

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 2

QP Code: 100192

PAPER - 2

Time Allotted: 3 Hours

Maximum Marks: 195

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections**.
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. All the section can be filled in **PART-A** of OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with **Blue/Black Ball Point Pen** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For Only One Part.

- (i) **Part-A (01-07)** – Contains seven (07) multiple choice questions which have **One or More** correct answer.
Full Marks: +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Partial Marks: +1 For darkening a bubble corresponding to **each correct option**, provided NO incorrect option is darkened.
Zero Marks: 0 If none of the bubbles is darkened.
Negative Marks: –2 In all other cases.
For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only (A) and (D) will result in **+2 marks**; and darkening (A) and (B) will result in **–2 marks**, as a wrong option is also darkened.
- (ii) **Part-A (08-14)** – Contains seven (07) multiple choice questions which have **ONLY ONE CORRECT** answer. Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.
- (iii) **Part-A (15-18)** - This section contains Two paragraphs. Based on each paragraph, there are Two multiple choice questions. Each question has only one correct answer and carries **+4 marks** for the correct answer and **– 2 marks** for wrong answer.

Name of the Candidate : _____

Batch : _____ Date of Examination : _____

Enrolment Number : _____

BATCHES – Two Yr CRP2123(AII)

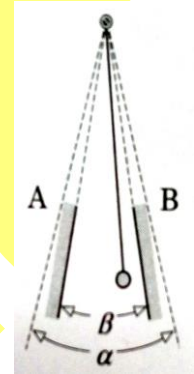
SECTION-1 : PHYSICS

PART – A

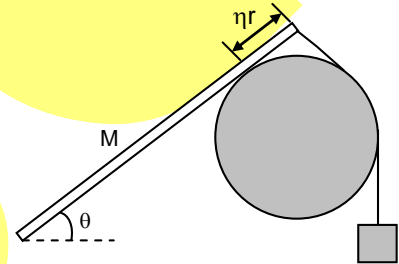
(Multi Correct Choice Type)

This section contains 7 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. A simple pendulum initially oscillating simple harmonically with angular amplitude α and period T_0 is symmetrically confined between two rigid fixed planes A and B making angle $\beta < \alpha$ with each other as shown in the figure.
- (A) If collisions at both the walls are elastic, period is $T_0(1 - \beta/\alpha)$.
- (B) If collisions at both the walls are inelastic, period is T_0 .
- (C) If collision at one wall is elastic and at the other is inelastic, the period is T_0 .
- (D) If collision at one wall is elastic and at the other is inelastic, the period is less than T_0 .



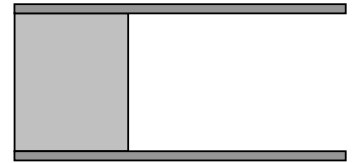
2. A load is suspended from one end of a uniform rod of mass $m = 10 \text{ kg}$ and length ℓ with the help of a light inextensible thin cord and the arrangement is placed on a fixed horizontal frictionless cylinder of radius r as shown in the figure. In equilibrium, the rod stays at angle $\theta = 30^\circ$ with the horizontal and distance between point of contact of the rod with the cylinder and top end of the rod is $\eta = 1/\sqrt{2}$ times the radius of the cylinder. Then



- (A) ratio of length of the rod to the radius of the cylinder = $\frac{4 + \sqrt{6}}{\sqrt{3}}$
- (B) ratio of length of the rod to the radius of the cylinder = $\frac{4 + \sqrt{3}}{\sqrt{6}}$
- (C) mass of the load = 15 kg
- (D) mass of the load = 10 kg

Space For Rough Work

3. A parallel plate capacitor of capacitance C_0 is charged to a voltage V and then the battery is disconnected. A dielectric covering one-third area of each plate is now inserted as shown in the figure. If charges on the capacitor plates get redistributed such that the portions covered with dielectric and not covered with the dielectric share equal amounts of charge, which of the following statements is/are true?



- (A) Dielectric constant of the dielectric is 2.0.
 (B) Charge appearing due to polarization on the surface of the dielectric is $0.25 C_0 V$.
 (C) Force of electrostatic interaction between portions of the plates covered with dielectric is equal to that between uncovered portions.
 (D) Force of electrostatic interaction between the plates after insertion of the dielectric becomes $9/8$ times of its value before insertion of the dielectric.

4. Four waves of the equation given by

$$y_1 = 5A \sin (wt - Kx + \pi/2)$$

$$y_2 = 2A \sin (wt - Kx + 3\pi/2)$$

$$y_3 = 6A \sin (wt - Kx)$$

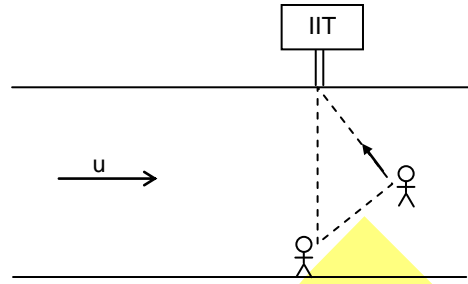
$$\text{and } y_4 = 2A \sin (wt - Kx + \pi)$$

- (A) The resulted wave will have amplitude $5A$.
 (B) It will make a phase difference of 37° with y_3 .
 (C) Its equation will be $y = 5A \sin (wt + 37^\circ)$.
 (D) Its equation will be $y = 5A \sin (wt - Kx + 37^\circ)$.

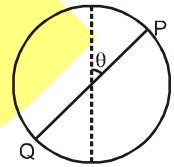
5. During an experiment, an ideal gas is found to obey a condition $\frac{P^2}{\rho} = \text{constant}$ [ρ = density of the gas]. The gas is initially at temperature T , pressure P and density ρ . The gas expands such that density changes to $\rho/2$
- (A) The pressure of the gas changes to $\sqrt{2}P$.
 (B) The temperature of the gas changes to $\sqrt{2}T$.
 (C) The graph of the above process on the P - T diagram is parabola.
 (D) The graph of the above process on the P - T diagram is hyperbola.

Space For Rough Work

6. There is a board on which 'IIT' is written. It is situated on the opposite side of a river while IIT-JEE aspirant swim the river such that he/she always approach the IIT board [i.e. he/she always try to swim towards the board]. If speed of river and speed of aspirants w.r.t. river are same and width of river is d .



