FIITJEE RBT-5 for (JEE-Advanced)

BATCHES – Two Yr CRP2123(All)

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 3 | QP CODE: 100187 | PAPER - 1

Time Allotted: 3 Hours **Maximum Marks: 204**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. All the section can be filled in **PART-A & B** of OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **Part-A (01-04) –** Contains **Four (04)** multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **PART–A** (**05–12**) contains Eight (8) Multiple Choice Questions which have **One or More Than One Correct** answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(iii) **PART – B (1 – 3):** This section contains **Three (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.** Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(iv) **Part-B (4 – 8)** contains **Five (05)** Numerical based questions, the answer of which maybe positive or negative numbers or decimals **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :____

Batch :____________________ **Date of Examination :**____________________

Enrolment Number :____

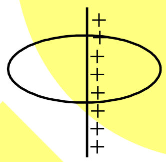
SECTION – I : PHYSICS

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

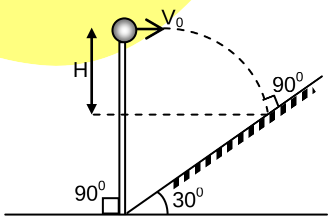
1. A paper ball (assumed to be carried easily by wind) is to be thrown in a trash can located $\frac{6\sqrt{3}}{5}$ m north of the point of projection. Wind is blowing towards east with speed 2 m/s. The paper ball is thrown at an angle 37° with the horizontal and it falls on the trash can. What is the speed with which it was thrown?
(A) 4 m/s (B) 5 m/s (C) 6 m/s (D) 7 m/s

2. A very long uniformly charged rod falls with a constant velocity *V* through the center of a circular loop making an angle 37° with axis of loop. Then the magnitude of induced emf in loop is
(charge per unit length of rod = λ)



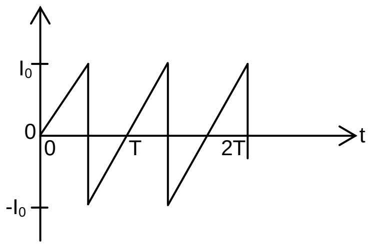
(A) $\frac{\mu_0}{2\pi}\lambda V^2$ (B) $\frac{\mu_0}{2}\lambda V^2$ (C) $\frac{\mu_0}{2\lambda}V$ (D) zero

3. In the given figure, the angle of inclination of the inclined plane is 30^0. The horizontal velocity V_0 so that the particle hits the inclined plane perpendicularly is



(A) $v_0 = \sqrt{\frac{2gH}{3}}$ (B) $V_0 = \sqrt{\frac{2gH}{7}}$
(C) $V_0 = \sqrt{\frac{gH}{5}}$ (D) $V_0 = \sqrt{\frac{gH}{7}}$

4. For a circuit, current variation with time is shown in figure, corresponding rms current is



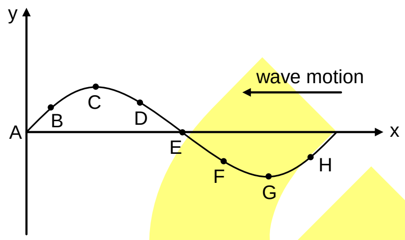
(A) I_0 (B) $I_0/\sqrt{2}$
(C) $I_0/\sqrt{3}$ (D) $I_0/2$

Space For Rough Work

PART – A (Maximum Marks: 32)

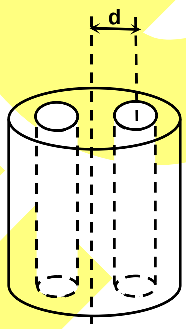
*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. A transverse wave is travelling along a stretched string from right to left. The figure shown represents the shape of the string (snap shot) at a given instant. At this instant:



(A) the particles at A, B and H have upward velocity.
(B) the particles at D, E and F have downward velocity.
(C) the particles at C, E and G have zero velocity.
(D) the particles at A and E have maximum velocity.

6. An infinitely long cylindrical conductor of radius R contains current of uniform density J along axis of the cylinder. Two infinitely long cylindrical holes of radius r are drilled symmetrically throughout the length of the cylinder. The axes of the holes are parallel to the axis of cylinder and at distance d from it in the same plane.



