FIITJEE RBT-7 for (JEE-Advanced)

BATCHES – Two Yr CRP2123(All)

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 3

QP CODE: 100202

PAPER - 1

Time Allotted: 3 Hours **Maximum Marks: 204**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. All the section can be filled in **PART-A & B** of OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **Part-A (01-04) –** Contains **Four (04)** multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **PART–A** (**05–12)** contains Eight (8) Multiple Choice Questions which have **One or More Than One Correct** answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(iii) **Part-B (1 – 8)** contains Eight **(08)** Numerical based questions, the answer of which maybe positive or negative numbers or decimals **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :____

Batch :____________________ **Date of Examination :**____________________

Enrolment Number :____

SECTION – I : PHYSICS

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. A ball is projected from a point in one of the two smooth parallel vertical walls against the other in a plane perpendicular to both. After being reflected at each wall impinge again on the second at a point in the same horizontal plane as it started from. If "e" is the coefficient of restitution, "a" is the distance between the walls and "R" is the free range on horizontal surface, then which of the following is correct?

(A) $Re^3 = a(1 + e + e^2)$
(B) $Re = a(1 + e^2 + e^3)$
(C) $Re^2 = a(1 + e + e^2)$
(D) $R = a(1 + e + e^2)$

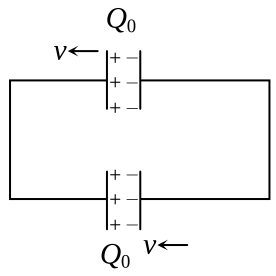
2. A hemisphere rests in equilibrium on a rough ground and against an equally rough wall, if the equilibrium is limiting, then the inclination θ of the base to the horizontal is

(A) $\theta = \sin^{-1}\left[\frac{3}{8}\left\{\frac{\mu+\mu^2}{1+\mu^2}\right\}\right]$
(B) $\theta = \sin^{-1}\left[\frac{8}{3}\left\{\frac{\mu+\mu^2}{1+\mu^2}\right\}\right]$
(C) $\theta = \cos^{-1}\left[\frac{8}{3}\left\{\frac{\mu+\mu^2}{1+\mu^2}\right\}\right]$
(D) $\theta = \cos^{-1}\left[\frac{3}{8}\left\{\frac{\mu+\mu^2}{1+\mu^2}\right\}\right]$

3. A cubical container of side 'a' and wall thickness x (x << a) is suspended in air and filled n moles of diatomic gas (adiabatic exponent γ) in a room where room temperature is T_0. If at time t = 0 gas temperature is T_1 ($T_1 > T_0$), the temperature of gas 'T' at time 't' is

(A) $T = T_0 + T_1 e^{-\frac{6ka^2(\gamma-1)}{nRx}t}$
(B) $T = T_0 - (T_1 - T_0)e^{-\frac{6ka^2(\gamma-1)}{nRx}t}$
(C) $T = T_0 + (T_1 + T_0)e^{-\frac{6ka^2(\gamma+1)}{nRx}t}$
(D) $T = T_0 + (T_1 - T_0)e^{\frac{6ka^2(\gamma-1)}{nRx}t}$

4. Two identical capacitor connected as shown and having initial charge Q_0 each. Separation between plates of capacitor is d. Suddenly the left plate of upper capacitor and right plate of lower capacitor start moving with speed v towards left while other plate of capacitor remains fixed. The current in the circuit is



(A) $\frac{Q_0v}{2d}$
(B) $\frac{Q_0v}{d}$
(C) $\frac{Q_0vt}{2d}$
(D) zero

Space For Rough Work

PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. A strip of wood of mass M and length ℓ is placed on a smooth horizontal surface. An insect of mass m starts at one end on the strip and walks to the other end in time t, moving with a constant speed.

(A) the speed of the insect as seen from the ground is < $\frac{\ell}{t}$.

(B) the speed of the strip as seen from the ground is $\frac{\ell}{t}\left(\frac{M}{M+m}\right)$

(C) the speed of the strip as seen from the ground is $\frac{\ell}{t}\left(\frac{m}{M+m}\right)$

(D) the total kinetic energy of the system is $\frac{1}{2}(m+M)\left(\frac{\ell}{t}\right)^2$

6. When photons of energy 4.25 eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50)$ eV. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$ then

(A) the work function of A is 2.25 eV
(B) the work function of B is 4.20 eV
(C) $T_A = 2.00$ eV
(D) $T_B = 2.75$ eV

7. A tube of length *l* and radius R carries a steady flow of fluid whose density is ρ and viscosity is η. The fluid flow velocity depends on the distance r from the axis of the tube as $v = v_0\left(1-\frac{r^2}{R^2}\right)$. Choose the correct option(s) from the following.

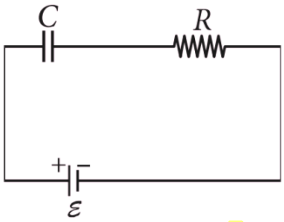
(A) the volume of the fluid flowing across the section of the tube per unit time is $\frac{1}{2}\pi^2 v_0 R^2$

(B) the kinetic energy of the fluid within the tube's volume is $\frac{1}{6}\pi\ell R^2\rho v_0^{\,2}$

(C) the friction force exerted on the tube the fluid is $\frac{4\pi\eta\ell}{v_0}$

(D) the pressure difference at the ends of the tube is $\frac{4\eta\ell v_0}{R^2}$

Space For Rough Work



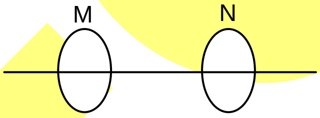
8. In the following $R-C$ circuit, the capacitor is in the steady state. The initial separation of the capacitor plates is x_0. If at $t=0$, the separation between the plates starts changing so that a constant current flows through R, find the velocity of the moving plates as a function of time. The plate area is A.

(A) $\left(\frac{I/\varepsilon_0 A}{IR-\xi}\right)\left[\left(\frac{I/\varepsilon_0 A}{\xi-/IR}\right)^2 t+\frac{1}{x_0}\right]^{-2}$

(B) $\left(\frac{I/\varepsilon_0 A}{IR-\xi}\right)\left[\left(\frac{I/\varepsilon_0 A}{\xi-IR}\right) t+\frac{1}{x_0}\right]^{-2}$

(C) $\left(\frac{I/\varepsilon_0 A}{IR-\xi}\right)\left[\left(\frac{I/\varepsilon_0 A}{\xi-IR}\right) t+\frac{1}{x_0}\right]^{2}$

(D) $\left(\frac{I/\varepsilon_0 A}{IR-\xi}\right)\left[\left(\frac{I/\varepsilon_0 A}{\xi-IR}\right)^{-1} t+\frac{1}{x_0}\right]^{2}$



9. Two identical circular coils M and N are arranged coaxially as shown in the figure. Separation between the coils is large as compared to their radii. The arrangement is viewed from left along the common axis. The sign convention adopted is that currents are taken to be positive when they appear to flow in clockwise direction. Then

(A) if M carries a constant positive current and is moved towards N, a positive current is induced in N

(B) if M carries a constant positive current and N is moved towards M, a negative current is induced in N

(C) if a positive current in M is switched off, a positive current is momentarily induced in N

(D) if both coils carry positive currents, they will attract each other

10. An external magnetic field is decreased to zero due to which a current is induced in a circular wire loop of radius r and resistance R placed in the field. This current will not become zero.

(A) at the instant when external magnetic field stops changing $(t=0)$, the current in the loop as a function of time for $t>0$ is given by $i_0 e^{-2Rt/\mu_2 \pi F}$

(B) at the instant when B stops changing $(t=0)$, the current in the loop as a function of time $t>0$ is given by $\frac{\mu_0 IR}{2r}$

(C) the time in which current in loop decreases to $10^{-3} I_0 (\text{fromt}=0)$ for $R=100\Omega$ and and $r=5$ cm is given by $\frac{3\pi^2 \ln 10}{10^{10}}$

(D) the time in which current in loop decreases to $10^3 I_0$ (from $t=0$)for $R=100\Omega$ and $r=5\text{cm}$ is given by $\frac{3\pi^2}{10^6}\text{s}$

Space For Rough Work

11. In the series L – C – R circuit, the voltage across resistance, capacitance and inductance are 30V each at frequency $f = f_0$.
(A) It the inductor is short-circuited, the voltage across the capacitor will be $30\sqrt{2}$ V.
(B) If the capacitor is short-circuited, the voltage drop across the inductor will be $\frac{30}{\sqrt{2}}$ V.
(C) If the frequency is changed to $2f_0$, the ratio of reactance of the inductor to that of the capacitor is 4 : 1.
(D) If the frequency is changed to $2f_0$, the ratio of the reactance of the inductor to that of the capacitor is 1 : 4.

12. A wave disturbance in a medium is described by

$$y(x, t) = 0.02 \cos (50 \pi t + \pi/2) \cos (10 \pi x),$$

where 'x' and 'y' are in metre and 't' in seconds.
(A) A node occurs at x = 0.15 m
(B) An antinode occurs at x = 0.3 m
(C) The speed of the component wave is 5.0 m/s
(D) The wavelength is 0.2 m.

