FIITJEE RBT-7 for (JEE-Advanced)

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 4 | **QP CODE: 100203** | **PAPER - 2**

Time Allotted: 3 Hours **Maximum Marks: 186**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & B** in the OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Part.

(i) **PART–A (01–08)** contains (8) Multiple Choice Questions which have **One or More Correct** answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(ii) **Part-A (09-12) –** This section contains Two (02) List-Match Sets, each List-Match set has Two (02) Multiple Choice Questions. Each List-Match set has two lists: List-I and List-II. FOUR options are given in each Multiple Choice Question based On List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question. Each question carries **+3 Marks** for correct combination chosen and **-1 marks** for wrong options chosen.

(iii) **Part-B (01-06)** contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :_____

Batch :_____ **Date of Examination :**_____

Enrolment Number :_____

BATCHES – Two Yr CRP2123(AII)

SECTION – I : PHYSICS

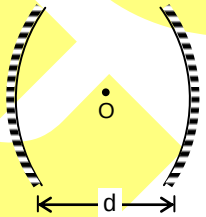
(PART – A)

(One or More Than One Options Correct Type)

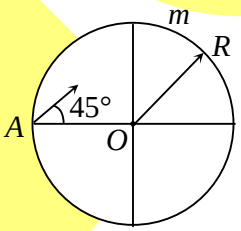
This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. The potential energy φ in joule of a particle of mass 1 kg moving in the x – y plane, obeys the law $\varphi = 3x + 4y$, where (x, y) are the coordinate of the particle in metre. If the particle is at rest at (6, 4) at time t = 0, then (only conservative force are acting on particle).
   (A) the particle has constant acceleration
   (B) the work done by the conservative forces from position of rest of the particle to the instant of the particle crossing x axis is 25 J
   (C) the speed of the a particle when it crosses the y axis is 10 ms^{-1}
   (D) the coordinates of the particle at time t = 4s are (–18, –28)

2. The distance d between two identical concave mirrors of radius of curvature R. So, as to coincide the image I on the object O itself, which is placed at the mid point can be
   (A) R (B) 2R
   (C) 3R (D) R/2



3. A ring of mass m and radius R is placed on a frictionless horizontal surface. A particle of mass m is projected from point A with velocity v at an angle of 45° with AO as shown. The correct statements are



   (A) The particle reaches the same point A on the ring after time $\frac{4R\sqrt{2}}{v}$.
   (B) Magnitude of impulse transformed during first collision is $\frac{mv}{\sqrt{2}}$.
   (C) Magnitude of impulse transformed during second collision is $\frac{mv}{\sqrt{2}}$.
   (D) Particle reaches diametrically opposite point on the ring in time $\frac{2R}{v}$.

Space For Rough Work

4. A point moves with deceleration along the circle of radius R so that at any time its tangential and normal acceleration are equal in magnitude. At the initial moment t =0, the velocity of the point is v_0. The velocity of the point will be

(A) $v = \dfrac{v_0}{1+\dfrac{v_0 t}{R}}$ at t second

(B) $v = v_0 e^{-S/R}$ where s is path travelled on circle

(C) $v = v_0 e^{-SR}$ where s is path travelled on circle

(D) $v = \dfrac{v_0}{1-\dfrac{v_0 t}{R}}$ at t second

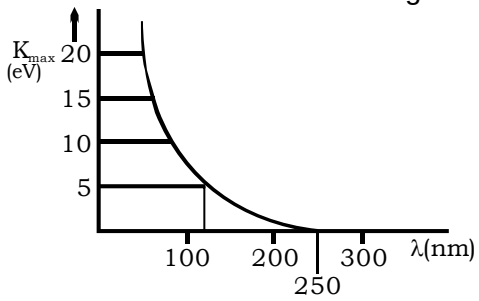
5. A horizontal disc rotates freely about a vertical axis through its centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc. After some time, they rotate with a common angular speed.
(A) some friction exists between the disc and the ring.
(B) the angular momentum of the disc plus ring is conserved.
(C) the final common angular speed is $\frac{2}{3}$rd of the initial angular velocity of the disc.
(D) $\frac{2}{3}$ rd of the initial kinetic energy changes to heat.

6. If the potential difference of Coolidge tube producing X-ray is increased, then choose the correct option(s).
(A) the interval between $\lambda_{k\alpha}$ and $\lambda_{k\beta}$ increases
(B) the interval between $\lambda_{k\alpha}$ and λ_0 increases
(C) the interval between $\lambda_{k\beta}$ and λ_0 increases
(D) λ_0 does not change
Here λ_0 is cutoff wavelength and $\lambda_{k\alpha}$ and $\lambda_{k\beta}$ are wavelength of k_α and k_β characteristic X-rays.

7. A coil of radius r(r << R) and mass m carries current i_2. Planes of two coils are mutually perpendicular and both the coils are free to rotate about common diameter.
Choose the correct statement/statements.
(A) The maximum Kinetic energy of smaller coil when both the coils are released, is
$K_{max} = \dfrac{\mu_0 \pi r^2 i_1 i_2 MR}{2(MR^2 + mr^2)}$
(B) Total energy of both the coils remains constant.
(C) Total angular momentum of both the coils remains constant.
(D) The potential energy of the system will be minimum when both the coils are in same plane and current in them are in same direction.

Space For Rough Work

8. In a photoelectric effect experiment, the maximum kinetic energy of the ejected photoelectrons is measured for various wavelengths of the incident light. Figure shows a graph of this maximum kinetic energy K_{max} as a function of the wavelength λ of the light falling on the surface of the metal. Which of the following statements is/are correct?



(A) Threshold frequency for the metal is 1.2×10^{15} Hz
(B) Work function of the metal is 4.968 eV.
(C) Maximum kinetic energy of photoelectrons corresponding to light of wavelength 100 nm is nearly 7.4 eV.
(D) Photoelectric effect takes place with red light.

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

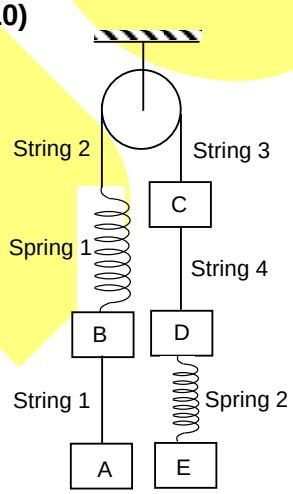
List Match Set (09-10)

The system shown below is initially in equilibrium

$m_A = m_B = 3kg$

$m_C = m_D = m_E = 2kg$

Take g = 10 m/s^2



List–I gives the four strings while List–II the value of the tension in the strings.

List – I		List – II	
(I)	String 1	(P)	10 N
(II)	String 2	(Q)	20 N
(III)	String 3	(R)	30 N
(IV)	String 4	(S)	40 N
		(T)	60 N
		(U)	0 N

9. If the spring 1 is cut, match the correct for the tension in the strings just after the cutting.

(A) I $\rightarrow$ R, II $\rightarrow$ T, III $\rightarrow$ T, IV $\rightarrow$ P
(B) I $\rightarrow$ U, II $\rightarrow$ T, III $\rightarrow$ T, IV $\rightarrow$ P
(C) I $\rightarrow$ U, II $\rightarrow$ U, III $\rightarrow$ U, IV $\rightarrow$ P
(D) I $\rightarrow$ R, II $\rightarrow$ S, III $\rightarrow$ S, IV $\rightarrow$ P

Space For Rough Work

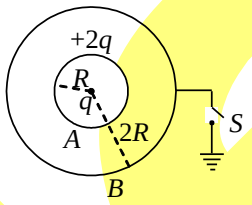
10. If the spring 2 is cut, match the correct for the tension in the strings just after the cutting.