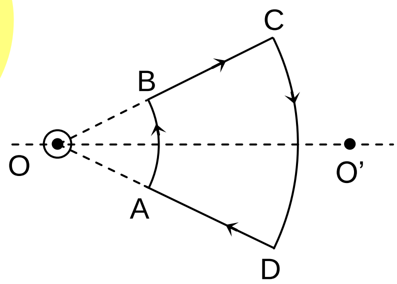
(A) Magnetic field on the axis of one of the hole will be $\frac{\mu_0 J}{2}\left(d - \frac{r^2}{2d}\right)$

(B) Magnetic field on the axis of one of the hole will be $\frac{\mu_0 J}{2}\left(d - \frac{r^2}{d}\right)$

(C) Magnetic field on the axis of cylinder will be $\frac{\mu_0 J}{2}\frac{r^2}{d}$

(D) Magnetic field on the axis of cylinder will be zero

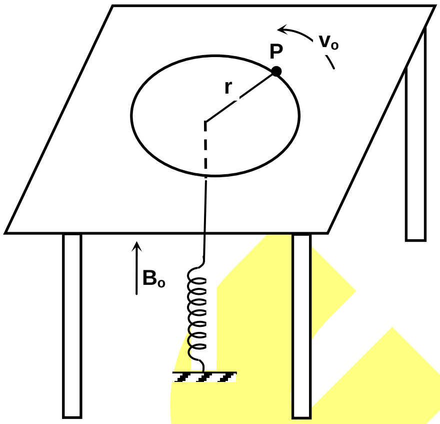
7. An infinite current carrying wire coming outward passes through point O and in perpendicular to the plane containing a current carrying loop ABCD as shown in the figure. Choose the correct option(s).



(A) net force on the loop is zero
(B) Net torque on the loop is zero
(C) As seen from O, the loop rotates clockwise
(D) As seen from O, the loop rotates anticlockwise

Space For Rough Work

8. A particle mass m charge q is moving on a circular path on the surface of a frictionless table with speed v_o where a magnetic field B_o exists uniformly over the whole region. It is attached by a string which passes through a hole in the table to a spring as shown. The spring is stretched by x_o. If now the magnetic field is increased slowly to $2B_o$,
(A) The extension in the spring will increase
(B) The speed of the particle will increase
(C) The speed of the particle will decrease
(D) The kinetic energy of the particle will decrease

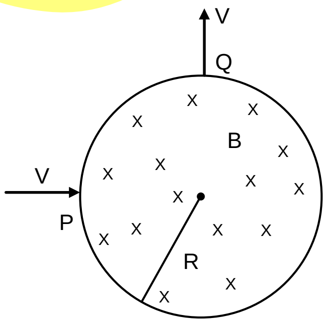


9. Two projectile are thrown at the same time from two different points. The projectile thrown from the origin has initial velocity $3\hat{i}+3\hat{j}$ with respect to earth. The projectile has initial velocity $a\hat{i}+b\hat{j}$ with respect to earth thrown from the point (10, 5). ($\hat{i}$ is a unit vector along horizontal, $\hat{j}$ along vertical). If the projectile collides after two second, then the

(A) value of a is -2
(B) value of a is $\frac{1}{2}$
(C) value of b is $\frac{1}{2}$
(D) value of b is –2

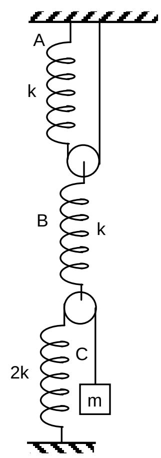
10. A particle of charge q mass m enters normally (at point P) in a region of magnetic field with speed v. It comes out normally from Q after time T as shown in figure. The magnetic field B is present only in region of radius R and is constant and uniform. Initial and final velocities are along radial direction and they are perpendicular to each other. For this to happen, which of the following expression(s) is/are correct?

(A) $B < \frac{2mv}{qR}$ (B) $T < \frac{\pi m}{Bq}$ (C) $T = \frac{\pi m}{2Bq}$ (D) $B < \frac{mv}{2qR}$



11. In the given figure, the block is attached with a system of three ideal springs A, B, C. The block is displaced by a small distance x from its equilibrium position vertically downwards and released. T represents the time period of small vertical oscillations of the block. Then (pulleys are ideal)

(A) $T = 2\pi\sqrt{\frac{11m}{2k}}$
(B) the deformation of the spring A is (2/11) times the displacement of the block.
(C) the deformation of the spring C is (1/11) times the displacement of the block.
(D) the deformation of the spring B is (4/11) times the displacement of the block.



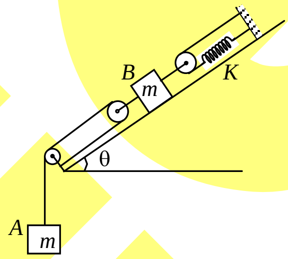
Space For Rough Work

12. The distance between a screen and an object is 120 cm. A convex lens is placed closed to the object and is moved along the line joining object and screen, towards the screen. Two sharp images of the object are found on the screen. The ratio of magnification of two real images is 1 : 9. Then,
(A) focal length of the lens is 22.5 cm.
(B) smaller image is brighter than the larger one.
(C) larger image is brighter than the smaller one.
(D) brightness of both the images is same.