- (A) drift suffered by the aspirant will be $\frac{d}{4}$
- (B) drift suffered by the aspirant will be $\frac{d}{2}$
- (C) If IIT board is taken away from the bank by distance ' d ' in perpendicular direction of river flow then drift suffered will be $\frac{4d}{3}$
- (D) If IIT board is taken away from the bank by distance ' d ' in perpendicular direction of river flow then drift suffered will be $\frac{3d}{4}$
7. A bead is sliding down a smooth, massless rod PQ attached on the circumference of a vertical circle of radius R . The bead starts from rest from P:
- (A) Time taken by bead to go from P to Q is $2\sqrt{\frac{R \cos \theta}{g}}$
- (B) Time taken by bead to go from P to Q is $2\sqrt{\frac{R}{g \cos \theta}}$
- (C) The velocity of bead when it reaches centre of circle is $\sqrt{2gR \cos \theta}$
- (D) The velocity of bead when it reaches centre of circle is $2\sqrt{gR \cos \theta}$



(Single Correct Choice Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

8. The pair(s) of physical quantities that do not have the same dimensions, is
- (A) Reynolds number and coefficient of friction
- (B) Latent heat and gravitational potential
- (C) Curie and frequency of a light wave
- (D) Planck's constant and torque

Space For Rough Work

9. In a LR decay circuit, the current at $t = 0$ is 10 A. After 5 s it reduces to 9 A then incorrect statement is
- (A) The time constant is $\frac{5}{\ln\left|\frac{10}{9}\right|}$ (B) The time constant is $5 \ln\left|\frac{10}{9}\right|$
- (C) At $t = 10$ s, the current will be 8.1 A (D) At $t = 15$ s, the current will be 7.3 A
10. A ball of mass m projected horizontally from the top of a tower with kinetic energy K flies in the air for a time interval τ before it hits the horizontal ground. If force of air drag is proportional to the speed, and the proportionality constant is k_0 , find horizontal range of the ball.
- (A) $\frac{\sqrt{Km}}{k_0} \left\{1 - e^{-\frac{k_0\tau}{m}}\right\}$ (B) $\frac{\sqrt{2Km}}{k_0} \left\{1 - e^{-\frac{k_0\tau}{m}}\right\}$
- (C) $\frac{\sqrt{Km}}{2k_0} \left\{1 - e^{-\frac{k_0\tau}{m}}\right\}$ (D) $\frac{\sqrt{4Km}}{k_0} \left\{1 - e^{-\frac{k_0\tau}{m}}\right\}$
11. Find suitable expression for the moment of inertia of a uniform ring of mass m and radius r about an axis that passes through the centre of the ring and makes an angle θ with the plane of the ring.
- (A) $\frac{mr^2}{2}$ (B) $\frac{mr^2}{2} (\sin^2 \theta)$
- (C) $mr^2(1 + \sin^2 \theta)$ (D) $\frac{mr^2}{2} (1 + \sin^2 \theta)$
12. If electric field in a region is radially outward with magnitude $E = Ar$. The charge contained in a sphere of radii r_0 centered at origin is
- (A) $\frac{4\pi\epsilon_0 A}{r_0^3}$ (B) $\frac{A}{4\pi\epsilon_0 r_0^3}$
- (C) $4\pi\epsilon_0 r_0^3 A$ (D) $2\pi\epsilon_0 r_0^3 A$
13. A ball is dropped from a height h above a massive platform that is moving upward with a constant speed u . If the ball collides with the platform elastically, find height above its initial position the ball will rebound. Acceleration of free fall is g .
- (A) $\frac{2u\sqrt{u^2 + gh}}{g}$ (B) $\frac{u\sqrt{u^2 + 2gh}}{g}$
- (C) $\frac{2u\sqrt{u^2 + 2gh}}{g}$ (D) $\frac{u\sqrt{u^2 + 2gh}}{2g}$

Space For Rough Work

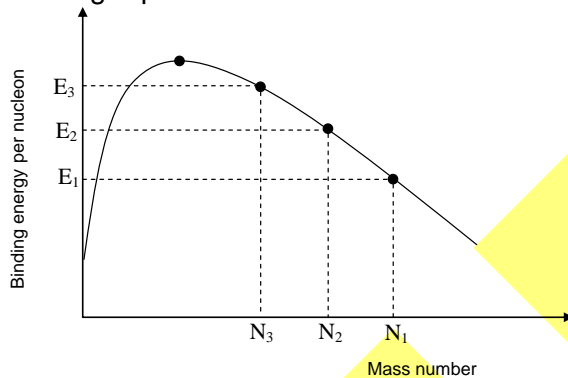
14. A person walking at the rate of 3km/hour, the rain appears to fall vertically when he increase his to speed 6 km/hr it appears to meet him at angle of 45° with vertical. The speed of rain is
- (A) $3\sqrt{2}$ km/hr (B) $\frac{3}{\sqrt{2}}$ km/hr
 (C) $6\sqrt{2}$ km/hr (D) $2\sqrt{3}$ km/hr

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the paragraphs **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question no. 15 to 16

Consider the nuclear fission reaction $W \rightarrow X + Y$. Using the graph given, answer the following equations:



15. What is the Q value of the reaction?
- (A) $E_1 N_1 - (E_2 N_2 + E_3 N_3)$ (B) $(E_2 N_2 + E_3 N_3 - E_1 N_1)$
 (C) $E_2 N_2 + E_1 N_1 + E_3 N_3 - E_3 N_3$ (D) $E_1 N_1 + E_3 N_3 - E_2 N_2$
16. If M_W is the mass of W, M_X is mass of X and M_Y is mass of Y nucleus, choose correct statement.
- (A) $\frac{M_W}{N_1} > \frac{M_Y}{N_2} > \frac{M_X}{N_3}$ (B) $\frac{M_W}{N_1} < \frac{M_Y}{N_2} < \frac{M_X}{N_3}$
 (C) $\frac{M_W}{N_1} < \frac{M_Y}{N_2} > \frac{M_X}{N_3}$ (D) $\frac{M_W}{N_1} > \frac{M_Y}{N_2} > \frac{M_X}{N_3}$

Space For Rough Work

Paragraph for Question no. 17 to 18

A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest. Its length is parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane. There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll off the edge without slipping

17. The angle θ through which the cylinder rotates before it leaves contact with the edge is
(A) $\tan^{-1} \frac{4}{7}$ (B) $\cos^{-1} \frac{7}{4}$ (C) $\cos^{-1} \frac{4}{7}$ (D) $\tan^{-1} \frac{7}{4}$
18. The speed of the centre of mass of the cylinder before leaving contact with the edge is
(A) $\sqrt{\frac{gR}{7}}$ (B) $\sqrt{\frac{4gR}{7}}$ (C) $\sqrt{\frac{9gR}{7}}$ (D) $\sqrt{\frac{3gR}{7}}$

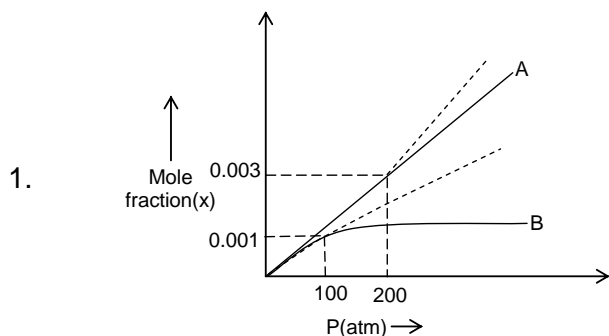
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SECTION-2 : CHEMISTRY

PART – A

(Multi Correct Choice Type)

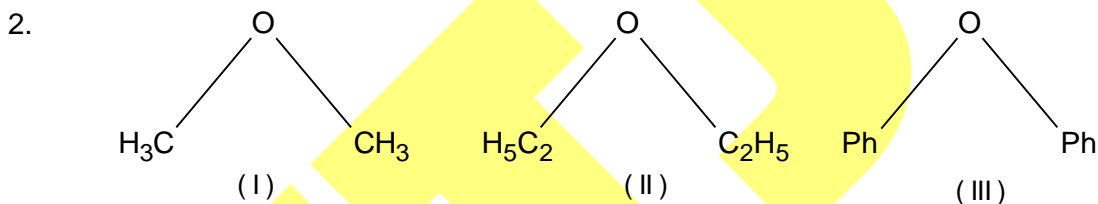
This section contains 7 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.