(A) I → R, II → T, III → T, IV → R
(B) I → R, II → U, III → U, IV → R
(C) I → U, II → R, III → R, IV → P
(D) I → P, II → Q, III → Q, IV → R

List Match Set (11-12)

Match the List-I with the List-II for question no. 11 and 12.

Two concentric conducting shells *A* and *B* have radii *R* and 2*R*. A charge *q* is placed at the centre of the shells and a charge 2*q* is given to shell *A*.



List – I		List – II	
(I)	Charge on inner surface of shell *A*	(P)	$-3q$
(II)	Charge on inner surface of shell *B*	(Q)	Zero
(III)	Charge on outer surface of shell *A*	(R)	$-q$
(IV)	Charge on outer surface of shell *B*	(S)	$3q$
		(T)	$2q$
		(U)	$-2q$

11. When switch is open

(A) I → R, II → P, III → S, IV → S
(B) I → T, II → P, III → S, IV → Q
(C) I → P, II → R, III → S, IV → T
(D) I → S, II → R, III → T, IV → U

12. When switch is closed

(A) I → P, II → R, III → S, IV → Q
(B) I → R, II → P, III → S, IV → Q
(C) I → T, II → U, III → R, IV → P
(D) I → P, II → Q, III → R, IV → S

(PART – B)

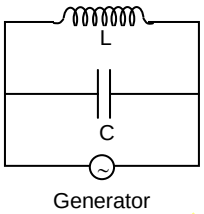
(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)**.**

1. A sphere of radius 0.1 m and mass 8π kg is attached to the lower end of a steel wire of length 5 m and diameter 10^{-3} m. The wire is suspended from 5.22 m high ceiling of a room. When the sphere is made to swing as a simple pendulum, it just grazes the floor at its lowest point. Calculate the velocity of the sphere at the lowest position (in m/s). Young's modulus of steel is 1.994×10^{11} N/m^2.

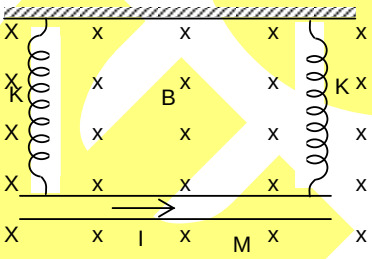
Space For Rough Work

2. For the circuit shown in the figure, the current through the inductor is 0.6 A, while the current through the capacitor is 0.4 A. Find the current (in A) drawn from the generator

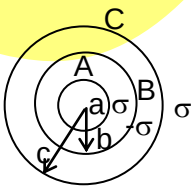


3. An ice cube of mass 0.1 kg at 0^0C is placed in an isolated container which is at 227^0C. The specific heat s of the container varies with temperature T according to the empirical relation $s = A + BT$, where $A = 100 \text{ cal kg}^{-1}K^{-1}$ and $B = 2\times10^{-2} \text{ cal kg}^{-1}K^{-2}$. The final temperature of the container is 27^0C, the mass of the container is $x \times 10^{-1}$ kg. What is the value of x? (Latent heat of fusion of water $= 8\times10^4 \text{ cal kg}^{-1}$, Specific heat of water $= 10^3 \text{ cal kg}^{-1}K^{-1}$).

4. A metal rod of mass 10gm and length 25 cm is suspended on two springs as shown in figure. The springs are extended by 4 cm. When a 20 ampere current passes through the rod it rises by 1 cm. The magnetic field is $x \times 10^{-2}$ T (g = 10 m/s^2). Find the value of 2x.



5. The adjacent figure shows charged spherical shells A, B and C having charge densities σ, -σ, σ and radii a, b, c respectively. If $V_A = V_c$ then c is equal to (in m) (assuming a = 0.10 m, b = 0.20 m)



6. In YDSE experiment if the screen is shifted by a distance of 0.5 m away from the slit, 3rd maxima is shifted by 3×10^{-4} m. The slit width is 2×10^{-3}m. If the wavelength (in 100 nm) used in the experiment is 'k', find the value of 'k'.

Space For Rough Work

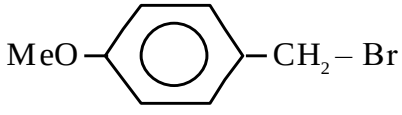
SECTION – II : CHEMISTRY

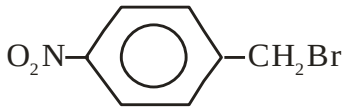
(PART – A)

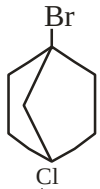
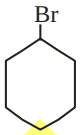
(One or More Than One Options Correct Type)

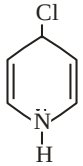
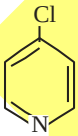
This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. Consider the following statements and pick up the correct statements:

(A)  will react more readily than

 for S_N1 reaction

(B)  will react more readily than  for S_N1 reaction

(C)  will react more readily than  for S_N1 reaction

(D) Generally S_N1 reaction occurs in polar protic solvent

2. Which of the following statement(s) is/are correct for PCl_5?
(A) Phosphorus undergoes sp^3d hybridization.
(B) Two sets of P–Cl bond length are observed in the molecule
(C) It's solid contains $[PCl_4]^+$ and $[PCl_6]^-$ ions
(D) The maximum value of $\angle ClPCl$ angle is 120°.

3 The standard electrode potential of a metal-metal ion $\left(Ag\,|\,Ag^+\right)$ and metal-sparingly soluble salt anion $(Ag\,|\,AgCl\,|\,Cl^-)$ are not related as

(A) $E^0_{Ag^+|Ag} = E^0_{Cl^-|Agcl|Ag} + \frac{RT}{F}\ln K_{sp}$

(B) $E^0_{Ag^+|Ag} = E^0_{Cl^-|Agcl|Ag} + \frac{RT}{F}\ln K_{sp}$

(C) $E^0_{Cl^-|AgCl|Ag} = E^0_{Ag^+|Ag} - \frac{RT}{F}\ln\frac{[Cl^-]}{K_{sp}}$

(D) $E^0_{Cl^-|AgCl|Ag} = E^0_{Ag^+|Ag} - \frac{RT}{F}\ln\frac{K_{sp}}{[Cl^-]}$

Space For Rough Work

4. A ($C_5H_{12}O$) ether, on reaction with PCl_5 form alkyl chloride B and C. B and C both on reaction with aqueous KOH form alcohol D and E. Both D and E give iodoform test. Identify correct answers.

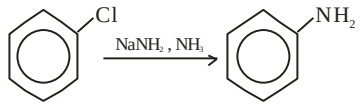
(A) A is $CH_3CH_2-O-CH(CH_3)_2$

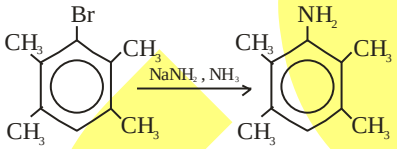
(B) C is $CH_3-\underset{\displaystyle Cl}{\underset{|}{CH}}-CH_3$

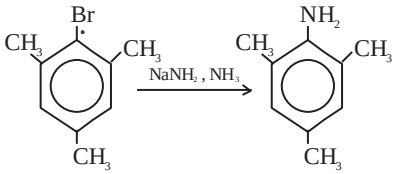
(C) C is $CH_3CH_2CH_2Cl$

(D) E is $CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3$

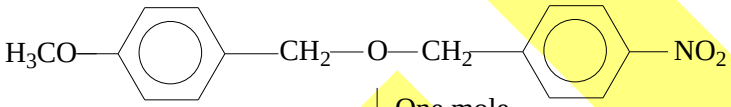
5. Which of the following reactions are not feasible?

