $\frac{3}{(1\times 2)^2} + \frac{5}{(2\times 3)^2} + \frac{7}{(3\times 4)^2} + ...$

$T_n = \frac{3+(n-1)\times 2}{\{n(n+1)\}^2} = \frac{(n+1)^2 - n^2}{n^2(n+1)^2}$

$T_n = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$\therefore$ Sum of series $= \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} ..... \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$= 1 - \frac{1}{(n+1)^2} = \frac{n(n+2)}{(n+1)^2}$

40. C

Sol. Case – I – $a < b < c < d < e < f = {}^9C_6$

Case – II – $a < b = c < d < e < f = {}^9C_5$

Case – III – $a < b = c < d < e = f = {}^9C_4$

Case – IV – $a < b < c < d < e = f = {}^9C_5$

Total number of ways = $({}^9C_6 + {}^9C_5) + ({}^9C_4 + {}^9C_5) = {}^{10}C_6 + {}^{10}C_5 = {}^{11}C_6$

One or More than one Correct Type (41 – 48) (+4, –1)

41. AB

Sol. We know, that A.M. $\geq$ G.M.

$$\frac{a_1+a_2+.....+a_{n-1}+2a_n}{n} \geq \left[a_1a_2.....2\ a_n\right]^{1/n}$$

$$\frac{a_1+a_2+.....+a_{n-1}+2a_n}{n} \geq (2c)^{1/n}$$

$$\Rightarrow a_1+a_2+.....+2a_n \geq n.(2c)^{1/n}a$$

$\therefore$ Minimum value of $a_1+a_2+.....+a_{n-1}+2a_n$ is $n.(2c)^{1/n}$

42. AB

Sol. $\frac{1}{16}$, a, b, are in G.P.

$\Rightarrow a=\frac{1}{16}r$ & $b=\frac{1}{16}r^2$

$\Rightarrow$ a, b, $\frac{1}{6}$ are in H.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, 6$ are in A.P.

$\Rightarrow \frac{16}{r}, \frac{16}{r^2}, 6$ are in A.P.

$\Rightarrow \frac{32}{r^2}=\frac{16}{r}+6$

43. AC

Sol. $A+B+C=101$

Total ways $\Rightarrow {}^{101+3-1}C_{3-1} = {}^{103}C_2$

Let A have 51 coins then total ways of distribution for B and C are ${}^{50+2-1}C_1 = {}^{51}C_1$

Similarly for A having 52. 53 ……101 coins for B and C we have

${}^{50}C_1, {}^{49}C_1, ..., {}^{1}C_1$

$\therefore$ Total ways

${}^{103}C_2-3\left({}^{51}C_1+{}^{50}C_1.....+{}^{1}C_1\right)$

$={}^{103}C_2-3\left({}^{52}C_2\right)=1275$

44. ABCD

Sol. a, b, c are 3 distinct number in H.P.

(a) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ ……..are in A.P.

$\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$..... are in A.P.

Subtract 2

$\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$..... are in A.P.

(b) Add 1 in previous result

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$..... are in A.P.

(c) For a^5, c^5

A.M. $\geq$ H.M.

$$\frac{a^5+c^5}{2} \geq \frac{2a^5c^5}{a^5+c^5} \geq \left(\frac{2ac}{a+c}\right)^5$$

$$\therefore \frac{a^5+c^5}{2} \geq b^5$$

$$\therefore a^5+c^5 \geq 2b^5$$

(d) $\frac{a-b}{b-c} = \frac{a}{c}$

Put $b = \frac{2ac}{a+c}$

$$= \frac{a-\frac{2ac}{a+c}}{\frac{2ac}{a+c}-c} = \frac{a^2+ac-2ac}{2ac-ac-c^2} = \frac{a^2-ac}{ac-c^2} = \frac{a}{c}$$

45. A,B,C,D

Sol. Let a_1 , $a_1 + d_1$, $a_1 + 2d_1$, ….. and b_1, $b_1 + d_2$, $b_1 + 2d_2$,….. be two A.P.'s