The solubility of two gases A and B in water at 50°C is plotted above. The variation of mole fraction and partial pressure of the gases is given in above graph.

Choose the correct statements

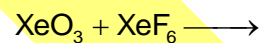
- (A) Gas 'A' follows Henry's law upto 200 atm
 (B) The Henry's law constant for gas B is 10^{-5} atm^{-1}
 (C) Gas 'A' is more soluble than gas 'B'
 (D) Gas 'B' does not follow ideal gas behaviour after 100 atm



Choose correct statements

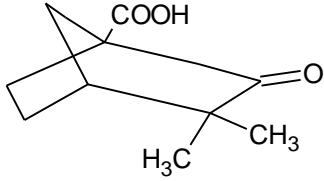
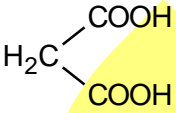
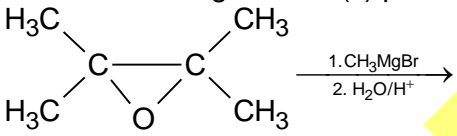
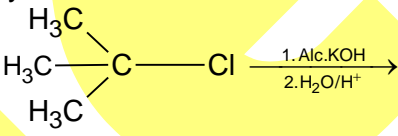
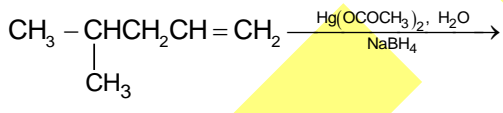
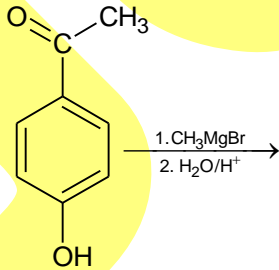
- (A) the dipole moment of (I) is higher than that of (II)
 (B) (II) is stable in basic medium whereas (III) is stable in acid as well as basic medium
 (C) (I) is more reactive than (III) towards Cl_2 in presence of sunlight
 (D) (II) is more reactive than (III) towards electrophilic substitution reaction

3. Which of the following oxo fluoride(s) is/are formed by the following reaction?



- (A) XeO_2F_2 (B) XeOF_4
 (C) XeOF_2 (D) XeO_3F_2

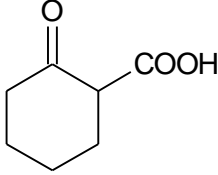
Space For Rough Work

4. An aqueous solution of 50 mL of a weak dibasic acid H_2A of concentration 0.02 M is titrated with 0.1 M NaOH solution $K_{a_1} = 10^{-2}$, $K_{a_2} = 10^{-5}$
Choose the correct statement [Assume $1 - \alpha = 1$]
(A) the pH of the solution before titration is approx 2
(B) the pH of solution at first equivalence point is 3.5
(C) the volume of NaOH solution consumed in complete titration is 180 mL
(D) the degree of dissociation of HA^- is greater than that of H_2A
5. Which of the following compound(s) undergo(es) decarboxylation on heating?
(A) $(CH_3)_2C(OOH)CH_2COOH$
(B) $HOOC-CH_2-C(=O)-CH_2-COOH$
(C) 
(D) 
6. Which of the following reaction(s) produce(s) tertiary alcohol?
(A) 
(B) 
(C) 
(D) 
7. Which of the following statement(s) is/are correct for zinc blend structure of ZnS ?
(A) The Zn^{2+} ions occupy half of the tetrahedral voids of the unit cell.
(B) Pure ZnS fluorescences when light falls on it.
(C) If the sulphide ions of ZnS are replaced by, X^{4-} ions, two cationic vacancy will be created
(D) It is more stable than the wurtzite structure of zinc sulphide.

Space For Rough Work

(Single Correct Choice Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

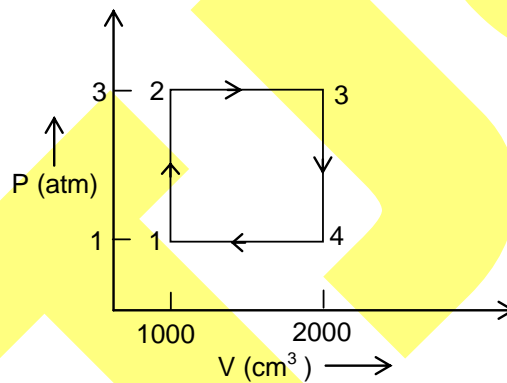
8. Reaction of an aqueous solution of $\text{Hg}(\text{NO}_3)_2$ with an aqueous solution of KNO_2 forms an eight-coordinate complex which is obtained as a yellow crystal. The correct statement for the complex is
- (A) it contains Hg – Hg bond (B) it contains eight NO_3^- ligands
(C) the oxidation state of Hg is +8 (D) it contains NO_2^- as a bidentate ligand
9. $\text{Ag}(\text{s}) \mid \text{AgCl}(\text{s}), \text{KCl}(0.2 \text{ M}) \parallel \text{KBr}(0.001 \text{ M}), \text{AgBr}(\text{s}) \mid \text{Ag}(\text{s})$
 K_{sp} of $\text{AgCl} = 2.8 \times 10^{-10}$
 K_{sp} of $\text{AgBr} = 3.3 \times 10^{-13}$
 What is the emf of the cell?
 (A) -0.41 V (B) -0.037 V
 (C) 0.037 V (D) +0.41 V
10. Which of the following substance undergoes dehydration on heating?
- (A) CH_3CHCOOH
 $\begin{array}{c} | \\ \text{OH} \end{array}$
 (B) $\text{CH}_3\text{CH} = \text{NOH}$
 (C) 
 (D) $\text{CH}_3\text{CH}_2\text{OH}$
11. Which of the following substance is not needed in order to elucidate the open chain structure of glucose?
- (A) $\text{Br}_2/\text{H}_2\text{O}$ (B) $(\text{CH}_3\text{CO})_2\text{O}/\text{catalyst}$
 (C) Conc. $\text{HI}/\text{red 'P'}$ (D) $\text{LiAlH}_4/\text{ether}$
12. $\text{XY}_2(\text{s}) \rightleftharpoons \text{X}(\text{g}) + 2\text{Y}(\text{g})$
 Above reaction takes place by taking only XY_2 in container. The reaction attains equilibrium at 1.2 atm.
 Choose the correct statement
 (A) if the reaction quotient Q_P is 0.12 atm^3 , then the forward reaction will take place
 (B) if the partial pressure of $\text{X}(\text{g})$ is doubled then the partial pressure of $\text{Y}(\text{g})$ will change to $\frac{p_y}{2\sqrt{2}}$
 (C) if the reaction takes place in a larger container at constant temperature then the equilibrium constant K_p will decrease
 (D) addition of XY_2 favours forward reaction