(A) 

(B) 

(C) 

(D) $R-OH+NaOH \rightarrow R-\bar{O}Na^{+}+H_2O$
(R = Aliphatic alcohol)

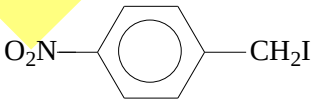
6. 

One mole of HI

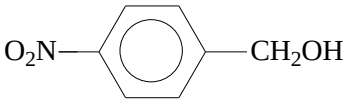
Product

Major products are

(A) 

(B) 

(C) 

(D) 

7. For a non-ideal solution with a negative deviation

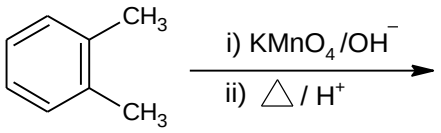
(A) $\Delta H_{mix} = -ve$

(B) $\Delta V_{mix} = -ve$

(C) $\Delta S_{mix} = -ve$

(D) $\Delta G_{mix} = -ve$

Space For Rough Work

8. 

Which of the following is correct regarding product 'B'?
(A) 'B' is colourless in acidic medium
(B) 'B' is pink colour in basic medium
(C) 'B' has Benzenoid form in Basic medium
(D) 'B' exists in Ionic form in Basic medium

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

Match the reversible reactions mentioned in List – I with the effect of Le – Chatelier principles on them mentioned in List - II and answer the question.

	List – I		List - II
(I)	$N_2(g)+O_2(g)\rightleftharpoons 2NO(g);\Delta H=+ve$	(P)	The concentration of NO_2 increases by increasing temperature.
(II)	$2NO(g)+O_2(g)\rightleftharpoons 2NO_2(g);\Delta H=+ve$	(Q)	The concentration of NO_2 increases by decreasing temperature
(III)	$N_2O_3(g)\rightleftharpoons NO_2(g)+NO(g);\Delta H=+ve$	(R)	The concentration of NO_2 increases by increasing the volume of reaction vessel.
(IV)	$2NO_2(g)\rightleftharpoons N_2O_4(g);\Delta H=-ve$	(S)	The concentration of NO_2 increases by increasing pressure.
		(T)	Inert gas addition does not have any result at constant pressure or at constant volume.

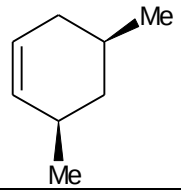
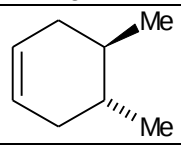
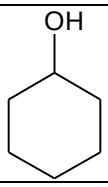
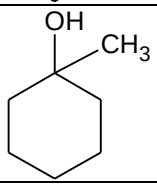
9. The correct matching according to List-I & List-II is
(A) I → PQ (B) II → T
(C) III → P (D) IV → Q

10. The correct matching according to List-I & List-II is
(A) I → R (B) II → Q
(C) III → T (D) IV → S

Space For Rough Work

List Match Set (11-12)

Match the compound(s) mentioned in column-I with their properties mentioned in column-II.

List – I		List– II	
(I)		(P)	Shows optical isomerism before hydrogenation with H_2/Ni
(II)		(Q)	Shows optical isomerism after hydrogenation with H_2/Ni
(III)		(R)	Shows optical isomerism before dehydration
(IV)		(S)	Shows optical isomerism after dehydration
		(T)	Shows optical isomerism before addition of HBr
		(U)	Shows optical isomerism after addition of HBr

11. The correct matching according to above list is
(A) I → Q (B) II → U
(C) III → R (D) IV → T

12. The correct matching according to above list is
(A) I → P (B) IV → S
(C) III → U (D) IV → R

Space For Rough Work

(PART – B)

(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)**.**

1. How many of them are correctly matched for purification process(refining of metal)?
   (a) Liquation – Sn
   (b) Distillation – Zn
   (c) Parke's process – Pb
   (d) Zone refining – Ge
   (e) Cupellation – Cu
   (f) Mond's process – Ni
   (g) Electrolytic refining – Ag
   (h) Baeyer's process – Al
   (i) Parting process – Au
   (j) Poling process – Cu

2. A dipositive metal ion is oxidized to it's tripositive ion by acidified permangante solution $\left(MnO_4^- / H^+\right)$. A solution containing 134.04 g of dipositive ion, required 1.2 L of 0.4 M acidified solution of MnO_4^- for complete oxidation. What is the ionic mass of the ion?

3. Which of the following is/are correctly matched?

	Drugs used for treatment	**Class of drug**
(a)	Ranitidine	Antacid
(b)	Histamine	Antacid
(c)	Seldane	Anti histamines
(d)	Seretonin	Tranquilizers
(e)	Aspirin	Analgasic & antipyretic
(f)	Heroin	Analgasic(Narcotic)
(g)	Pencillin	Antibiotics(Bactericidal)
(h)	1% Phenol	Antiseptic
(i)	Norethindrone	Antifertility

4. A radiation travels with an uniform velocity of 2116.52×10^4 m s^{-1}. What is the wavelength of the radiation in nm unit if it's energy is 3.3×10^{-20} Joule?

5. $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

   A container contains 6 moles of PCl_5 and 4 moles of PCl_3. On heating, PCl_5 decomposes to PCl_3 and Cl_2 according to above reaction. The total number of gaseous moles present at equilibrium is 14. What will be the volume of the reaction container in liter unit, if the equilibrium constant, K_C is $\frac{1}{35.45}$ mol L^{-1}.

6. What is the maximum limit of NO_3^- (Nitrate ion) in drinking water prescribed, otherwise excess nitrate in drinking water can cause disease Methemoglobinemia(Blue Baby Syndrome)
   [Give answer in ppm or mg dm^{-3}]

Space For Rough Work

SECTION – III : MATHEMATICS

(PART – A)

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. Let a and b be two complex numbers such that $|a+b|=20$ and $|a^2+b^2|=16$ then

   (A) maximum value of $|a^3+b^3|$ is 4480 (B) minimum value of $|a^3+b^3|$ is 2022

   (C) minimum value of $|a^3+b^3|$ is 3520 (D) maximum value of $|a^3+b^3|$ is 4044

2. If $y=\alpha x^2+\alpha x+\frac{1}{24}$ and $x=\alpha y^2+\alpha y+\frac{1}{24}$ are tangent to each other, then

   (A) $\alpha=\frac{2}{3}$ (B) $\alpha=\frac{3}{2}$

   (C) $\frac{13-\sqrt{601}}{12}$ (D) $\frac{13+\sqrt{601}}{12}$

3. A parabola touches x – axis at A (4, 0) and y – axis at B (0, 3)

   (A) Equations of directrix is $4x+3y=0$ (B) Co – ordinates of focus are $\left(\frac{36}{25},\frac{48}{25}\right)$

   (C) Equation of directrix is $3x+4y=0$ (D) Co – ordinates of focus are $\left(\frac{48}{25},\frac{36}{25}\right)$

4. If $8\sin x=\frac{\sqrt{3}}{\cos x}+\frac{1}{\sin x}$ then x = _____

   (A) $n\pi\pm\frac{\pi}{4}$ (B) $n\pi\pm\frac{\pi}{6}$

   (C) $n\pi+\frac{\pi}{6}$ (D) $\frac{n\pi}{2}-\frac{\pi}{12}$

5. Let a_n be the number of non-empty subsets of $S=\{1,2,3,4....,n-1,n\}$ such that there are no two consecutive numbers in the same set, then