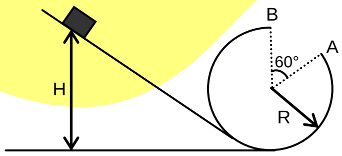
PART – B (Maximum Marks: 9)

This section contains ***THREE (03)*** *questions. The answer to each question is a* ***NON-NEGATIVE INTEGER.***

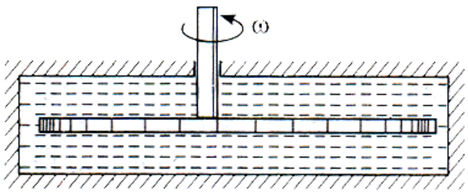
1. Two blocks *A* and *B*, each of mass *m* are connected by means of a pulley-spring system on a smooth inclined plane of inclination θ as shown in the figure. All the pulleys and spring are ideal. Now, *B* is slightly displaced from its equilibrium position. It starts to oscillate. Time period of oscillation of *B* is $n\pi$, then find n. (Take $m = 4$ kg, $K = 5$ N/m, $\pi = 3.14$)



2. A small particle slides from height H = 45 cm as shown and then loops inside the vertical loop of radius R from where a section of angle $\theta = 60°$ has been removed. If R = (1/N) meter, such that after losing contact at A and flying through the air, the particle will reach at the point B. Find N. Neglect friction everywhere.



3. A thin circular disc of radius R is made to rotate with a constant angular speed ω within a oil filled (coeff. of viscosity η) cylindrical box as shown in the figure. The clearance between the disc & the horizontal planes of the cylindrical box is very small & is equal to h. Considering that the vertical side of the cylindrical box is almost in contact with the disc, the power to be supplied to the system to maintain the constant angular speed is $\frac{\pi\eta\omega^x R^y}{h}$. Then x + y is



Space For Rough Work

PART – B (Maximum Marks: 15)

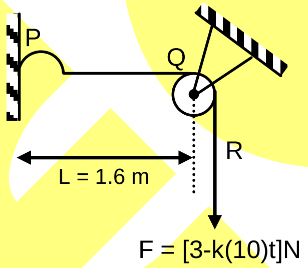
(Numerical **Type)**

This section contains ***Five (05)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

4. A ring of mass 3.15 kg is rolling without slipping with linear velocity 1 m/sec on a smooth horizontal surface. A rod of same mass is fitted along its one diameter. Find total kinetic energy of the system (in J).

5. Potential energy of a particle moving along x-axis is given by U = $\frac{x^3}{3} - \frac{9x^2}{2} + 20x$. Find out position of stable equilibrium state.

6. As shown in figure light string PQR is stretched by force F = (3 – 10kt) N, where k is a constant and t is time in second. At time t = 0, a pulse is generated at the end P of the string. For the value of k (in N/s) if the value of force becomes zero as the pulse reaches point Q. Given mass per unit length of string is 0.03 kg/m.



7. In a moving coil galvanometer, torque on the coil can be expressed as $\tau \propto i$, where i is current through the wire. The rectangular coil of the galvanometer having number of turns N, area A and moment of inertia I is placed in magnetic field B. The torsion constant of spring in the galvanometer is C and charge Q is passed through the coil almost instantaneously. If the maximum angle through which coil deflected is $\frac{KNABQ}{\sqrt{3CI}}$. (Ignore the damping in mechanical oscillations). Then the value of K is

8. 2 kg of ice at –22.5^{0}C is mixed with 2.5 kg of water at 25^{0}C in an insulating container. If the specific heat of ice and water are 0.5 cal/gm°C and 1 cal/gm°C respectively. Latent heat of fusion of ice = 80 cal/gm. Find the amount of water present in the container (In kg).