Space For Rough Work

13. $\text{Co}_2(\text{CO})_8 \xrightarrow{\text{NO}} (\text{X}) + \text{CO}$
 In above reaction 'X' may be
 (A) $\text{Co}(\text{NO})_2(\text{CO})_2$ (B) $\text{Co}(\text{NO})(\text{CO})_3$
 (C) $\text{Co}(\text{NO})_3(\text{CO})_2$ (D) $\text{Co}(\text{NO})(\text{CO})_4$
14. $\text{CH}_3\text{CH}_2\text{CH}_2\text{Cl} \xrightarrow[\text{Heat}]{\text{Anhy. AlCl}_3} \text{Product}$
 The product and type of above reaction is
 (A) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{CH}_2\text{CH}_3$, dimerisation
 (B) $\text{CH}_3\underset{\text{Cl}}{\text{CH}}\text{CH}_3$, isomerisation
 (C) $\text{CH}_3\text{CH}_2\underset{\text{Cl}}{\overset{\text{Cl}}{\text{C}}}$, nucleophilic substitution
 (D) $\text{CH}_3\text{CH}_2\text{Cl}$, CH_3Cl , pyrolysis

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the paragraphs **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question no. 15 to 16

0.1 mole of an ideal gas having $C_{V, M} = \frac{3R}{2}$

(independent of temperature) undergoes a reversible cyclic process $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ as shown above. Each step is accompanied with either constant P or constant V.

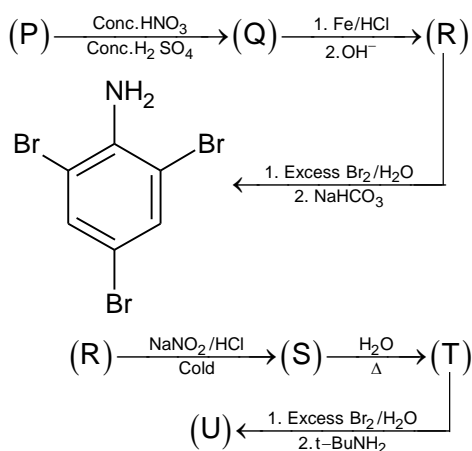
Answer the following questions on the basis of above write up.

15. How much work is done for the complete cycle in Joule unit?
 (A) -281 (B) -408 (C) -203 (D) -513

Space For Rough Work

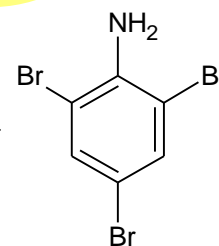
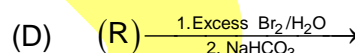
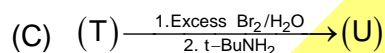
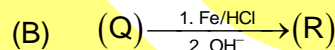
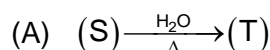
16. What is the sum of the heat absorbed in the steps $1 \rightarrow 2$ and $2 \rightarrow 3$ ($q_{1 \rightarrow 2} + q_{2 \rightarrow 3}$) in joule unit?
 (A) 1218 (B) 1120 (C) 1065 (D) 975

Paragraph for Question no. 17 to 18

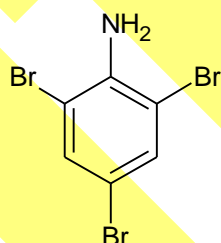
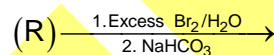


Answer the following question on the basis of above write up

17. Which of the following reaction is accompanied with formation of aromatic side products?



18.



Which of the following change(s) is/are observed if NaHCO_3 is not used in the reaction?

- (I) Rate of reaction will decrease
 (II) Meta orientation products are formed
 (III) Nucleophilic substitution will take place
 (IV) Dehydrogenation reaction will take place

The correct answer(s) is/are

(A) II, III

(B) I, II

(C) I, II, III

(D) III, IV

Space For Rough Work

SECTION-3 : MATHEMATICS

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. The sum of two non-negative numbers is $2a$. If P is the probability that the product of these numbers is not less than m times their greatest possible product, then

(A) $P = \frac{2}{3}, m = \frac{4}{5}$

(B) $P = \frac{1}{3}, m = \frac{3}{5}$

(C) $P = \frac{1}{2}, m = \frac{3}{4}$

(D) $P = \frac{3}{4}, m = \frac{7}{16}$

2. If $\theta \in \left[-\frac{\pi}{9}, -\frac{\pi}{36}\right]$, such that $f(\theta) = \tan\left(\theta + \frac{5\pi}{18}\right) + \cos\left(\theta + \frac{5\pi}{18}\right) - \tan\left(\theta + \frac{7\pi}{9}\right)$ has maximum and minimum values as a and b respectively, then

(A) $a = \frac{11}{\sqrt{3}}$

(B) $b = \frac{-3 + 2\sqrt{2}}{2}$

(C) $b = \frac{2\sqrt{2} + 1}{\sqrt{2}}$

(D) $a = \frac{11}{2\sqrt{3}}$

3. If $a_0 = \sqrt{2} - \sqrt{3} + \sqrt{6}$ and $a_n = \tan\left(\frac{2^{n-3}\pi}{3}\right) + 2$ then the value of

$M = \sum_{n=0}^3 \tan\left(\frac{2^{n-3}\pi}{3}\right) \cdot \sec\left(\frac{2^{n-2}\pi}{3}\right)$ is equal to

(A) $a_4 - a_0$

(B) $a_3 - a_1$

(C) $a_1 - a_0$

(D) $a_1 - a_0 - 2$

Space For Rough Work

4. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} \lim_{n \rightarrow \infty} \left(\frac{[x]}{1+n^2} + \frac{3[x]}{2+n^2} + \frac{5[x]}{3+n^2} + \dots + \frac{(2n-1)[x]}{n+n^2} \right) & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}, \text{ where } [.] \text{ denotes GIF, then}$$

which of the following statement(s) is (are) correct?