PART – B (Maximum Marks: 24)

(Numerical **Type)**

This section contains ***Eight (08)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

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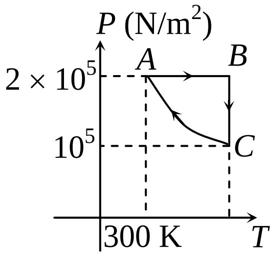
1. A machine is blowing spherical soap bubbles of different radii filled with helium gas. It is found that if the bubbles have a radius smaller than 1 cm, then they sink to the floor in still air. Larger bubbles float in the air. Assume that the thickness of the soap film in all bubbles is uniform and equal. Assume that the density of soap solution is same as that of water (=1000 kg m^{-3}). The density of helium inside the bubbles and air are 0.18 kg m^{-3} and 1.23 kg m^{-3}, respectively. If d is the thickness (in μm) of the soap film of the bubbles which are just floating, find the value of 2d.

2. Find the mass ratio M_D/M_H of deuterium and hydrogen (in near integer value) if their Hα lines have wavelengths of 6561.01 Å and 6562 Å respectively.

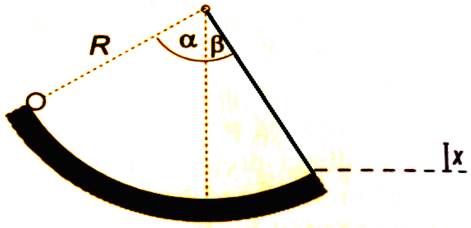
3. Two moles of a monatomic ideal gas is taken through a cyclic process shown on pressure(P) - temperature(T) diagram in figure. Process CA is represented as PT = Constant. If efficiency of given cyclic process is $1-\frac{3x}{12\ln 2+15}$, then find the value of 'x'.



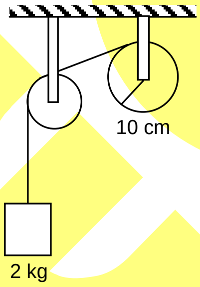
Space For Rough Work

4. A solid sphere of radius R = 0.2 m and of negligible mass is swimming in the water tank of base surface area A = 0.5 m^2. The depth of the water in the tank is h = 1 m. Determine the work (in joules) needed to push to push the sphere down the bottom of tank. (Assume that no water flows out of the tank. The density of water is ρ = 1000 kg/m^3 and g = 9.81 m/s^2).

5. A small wheel, initially at rest, rolls down a ramp in the shape of a quarter circle without slipping. The radius of the circle is R = 1 m and $\alpha = 60°$, $\beta = 30^o$. Find the height ‘x’ (in cm) reached by the wheel after leaving the track.

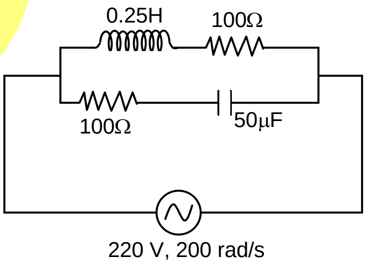


6. A string is wrapped on a wheel of moment of inertia 0.20 kg m^2 and radius 10 cm and goes through a light pulley to support a block of mass 2.0 kg as shown in figure. Find the acceleration of the block. (in m/s^2)



7. A battery is made by joining identical cells in m rows each row having m cells. The current flowing in an external resistance R is i_1. Now if the number of rows ‘m’ and number of cells in each row ‘n’ are interchanged, the current in external resistance is i_2. If $\frac{m}{2} = 2$ and $\frac{R}{r} = 4$ then the value of $\frac{i_2}{i_1}$ is

8. In the given Ac circuit, what is the power developed in watts?



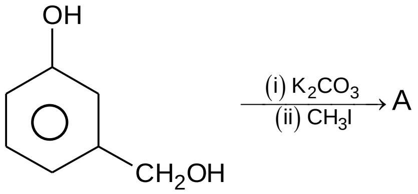
Space For Rough Work

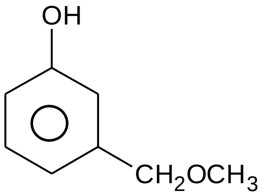
SECTION – II : CHEMISTRY

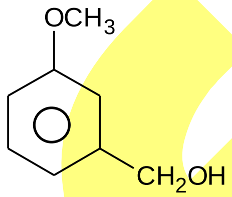
PART – A (Maximum Marks: 12)

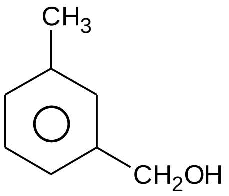
This section contains ***FOUR (04)*** *questions. Each question has* ***FOUR*** *options.* ***ONLY ONE*** *of these four options is the correct answer.*

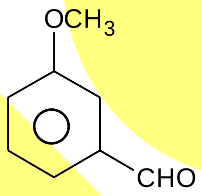
1. The product A is



(A)


(B)


(C)


(D)


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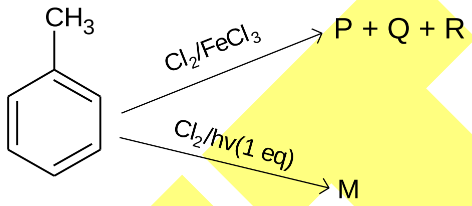


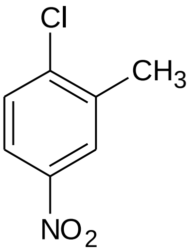
Which product can easily undergoes nucleophilic substitution reaction(S_N1 and S_N2 with equal ease).

(A) P
(B) Q
(C) R
(D) M

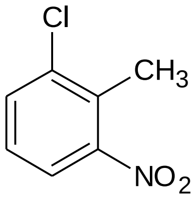
Space For Rough Work

3. Which is most reactive towards aromatic nucleophilic substitution reaction towards NaOH?

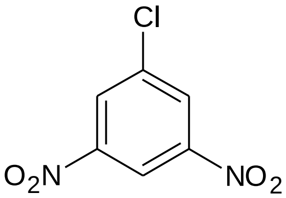
(A)



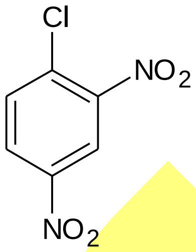
(B)



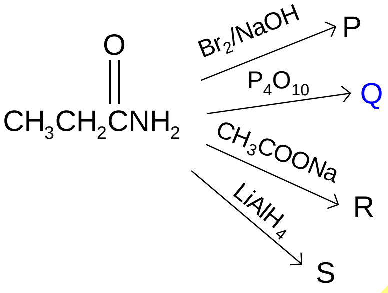
(C)



(D)



4.



Choose correct statement

(A) (P) and (S) are chain isomers
(B) hydrolysis of (Q) produces carboxylic acid
(C) (R) is obtained by the fastest reaction
(D) (S) contains four carbon atom

PART – A (Maximum Marks: 32)

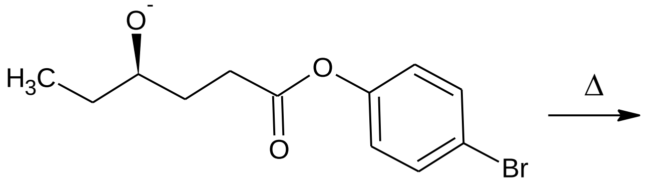
*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. Choose incorrect statements:

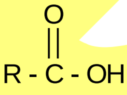
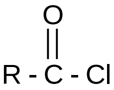
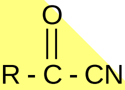
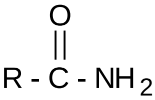
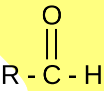
(A) $AlCl_3$ undergoes hydrolysis to form $\left[Al(H_2O)_6\right]^{3+}$
(B) Amalgamated Al reacts with cold water to liberate H_2
(C) The ash formed on burning magnesium in air fumes on wetting and exposing to HCl
(D) B^{3+} ion can exist in aqueous solution in the hydrated form

Space For Rough Work

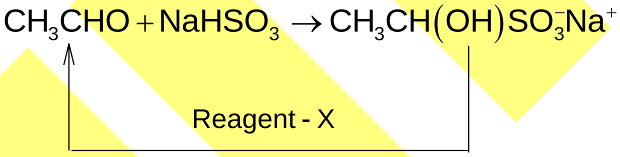
6. Given $\Delta H_f^0(C_2H_6, g) = -85\,kJ/mole$,
$\Delta H_f^0(C_3H_8, g) = -104\,kJ/mole$,
$\Delta H_{sub}^0(C, s) = 718\,kJ/mole$ & B.E. of $(H-H) = 436$ kJ/mole.
Then, in kJ/mole, the
(A) C – C bond enthalpy is 218
(B) C – H bond enthalpy is 414
(C) C – C bond enthalpy is 345
(D) C – H bond enthalpy is 448

7.