Space For Rough Work

SECTION – II : CHEMISTRY

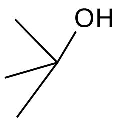
PART – A (Maximum Marks: 12)

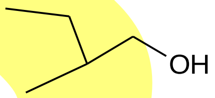
*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

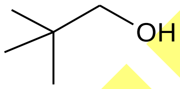
1. Choose correct statement from the following
(A) Bond angle: $OCl_2 > BCl_3 > NCl_3$
(B) Strength of backbonding: $BCl_3 > OCl_2 > NCl_3$
(C) Bond order: $NCl_3 > OCl_2 > BCl_3$
(D) Dipole moment: $NCl_3 > BCl_3 > OCl_2$

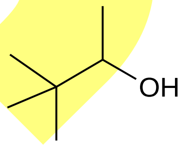
2. $R-C\equiv C-R' \xrightarrow{LiAlH_4} RCH=CHR'$
In the above reaction, R and R' should be
(A) R = CHO and R' = CH_3
(B) R = CH_3 and R' = CH_2OH
(C) R = C_2H_5 and R' = CO
(D) R = CH_3 and R' = COOH

3. The S – O bond order of SOF_2 is greater than that of $SOCl_2$. This is due to
(A) polarity of O – F bond is greater than that of O – Cl bond
(B) the O $\rightarrow$ S back bond in SOF_2 is stronger than that in $SOCl_2$
(C) the S $\rightarrow$ O back bond in SOF_2 is stronger than that in $SOCl_2$
(D) the $\angle$FSF is greater than $\angle$ClSCl

4. Which alcohol can't be formed by acidic hydration of any alkene?
(A) 

(B) 

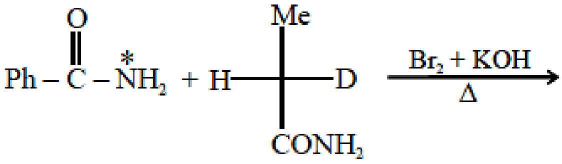
(C) 

(D) 

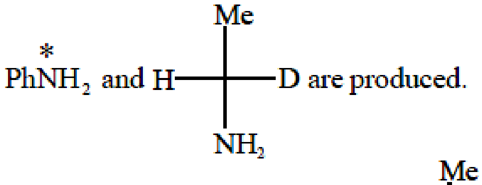
Space For Rough Work

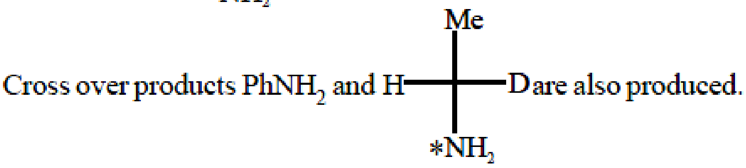
PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

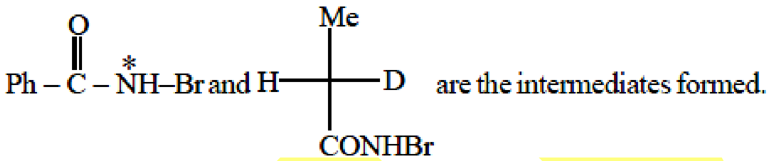
5. 

Which of the following statement(s) is/are correct regarding the given reaction?

(A) 

(B) 

(C) Migration of –R(alkyl group or aryl) of R–$CONH_2$ takes place with retention of configuration.

(D) 

6. Compound A(C_5H_7OCl) reacts rapidly with ethanol and catalytic amount of acid to form a pleasant smelling substance B($C_7H_{12}O_2$). A also reacts with H_2O to form C with neutralization equivalent of 100. A, B and C all react with Br_2 water. Acid C, which can also be obtained by acidic hydrolysis of B is oxidized to new acid D($C_4H_6O_3$) and CO_2. D on heating with soda lime gives acetone. Identify A to D.

(A) A = $CH_3-C(=CH_2)-CH_2-C(=O)-Cl$

(B) B = $CH_3-C(=CH_2)-CH_2-C(=O)-OEt$

(C) C = $CH_3-C(=CH_2)-CH_2-C(=O)-COOH$

(D) D = $CH_3-C(=O)-CH_2-COOH$

7. Choose the correct statement

(A) d_{yz} orbital lies in the xz plane

(B) p_z orbital lies along the x axis

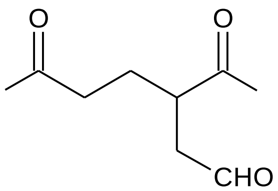
(C) lobes of $d_{x^2-y^2}$ orbital are at 90º with the z axis

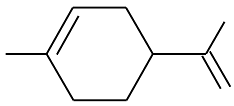
(D) lobes of d_{xy} orbital are at 90º with the z axis

Space For Rough Work

8. The spectra of which complex(s) is/are spin forbidden as well as Laporte forbidden?
(A) $[Mn(H_2O)_6]^{2+}$
(B) $[Fe(H_2O)_6]^{3+}$
(C) $[Co(CN)_6]^{3-}$
(D) $[Cr(NH_3)_6]^{3+}$

9. A hydrocarbon X ($C_{10}H_{16}$) upon catalytic hydrogenation gives 4-methyl-1-isopropyl cyclohexane. Also (X) upon ozonolysis followed by hydrolysis in the presence of Zn gives CH_2O and



The correct statement(s) concerning (X) is/are
(A) Structure of (X) is 
(B) (X) has two chiral carbon.
(C) (X) has one chiral carbon.
(D) With excess of HCl, (X) gives racemic dichloride.