- (A) $f(x)$ is injective but not surjective
- (B) $f(x)$ is non-differentiable at $x = \frac{\pi}{2}$
- (C) $f(x)$ is discontinuous at all integers and continuous at $x = \frac{\pi}{2}$
- (D) $f(x)$ is unbounded function.
5. If $x \in \mathbb{R}$, then the roots of the equation $x^4 + 4x^3 - 8x^2 + k = 0$ with respect to the values of k , ($k \in [0, 2014]$), are:
- (A) for $k \in [0, 3]$: 4 real and distinct roots
- (B) for $k \in (3, 128)$: two real and two imaginary roots
- (C) for $k \in (128, 583)$: no real root
- (D) for $k \in (0, 3)$: four real and distinct roots
6. If $f(x) = \int_0^x \frac{\sin t}{t} dt$, $x > 0$ then
- (A) $f(x)$ has a local maxima at $x = n\pi$ ($n = 2k, k \in \mathbb{I}^+$)
- (B) $f(x)$ has a local minima at $x = n\pi$ ($n = 2k, k \in \mathbb{I}^+$)
- (C) $f(x)$ has neither maxima nor minima at $x = n\pi$ ($n \in \mathbb{I}^+$)
- (D) $f(x)$ has local maxima at $x = n\pi$ ($n = 2k - 1, k \in \mathbb{I}^+$)
7. Consider the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z = 3$. The plane $67x - 162y + 47z + 44 = 0$ bisects the angle between the given planes which
- (A) contains the origin
- (B) is acute
- (C) is obtuse
- (D) none of these

Space For Rough Work

(Single Correct Choice Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

8. The maximum value of n for which the quadratic equation $a_{n+1}x^2 - 2x\sqrt{\sum_{k=1}^{n+1} a_k^2} + \sum_{k=1}^n a_k = 0$ has real roots for every choice of real numbers a_1, a_2, \dots, a_{n+1} is
 (A) 2 (B) 3
 (C) 4 (D) 5
9. Let $f : \mathbb{R} \rightarrow \mathbb{R} - \{3\}$ be a function which satisfies $f(x+10) = \frac{f(x)-5}{f(x)-3}$, then $\int_{10}^{20} f(x) dx$ is equal to
 (A) $\int_{100}^{110} f(x) dx$ (B) $\int_{110}^{120} f(x) dx$
 (C) $\int_{120}^{130} f(x) dx$ (D) $\int_{130}^{140} f(x) dx$
10. C the centre of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The tangents at any point P on this hyperbola meets the straight lines $bx - ay = 0$ and $bx + ay = 0$ in the points Q and R respectively. Then $CQ \cdot CQ \cdot CR =$
 (A) $a^2 + b^2$ (B) $a^2 - b^2$
 (C) $\frac{1}{a^2} + \frac{1}{b^2}$ (D) $\frac{1}{a^2} - \frac{1}{b^2}$
11. The ratio of the areas of two regions in which the curve $C_1 \equiv 4x^2 + \pi^2 y^2 = 4\pi^2$ is divided by the curve $C_2 \equiv y = -\left(\operatorname{sgn}\left(x - \frac{\pi}{2}\right)\right) \cos x$ is equal to (where sgn denotes signum function)
 (A) $\frac{\pi^2 - 2}{\pi^2 + 2}$ (B) $\frac{\pi^2 + 4}{\pi^2 - 2\sqrt{2}}$
 (C) $\frac{\pi^2 + 3\sqrt{2}}{\pi^2 + 6}$ (D) $\frac{\pi^2 - \sqrt{2}}{\pi^2 + 1}$

Space For Rough Work

12. The plane $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$ intersects x-axis, y-axis and z-axis at points A, B and C respectively. The distance between origin and orthocentre of $\triangle ABC$ is equal to
- (A) $\frac{1}{\sqrt{7}}$ units (B) $\frac{6}{\sqrt{7}}$ units
 (C) $\frac{6}{7}$ units (D) none of these
13. If the equation $4x^4 - ax^3 + bx^2 - cx + 6 = 0$, $a, b, c \in \mathbb{R}^+$ has four distinct real roots say $x_1 < x_2 < x_3 < x_4$ such that $\frac{1}{x_1} + \frac{2}{x_2} + \frac{3}{x_3} + \frac{4}{x_4} = 8$, then $\frac{x_4}{x_1}$ is equal to
- (A) 2 (B) 3
 (C) 4 (D) 6
14. The perpendicular bisectors $x + y + 2 = 0$ and $x - y - 1 = 0$ of sides AB and AC of a triangle ABC intersect them at $(-1, -1)$ and $(2, 1)$ respectively. If the midpoint of side BC is P, then the distance of P from the orthocentre of triangle ABC is
- (A) $\sqrt{85}$ (B) $\sqrt{41}$
 (C) $\sqrt{13}$ (D) none of these

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the paragraphs **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question no. 15 to 16

Consider the equation $x^2 - [x^2] = (x - [x])^2$ where $[.]$ denotes Greatest integer function.

For $n \in \mathbb{N}$, let $f(n)$ denote the number of roots of the above equation in the interval $1 \leq x \leq n$. Answer the following questions:

15. The number of solutions of the equation $f(n) = n$ is
- (A) 0 (B) 1
 (C) 2 (D) none of these

Space For Rough Work

16. The value of $\lim_{n \rightarrow \infty} \left(\sqrt{f(n)} - [\sqrt{f(n)}] \right)$ (where $[.]$ denotes GIF) is
- (A) 1 (B) $\frac{1}{4}$
 (C) $\frac{1}{2}$ (D) none of these

Paragraph for Question no. 17 to 18

Let $\vec{a}, \vec{b}, \vec{c}$ and \vec{d} be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = \cos \theta$ and $\vec{d} \cdot \vec{a} = \vec{d} \cdot \vec{b} = \vec{d} \cdot \vec{c} = \cos \alpha$.

17. If $\theta = \frac{\pi}{3}$ then α is equal to
- (A) $\frac{\pi}{4}$ (B) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
 (C) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$ (D) $\frac{\pi}{2}$
18. If $\theta = \cos^{-1}\left(\frac{1}{4}\right)$ then α is equal to
- (A) $\frac{\pi}{4}$ (B) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
 (C) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$ (D) $\frac{\pi}{2}$

Space For Rough Work

Q.P. Code: 100192**Answers****SECTION-1 : PHYSICS****PART – A**

- | | | | |
|-------|-------|--------|--------|
| 1. BC | 2. AC | 3. ABD | 4. ABD |
| 5. BD | 6. BD | 7. BC | 8. D |
| 9. B | 10. B | 11. D | 12. C |
| 13. C | 14. A | 15. B | 16. A |
| 17. C | 18. B | | |

SECTION-1 : CHEMISTRY**PART – A**

- | | | | |
|---------|--------|-------|-------|
| 1. ABCD | 2. ABC | 3. AB | 4. AB |
| 5. ABD | 6. AB | 7. AD | 8. D |
| 9. B | 10. A | 11. D | 12. A |
| 13. B | 14. B | 15. C | 16. C |
| 17. A | 18. B | | |

SECTION-1 : MATHEMATICS**PART – A**

- | | | | |
|--------|-------|-------|-------|
| 1. CD | 2. CD | 3. AD | 4. CD |
| 5. BCD | 6. BD | 7. AB | 8. C |
| 9. D | 10. A | 11. A | 12. C |
| 13. C | 14. C | 15. B | 16. C |
| 17. C | 18. A | | |

Answers & Solutions

SECTION-1 : PHYSICS

PART – A

1. **BC**

Sol. If collisions at both the walls are inelastic, period is T_0 .
If collision at one wall is elastic and at the other is inelastic, the period is T_0 .