Which of the statements regarding the product formed in the above reaction is/are correct?
(A) If Br is replaced with NO_2 rate of reaction increases
(B) Inversion of configuration takes place.
(C) Nucleophillic acyl substitution takes place.
(D) Configuration at chiral carbon does not change

8. Which of the following compounds are more reactive than  $R-\overset{\overset{O}{||}}{C}-OH$ towards attack of nucleophile?
(A)  $R-\overset{\overset{O}{||}}{C}-Cl$
(B)  $R-\overset{\overset{O}{||}}{C}-CN$
(C)  $R-\overset{\overset{O}{||}}{C}-NH_2$
(D)  $R-\overset{\overset{O}{||}}{C}-H$

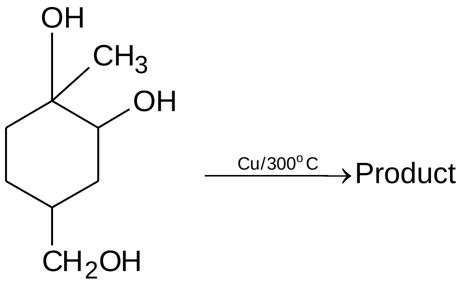
9. Compound/s which contain 3c – 2e⁻ bond.
(A) $(BeH_2)_n$
(B) $Al_2(CH_3)_6$
(C) B_2H_6
(D) Al_2Cl_6

10.

$$CH_3CHO + NaHSO_3 \rightarrow CH_3CH(OH)SO_3^-Na^+$$

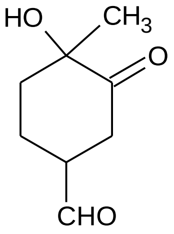
In the above change, the reagent (X) can be
(A) CCl_4
(B) HCl
(C) NaOH
(D) DMF

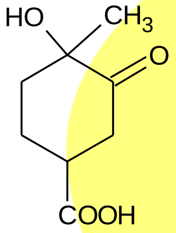
Space For Rough Work

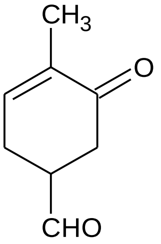
11. Acetone can form an aromatic compound when treated with conc. H_2SO_4. Which of the following reaction(s) do(es) not take place?
(A) oxidation (B) dehydration
(C) sulphonation (D) dehydrogenation

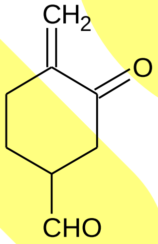
12.


Which of the following(s) is/are minor product(s) or product(s) that are not formed at all?

(A)


(B)


(C)


(D)


PART – B (Maximum Marks: 24)

(Numerical **Type)**

This section contains ***Eight (08)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

1. 4 moles of a liquid A (V.P = 100 mm of Hg) was mixed with 6 moles of B(V.P = 400 mm of Hg). If the mole fraction of A in the vapour above the solution is expressed as $\frac{x}{y}$, what is the value of (x + y)?

2. How many moles of O_2 gas can be produced by passing eight Faraday of electricity into sufficient water?

Space For Rough Work

3. $NH_4NO_3 \longrightarrow N_2O + 2H_2O$

What is the n-factor of NH_4NO_3 in above reaction?

4. An organic compound(X) upon ozonolysis reaction, produces 2-heptanone and formaldehyde. Reaction of (X) with HCl forms major product(Y) and minor product (Z). Reaction of (Y) with sodium metal in presence of dry ether(Wurtz reaction) forms the only product(P).

If x = the number of quartenary carbon atoms present in (P)

y = the number of monochloro products formed by (P) excluding stereoisomers

z = number of allylic hydrogen atoms present in (X)?

What is the value of $\left(\frac{x+y+z}{10}\right)$?

5. 11.7 g of NaCl solid is added to 176.4 g of H_2O. The degree of dissociation of NaCl in the solution is 0.8. If the relative lowering of vapour pressure of water in the solution is expressed as $x \times 10^{-2}$, what is the value of x?

6. In a mixed metallic oxide, the oxide ion forms the hexagonal close packing(hcp) lattice. The M^{2+} ions occupy ½ of the octahedral voids and the N^{3+} ions occupy 1/6th of the tetrahedral voids. If the formula of the solid is written as $M_xN_yO_z$, what is the value of $\left(\frac{x+y+z}{10}\right)$?

7. A 0.5 M aqueous solution of $Al_2(SO_4)_3$ is isotonic to a 0.2 M aqueous solution of glucose. If the solute undergoes completely ionization, then the ratio of their osmotic pressure is expressed as x : y. What is the value of $\frac{x}{y}$ in decimal form?

8. 5.35 g of a salt of a strong acid and weak base is dissolved in water to make one litre solution. The pH of the solution is found to be 5. The crystal of the salt contains NaCl type unit cell. What is the mass of one mole of unit cell of the salt in gram unit?

[K_b of the weak base = 10^{-5}]

Space For Rough Work

SECTION – Iii : MATHEMATICS

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. A varying parabola ‘S’ of latus rectum ‘4a’ touches $y^2 = 4ax$, the axis of ‘S’ being parallel to x – axis. The locus of vertex of ‘S’ is

(A) $x + y + a = 0$ (B) $y^2 = 16ax$

(C) $x + y - a = 0$ (D) $y^2 = 8ax$

2. On the open interval (–c, c), where c is a positive real number, y(x) is an infinitely differentiable solution of the differential equation $\frac{dy}{dx} = y^2 - 1 + \cos x$, with the initial condition $y(0) = 0$. Then which one of the following is correct?

(A) $y(x)$ has a local maximum at the origin

(B) $y(x)$ has a local minimum at the origin.

(C) $y(x)$ is strictly increasing on the open interval $(-\delta, \delta)$ for some positive real number δ

(D) $y(x)$ is strictly decreasing on the open interval $(-\delta, \delta)$ for some positive real number δ.

3. Let $f:(0, \infty) \to (0, \infty)$, $f(1) = \sqrt{2}$ be a twice differentiable function such that $f'\left(\frac{1}{x}\right) = \frac{1}{f(x)}$ for all x in given domain, then $f(3) =$

(A) $\sqrt{2}$ (B) $\sqrt{10}$

(C) $3\sqrt{3}$ (D) $\sqrt{3}$

4. The values that ‘a’ can take so that the function $f:R \to R$ $f(x) = x^3 + x^2 + a\cos x$ is injective

(A) $a \in \left(\frac{1}{3}, \infty\right)$ (B) $a \in \left(-\infty, -\frac{1}{3}\right)$

(C) $a = 2$ (D) No value of a

Space For Rough Work

PART – A (Maximum Marks: 32)

This section contains ***EIGHT (08)*** *questions. Each question has* ***FOUR*** *options (A), (B), (C) and (D).* ***ONE OR MOER THAN ONE*** *of these four option(s) is (are) correct answer(s).*

5. Equation of ellipse E is $\frac{x^2}{16}+\frac{y^2}{4}=1$. Suppose C be a circle concentric with E intersecting the ellipse in four points. Let A be a point on E and B be a point on C such that line joining A and B is common tangent of ellipse E and circle C. If ℓ be maximum value of length AB and m be its corresponding slope, then

(A) $\ell=2$
(B) $m=\sqrt[\pm]{2}$
(C) $\ell=6$
(D) $m=\frac{\pm1}{\sqrt{2}}$

6. Let $S=\{w_1,w_2,....\}$ be a sample space associated to a random experiment. Let $P(w_n)=\frac{P(w_{n-1})}{2}, n\ge2$. Let $A=\{2k+3\ell:k,\ell\in N\}$ and $B=\{w_n:n\in A\}$. Then

(A) $P(w_n)=\frac{1}{2^{n-1}}$
(B) $P(B)=\frac{3}{32}$
(C) $P(w_n)=\frac{1}{2^n}$
(D) $P(B)=\frac{3}{64}$

7. If $|2z+3|=|3z+4|$, then

(A) $|z|_{min}=2$
(B) $|z|_{max}=\frac{7}{5}$
(C) $|z|_{max}=2$
(D) $|z|_{min}=1$

8. Let $f:X\to Y$ be a function. If $P\subset X$ and $Q\subset Y$, define $f(p)=\{f(x):x\in P\}$ and $f^{-1}(Q)=\{x:f(x)\in Q\}$. Then the true statement(s) is/are:

(A) $f(f^{-1}(Q))=Q$
(B) $f^{-1}(Q)\cup f^{-1}(R)=f^{-1}(Q\cup R)$
(C) $f^{-1}(f(P))=P$
(D) $f^{-1}(Q)\cap f^{-1}(R)=f^{-1}(Q\cap R)$

9. Consider function $f:A\to\bar{B}$ and $g:B\to C(A,B,C\subseteq R)$ such that $(gof)^{-1}$ exists then

(A) f is ONTO and g is ONE – ONE
(B) f is ONE – ONE
(C) g is ONTO
(D) f and g are both ONTO

Space For Rough Work

10. The function $f(x)$ is differentiable, continuous and $f(x) \neq 0$ for all x on the interval $[4, 8]$. Also $f(4) = \frac{1}{4}$, $f(8) = \frac{1}{2}$ and $\int_4^8 \frac{(f'(x))^2}{(f(x))^4} dx = 1$. Find $f(6)$.