$\therefore a_{100} = a_1 + 99d_1, b_{100} = b_1 + 99d_2$

Adding $a_{100} + b_{100} = a_1 + b_1 + 99(d_1 + d_2)$

Or $100 = 100 + 99(d_1 + d_2)$

$\Rightarrow d_1 + d_2 = 0$ or $d_1 = -d_2$

$\therefore$ option (B) gives $a_n + b_n$

$= a_1 + (n-1)d_1 + b_1 + (n-1)d_2$

$= a_1 + b_1 = 100$

Option C is obviously true

Now $\sum_{r=1}^{100}(a_r + b_r) = 100(a_1 + b_1) = 10^4$

46. A, B, C

Sol. Let x_0 be the number of objects to the left of the first object chosen, x_1 the number of objects between the first and the second, x_2 the number of objects between the second and the third and x_3 the number of objects to the right of the third object, we have

x_0 , $x_3 \geq 0$, x_1, $x_2 \geq 1$ and $x_0 + x_1 + x_2 + x_3 = n - 3$ …(1)

put $x_1 = y_1 + 1$ and $x_2 = y_2 + 1$

then (1) $\Rightarrow$ $x_0 + y_1 + y_2 + x_3 = n - 5$ …(2)

where x_0 , y_1, y_2, $x_3 \geq 0$

the number of non-negative integral solution of (2) is

${}^{n-5+3}C_3 = {}^{n-2}C_3$

we have ${}^{n-2}C_3 = \frac{1}{6}(n-2)(n-3)(n-4)$

also ${}^{n-3}C_3 + {}^{n-3}C_2 = {}^{n-2}C_3$.

47. BCD

Sol. Apply distribution of different kind of objects in equal group size

48. ABCD

Section – C

Numerical Based (XXXXX.XX) (49 – 54) (+3, 0)

49. 77

Sol. In a 7 – digit number formed by using 1, 2 and 3, suppose that 1 appears x times, 2 appear y times and 3 appears z times. Then by hypothesis $x+2y+3z=10$ and $x+y+z=7$

Solving these equations we get $y+2z=3$ from which we get either

$y=1, z=1$ and $x=5$ or $y=3, z=0$ and $x=4$

Therefore, the total number is $\frac{7!}{5!}+\frac{7!}{4!3!}=42+35=77$

50. 2600

Sol. Let $t\geq 0$ be such that $x+y+z+t=29$. Put $x=x_1+1, y=y_1+2, z=z_1+3$ where $x_1, y_1\geq 0$, the equation becomes $x_1+y_1+z_1+t=23$.

Its number of solutions is ${}^{26}C_3=2600$.

51. 12.00

Sol. Let first term of G.P. be a then

$t_1+t_2+.....+t_{109}=t_1+t_2+.....+t_{100}+12$

$t_{101}+t_{102}+.....+t_{109}=12$

$aq^{100}+aq^{101}+.....+aq^{108}=12$ ……(i)

$a+aq+aq^2+.....+aq^8=\frac{\lambda}{q^{100}}$

$a\left[1+q+q^2+.....+q^8\right]=\frac{\lambda}{q^{100}}$ …….(ii)

$\frac{\lambda}{q^{100}}=\frac{12}{q^{100}}\lambda=12$

52. 62

Sol. Two circles intersect at two distinct points. Two straight lines intersect at one point. One circle and one straight line intersect at two distinct points. Then the total numbers of points of intersections are as follows:

Number of ways of section	Points of intersection
Two straight lines : ${}^5C_2$	${}^5C_2\times 1=10$
Two circles : ${}^4C_2$	${}^4C_2\times 2=12$
One line and one circle : ${}^5C_1\times{}^4C_1$	${}^5C_1\times{}^4C_1\times 2$
Total	62

53. 2.50

Sol. Let three numbers in G.P. be a, ar, ar^2

Now, $a\left(1+r+r^2\right)=21$ and $a^2\left(1+r^2+r^4\right)=189$

Now $\frac{a^2\left(1+r^2+r^4\right)}{a^2\left(1+r+r^2\right)^2}=\frac{189}{21\times 21}$

$$\Rightarrow \quad \frac{\left(r^2+r+1\right)\left(r^2-r+1\right)}{\left(r^2\right)}=\frac{3}{7}$$

$$\Rightarrow \quad 7r^2-7r+7=3r^2+3r+3$$

$$\Rightarrow \quad 4r^2-10r+4=0$$

$$\Rightarrow \quad (4r-2)(r-2)=0$$

Hence, $r=2, \frac{1}{2}$

54. 02019.00

Sol. $T_r=\frac{r+2}{r!+(r+1)!+(r+2)!}=\frac{(r+2)}{r!\{1+(r+1)+(r+1)(r+2)\}}=\frac{(r+2)}{r!(r+2)^2}=\frac{1}{r!(r+2)}$

$$=\frac{r+1}{(r+2)!}=\frac{(r+2)-1}{(r+2)!}$$

$$T_r=\frac{1}{(r+1)!}-\frac{1}{(r+2)!}$$

$$T_1=\frac{1}{2!}-\frac{1}{3!}$$

$$\vdots$$

$$T_n=\frac{1}{(n+1)!}-\frac{1}{(n+2)!}$$

$$\text{Sum}=\frac{1}{2!}-\frac{1}{(n+2)!}$$

$$\Rightarrow \frac{1}{2!}-\frac{1}{(2021)!}$$