2. **AC**

Sol. $\frac{\ell}{r} = \frac{4 + \sqrt{6}}{\sqrt{3}}$

$$\text{Mass of load} = \frac{m(1 + \eta^2)}{(1 - \eta^2)} \sin \theta = 15 \text{ kg.}$$

3. **ABD**

Sol. Dielectric constant = $\frac{\ell - \frac{\ell}{3}}{\frac{\ell}{3}} = 2$.

Charge appearing due to polarization on the surface of the dielectric is $0.25 C_0V$.

4. **ABD**

Sol. Resultant amplitude = $\sqrt{(5A - 2A)^2 + (6A - 2A)^2} = 5A$

$$\tan \phi = \frac{3A}{4A} ; \phi = 37^\circ$$

$$\text{Resultant wave} = 5A \sin(\omega t - kx + 37^\circ)$$

5. **BD**

Sol. $PT = \text{constant}$

$$P_i = P \sqrt{\frac{\rho}{2}} = \frac{P}{\sqrt{2}}$$

$$T_i = T \sqrt{\frac{\rho}{2}} = T\sqrt{2}$$

6. **BD**

Sol. Drift = $\frac{d}{2}$, when board at bank of river.

Say drift = x , when board taken away from bank by a distance 'd'.

$$2d - x = \sqrt{d^2 + x^2}$$

$$4d^2 + x^2 - 4dx = d^2 + x^2$$

$$4dx = 3d^2$$

$$x = \frac{3d}{4}$$

7. **BC**

Sol. $PQ = 2R$
 $a = g \cos \theta$
 $u^2 = 2(g \cos \theta)(2R)$
 $u = 2\sqrt{gR \cos \theta}$

$$t = \frac{u}{a} = \frac{2\sqrt{gR \cos \theta}}{g \cos \theta} = 2\sqrt{\frac{R}{g \cos \theta}}$$

In reaching upto centre.

$$u^2 = 2(g \cos \theta) \cdot R = \sqrt{2gR \cos \theta}.$$

8. **D**

Sol. Reynolds number and coefficient of friction are dimensionless.

Latent heat and gravitational potential both have dimension $[L^2T^{-2}]$

Curie and frequency of a light wave both have dimension $[T^{-1}]$. But dimensions of Planck's constant is $[ML^2T^{-1}]$ and torque is $[ML^2T^{-2}]$.

9. **B**

Sol. $I = I_0 (e^{-Rt/L})$

10. **B**

Sol. $x = \frac{\sqrt{2Km}}{k_0} \left\{ 1 - e^{-\frac{k_0 x}{m}} \right\}$

At $t = \tau$, $x = R$

$$R = \frac{\sqrt{2Km}}{k_0} \left\{ 1 - e^{-\frac{k_0 R}{m}} \right\}$$

11. **D**

Sol. $I = \frac{mr^2}{2} + \frac{mr^2 \sin^2 \theta}{2}$
 $= \frac{mr^2}{2} (1 + \sin^2 \theta)$

12. **C**

Sol. $\oint \vec{E} \cdot d\vec{s} = \frac{q_{en}}{\epsilon_0}$

$$Ar_0 \cdot 4\pi r_0^2 = \frac{q}{\epsilon_0}$$

$$q = 4\pi\epsilon_0 A r_0^3$$

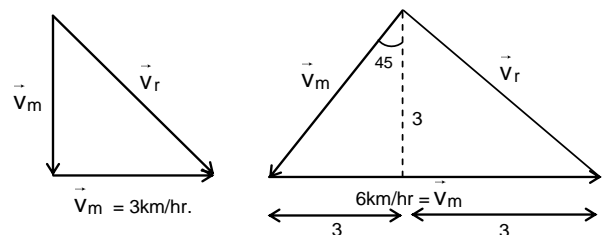
13. **C**

Sol. $H = \frac{(u + \sqrt{u^2 + 2gh})^2}{2g} - h + \frac{\sqrt{u^2 + 2gh} - u}{g}$
 $= \frac{2u\sqrt{u^2 + 2gh}}{g}$

14. **A**

Sol. \vec{v}_r should be same in both the situations.

Thus, $|\vec{v}_r| = 3\sqrt{2} \text{ km/hr.}$



15. **B**

Sol. W correspond to N_1 , X and Y correspond to N_2 and N_3 and Q value = $E_2N_2 + E_3N_3 - E_1N_1$

16. **A**

Sol. Binding energy per nucleon for N_1 is least hence mass of nucleus of W is greatest.

17. **C**

Sol. $\frac{3}{4}MV^2 = mgr(1 - \cos \theta)$ and $Mg \cos \theta = \frac{MV^2}{r} \Rightarrow \cos \theta = \frac{4}{7}$

18. **B**

Sol. $V = \sqrt{gR \cos \theta} = \sqrt{\frac{4}{7}gR}$

SECTION-2 : CHEMISTRY

PART – A

1. ABCD
Sol. Henry's law is followed by ideal gas for which mole fraction is directly proportional to partial pressure.
 $x = K_H P$
 $\therefore k_h = \frac{x}{p} = \frac{10^{-3}}{100} = 10^{-5} \text{ atm}^{-1}$
2. ABC
3. AB
Sol. $2 \text{XeO}_3 + \text{XeF}_6 \longrightarrow 3 \text{XeO}_2\text{F}_2$
 $\text{XeO}_3 + 2 \text{XeF}_6 \longrightarrow 3 \text{XeOF}_4$
4. AB
Sol. Before titration, the pH of acid is calculates as
 $[\text{H}^+] = C\alpha = 0.02 \times 0.5 = 0.01$
 $\therefore \text{pH} = 2$
At first equivalence point the solution contains HA^-
So $\text{pH} = \frac{1}{2}(\text{p}^{K_{a1}} + \text{p}^{K_{a2}}) = \frac{1}{2}(2 + 5) = 3.5$
5. ABD
Sol. (A) forms aldehyde on decarboxylation reaction (C) does not undergo decarboxylation as six-membered ring is not formed.
6. AB
Sol. In (C) 3° -alcohol is not formed
(D) CH_4 is formed
7. AD
Sol. Zn^{2+} ions are present in half of the tetrahedral voids which are present on body diagonals. Pure ZnS does not fluoresce it does so in presence of impurities like Ag^+ and Cd^{2+}
Anionic vacancy will be created not cationic vacancy.
8. D
Sol. The complex is:
 $\text{K}_2[\text{Hg}(\text{NO}_2)_4] \cdot \text{KNO}_3$
9. B
Sol. $E = E^\circ - \frac{0.0591}{1} \log \frac{[\text{Ag}^+]_{\text{LHS}}}{[\text{Ag}^+]_{\text{RHS}}}$
 $= 0 - 0.0591 \log \frac{(2.8 \times 10^{-10}) / 0.2}{(3.3 \times 10^{-13}) / 10^{-3}} = -0.03 \text{ V}$
10. A
Sol. α -hydroxy acids undergoes dehydration on heating to form a cyclic compound.
11. D
Sol. $\text{Br}_2/\text{H}_2\text{O}$ confirms CHO group
 $(\text{CH}_2\text{CO})_2\text{O}$ confirms OH group
Conc. HI/red P confirm six membered carbon chain