   (A) $a_n=a_{n-1}+a_{n-2}$ (B) $a_n=a_{n-1}+a_{n-2}+1$

   (C) $a_7=33$ (D) $a_7=21$

Space For Rough Work

6. Suppose ***100*** students appeared for IIT Genius Test. The test carries a maximum of ***100*** marks and there is no negative marking. The arithmetic mean of their scores is ***40*** with standard deviation ***20***. Let p be the number of students who have scored marks equal to or more than ***80***. Then the possible value(s) of p can be

(A) 10 (B) 20

(C) 22 (D) 18

7. If ΔABC, let G be the centroid, and let I be the centre of the inscribed circle. If IG be parallel to AB and $\tan\frac{B}{2}=\frac{1}{3}$. Then

(A) ΔABC is obtuse angled (B) ΔABC is right angled but not isosceles

(C) $3c=4b$ (D) $5b=3a$

8. Let $a_1,a_2,a_3,.....,a_n$ be first 'n' terms of a G.P. with first term 'a' and common ratio 'r' then

(A) $\frac{1}{a_1^2-a_2^2}+\frac{1}{a_2^2-a_3^2}+.....+\frac{1}{a_{n-1}^2-a_n^2}=\frac{\left(1-r^{2n}\right)}{a^2r^{2n-4}\left(1-r^2\right)^2}$

(B) $\frac{1}{a_1^2-a_2^2}+\frac{1}{a_2^2-a_3^2}+.....+\frac{1}{a_{n-1}^2-a_n^2}=\frac{\left(1-r^{2n-2}\right)}{a^2r^{2n-4}\left(1-r^2\right)^2}$

(C) $\frac{1}{a_1^m+a_2^m}+\frac{1}{a_2^m+a_3^m}+.....+\frac{1}{a_{n-1}^m+a_n^m}=\frac{\left(r^{mn-m}-1\right)}{a^m\left(1+r^m\right)\left(r^{mn-m}-r^{mn-2m}\right)}$

(D) $\frac{1}{a_1^m+a_2^m}+\frac{1}{a_2^m+a_3^m}+.....+\frac{1}{a_{n-1}^m+a_n^m}=\frac{\left(r^{mn-m}-1\right)}{a^m\left(1-r^m\right)\left(r^{mn-m}-r^{mn-2m}\right)}$

Space For Rough Work

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

Three players A, B and C alternatively throw a die in that order, the first player to throw a being deemed the winner. A's die is fair whereas B and C throw dice with probabilities p_1 and p_2 respectively, or throwing a 6.

Column-I		Column-II	
(I)	If $p_1 = \frac{1}{5}, p_2 = \frac{1}{4}$ and probability that A wins the game is $\frac{1}{\lambda_1}$, then λ_1 is divisor of	(P)	6
(II)	If $p_1 = \frac{1}{5}, p_2 = \frac{1}{4}$ and probability that C wins the game is $\frac{1}{\lambda_2}$, then λ_2 is divisor of	(Q)	8
(III)	If P (A wins) = P (B wins) and $\frac{1}{p_1} = \lambda_3$, then λ_3 is divisor of	(R)	12
(IV)	If game is equiprobable to all the three players and $\frac{1}{p_1} = \lambda_4$, then λ_4 is divisor of	(S)	15

9. Which of the following is the correct combination?
(A) I $\rightarrow$ Q (B) II $\rightarrow$ PRS
(C) III $\rightarrow$ R (D) IV $\rightarrow$ P

10. Which of the following is the correct combination?
(A) I $\rightarrow$ PQS (B) II $\rightarrow$ Q
(C) III $\rightarrow$ S (D) IV $\rightarrow$ P

Space For Rough Work

List Match Set (11-12)

Matching Type Questions

Column-I		Column-II	
(I)	If number of dissimilar terms in the expansion of $(x+2y+3z)^n (n \in N)$ is am^2+bn+c, then	(P)	$a+b+c=3$
(II)	If number of dissimilar terms in the expansion of $(x+y+z)^{2n+1}-(x+y-z)^{2n+1} (n \in N)$ is an^2+bn+c, then	(Q)	$a+b+c=4$
(III)	If number of dissimilar terms in the expansion $(x-y+z)^n+(x+y-z)^n$ ($n \in$ is even natural number) is an^2+bn+c, then	(R)	$a+b=2c$
(IV)	If number of dissimilar terms in the expansion of $\left(\frac{x^2+1+x^4}{x^2}\right)^{\sum n} (n \in N)$ is an^2+bn+c, then	(S)	$b+c=8a$

11. Which of the following is the correct combination?
(A) I $\rightarrow$ RS (B) I $\rightarrow$ PRS
(C) II $\rightarrow$ P (D) IV $\rightarrow$ PR

12. Which of the following is the correct combination?
(A) I $\rightarrow$ QS (B) I $\rightarrow$ R
(C) II $\rightarrow$ Q (D) III $\rightarrow$ Q

(PART – B)

(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)**.**

1. Let A and B be two square matrices having real elements satisfying $A^2+B^2=2AB$ then value of $\det(AB-BA)=$

2. If $p(t)=\frac{t(t-1)...(t-2023)}{2023!}$, then the value of
$\int_0^1\left(\frac{1}{t+1}+\frac{1}{t+2}+...+\frac{1}{t+2024}\right)p(-t-1)dt=k^2$ then k – 2000 equals

Space For Rough Work

3. If $\cot k^\circ = \tan 78^\circ + \tan 72^\circ + \tan 60^\circ$ then k=

4. 6 given points on a circle are joined by line segments of which no three are concurrent. The number of triangles inside the circle equals _____

5. The number of continuous functions $f:\left[0, \frac{3}{2}\right] \to (0, \infty)$ satisfy the equation $4\int_0^{3/2} f(x)dx + 125\int_0^{3/2} \frac{dx}{\sqrt{f(x)+x^2}} = 108$ is _____.

6. If $S = \frac{1}{1.2.3} + \frac{1}{5.6.7} = \frac{1}{9.10.11} + \ldots$ (value of $\ell n 2 \approx 0.693$) then value of S = _____

Space For Rough Work

Q. P Code: 100203

Answers

SECTION – I : PHYSICS

(PART – A)

1.	ABCD	2.	AB	3.	ABC	4.	AB
5.	ABD	6.	BC	7.	ABCD	8.	ABC
9.	C	10.	A	11.	A	12.	B

(PART – B)

1.	8.74	2.	0.20	3.	4.95	4.	3
5.	0.30	6.	4				

SECTION – II : CHEMISTRY

(PART – A)

1.	ACD	2.	ABD	3.	ACD	4.	ABD
5.	BC	6.	AD	7.	ABD	8.	ABD
9.	C	10.	D	11.	B	12.	A

(PART – B)

1.	8	2.	55.85 (Range 55 to 56)	3.	7
4.	423.30 (Range 421 to 425)	5.	567.2(range 566 to 568)		
6.	50 (Range 49 to 51)				

SECTION – III : MATHEMATICS

(PART – A)

1.	AC	2.	ABCD	3.	AB	4.	CD
5.	BC	6.	ABD	7.	BCD	8.	BC
9.	B	10.	C	11.	D	12.	C

(PART – B)

1.	0	2.	24	3.	6	4.	111
5.	1	6.	0.17				

Answers & Solutions
SECTION – I : PHYSICS

(PART – A)

1. **ABCD**

Sol. $\vec{F} = 3\hat{i} - 4\hat{j}, W_{con.f} = U_i - U_f = 34 - 9 = 25$ & $\frac{1}{2}mv^2 - 16 = 34$

2. **AB**

Sol. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

3. **ABC**

Sol. Assume x-axis along OA and y-axis perpendicular to it in the plane of ring.
After 1st collision