(A) $f(6) = \frac{1}{9}$
(B) $f(6) = \frac{1}{3}$
(C) $f(5) = \frac{2}{7}$
(D) $f(5) = 3$

11. A relation R on the set of complex numbers is defined by $z_1 R\ z_2$ if and only if $\frac{z_1 - z_2}{z_1 + z_2}$ is real. Then

(A) R is reflexive relation
(B) R is symmetric relation
(C) R is transitive relation
(D) R is equivalence relation

12. Consider 4 persons A, B, C, D

(A) If A, B, C, D each have four houses then number of ways in which they can enter these houses so that none of them enter their own house is **9**
(B) If A has two houses whereas B, C, D have one house each then number of ways in which they can enter these houses such that no one enters their own house is **44**
(C) If A, B, C, D each have four houses then number of ways in which they can enter these houses so that none of them enter their own house is **16**
(D) If A has two houses whereas B, C, D have one house each then number of ways in which they can enter these houses such that no one enters their own house is **42**

PART – B (Maximum Marks: 24)

(Numerical **Type)**

This section contains ***Eight (08)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

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1. If number of arrangements of letters of the word MISSISSIPPI in which the first S precedes the first I equals α, then $\frac{\alpha}{10}$ equals

2. For all real numbers x, $f(x)$ is an increasing function that is differentiable and satisfies the following conditions

(i) $f(1) = 1, \int_1^2 f(x)dx = \frac{5}{4}$

(ii) Let $g(x)$ be inverse of $f(x)$, then for all $x \geq 1, g(2x) = 2f(x)$.

If $\int_1^8 x f'(x)dx = \frac{p}{q}$; $HCF(p,q) = 1$, then $p + q =$

Space For Rough Work

3. The value of $\left[\dfrac{\sum\limits_{n=1}^{99}\sqrt{10+\sqrt{n}}}{\sum\limits_{n=1}^{99}\sqrt{10-\sqrt{n}}}\right]$ is equal to (here [.] represents greatest integer function) is equal to

4. If the length of the shortest distance between any two opposite edges of the tetrahedron formed by the planes whose equations are $y+z=0,\ z+x=0,\ x+y=0$ and $x+y+z=\sqrt{3}$ is λ, then the value of 2λ is equal to

5. f is differentiable function such that $f\left(f(x)\right)=x$ where $x\in[0,1]$. Also, $f(0)=1$. Then the value of $2024\int_0^1\left(x-f(x)\right)^{2023}dx$ is

6. Let positive vector of points A, B and C of triangle ΔABC respectively be $\hat{i}+\hat{j}+2\hat{k},\ \hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$. Let l_1, l_2 and l_3 be the lengths of perpendiculars drawn from the orthocenter 'O' on the sides AB, BC and CA, then $\left(l_1+l_2+l_3\right)$ equals.

7. Let $f(x)=\frac{1}{2}x\sin x-(1-\cos x)$. The smallest positive integer k such that $\lim\limits_{x\to 0}\frac{f(x)}{x^k}\neq 0$ is

8. Consider the real valued function $h:\{0,1,2,......,100\}\to R$ such that $h(0)=5,\ h(100)=20$ and satisfying $h(i)=\frac{1}{2}\left(h(i+1)+h(-1)\right)$ for every $i=1,2,.......,99$. Then the value of h(1) is

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Space For Rough Work

Q. P. Code: 100202

Answers

SECTION – I : PHYSICS

PART – A

1.	C	2.	B	3.	D	4.	B
5.	AC	6.	ABC	7.	BCD	8.	B
9.	BCD	10.	AC	11.	BC	12.	ABCD

PART – B

1.	7	2.	2	3.	7	4.	274.00
5.	4.58	6.	0.89	7.	1.50	8.	629.20

SECTION – II : CHEMISTRY

PART – A

1.	B	2.	D	3.	D	4.	B
5.	AD	6.	BC	7.	ACD	8.	ABD
9.	ABC	10.	BC	11.	ACD	12.	ABD

PART – B

1.	8	2.	2	3.	4	4.	1.4
5.	3.6	6.	1.1	7.	12.5	8.	214

SECTION – III : MATHEMATICS

PART – A

1.	D	2.	D	3.	B	4.	C
5.	AD	6.	CD	7.	BD	8.	BD
9.	BC	10.	BC	11.	ABCD	12.	AD

PART – B

1.	924	2.	143	3.	2	4.	2.82
5.	1	6.	1.22	7.	4.00	8.	5.15

Answers & Solutions

SECTION – I : PHYSICS

PART – A

1. **C**

Sol. $\frac{a}{u_x} + \frac{q}{eu_x} + \frac{a}{e^2u_x} = \frac{2u_y}{g}$ …(1)

Horizontal range:

$$R = \frac{2.u_xu_y}{g} \quad \text{…(2)}$$

2. **B**

Sol. $N_1 = \mu N_2$ …(1)

$W = \mu N_1 + N_2$ …(2)

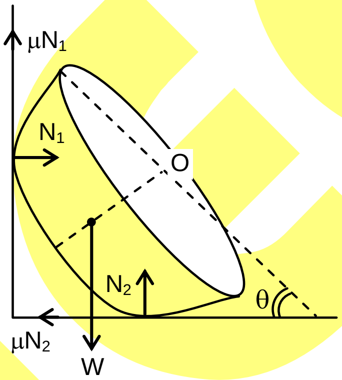
By (1) and (2)

$N_1 = \frac{\mu W}{1+\mu^2}$ …(2)

Balancing torque about 'O'

$\mu N_1 \times R + \mu N_2 \times R = W \times \frac{3R}{8} \sin\theta$

$\text{Sin }\theta = \frac{8}{3}\left[\frac{\mu+\mu^2}{1+\mu^2}\right]$



3. **D**

Sol. $\frac{dQ}{dt} = \frac{K(6a^2)(T-T_0)}{x} = n\left(\frac{R}{\gamma-1}\right)\frac{dT}{dt}$

$\int_{T_1}^{T} \frac{dT}{T-T_0} = \frac{6Ka^2(\gamma-1)}{nRx}\int_{o}^{t} dt$

$T = T_0 + (T_1 - T_0)e^{\frac{6Ka^2(\gamma-1)t}{nRx}}$

4. **B**

Sol. $\frac{q_1}{C_1} = \frac{q_2}{C_2}$ **;** $q_1 + q_2 = 2Q_0$

$C_1 = \frac{\varepsilon_0 A}{d_0 + vt}$; $C_2 = \frac{\varepsilon_0 A}{d_0 - vt}$

$\frac{q_1}{q_2} = \frac{d_0 - vt}{d_0 + vt}$

$q_2\left(\frac{d_0 - vt}{d_0 + vt}\right) + q_2 = 2Q_0$

$q_2\left[\frac{2d_0}{d_0 + vt}\right] = 2Q_0$

$q_2 = \frac{2Q_0}{2d_0}(d_0 + vt)$

$I = \frac{dq_2}{dt} = \frac{Q_0 v}{d_0}$

5. **AC**

Sol. Velocity of insect w.r.t. strip = $\frac{\ell}{t}$

Let strip moves with speed ν

Initial momentum was 0

$$\Rightarrow \quad 0 = m\left(\frac{\ell}{t} + v\right) + Mv$$

$$v = -\frac{m\ell / t}{M+m} \text{ i.e. } \frac{m\ell / t}{M+m} \text{ toward left}$$

$\therefore$ Velocity of insect w.r.t. ground = $\frac{\ell}{t} - \frac{m\ell / t}{M+m} < \frac{\ell}{t}$.

6. **ABC**

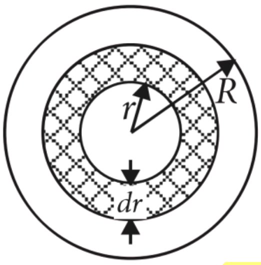
Sol. KE = hν – ϕ

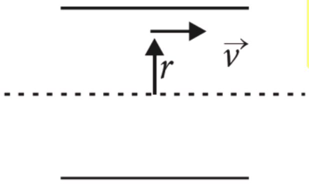
7. **BCD**

Sol. Let dV be the volume flowing per second through the cylindrical shell of thickness dr then,

$$dV = (2\pi r dr)v_0\left(1 - \frac{r^2}{R^2}\right) = 2\pi v_0\left(r - \frac{r^3}{R^2}\right)dr \text{ and the total volume,}$$

$$V = 2\pi v_0 \int_0^R \left(r - \frac{r^3}{R^2}\right)dr = 2\pi v_0 \frac{R^2}{4} = \frac{\pi R^2 v_0}{2}$$





Let, dK be the kinetic energy, within the above cylindrical shell. Then

$$dK = \frac{1}{2}(dm)v^2 = \frac{1}{2}(2\pi r \ell \rho)v^2 dr$$

$$= \frac{1}{2}(2\pi \ell \rho) r v_0^2\left(1 - \frac{r^2}{R^2}\right)dr$$

$$= \pi l \rho v_0^2\left[r - \frac{2r^3}{R^2} + \frac{r^5}{R^4}\right]dr$$

Hence, kinetic energy of the fluid,

$$K = \pi l \rho v_0^2 \int_0^R \left(r - \frac{2r^3}{R^2} + \frac{r^5}{R^4}\right)dr = \frac{\ell \pi R^2 \rho v_0^2}{6}$$

Hence frictional force is the shearing force on the tube, exerted by the fluid, which equals $-\eta S \frac{dv}{dr}$, where S is the surface area of tube.