12. A

Sol. $P_{\text{equ}^m} = 1.2 \text{ atm}$

$$\therefore p_x + p_y = 1.2 \text{ atm}$$

$$\text{or, } p_x + 2p_x = 1.2 \text{ atm}$$

$$\therefore p_x = 0.4 \text{ atm}$$

$$K_p = p_x \times p_y^2 = 0.4 \times (0.8)^2 = 0.256$$

$Q_p < K_p$, so forward reaction takes place

$$K_p = p_x \times p_y^2 = 2p_x \times p_y'^2$$

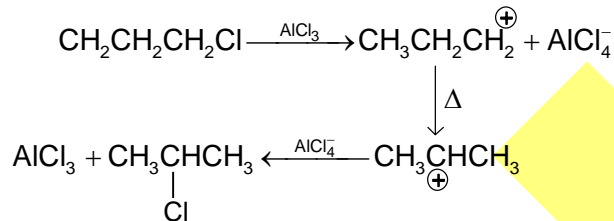
$$p_y'^2 = \frac{p_y^2}{2} \Rightarrow p_y' = \sqrt{\frac{p_y^2}{2}} = \frac{p_y}{\sqrt{2}}$$

13. B

Sol. One NO can substituted three CO groups.

14. B

Sol.



15. C

Sol.

$$\begin{aligned} W_T &= W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 4} + W_{4 \rightarrow 1} \\ &= 0 + \left[-\int_2^3 Pdv \right] + 0 + \left[-\int_4^1 Pdv \right] \\ &= 0 + (-304 \text{ J}) + 0 + 101 \text{ J} = -203 \text{ J} \end{aligned}$$

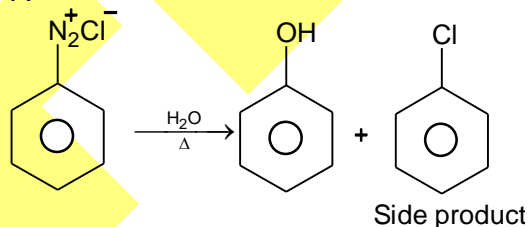
16. C

Sol.

$$\begin{aligned} q_{1 \rightarrow 2} &= \int_1^2 C_v dT = nC_{v,m} \int_1^2 (T_2 - T_1) = 304 \text{ J} \\ q_{2 \rightarrow 3} &= \int_2^3 C_p dT = nC_{p,m} \int_2^3 (T_3 - T_2) = 761 \text{ J} \\ \therefore q_T &= 304 + 761 = 1065 \text{ J} \end{aligned}$$

17. A

Sol.



18. B

Sol.

HBr will be formed during reaction. It is consumed by NaHCO_3 . Unless it is consumed, 'H⁺' will change $-\text{NH}_2$ group to $-\overset{+}{\text{N}}\text{H}_3$ which decreases the reaction rate and direct meta substitution.

SECTION-3 : MATHEMATICS**PART - A**

1. CD

Sol: Let numbers be x and y and let $z = xy$

$$z_{\max} = a^2$$

$$\text{Now, } P(xy \geq ma^2) = P(x(2a-x) \geq ma^2) \Rightarrow x^2 - 2ax + ma^2 \leq 0$$

$$\text{For } m = \frac{3}{4}, P = \frac{1}{2}, \text{ and for } m = \frac{7}{16}, P = \frac{3}{4}.$$

2. CD

$$\text{Sol: Let } A = \theta + \frac{5\pi}{18} \Rightarrow A \in \left[\frac{\pi}{6}, \frac{\pi}{4} \right]$$

$$f(A) = \tan A + \cos A + \cot A \Rightarrow a = \frac{11}{2\sqrt{3}}, b = \frac{2\sqrt{2}+1}{\sqrt{2}}$$

3. AD

$$\text{Sol: } M = \sum_{n=0}^3 \tan\left(\frac{2^{n-3}\pi}{3}\right) \cdot \sec\left(\frac{2^{n-2}\pi}{3}\right)$$

$$= \frac{\sin\left(\frac{\pi}{12} - \frac{\pi}{24}\right)}{\cos\frac{\pi}{24} \cdot \cos\frac{\pi}{24}} + \dots + \frac{\sin\left(\frac{2\pi}{3} - \frac{\pi}{3}\right)}{\cos\frac{\pi}{3} \cdot \cos\frac{2\pi}{3}}$$

$$= \tan\frac{2\pi}{3} - \tan\frac{\pi}{24}$$

$$= -\sqrt{3} - \sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$$

$$M = 2 - \sqrt{2} - \sqrt{6}$$

$$a_0 = \sqrt{2} - \sqrt{3} - \sqrt{6}$$

$$a_1 = \tan\frac{\pi}{12} + 2 = 4 - \sqrt{3}$$

$$a_4 = \tan\left(\frac{2\pi}{3}\right) + 2 = 2 - \sqrt{3}$$

$$\text{then } a_1 - a_0 - 2 = 2 - \sqrt{2} - \sqrt{6} \text{ and } a_4 - a_0 = 2 - \sqrt{2} - \sqrt{6}$$

4. CD

$$\text{Sol: } f(x) = \begin{cases} \lim_{n \rightarrow \infty} \left(\frac{1}{1+n^2} + \frac{3}{2+n^2} + \dots + \frac{(2n-1)}{n+n^2} \right) [x], & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$$

$$\therefore \frac{1}{n+n^2} + \frac{3}{n+n^2} + \dots + \frac{(2n-1)}{(n+n^2)} < \frac{1}{1+n^2} + \frac{3}{2+n^2} + \dots + \frac{(2n-1)}{(n+n^2)} < \frac{1}{1+n^2}$$

$$+ \frac{3}{1+n^2} + \dots + \frac{(2n-1)}{1+n^2}$$

$$\Rightarrow \frac{n^2}{n+n^2} < \frac{1}{1+n^2} + \frac{3}{2+n^2} + \dots + \frac{(2n-1)}{n+n^2} < \frac{n^2}{1+n^2}$$

$$\begin{aligned} \therefore \lim_{n \rightarrow \infty} \frac{n^2}{n+n^2} &= 1 \text{ and } \lim_{n \rightarrow \infty} \frac{n^2}{1+n^2} = 1 \\ \therefore \lim_{n \rightarrow \infty} \left(\frac{1}{1+n^2} + \frac{3}{2+n^2} + \dots + \frac{(2n-1)}{n+n^2} \right) &= 1 \\ \therefore f(x) &= \begin{cases} [x], & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases} \Rightarrow f(x) = [x] \forall x \in \mathbb{R}. \end{aligned}$$

Hence range of $f(x)$ is set of all integers.
 $\Rightarrow f(x)$ is unbounded.