Velocity of particle = $\frac{v}{\sqrt{2}}$ along x-axis

And velocity of ring = $\frac{v}{\sqrt{2}}$ along y-axis

After second collision velocity of ring = $\frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}$

Velocity of particle = 0

Time to return at point A = $4 \times \frac{R\sqrt{2}}{v}$

4. **AB**

Sol. $-\frac{dv}{dt} = \frac{v^2}{R}$ & $-v\frac{dv}{ds} = \frac{v^2}{R}$

5. **ABD**

Sol. By conservation of angular momentum,

$I_1\omega_i = (I_1 + I_2)\omega_f$

$\omega_f = \omega_1 / 3$

$E_i = \frac{1}{2}I_1\omega_i^2$

$E_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$

Ratio of the heat produced to initial kinetic energy $= \frac{E_i - E_f}{E_i} = \frac{2}{3}$

6. **BC**

Sol. As $\lambda_0 = \frac{hc}{eV}$ and as V increases. λ_0 decreases but characteristic wavelengths do not change so interval between $\lambda_{k\alpha}$ & λ_0 increases and the same for the interval between $\lambda_{k\beta}$ & λ_0

7. **ABCD**

Sol. MB(cos 0° – Cos 90°) = $\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$ …(1)

Also, $I_1\omega_1 = I_2\omega_2$ …(2)

By (1) and (2) we get

$$K_{max} = \frac{\mu_0 \pi r^2 i_1 i_2 MR}{2(MR^2 + mr^2)}$$

8. **ABC**

Sol. From graph,

$\lambda_{th} = 250\ \text{nm}$

$$f_{th} = \frac{C}{\lambda_{th}} = \frac{3\times10^8\,\text{m/s}}{250\times10^{-9}\,\text{m}} = 1.2\times10^{15}\,\text{Hz}$$

Work function $\phi = \frac{\lambda f_{th}}{e}(eV)$

$= 4.968\,eV$

$K.E_{max} = h\upsilon - h\upsilon_{th}(\phi)$

$= 7.4\,eV$

9. **C**

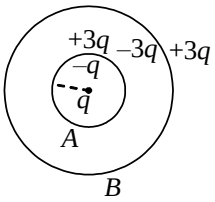
Sol. Just after cutting force of cut spring will be zero whereas the force of other spring will be unchanged.

10. **A**

Sol. Just after cutting force of cut spring will be zero whereas the force of other spring will be unchanged.

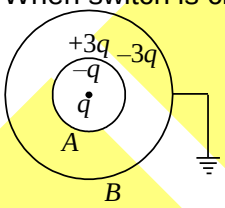
11. **A**

Sol. When switch is open



12. **B**

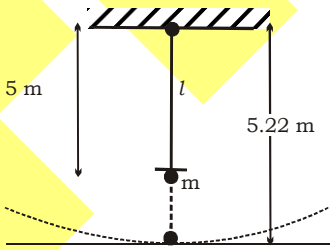
Sol. When switch is closed



(PART – B)

1. **8.74**

Sol.



$$T = mg + \frac{m\upsilon^2}{R} \qquad \upsilon = \sqrt{\left(\frac{T}{m} - g\right)R}$$

$$\frac{mv^2}{R} = T - mg \qquad \frac{T}{m} = \frac{yA\Delta L}{mL}, g = 10$$

$$mv^2 = TR - mgR \qquad R = 5.22 - 0.1$$

$$v^2 = \frac{TR}{m} - gR \qquad (R = 5.12)$$

$$v^2 = \left(\frac{T}{m} - g\right)R \qquad \frac{T}{m} = \frac{1.994\times10^{11}}{1000\ 8A} \times \frac{A\times10^{-6}}{4} \times \frac{0.02}{5100}$$

$$= \frac{1994}{84} \times \frac{2}{20}$$

$$v = \sqrt{(24.925 - 10)5.12} \qquad \frac{T}{m} = \frac{1994}{80}$$

$$= 8.74\, m/s$$

2. **0.20**

Sol. I_L and I_C will be in opposite phases.

$I_{net} = I_L - I_C$

$= 0.6 - 0.4 = 0.2$ A

3. **4.95**

Sol. Heat last by contains $= msdt.$

$$fdQ = \int_{500}^{300} m(A + Bt)dt$$

$$-Q = m\left[AT + \frac{BT^2}{\alpha}\right]_{500}^{300}$$

$$+Q = m[+21600]cal$$

Heat gained by ice at 0^o $\quad Q_1 = mL$

$$Q_1 = 8000cal$$

Heat gained by water to raise temp. $= ms\Delta 7$

$$Q_2 = 0.1\times1000\times27$$

$$= 2700$$

Total heat gained $= Q_1 + Q_2$

$$= 8000 + 2700$$

$$= 10700\, cal$$

Heat lost = Heat gained

$$m21600 = 10700$$

$$m = 0.495\, kg. = 4.95\times10^{-1} kg$$

4. **3**

Sol. Initially the rod will be in equilibrium if

$2T_o = Mg$ with $T_o = kx_o$ …(i)

when the current I is passed through the rod, it will experience a force F = BIL vertically up,

In equilibriums

2T + BIL = Mg with T = kx …(ii)

from (i) & (ii)

$$\frac{T}{T_o} = \frac{Mg - BIL}{Mg} \text{ i.c. } \frac{x}{x_o} = 1 - \frac{BIL}{Mg}$$

or, $B = \dfrac{Mg(x_o - x)}{I\ \ L\ \ x_0}$

Putting the values we get $B = 1.5\times10^{-2}$ T.

5. **0.30**

Sol. $V_A=V_C$

$c=a+b$

6. **4**

Sol. Here 3rd maxima is shifted by 3×10^{-4} m. It indicates fringe width increases by 1×10^{-4} m.

Hence $\beta = \frac{\lambda(D+0.5)}{d} = \frac{\lambda D}{d} + 1\times10^{-4}$

or $\frac{0.5\lambda}{d} = 1\times10^{-4}\,m$ or $\lambda = \frac{2\times10^{-3}\times1\times10^{-4}}{0.5} = 4\times10^{-7}\,m = 400\ nm.$

SECTION – II : CHEMISTRY

(PART – A)

1. ACD

Sol. S_N1 Rate α Steric α Stability of C^+

S_N2 Rate $\alpha \frac{1}{\text{Steric}}$

2. ABD

Sol. Basic hybridization.

3. ACD

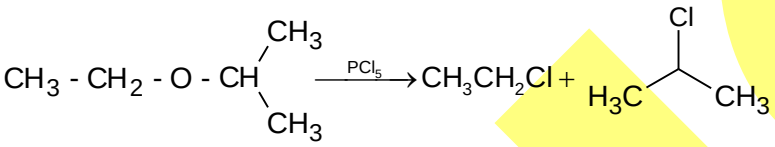
Sol. $E^0_{AgCl/Ag} = E^0_{AgCl/Ag} + \frac{RT}{F}\ell n\left[Cl^-\right]$

$AgCl + e^- \longrightarrow Ag + Cl^-$

$K_{sp} = [Ag^+][Cl^-]$

4. ABD

Sol.



5. BC

Sol. Benzyne mechanism, $K_{eq} < 1$

6. AD

Sol. S_N1 mechanism

7. ABD

Sol. Conceptual

8. ABD

Sol. A = Pthalic anhydride

B = Phenol/ohthalien

9. C

Sol. ΔH = +ve, T↑ = K_{eq}↑

10. D

Sol. P↑ = $Conc^n$↑

11. B

Sol. Chiral centre generated, without symmetry.