Given, $v = v_0\left(1 - \frac{r^2}{R^2}\right)$ So $\frac{dv}{dr} = -2v_0 \frac{r}{R^2}$

and at $r = R, \frac{dv}{dr} = -\frac{2v_0}{R}$

Then, viscous force is given by,

$$dF = -\eta(2\pi R l)\left(\frac{dv}{dr}\right)_{r=R}$$

$$= -2\pi R \eta l\left(-\frac{2v_0}{R}\right) = 4\pi \eta l v_0$$

Taking a cylindrical shell of thickness dr and radius r Viscous force, $F = -\eta(2\pi r l)\frac{dv}{dr}$, Let, ΔP be the pressure difference, the net force on the element $= \Delta P \pi r^2 + 2\pi\eta l r \frac{dv}{dr}$

But, since the flow is steady, $F_{net} = 0$

$$\text{or } \Delta P = \frac{-2\pi l \eta r \frac{dv}{dr}}{\pi r^2} = \frac{2\pi l \eta r \left(2v_0 \frac{r}{R^2}\right)}{\pi r^2} = \frac{4\eta l v_0}{R^2}$$

8. **B**

Sol. Let q be the instantaneous charge on the capacitor when a steady current I flows through the circuit. Appling KVL on the circuit, we have

$$\xi = \frac{q}{C} + IR \text{ or } \xi = \frac{qx}{\varepsilon_0 A} + IR \left(\because C = \frac{\varepsilon_0 A}{x}\right) \quad .....(i)$$

Differentiating eqn. (i) with respect to time, we get

$$0 = \frac{q}{\varepsilon_0 A}\left(\frac{dx}{dt}\right) + \frac{Ix}{\varepsilon_0 A} + 0 \qquad (\because q = It)$$

$$\text{or } q = -\frac{Ix}{(dx/dt)} \left(\text{where } \frac{dx}{dt} = \text{velocity}\right) \quad .....(ii)$$

From equationn (ii) substituting the value of q in equation (i). we have

$$\xi = -I\frac{x^2}{\varepsilon_0 A(dx/dt)} + IR$$

$$\text{or } \frac{dx}{dt} = v = \left(\frac{-I/\varepsilon_0 A}{\xi - IR}\right)x^2 \quad ....(iii)$$

$$\text{or } -\frac{dx}{x^2} = \left(\frac{I/\varepsilon_0 A}{\xi - IR}\right)dt \quad ....(iv)$$

Integrating the above expression w.r.t. time, we get

$$\frac{1}{x} - \frac{1}{x_0} = \left(\frac{I/\varepsilon_0 A}{\xi - IR}\right)t \quad ....(v)$$

From eqn (iii) and (v), we get

$$v = \left(\frac{I/\varepsilon_0 A}{IR - \xi}\right)\left[\left(\frac{I/\varepsilon_0 A}{\xi - IR}\right)t + \frac{1}{x_0}\right]^{-2}$$

9. **BCD**

Sol. Apply Faraday law for direction of induced current.

10. **AC**

Sol. Flux linked with loop due to its own magnetic field,

$$\phi = \frac{\mu_0 I}{2r}(\pi r^2) = \frac{\mu_0 \pi r I}{2}$$

emf induced $= \frac{d\phi}{dt} = \varepsilon = -\frac{\mu_0 \pi r}{2}\frac{dI}{dt}$

$$I = \frac{\varepsilon}{R} = -\frac{\mu_0 \pi r}{2R}\frac{dI}{dt}$$

$$\int_{I_0}^{1}\frac{dI}{I} = -\int_0^t \frac{2R}{\mu_0 \pi r}.dt; I = I_0 e^{-2Rt/\mu_0 \pi r}$$

Now, $10^{-3} I_0 = I_0 e^{-\frac{2Rt}{\mu_0 \pi r}} = i_0 e^{-2Rt/\mu_0 \pi r}$

which gives $t = \frac{3\pi^2 \ln 10}{10^{10}}$s.

11. **BC**

$v_L = v_C = v_R$;

$\Rightarrow x_L = x_C = R$

when inductor is short circuited

$Z = \sqrt{R^2 + x_C^2} = \sqrt{2}R$

$\therefore I = \frac{30}{Z} = \frac{30}{\sqrt{2}R}$

$\therefore V_L = ix_L = \frac{30}{\sqrt{2}R} \times R = \frac{30}{\sqrt{2}}$

$\therefore$ (A) is incorrect and with similar calculations (B) will be correct.

Here f_0 is the resonance frequency as $v_L = v_C$

$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$

$\frac{x_L}{x_C} = \frac{\omega L}{1/\omega C} = \omega^2 LC$

Given $f = 2f_0$

$\Rightarrow \omega = 2\omega_0$

$\therefore \frac{x_L}{x_C} = 4$

$\therefore$ (C) is also correct.

12. **ABCD**

Sol. $y = 2a \cos(\omega t + \phi)\cos(Kx)$ comparison with given equation gives

$$K = \frac{2\pi}{\lambda} = 10\pi \Rightarrow \lambda = \frac{1}{5}m$$

$\omega = 2\pi f = 50\pi \Rightarrow f = 25$ Hz

$\therefore$ $v = 5\ ms^{-1}$

at x = 0.15 m

$\cos(10\pi \times 0.15) = \cos(1.5\pi)$

= –1 for all t

at x = 0.3

$\cos(10\pi \times 0.3) = \cos 3\pi = -0$ for all t

PART – B

1. **7**

Sol. For equilibrium of bubble.

$$4\pi R^2 d\rho_{sol}g + \frac{4}{3}\pi R^3 \rho_{He}g = \frac{4}{3}\pi R^3 \rho_{air}g$$

$d \rightarrow$ Thickness of soap solution.

$\therefore d = 3.5\ \mu m.$

2. **2**

Sol. Use formula in terms of reduced mass

$$\frac{1}{\lambda} = \frac{R_\infty z^2}{\left(1+\frac{m}{M}\right)}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore \frac{\lambda_D}{\lambda_H} = \frac{1+\frac{m}{M_D}}{1+\frac{m}{M_H}}$$

$$\frac{M_D}{M_H} \approx 2.0$$

3. **7**

Sol. For process AB $T_A = 300\text{ K},\ T_B = 600\text{ K}$

$$W = nR\Delta T = nR(T_B - T_A) = 300\ nR\ = 600R.$$

$$Q = n\ Cp\,\Delta T = 2\times\frac{5}{2}R\ (300) = 1500R.$$

For process BC $W = nRT\ln\frac{v_f}{v_c} = nRT\ln\frac{p_i}{p_f}\ = nRT\ln 2\ = 1200R\ln 2$

$$Q = W = 1200R\ln 2$$

For process CA $W = \int P\ dV\ = \int_{600}^{300}\frac{K}{T}\,\frac{2nRT}{K}\,dT.$

$$= -2nR(300) = -1200R.$$

$$Q = nC_V\ \Delta\ T + W$$

$$= 2\times\frac{3}{2}R(-300) - 1200R.$$

$$= -900R - 1200R = -2100R$$

$$\eta = \frac{600R + 1200R\ln 2 - 1200R}{1500R + 1200R\ln 2}$$

$$= 1 - \frac{21}{12\ln 2 + 15} \quad \Rightarrow\ x = 7.$$

4. **274.00**

Sol. $W = \frac{4}{3}\pi R^3\rho g\left[h - R + \frac{x}{2}\right] = \frac{4}{3}\pi R^3\rho g\left[h - R + \frac{\frac{4}{3}\pi R^3}{2A}\right]$

5. **4.58**

Sol. h = (R – r) [cos 30° – cos 60°]

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\omega^2$$

$$x = \frac{v^2\sin^2 30^o}{2g}$$

6. **0.89**

Sol. $2g - T = 2a$ …(i)

$TR = I\alpha$ …(ii)

$a = R\alpha$ …(iii)

From (ii) and (iii) $T = \frac{Ia}{R^2}$

$\therefore\quad 2g = a\left(2 + \frac{I}{R^2}\right)$

$\Rightarrow\quad a = \frac{2g}{\left(2 + \frac{I}{R^2}\right)} = \frac{2\times 9.8}{2 + \frac{0.2}{0.01}} = \frac{9.8}{11} \approx 0.89\ m/s^2$

7. **1.50**

Sol. $i_1 = \frac{E}{\frac{r}{m} + \frac{R}{n}}$ & $i_2 = \frac{E}{\frac{r}{n} + \frac{R}{m}}$

$$\frac{i_2}{i_1} = \frac{\frac{r}{m} + \frac{R}{n}}{\frac{r}{n} + \frac{R}{m}} = \frac{1 + \frac{m}{n}\cdot\frac{R}{r}}{\frac{m}{n} + \frac{R}{r}} = \frac{3}{2} = 1.5$$

8. **629.20**

Sol. $X_L = 50\ \Omega$, $X_C = 100\ \Omega$

$Z_1 = 100 - 50\ I$; $Z_2 = 100 + 100\ i\ (i = \sqrt{-1})$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{100}{17}(13 + i)$$

$$|Z| = \frac{100}{17}\sqrt{170}\ ;\ \cos\theta = \frac{13}{\sqrt{170}}$$

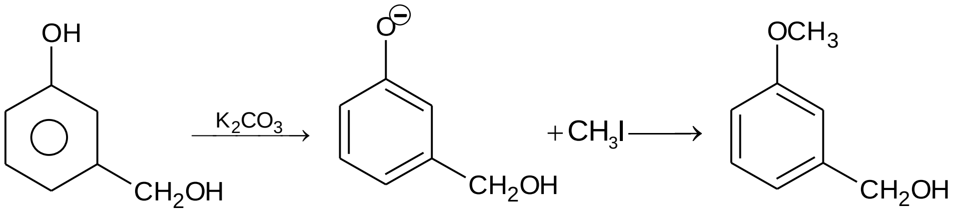
$$P = \frac{V^2}{|Z|}.\cos\phi\ = 629.20$$

SECTION – II : CHEMISTRY

PART – A

1. B

Sol.