Now, at $x = \frac{\pi}{2}$

Clearly, L.H.L = R.H.L = 1 = $f\left(\frac{\pi}{2}\right)$

But $f(x)$ is discontinuous at all integers as $[x]$ is discontinuous at integers.

5. BCD

Sol:

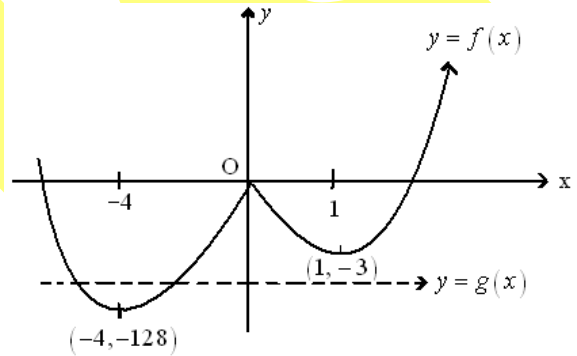
$$\begin{aligned} x^4 + 4x^3 - 8x^2 &= -k \\ \Rightarrow f(x) &= -k \end{aligned}$$

Where

$$f(x) = x^4 + 4x^3 - 8x^2 = x^2(x^2 + 4x - 8)$$

Let $g(x) = -k$

From the graph the following cases arise :



1. When $-3 \leq -k \leq 0, \Rightarrow 0 \leq k \leq 3$
 In this case, $y = -k$ intersect at four points.
2. When $-4 \leq -k < -3, \Rightarrow 3 < k \leq 4$
 In this case, $y = -k$ intersect at two points, the given equation has two real roots.
3. When $k < 0, \Rightarrow -k > 0$
 In this case, there are two points of intersection. So, the equation has two real roots.

6. BD

Sol. $f'(x) = \frac{\sin x}{x}$

For $f'(x) = 0, \frac{\sin x}{x} = 0 \Rightarrow x = n\pi (n \in \mathbb{I}, n \neq 0)$

$$f''(x) = \frac{x \cos x - \sin x}{x^2}$$

$$f''(n\pi) = \frac{\cos n\pi}{n\pi} < 0 \text{ if } n = 2k - 1 \text{ and } > 0 \text{ if } n = 2k, k \in \mathbb{I}^+$$

Hence, $f(x)$ has local maxima at $x = n\pi$, where $n = 2k - 1$ and local minima at $x = n\pi, n = 2k$, where $k \in \mathbb{I}^+$.

7. AB

Sol: $3x - 6y + 2z + 5 = 0$
 $-4x + 12y - 3z + 3 = 0$

$$\text{Bisectors are } \frac{3x - 6y + 2z + 5}{\sqrt{9 + 36 + 4}} = \pm \frac{-4x + 12y - 3z + 3}{\sqrt{16 + 144 + 9}}$$

The plane which bisects the angle between the planes that contains the origin is

$$13(3x - 6y + 2z + 5) = 7(-4x + 12y - 3z + 3)$$

$$67x - 162y + 47z + 44 = 0$$

$$\text{Further, } 3x(-4) + (-6)(12) + 2(-3) < 0$$

Hence, the origin lies in the acute angle.

8. C

$$\text{Sol: Clearly } D_{\min} = 4a_{n+1}^2 \left[1 - \frac{n}{4} \right]$$

$$\text{For real roots, } D_{\min} \geq 0 \Rightarrow n \leq 4.$$

9. D

Sol: $f(x)$ is periodic with period 40.

10. A

Sol: P is $(a \sec \theta, b \tan \theta)$

$$\text{Tangent at P is } \frac{x \sec \theta}{a} - \frac{y \tan \theta}{b} = 1$$

$$\text{It meets } bx - ay = 0 \text{ i.e. } \frac{x}{a} = \frac{y}{b} \text{ in Q}$$

$$\therefore \text{Q is } \left(\frac{a}{\sec \theta - \tan \theta}, \frac{-b}{\sec \theta - \tan \theta} \right)$$

$$\text{It meets } bx + ay = 0 \text{ i.e. } \frac{x}{a} = -\frac{y}{b} \text{ in R.}$$

$$\therefore \text{R is } \left(\frac{a}{\sec \theta + \tan \theta}, \frac{-b}{\sec \theta + \tan \theta} \right)$$

$$\therefore \text{CQ} \cdot \text{CR} = \frac{\sqrt{a^2 + b^2}}{(\sec \theta - \tan \theta)} \cdot \frac{\sqrt{a^2 + b^2}}{(\sec \theta + \tan \theta)}$$

$$= a^2 + b^2, (\because \sec^2 \theta - \tan^2 \theta = 1).$$

11. A

Sol: The areas of the regions are respectively $\pi^2 - 2$ and $\pi^2 + 2$.

12. C

Sol: Orthocentre is the foot of perpendicular from origin to the plane.

13. C

Sol: All roots are positive. Applying $AM \geq GM$ on $\frac{1}{x_1}, \frac{2}{x_2}, \frac{3}{x_3}, \frac{4}{x_4}$ we get $x_1 = \frac{1}{2}$ and $x_4 = 2$.

14. C

Sol: Triangle is right angled. Distance AP is equal to distance between $(-1, -1)$ and $(2, 1)$ which is equal to $\sqrt{13}$.

15. B

16. C

Sol: (15 & 16)

It is obvious that $x = n$ is a required root. Let $1 \leq \alpha < n$ be another root of the given equation.

Let $m = [\alpha]$ and $t = \{\alpha\}$. Then the equation becomes: $2mt - [2mt + t^2] = 0$.

Therefore $2mt$ is a non-negative integer, i.e. $t = 0, \frac{1}{2m}, \frac{2}{2m}, \dots, \frac{2m-1}{2m}$ for

$m = 1, 2, 3, \dots, n-1$. Hence the number of required roots is

$$f(n) = 2[1+2+3+\dots+(n-1)] + 1 = n^2 - n + 1$$

17. **C**18. **A****Sol 17 & 18:**

Obviously, $\vec{d} \parallel (\vec{a} + \vec{b} + \vec{c})$ and since \vec{d} is a unit vector $\therefore \vec{d} = \frac{\vec{a} + \vec{b} + \vec{c}}{|\vec{a} + \vec{b} + \vec{c}|}$

$$\text{Now, } |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3 + 6\cos\theta}$$

$$\text{And } \vec{d} \cdot \vec{a} = \frac{\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}}{\sqrt{3 + 6\cos\theta}}$$

$$\Rightarrow \frac{1 + 2\cos\theta}{\sqrt{3 + 6\cos\theta}} = \cos\alpha$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}} \sqrt{1 + 2\cos\theta}$$

$$\therefore \theta = \frac{\pi}{3} \Rightarrow \alpha = \cos^{-1} \frac{\sqrt{2}}{\sqrt{3}} \text{ and } \theta = \cos^{-1} \frac{1}{4} \Rightarrow \alpha = \frac{\pi}{4}$$