12. A

Sol. POS X, COS X } Optically active

(PART – B)

1. 8

Sol. Refining of metals

2. 55.85 (Range 55 to 56)

Sol. $\frac{134.04}{M} \times 1 = 1.2 \times 0.4 \times 5$

3. 7
Sol. Theoretical

4. 423.30 (Range 421 to 425)
Sol. Speed = $\lambda\nu$

5. 567.2(range 566 to 568)
Sol. $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

6	4	-	
6 - x	4 + x	x	= 14

x = 4

$\frac{1}{35.45} = \frac{8.4}{2} \cdot \frac{1}{V} \Rightarrow V = 567.2$

6. 50 (Range 49 to 51)
Sol. Theoretical

SECTION – III : MATHEMATICS

(PART – A)

1. AC

Sol. $|a+b|=20 \Rightarrow a+b=20e^{i\theta}$

Assume $a+b=20$ (Coordinate axes can be related so that $\theta=0$)

$|a^2+b^2|=16 \Rightarrow a^2+b^2=16e^{i\alpha}$

$|a^3+b^3|=|(a+b)^3-3ab(a+b)|$

$=\left|(a+b)^3-\frac{3}{2}\left\{(a+b)^2-(a^2+b^2)\right\}(a+b)\right|$

$=\left|\frac{3}{2}(a^2+b^2)(a+b)-\frac{1}{2}(a+b)^3\right|$

$=|480e^{i\alpha}-4000|$

$=160|3e^{i\alpha}-25|$

$=160\sqrt{634-150\cos\alpha}$

$=4480(\cos\alpha=-1)$ and $3520(\cos\alpha=1)$

2. ABCD

Sol. Let (x, y) be point common to both parabolas at point of tangency.

$y-x=\alpha(x^2-y^2)+\alpha(x-y)$

$\Rightarrow (x-y)\{\alpha(x+y)+\alpha+1\}=0$

Either x = y OR $\alpha(x+y)+\alpha+1=0$

Solving $y=x$ with $y=\alpha x^2+\alpha x+\frac{1}{24}$

$x=\alpha x^2+\alpha x+\frac{1}{24} \Rightarrow \alpha x^2+(\alpha-1)x+\frac{1}{24}=0$

Since the curves touch $\Rightarrow$ above quadratic should have one root only

$\Rightarrow$ Disc $=0 \Rightarrow (\alpha-1)^2-4\alpha\left(\frac{1}{24}\right)=0 \Rightarrow 6\alpha^2-13\alpha+6=0$

$\Rightarrow \alpha=\frac{2}{3},\frac{3}{2}$

Solving $\alpha(x+y)+\alpha+1=0$ and $y=\alpha x^2+\alpha x+\frac{1}{24}$

$\alpha x^2+(\alpha+1)x+\frac{\alpha+1}{\alpha}+\frac{1}{24}=0$

$\Rightarrow$ Disc. $=0 \Rightarrow (\alpha+1)^2-4\alpha\left(\frac{25}{24}+\frac{1}{\alpha}\right)=0 \Rightarrow \alpha=\frac{13\pm\sqrt{601}}{12}$

3. AB

Sol. By default, x – axis and y – axis (both tangents) are perpendicular

$\Rightarrow$ Origin lies on the directrix

Also, line joining point of intersection of tangents at A and B and mid – point of chord AB is parallel to axis or perpendicular to directrix.

$\Rightarrow$ midpoint of AB $\equiv\left(2, \frac{3}{2}\right)\equiv M$

$\Rightarrow$ slope of OM $=\frac{\frac{3}{2}-0}{2-0}=\frac{3}{4}\Rightarrow$ slope of directrix x $=\frac{-4}{3}$

$\Rightarrow$ Equation of directrix : $y=\frac{-4}{3}x$ i.e. $4x+3y=0$

Line segment joining point of contact (0, 3) and (0, 0) subtend a right angle at focus (h, k)

Slope of AB(focal chord) $=-\frac{3}{4}$

Slope of OS $=\frac{4}{3}=\frac{k-0}{h-0}\Rightarrow k=\frac{4}{3}h$

Since tangents at A and B are perpendicular AB $\left(\frac{x}{4}+\frac{y}{3}=1\right)$ is the focal chord so

$\frac{h}{4}+\frac{k}{3}=1$

Solving for h and k, we get $h=\frac{36}{25}, k=\frac{48}{25}$

4. CD

Sol. $8\sin^2 x\cos x=\sqrt{3}\sin x+\cos x$

$4\sin 2x\sin x=\sqrt{3}\sin x+\cos x$

$2(\cos x-\cos 3x)=\sqrt{3}\sin x+\cos x$

$2\cos 3x=\cos x-\sqrt{3}\sin x$

$\cos 3x=\cos x\frac{1}{2}-\sin x.\frac{\sqrt{3}}{2}$

$\cos 3x=\cos\left(x+\frac{\pi}{3}\right)$

$3x=2n\pi\pm\left(x+\frac{\pi}{3}\right)$

$\Rightarrow x=n\pi+\frac{\pi}{6}$ or $x=\frac{n\pi}{2}-\frac{\pi}{12}$

5. BC

Sol. a_n = Number of sets containing the element 'n' + Number of sets that do not contain 'n'

Number of subsets containing 'n'= set containing 'n' itself + number of subsets to be chosen from $\{1, 2, 3, 4....,n-2\}$

$= 1 + a_{n-2}$

Number of subsets not containing 'n'= subset to be chosen from $\{1, 2, 3, 4....., n-2, n-1\}$

$= a_{n-1}$

So, $a_n = (1 + a_{n-2}) + a_{n-1}$

6. ABD

Sol. Let $\bar{x} = 40$ be the mean of the scores and let $x_1, x_2, ..., x_p$ be the scores higher than or equal to 80. Then

$$(\bar{x} - x_{p+1}) + (\bar{x} - x_{p+2}) + ... + (\bar{x} - x_{100}) = (x_1 - \bar{x}) + ... + (x_p - \bar{x}) \geq (80 - 40)p = 40p$$

$$100.20^2 = (x_1 - \bar{x})^2 + ... + (x_p - \bar{x})^2 + (x_{p+1} - \bar{x})^2 + ... + (x_{100} - \bar{x})^2$$

By RMS $\geq$ AM inequality $\sqrt{\frac{a_1^2 + a^2_2 + ... + a^2_n}{n}} \geq \frac{a_1 + a_2 + ... + a_n}{n}$

We have

$$\sqrt{\frac{(\bar{x} - x_{p+1})^2 + (\bar{x} - x_{p+2})^2 + ... + (\bar{x} - x_{100})^2}{100 - p}} \geq \frac{(\bar{x} - x_{p+1}) + (\bar{x} - x_{p+2}) + ... + (\bar{x} - x_{100})}{100 - p} \geq \frac{(40 - 0)p}{100 - p}$$

So $(x_{p+1} - \bar{x})^2 + (x_{p+2} - \bar{x})^2 + ... + (x_{100} - \bar{x})^2 \geq \frac{(40p)^2}{100 - p}$

Similarly,

$$\sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ... + (x_p - \bar{x})^2}{p}} \geq \frac{(x_1 - \bar{x}) + (x_2 - \bar{x}) + ... + (x_p - \bar{x})}{p} \geq \frac{(80 - 40)p}{p} = 40$$

$$(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ... + (x_p - \bar{x})^2 \geq (40)^2 p$$

$$100.20^2 = (x_1 - \bar{x})^2 + ... + (x_p - \bar{x})^2 + (x_{p+1} - \bar{x})^2 + ... + (x_{100} - \bar{x})^2 \geq (40)^2 p + \frac{(40p)^2}{100 - p}$$

On simplifying we get

$100 - p \geq 4p$

$p \leq 20$

7. BCD

Sol. Let AB lie along the x – axis.