2. D

Sol. P, Q and R are formed by EAS reaction M is formed by free radical chlorination.

3. D

Sol. Electron withdrawing groups favour nucleophilic substitution when they are ortho and para to leaving group.

4. B

Sol. P = $CH_3CH_2NH_2$, Q = CH_3CH_2CN, R = Not formed, S = $CH_3CH_2CH_2NH_2$.

5. AD

Sol. $AlCl_3 + 3H_2O \xrightarrow{\text{Hydrolysis}} Al(OH)_3 + 3HCl$

B^{3+} cannot exist in aqueous media

6. BC

Sol. $\underset{(s)}{2C} + \underset{(g)}{3H_2} \rightarrow \underset{(g)}{C_2H_6}$

$\Delta H_f^0 = \left[2\times\Delta H^0_{sub} + 3\times B.E(H-H)\right] - \left[B.E.(C-C) + 6\times B.E(C-H)\right]$

$\Rightarrow -85 = \left[(2\times718) + (3\times436)\right] - (x+6y)$

$\therefore x + 6y = 2829$ …..1

Similarly for $C_3H_8(g)$

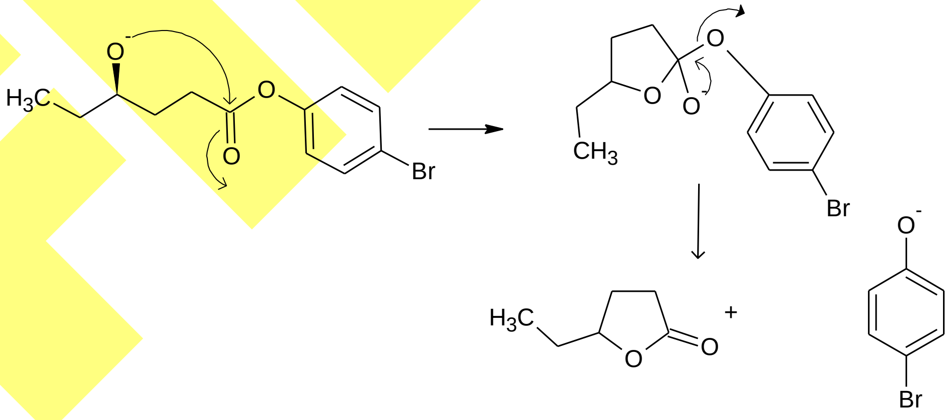
$2x + 8y = 4002$ …..2

Solving (1) & (2), x = 345

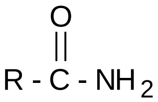
y = 414

7. ACD

Sol.



8. ABD



Sol. $R-\overset{\overset{\Large O}{||}}{C}-NH_2$ has least +ve charge density among all of the given compound.

9. ABC

Sol. Bridge bonding

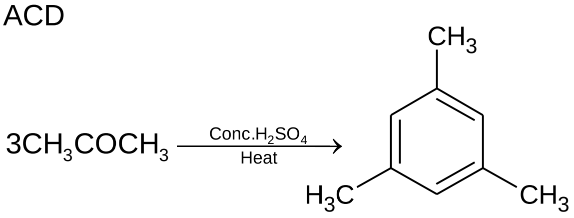
$Al_2Cl_6 \Rightarrow 3c-4e^-$ Bond

10. BC

Sol. Original carbonyl compound can be generated from product by weak acid or base treatment.

11. ACD

Sol.



12. ABD

Sol. Dehydrogenation reaction takes place.

PART – B

1. 8

Sol. $P_T = p_A^0 x_A + p_B^0 x_B = 100 \times \left(\frac{4}{6+4}\right) + 400 \times \left(\frac{6}{6+4}\right)$ = 280 mm of Hg

$p_A = y_A P_T$

or, $p_A^0 x_A = y_A P_T$ or, $100 \times 0.4 = y_A \times 280$

or, $y_A = \frac{40}{280} = \frac{1}{7}$

$\therefore \frac{x}{y} = \frac{1}{7}$

so, (x + y) = 8

2. 2

Sol. $2O^{2-} \longrightarrow O_2 + 4e^-$

One mole of O_2 produced by passing 4F electricity.

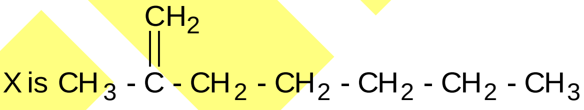
$\therefore$ Two moles of O_2 will be produced by passing 8F electricity.

3. 4

Sol. The oxidation number of N changes from -3 to +1 and +5 to +1.

4. 1.4

Sol. x = 2, y = 7, z = 5



$$\text{X is } CH_3-\overset{\overset{\displaystyle CH_2}{\|}}{C}-CH_2-CH_2-CH_2-CH_2-CH_3$$

5. 3.6

Sol. $\frac{p^o - p}{p^o} = iX_{solute} = (1+\alpha)\left(\frac{0.2}{10}\right)$

= (1 + 0.8) (0.02) = 0.036

$\therefore x \times 10^{-2} = 0.036 = 3.6 \times 10^{-2}$

$\therefore$ x = 3.6

6. 1.1

Sol. Formula of solid $M_3N_2O_6$

$\therefore \frac{x+y+z}{10} = \frac{3+2+6}{10} = 1.1$

7. 12.5

Sol. π of $Al_2(SO_4)_3$ = π of glucose

or, $iC_1RT = C_2RT$

or, $iC_1 = C_2$

or, $5 \times 0.5 = 0.2$

Ratio = $\frac{5 \times 0.5}{0.2} = 12.5$

8. 214

Sol. $pH = \frac{1}{2}\left[p^{k_w} - p^{k_b} - \log C\right] = \frac{1}{2}\left[14 - 5 - \log C\right] = 5$

or logC = -1 or C = $10^{-1} = 0.1$ M

Mass of 0.1 mole of salt = 5.35 g

Mass of one mole of salt = 53.5 g

NaCl unit cell contains 4 units of NaCl

The unit cell of the salt contains 4 units of the salt

Mass of one mole of unit cell = 53.5 × 4 = 214

SECTION – III : MATHEMATICS

PART – A

1. D

Sol. Let the vertex he (h, k).

So, 'S' can be assumed as $(y-k)^2 = -4a(x-h)$

Solving with $y^2 = 4ax$

$y^2 - 2ky + k^2 = -y^2 + 4ah$

$\Rightarrow 2y^2 - 2ky + k^2 - 4ah = 0$

Disc. = 0 ($\because$ the parabolas touch each other)

$4k^2 - 8\left(k^2 - 4ah\right) = 0$

$-k^2 + 8ah = 0$

$\Rightarrow y^2 = 8ax$

2. D

Sol. $y(0) = y'(0) = y''(0) = 0$

$y'''(0) = -1$

$\Rightarrow y'''(x)$ is negative in the neighbourhood of $x = 0$

$\Rightarrow y''(x)$ is a decreasing function in the neighbourhood of x = 0.

$\Rightarrow$ For $x > 0$	For $x < 0$
$y''(x) < y''(0)$	$y''(x) > y''(0)$
$\Rightarrow y''(x) < 0$	$y''(x) > 0$

Clearly, the concavity of curve changes from positive to negative across x = 0

So $x = 0$ is point of inflection

So the answer should therefore be (D).

Also, by n – th derivative test if $f^n(c) \neq 0$ (n is odd) then $x = c$ is neither point of maxima nor minima.

3. B

Sol. $f'\left(\frac{1}{x}\right)f(x) = 1$

Replace x by $\frac{1}{x}$, we get $f'(x)f\left(\frac{1}{x}\right) = 1$

Differentiating $f''(x)f\left(\frac{1}{x}\right) + f'(x)f'\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) = 0$

$$\frac{f''(x)}{f'(x)} - \frac{f'(x)}{f(x)}.\frac{1}{x^2} = 0$$

$$\Rightarrow \int \frac{f''(x)f(x)}{\left(f'(x)\right)^2} f(x)dx = \int \frac{1}{x^2}dx$$

$$-\frac{1}{f'(x)}f(x) - \int\left(-\frac{1}{f'(x)}\right)f'(x)dx = -\frac{1}{x} + c$$

$$-\frac{f(x)}{f'(x)} + x = -\frac{1}{x} + c$$

Put $x = 1 \Rightarrow -\frac{f(1)}{f(1)} + 1 = -1 + c \Rightarrow c = 2 - \frac{\sqrt{2}}{\frac{1}{\sqrt{2}}} = 2 - 2 = 0$

$$\Rightarrow \frac{f(x)}{f'(x)} = x + \frac{1}{x}$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{x}{x^2+1} \Rightarrow \ell n(f(x)) = \frac{1}{2}\ell n(1+x^2) + c'$$

$$\ell n\sqrt{2} = \frac{1}{2}\ell n 2 + c' \Rightarrow c' = 0$$

$$\Rightarrow f(x) = \sqrt{1+x^2} \Rightarrow f(3) = \sqrt{10}$$

4. C

Sol. $f'(x) = 3x^2 + 2x - a\sin x;\ f''(x) = 6x + 2 - a\cos x$

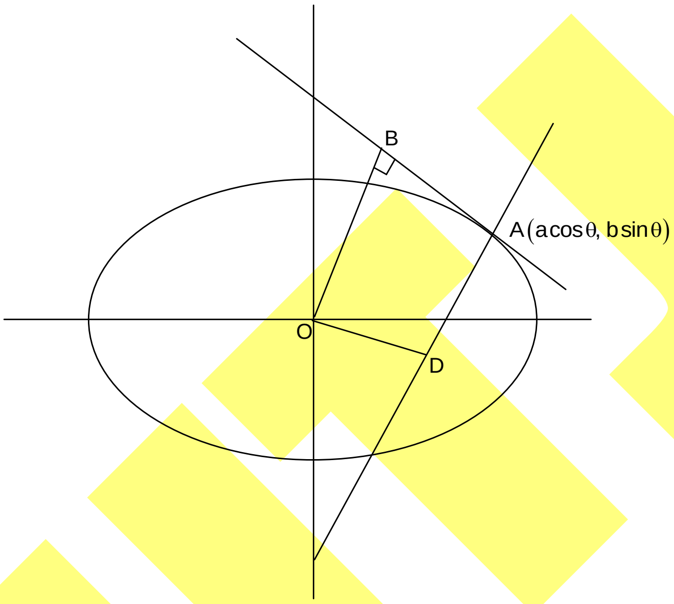
Clearly $f'(0) = 0$

So, for f to be injective $f'(x)$ must attain minima at $x = 0$

$\Rightarrow f''(0) = 0 \Rightarrow 6(0) + 2 - a = 0 \Rightarrow a = 2$

5. AD

Sol.



AD is normal to ellipse at A $\Rightarrow$ Equation of AD is $a\sec\theta - by\,\text{cosec}\,\theta = a^2 - b^2$

$$\Rightarrow AB = OD = \frac{a^2-b^2}{\sqrt{a^2\sec^2\theta + b^2\,\text{cosec}^2\theta}} = \frac{a^2-b^2}{\sqrt{(a+b)^2 + (a\tan\theta - b\cot\theta)^2}} \le \frac{a^2-b^2}{a+b}$$

$\Rightarrow AB \le a - b \Rightarrow AB \le 4 - 2 \Rightarrow AB \le 2 \Rightarrow \ell = 2$

$\ell = 2$ happens when $a\tan\theta - b\cot\theta = 0 \Rightarrow \tan^2\theta = \frac{b}{a} = \frac{1}{2} \Rightarrow \tan\theta = \frac{1}{\sqrt{2}}$

Slope of tangent at A $= -\frac{\cot\theta}{2} = -\frac{1}{\sqrt{2}}$

6. CD

Sol. Since P is a probability distribution, we have $\sum_{n=1}^{\infty} P(w_n) = 1$

$$P(w_n) = \frac{1}{2}(w_{n-1}) = \frac{1}{2^2}P(w_{n-2}) = \ldots = \frac{1}{2^{n-1}}P(w_1)$$

$$\sum_{n=1}^{\infty} P(w_n) = 1 \Rightarrow \sum_{n=1}^{\infty}\frac{1}{2^{n-1}}P(w_1) = 1 \Rightarrow P(w_1)\sum_{n=1}^{\alpha}\frac{1}{2^{n-1}} = 1 \Rightarrow P(w_1)(2) = 1$$

So, $P(w_1)=\dfrac{1}{2}$

Clearly, the set A contains all natural numbers greater than 4 (excluding 6)

So, $P(B)=1-\{P(w_1)+P(w_2)+P(w_3)+P(w_4)+P(w_6)\}$

$$=1-\left\{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{64}\right\}=\frac{3}{64}$$

7. BD

Sol. $|2z+3|^2=|3z+4|^2\Rightarrow 4|z|^2+9+6(z+\bar{z})=9|z|^2+16+12(z+\bar{z})$

$\Rightarrow 5|z|^2+7=-6(z+\bar{z})=-12\text{Re}(z)\le 12|z|$

$\Rightarrow 5|z|^2-12|z|+7\le 0\Rightarrow (5|z|-7)(|z|-1)\le 0\Rightarrow 1\le |z|\le \dfrac{7}{5}$

8. BD

Sol.

(B) If $x\in f^{-1}(Q)\cup f^{-1}(R)\Rightarrow x\in f^{-1}(Q)$ OR $x\in f^{-1}(R)\Rightarrow f(x)\in Q$ OR $f(x)\in R$

$\Rightarrow f(x)\in Q\cup R\Rightarrow x\in f^{-1}(Q\cup R)$

If $x\in f^{-1}(Q\cup R)\Rightarrow f(x)\in Q$ or $f(x)\in R\Rightarrow x\in f^{-1}(Q)$ or $x\in f^{-1}(R)\Rightarrow x\in \left(f^{-1}(Q)\cup f^{-1}(R)\right)$

$\Rightarrow f^{-1}(Q)\cup f^{-1}(R)=f^{-1}(Q\cup R)$

(D) If $x\in f^{-1}(Q)\cap f^{-1}(R)$ then $x\in f^{-1}(Q)$ and $x\in f^{-1}(R)\Rightarrow f(x)\in Q$ and $f(x)\in R$.

$\Rightarrow f(x)\in Q\cap R\Rightarrow x\in f^{-1}(Q\cap R)$

If $x\in f^{-1}(Q\cap R)$ then $f(x)\in Q\cap R\Rightarrow f(x)\in Q$ and $f(x)\in R\Rightarrow x\in f^{-1}(Q)$ and $x\in f^{-1}(R)$

$\Rightarrow x\in \left(f^{-1}Q\right)\cap f^{-1}(R)$

$\Rightarrow f^{-1}(Q)\cap f^{-1}(R)=f^{-1}(Q\cap R)$

(A) Consider $f:\{1,2,3\}\to\{1,2,3\}$
where $f(1)=f(2)=f(3)=1$

If $y\in f\left(f^{-1}(Q)\right)$
the $f^{-1}(\gamma)\in f^{-1}(Q)$
not possible for $y=2,3$

If $y\in Q$
then $f^{-1}(\gamma)$ is not possible
since f is not surjective

(C) Same counter example as in A can be used

9. BC

Sol. If $(gof)^{-1}$ exists then gof must be one – one and onto

gof is one – one ⇒ f is one – one

gof is onto ⇒ g is onto

Consequently, since $(gof)^{-1}$ exists, then gof is one – one and onto, hence f is one – one and g is onto

Claim 1: gof if one – one ⇒ f is one – one

Proof Let a_1 and a_2 be two elements of the set A.

$f(a_1)=f(a_2)\Rightarrow gof(a_1)=gof(a_2)$ since gof is one – one we get that $a_1=a_2$

So, we have proved that $f(a_1)=f(a_2)$ implies f is one – one

Claim 2 : gof is ONTO ⇒ g is ONTO

Proof Since gof is onto, it follows that for any $c\in C$, there exists $a\in A$ such that $gof(a)=c$

Consequently, for any $c\in C$ there exists $b=f(a)\in B$ such that $g(b)=gof(a)=c$

Hence g is ONTO

10. BC

Sol. Let $g(x)=\frac{f'(x)}{(f(x))^2} \Rightarrow \int g(x)dx=-\frac{1}{f(x)}+c$

$$\Rightarrow \int_4^8 g(x)dx=-\frac{1}{f(8)}+\frac{1}{f(4)}=-2+4=2$$

Also, $\int_4^8 (g(x))^2 dx=1$

$$\Rightarrow \int_4^8 \left(g(x)-\frac{1}{2}\right)^2=0$$

$$\Rightarrow g(x)=\frac{1}{2}$$

$$\Rightarrow \frac{f'(x)}{(f(x))^2}=\frac{1}{2}$$

$$\Rightarrow -\frac{1}{f(x)}=\frac{x}{2}+d$$

$\Rightarrow d=-6$ using $f(4)=\frac{1}{4}$

$$\Rightarrow f(x)=\frac{2}{12-x}$$

$$\Rightarrow f(6)=\frac{1}{3}$$

11. ABCD

Sol. $z_1 R z_1 \Rightarrow \frac{z_1-z_2}{z_1+z_2}=0=$ a real number $\Rightarrow R$ is reflexive