Let $A \equiv (0,0)$

$\tan\frac{B}{2} = \frac{1}{3} \Rightarrow \tan B = \frac{3}{4}$

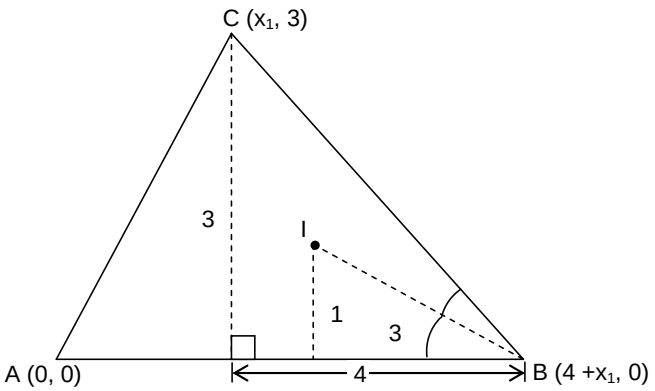
Assume, $B \equiv (4 + x_1, 0)$, so that

tanB can be equal to $\frac{3}{4}$

and $C \equiv (x_1, 3)$

Centroid $\equiv \left(\frac{4 + 2x_1}{3}, 1\right)$

Since IG || AB, we have inradius, r = 1



$r = \frac{\Delta}{s}$

$\Rightarrow \Delta = S$

$$\Rightarrow \frac{1}{2}(3)(4+x_1)$$

$$= \frac{(4+x_1)+5+\sqrt{9+x_1^2}}{2}$$

$$\Rightarrow 3x_1^2+12x_1=0$$

$\Rightarrow x_1=0$ (–4 is rejected otherwise area of triangle will become zero) .

So the foot of perpendicular from C on AB coincides with A

$$\Rightarrow \angle CAB = 90^o$$

$$\Rightarrow \sin B = \frac{b}{a}=\frac{3}{5};\ \tan B=\frac{3}{4}=\frac{b}{c}$$

8. BC

Sol. (B) $$\frac{1}{a^2\left(1-r^2\right)}\left[1+\frac{1}{r^2}+\frac{1}{r^4}+.....\frac{1}{r^{2(n-2)}}\right]=\frac{1-\frac{1}{r^{2(n-1)}}}{\left(1-\frac{1}{r^2}\right)a^2\left(1-r^2\right)}$$

$$=\frac{\left(1-r^{2n}\right)}{a^2\left(1-r^2\right)^2r^{2n-4}}$$

(C) $$\frac{1}{a^m\left(1+r^m\right)}\left[1+\frac{1}{r^m}+\frac{1}{r^{2m}}+.....+\frac{1}{r^{(n-2)m}}\right]=\frac{\left(1-\frac{1}{r^{(n-1)m}}\right)}{\left(1-\frac{1}{r^m}\right)a^m\left(1+r^m\right)}$$

$$=\frac{\left(1-r^{mn-m}\right)}{\left(1-r^m\right)\left(r^{mn-m}-r^{mn-2m}\right)}$$

9. B

10. C

Sol.

(I) Let $q_1=1-p_1$ and $q_2=1-p_2$

$\because$ A can win the game at the 1st , 4th, 7th… trials.

$$\therefore \text{ P (A wins) } = \frac{1}{6}+\frac{5}{6}(q_1)(q_2)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^2$$

$$(q_1)^2(q_2)^2\left(\frac{1}{6}\right)+...$$

$$=\frac{\frac{1}{6}}{1-\frac{5}{6}q_1q_2}=\frac{1}{6-5q_1q_2}=\frac{1}{6-5\left(\frac{4}{3}\right)\left(\frac{3}{4}\right)}=\frac{1}{\lambda_1}\text{ [given]}$$

$\therefore \lambda_1=3$

(II) $$\text{P (C wins) } = \frac{5}{6}.q_1.p_2+\left(\frac{5}{6}\right)^2.q_1^2.q_2p_2+...$$

$$= \frac{\frac{5}{6}q_1 . p_2}{1-\frac{5}{6}.q_1q_2} = \frac{5q_1p_2}{6-5q_1q_2} = \frac{5\times\frac{4}{5}\times\frac{1}{4}}{6-5\times\frac{4}{5}\times\frac{3}{4}} = \frac{1}{3}$$

$$= \frac{1}{\lambda_2} \qquad \text{[given]]}$$

$$\therefore \lambda_2 = 3$$

(III) $\because$ P (A wins) = P (B wins)

$$\Rightarrow \frac{1}{6-5q_1q_2} = \frac{5p_1}{6-5q_1q_2}$$

$$\therefore \quad p_1 = \frac{1}{5} = \frac{1}{\lambda_3} \qquad \text{[given]}$$

$$\therefore \quad \lambda_3 = 5$$

(IV) P (Q wins) = P (B wins) = P (C wins)

$$\Rightarrow \quad \frac{1}{6-5q_1q_2} = \frac{5p_1}{6-5q_1q_2} = \frac{5q_1p_2}{6-5q_1q_2}$$

$$\Rightarrow \quad 1 = 5p_1 = 5q_1p_2$$

$$\Rightarrow \quad p_1 = \frac{1}{5}, \frac{1}{p_2} = 5q_1 = 5\left(1-\frac{1}{5}\right) = 4 = \lambda_4 \qquad \text{[given]}$$

$$\therefore \quad \lambda_4 = 4$$

11. D

12. C

(I) Number of dissimilar terms in the expansion of $(x+2y+3z)^n (n \in N)$

$$= {}^{n+3-1}C_{3-1} = {}^{n+2}C_2 = \frac{1}{2}n^2 + \frac{3}{2}n + 1$$

$$\therefore \quad a = \frac{1}{2}, b = \frac{3}{2}, c = 1$$

Hence, $a+b+c = \frac{1}{2}+\frac{3}{2}+1 = 3$ and $a+b = \frac{1}{2}+\frac{3}{2} = 2 = 2c$

(II) We have $(x+y+z)^{2n+1} = \{(x+y)+z\}^{2n+1}$

$$= (x+y)^{2n+1} + {}^{2n+1}C_1(x+y)^{2n} z + {}^{2n+1}C_2(x+y)^{2n-1} z^2 + {}^{2n+1}C_3(x+y)^{2n-2} z^3 + \ldots$$

${}^{2n+1}C_{2n}(x+y)z^{2n} + {}^{2n+1}C_{2n+1}z^{2n+1}$ and $(x+y-z)^{2n+1} = \{(x+y)-z\}^{2n+1}$

$$= (x+y)^{2n+1} - {}^{2n+1}C_1(x+y)^{2n} z + {}^{2n+1}C_2(x+y)^{2n-1} z^2$$

$$-{}^{2n+1}C_3(x+y)^{2n-2} z^3 + \ldots + {}^{2n+1}C_{2n}(x+y)z^{2n} - {}^{2n+1}C_{2n+1}z^{2n+1}$$

$$\therefore (x+y+z)^{2n+1} - (x+y-z)^{2n+1}$$

$$= 2\left\{ {}^{2n+1}C_1(x+y)^{2n} z + {}^{2n+1}C_3(x+y)^{2n-2} z^3 + \ldots + z^{2n+1} \right\}$$