If $z_1 R z_2$ holds i.e. $\frac{z_1-z_2}{z_1+z_2}$ is real then clearly $\frac{z_2-z_1}{z_2+z_1}=-\left(\frac{z_1-z_2}{z_1+z_2}\right)$ is also real

$\Rightarrow$ R is symmetric

If $\frac{z_1-z_2}{z_1+z_2}$ is real i.e. $\frac{z_1-z_2}{z_1+z_2}=\frac{\bar{z}_1-\bar{z}_2}{\bar{z}_1+\bar{z}_2}$

$$z_1\bar{z}_1+z_1\bar{z}_2-\bar{z}_1 z_2-z_2\bar{z}_2=z_1\bar{z}_1-z_1\bar{z}_2+\bar{z}_1 z_2-z_2\bar{z}_2$$

$$z_1\bar{z}=\bar{z}_1 z_2$$

$$\Rightarrow \frac{z_1}{\bar{z}_1}=\frac{z_2}{\bar{z}_2}$$

So, if $\frac{z_2-z_3}{z_2+z_3}$ is real, in same manner we have $\frac{z_2}{\bar{z}_2}=\frac{z_3}{\bar{z}_3}$

$\Rightarrow \frac{z_1}{\bar{z}_1}=\frac{z_2}{\bar{z}_2}=\frac{z_3}{\bar{z}_3} \Rightarrow R$ is transitive.

12. AD

Sol. Derangement of 4 persons in 4 houses $=D_4=9$ ways. Hence option A.

If A has two houses. Then let there be a hypothetical person E. So that A, B, C, D, E have one house each now.

Among the total derangements $(D_5 = 44)$. A is equally likely to go to B, C, D, E houses. So A does not go to E's house in $\frac{1}{4}D_5$ ways.

$\therefore$ Number of ways $= D_5 - \frac{1}{4}D_5 = 44 - \frac{1}{4}(44) = 33$ ways

But it is OK, for E to go to A'C house, it means one of A's houses is occupied.

So, number of derangements in this case $= D_4 = 9$

$\therefore$ Required number of ways $= 33 + 9 = 42$ ways

PART – B

1. 924

Sol. Consider the first S, I, P

* S * I * P *

The second P can only be placed in the last gap and hence we have * S * I * P *

The three I's can be placed only in the last three gaps.

Choosing the places for I's is put 3 identical object into 3 boxes i.e.

${}^{3+3-1}C_{3-1} = {}^5C_2 = 10$ ways $(x_1 + x_2 + x_3 = 3)$

We have so far placed 7 letters. We have to put 3S's in the gaps created by these. However, for placing the 3 – S's, we cannot choose the gap before the first S and hence we have 7 – gaps to choose from

This can be done in ${}^{7+3-1}C_{2-1} = {}^9C_6 = 84$ ways.

Only, the letter M remains to be placed. M can go in any of these 11 – gaps.

$\Rightarrow$ Required number of ways $= 10 \times 84 \times 11 = 9240$

2. 143

Sol. $f(2) = g(2), f(4) = g(4), f(8) = g(8)$

$$\int_1^8 x f'(x) = x f(x)\Big|_1^8 - \int_1^8 f(x)dx = 8f(8) - f(1) = 63 - \int_1^8 f(x)dx$$

$$\int_1^2 f(x)dx = \int_1^2 \frac{g(2x)}{2}dx = \int_2^4 \frac{g(t)}{4}dt \quad (t = 2x)$$

$$\int_1^2 f(x)dx = \frac{5}{4} \Rightarrow \frac{5}{4} = \frac{1}{4}\int_2^4 g(t)dt \Rightarrow \int_2^4 g(x)dx = 5$$

$$\int_2^4 f(x)dx + \int_2^4 g(x)dx = 16 - 4 = 12 \Rightarrow \int_2^4 f(x)dx = 7$$

$$\int_2^4 \frac{g(2x)}{2}dx = 7 \Rightarrow \int_2^4 \frac{g(t)}{4}dt = 7 \Rightarrow \int_4^8 g(x)dx = 28$$

$$\Rightarrow \int_4^8 f(x)dx + \int_4^8 g(x)dx = 64 - 16 = 48$$

$$\Rightarrow \int_4^8 f(x)dx = 48 - 28 = 20$$

$$\Rightarrow \int_1^8 x f'(x)dx = 63 - \left[\int_1^2 f(x)dx + \int_2^4 f(x)dx + \int_4^8 f(x)dx\right]$$

$$= 63 - \left[\frac{5}{4} + 7 + 20\right] = \frac{139}{4} = \frac{p}{q}$$

$\Rightarrow p+q=143$

3. 2

Sol. $A=\sqrt{10+\sqrt{1}}+\sqrt{10+\sqrt{2}}+......+\sqrt{10+\sqrt{99}}$

$B=\sqrt{10-\sqrt{1}}+\sqrt{10-\sqrt{2}}+......+\sqrt{10-\sqrt{99}}$

$$\frac{A}{B}=1+\frac{A-B}{B}$$

$$A-B=\sum_{l=1}^{99}\left(\sqrt{10+\sqrt{i}}-\sqrt{10-\sqrt{i}}\right)$$

Consider, for each i

$$\left(\sqrt{10+\sqrt{i}}-\sqrt{10-\sqrt{i}}\right)^2=10+\sqrt{i}+10-\sqrt{i}-2\sqrt{10^2-i}$$

$$=20-2\sqrt{100-i}$$

$$\therefore\sqrt{10+\sqrt{i}}-\sqrt{10-\sqrt{i}}=\sqrt{20-2\sqrt{100-i}}=\sqrt{2}\sqrt{10-\sqrt{100-i}}$$

$$A-B=\sum_{i=1}^{99}\left(\sqrt{10+\sqrt{i}}-\sqrt{10-\sqrt{i}}\right)=\sqrt{2}\sum_{i=1}^{99}\sqrt{10-\sqrt{100-i}}=\sqrt{2}\sum_{i=1}^{99}\sqrt{10-\sqrt{i}}=B\sqrt{2}$$

$$\frac{A-B}{B}=\sqrt{2}$$

4. 2.82

Sol. The equation of two edges can be taken to be

$$\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-0}{-1}\text{ and }\frac{x-0}{1}=\frac{y-0}{-1}=\frac{z-\sqrt{3}}{0}$$

The shortest distance between them is $\sqrt{2}$.

5. 1

Sol. $x=f(t)\Rightarrow dx=f'(t)dt$

$$I=\int_1^0\left(f(t)-t\right)^{2023}f'(t)dt$$

$$=-\int_0^1\left(t-f(t)\right)^{2023}f'(t)dt$$

$$=-\int_0^1\left(t-f(t)\right)^{2023}\left(-1+f'(t)+1\right)dt$$

$$=+\int_0^1\left(t-f(t)\right)^{2023}\left(1-f'(t)\right)dt-I$$

$$2I=\int_0^1\left(t-f(t)\right)^{2023}\left(1-f'(t)\right)dt=\left.\frac{\left(f-f(t)\right)^{2024}}{2024}\right|_0^1=\frac{2}{2024}$$

6. 1.22

Sol. Clearly, triangle formed by the given points $\hat{i}+\hat{j}+2\hat{k},\ \hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ is equilateral as $AB=BC=AC=\sqrt{2}$.

$\therefore$ Distance of orthcentre ‘O’ from the sides is equal to inradius of the triangle.

$$\therefore l_1 = l_2 = l_3 = \text{inradius } = r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}}{4}\left(\sqrt{2}\right)^2}{\frac{4}{2}\left(\sqrt{2}\right)} = \frac{1}{\sqrt{6}}$$

$$\Rightarrow \qquad \left(l_1 + l_2 + l_3\right) = \frac{3}{\sqrt{6}} = \frac{\sqrt{6}}{2}$$

7. 4.00

Sol. $$\lim_{x\to 0}\frac{f(x)}{x^k} = \lim_{x\to 0}\frac{\frac{1}{2}x\sin x - 1 + \cos x}{x^k}$$

$$= \lim_{x\to 0}\frac{\frac{1}{2}\sin x + \frac{1}{2}x\cos x - \sin x}{k\,x^{k-1}}$$

$$= \lim_{x\to 0}\frac{\frac{1}{2}\cos x + \frac{1}{2}\cos x - \frac{1}{2}\times\sin x - \cos x}{k(k-1)x^{k-2}}$$

$$= \lim_{x\to 0}\frac{-\frac{1}{2}\sin x}{k(k-1)x^{\kappa-3}}$$

If $k-3=1$ i.e. $k=4$, $\lim_{x\to 0}\frac{f(x)}{x^k} = \frac{-\frac{1}{2}}{4\times 3} = \frac{-1}{24}$

8. 5.15

Sol. Clearly $h(i)$ is an A.P.

$h(100) = h(0) + 100d \Rightarrow 100d = 20 - 5 = 15 \Rightarrow d = 0.15$

$\Rightarrow h(1) = 5.15$