$\therefore$ The number of dissimilar terms in the expansion of $(x+y+z)^{2n+1} - (x+y-z)^{2n+1}$

$$= (2n+1) + (2n-1) + (2n-3) + \ldots 5 + 3 + 1$$

$$= \frac{(n+1)}{2}(2n+1+1) = (n+1)^2$$

$$= n^2 + 2n + 1$$

$\therefore\ a=1, b=2, c=1$

Hence, $a+b+c=1+2+1=4$

(III) We have, $(x-y+z)^n=\{x-(y-z)\}^n$

$=x^n-{}^nC_1x^{n-1}(y-z)+{}^nC_2x^{n-2}(y-z)^2+{}^nC_3x^{n-3}(y-z)^3$

$+\ldots-{}^nC_{n-1}x(y-z)^{n-1}+{}^nC_n(y-z)^n$ and $(x+y-z)^n=(x+y-z)^n$

$=x^n+{}^nC_1x^{n-1}(y-z)+{}^nC_2x^{n-2}(y-z)^2+{}^nC_3x^{n-3}(y-z)^3+\ldots+{}^nC_{n-1}x(y-z)^{n-1}+(y-z)^n$

$\therefore\ (x+y+z)^n+(x+y-z)^n$

$=2\left[x^n+{}^nC_2x^{n-2}(y-z)^2+{}^nC_4x^{n-4}(y-z)^n+\ldots+(y-z)^n\right]$

$\therefore$ The number of dissilmilar terms in the expansion of

$(x-y+z)^n+(x+y-z)^n=1+3+5+\ldots+(n+1)$

$=\dfrac{\frac{(n+2)}{2}}{2}(1+n+1)=\dfrac{(n+2)^2}{4}=\dfrac{1}{4}(n^2+n+1)$

$\therefore\ a=\dfrac{1}{4}, b=1, c=1$

Hence, $b+c=1+1=2=8a$

(IV) $\because\left(\dfrac{x^2+1+x^4}{x^2}\right)^{\sum n}$

$=\dfrac{a_0+a_2x^2+a_4x^4+\ldots+a_{2n(n+1)}x^{2n(n+1)}}{x^{n(n+1)}}$

$\therefore$ Number of terms $=\dfrac{1}{2}.2n(n+1)+1=n^2+n+1$

$\therefore\ a=1, b=1, c=1$

Hence, $a+b+c=1+1+1=3$ and $a+b=1+1=2=2c$

(PART – B)

1. 0

Sol. Assume $\det(AB-BA)\neq 0$

Since $A^2+B^2=2AB$ we have $A^2+B^2-AB-BA=2AB-AB-BA$

$\Rightarrow(A-B)^2=AB-BA$

$\Rightarrow\det(A-B)^2=\det(AB-BA)\neq 0$

$\Rightarrow\det(A-B)\neq 0\Rightarrow A-B$ is invertible.

Also, if $A^2+B^2=2AB$

then $A^2-AB=AB-B^2$

$A(A-B)=(A-B)B$

$\Rightarrow A(A-B)(A-B)^{-1}=(A-B)B(A-B)^{-1}$

$\Rightarrow A=(A-B)B(A-B)^{-1}$

Subtracting B from both sides.

$\Rightarrow A-B=(A-B)B(A-B)^{-1}-B$

$\Rightarrow(A-B)^{-1}(A-B)=(A-B)^{-1}(A-B)B(A-B)^{-1}-(A-B)^{-1}B$

$\Rightarrow I=B(A-B)^{-1}-(A-B)^{-1}B$

$\text{trace}(I)=\text{trace}\left\{B(A-B)^{-1}-(A-B)^{-1}B\right\}$

$\Rightarrow \text{trace}(I) = \text{trace}\left\{B(A-B)^{-1}\right\} - \text{trace}\left\{(A-B)^{-1}B\right\}$

$\Rightarrow \text{trace}(I) = 0 \qquad \left(\because \text{trace}(AB) = \text{trace}(BA)\right)$

which is a contradiction.

Hence, $\det(AB-BA) = O$

2. 24

Sol. $p(-t-1) = \dfrac{(-t-1)(-t-1-1)(-t-1-2)...(-t-1-2023)}{2023!}$

$= \dfrac{(t+1)(t+2)(t+3)...(t+2024)}{2023!}$

$\int_0^1 \left\{\dfrac{1}{t+1} + \dfrac{1}{t+2} + ... + \dfrac{1}{t+2024}\right\} \dfrac{(t+1)(t+2)(t+3)...(t+2024)}{2023!} dt$

$= \dfrac{1}{2023!}\left|(t+1)(t+2)(t+3)....(t+2024)\right|_0^1$

$= \dfrac{1}{2023!}2025! - 2024!$

$= \dfrac{(2024)!}{2023}\{2025-1\}$

$= (2024)^2$

$\Rightarrow k = 2024$

3. 6

Sol. $\tan 78^\circ + \tan 72^\circ + \tan 60^\circ$

$= \dfrac{1}{2\cos 72^\circ \cos 75^\circ} + \cot 30^\circ$

$= \dfrac{2\cos 36^\circ}{\cos 78^\circ} + 2\cos 30^\circ \qquad 2\cos 72^\circ = 2\sin 18^\circ = \dfrac{\sqrt{5}-1}{2}$

$= \dfrac{2}{\sqrt{5}+1}$

$= \dfrac{2\cos 36^\circ + 2\cos 30^\circ \cos 78^\circ}{\cos 78^\circ} \qquad = \dfrac{1}{2\cos 36^\circ}$

$= \dfrac{2\cos 36^\circ + \cos 108^\circ + \cos 48^\circ}{\cos 78^\circ}$

$= \dfrac{2\cos 36^\circ - \cos 72^\circ + \cos 48^\circ}{\cos 78^\circ} \qquad \cos 36^\circ - \cos 72^\circ = \dfrac{\sqrt{5}+1}{4} - \dfrac{\sqrt{5}-1}{4} = \dfrac{1}{2}$

$= \dfrac{1+\cos 72^\circ + \cos 48^\circ}{\cos 78^\circ} = \dfrac{1+2\cos 60^\circ \cos 12^\circ}{\sin 12^\circ} = \dfrac{1+\cos 12^\circ}{\sin 12^\circ} = \cot 6^\circ$

$\Rightarrow k = 6$

4. 111

Sol. In general, number of triangles is ${}^nC_3 + 4\left({}^nC_4\right) + 5\left({}^nC_5\right) + {}^nC_6$

So, putting $n = 6$, number of triangles is 111

5. 1

Sol. $4\int_0^{\pi/2}\left(f(x)+x^2\right)dx + 125\int_0^{3/2}\dfrac{dx}{\sqrt{f(x)+x^2}} = \dfrac{225}{2}$

Let $g(x)=f(x)+x^2$

$$4\int_0^{3/2} g(x)dx+125\int_0^{3/2}\frac{1}{2\sqrt{g(x)}}dx+125\int_0^{3/2}\frac{1}{2\sqrt{g(x)}}dx$$

$$\geq 3\int_0^{3/2}\left(4g(x).\frac{125}{2\sqrt{g(x)}}.\frac{125}{2\sqrt{g(x)}}\right)^{1/3}dx$$

$$=\frac{225}{2}$$

Equality occurs if $4g(x)=\frac{125}{2\sqrt{g(x)}}\Rightarrow (g(x))^{\frac{3}{2}}=\left(\frac{5}{2}\right)^3$

$$\Rightarrow g(x)=\frac{25}{4}$$

6. 0.17

Sol. $S=\sum_{i=0}^{\infty}\int_0^1\left(x^{4i}-2x^{4i+1}+x^{4i+2}\right)dx=\frac{1}{2}\int_0^1\left(\frac{1}{1-x^4}-\frac{2x}{1-x^4}+\frac{x^2}{1-x^4}\right)dx=\frac{1}{2}\int_0^1\frac{(1-x)^2}{1-x^4}dx]$

So, $S=\frac{1}{2}\int_0^1\left(\frac{1}{1+x}-\frac{x}{1+x^2}\right)dx=\frac{\ln 2}{4}$