10. Which of the following reactant(s) produce(s) racemic mixtures when treated with HBr?
(A) Cis-2-butene
(B) Trans-2-butene
(C) 1-butene
(D) Isobutene

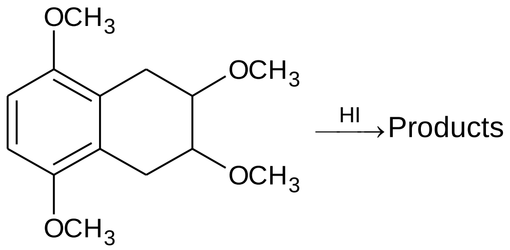
11. Which of the following increases by increasing the temperature of a chemical reaction?
(A) Rate constant
(B) Half-time
(C) Rate of reaction
(D) Temperature coefficient

12. Which reaction produces cyclic products?
(A) $CH_3COCH_3 \xrightarrow[\text{Heat}]{\text{Conc.}H_2SO_4}$
(B) $COONa-CH_2-CH_2-CH_2-COONa \xrightarrow{\text{Kolbe's electrolysis}}$
(C) $C_6H_5COONa \xrightarrow[\text{Heat}]{\text{Soda lime}}$
(D) $CH_3-\underset{CH_3}{\overset{OH}{C}}-CH_3 \xrightarrow{Cu/300^{\circ}C}$

Space For Rough Work

PART – B (Maximum Marks: 9)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

1. 

If x moles of HI can be completely consumed by one mole of the above compound, what is the value of x?

2. $FeCl_3 + K_4[Fe(CN)_6] \rightarrow Fe_4[Fe(CN)_6]_3$
If 'x' moles of KSCN will be completely consumed by one mole of product in above reaction, to produce blood red colouration, what is the value of x/3?

3. If the pH of 0.1 M aqueous solution of NH_4NO_3 is x, what is the value of x? [K_b of $NH_4OH = 10^{-5}$]

PART – B (Maximum Marks: 15)

(Numerical **Type)**

This section contains ***Five (05)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

4. 2.5 mole of KCl is added to one Kg of water at 1 atm. At what temperature in kelvin unit will the water boil if the degree of dissociation of KCl is 0.8? [K_b of H_2O = 0.52 K kg mol^{-1}]

5. 14 g of a metal oxide of formula MO is added to a container containing 500 mL of 1.25 M HCl solution. After complete reaction, the reaction mixture is treated with 250 mL of 0.5 M NaOH to neutralize the excess acid. If the atomic mass of M is expressed as (2x + 15). What is x?

6. The successive ionization energies of a s-block element are
8.6, 12.9, 1630.2, 2406.2 eV, etc
If atomic mass of the elements is 39.8 g mol^{-1}, what will be the molar mass of it's normal oxide?

Space For Rough Work

7. The molecular mass of a polyhydric aliphatic alcohol, increases by 252 unit if it reacts with $CH_3COCl/(CH_3CO)_2O$. if the number of OH groups present in the alcohol is x, what is the value of $\frac{x}{4}$?

8. $A_2(g) \longrightarrow A(g) + A(g)$

The heat of reaction of above decomposition of a diatomic molecule $A_2(g)$, into monoatomic gases A(g) is 40.5 kJ mol^{-1} at 320 K. What will be the enthalpy change of the reaction at 330 K in kJ mol^{-1} unit?

[C_P of monoatomic gases = $\frac{5R}{2}$]

[C_P of diatomic gases = $\frac{7R}{2}$]

[R = 8.3 J K^{-1} mol^{-1}]

Space For Rough Work

SECTION – Iii : MATHEMATICS

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. Let $a, b, p, q \in \mathbb{Q}$ and suppose that $f(x) = x^2 + ax + b = 0$ and $g(x) = x^3 + px + q = 0$ have a common irrational root, then
(A) $f(x)$ divides $g(x)$
(B) $g(x) \equiv xf(x)$
(C) $g(x) \equiv (x-b-q)f(x)$
(D) none of these

2. If $x = (7+4\sqrt{3})^{2n} = [x] + f$, then $x(1-f)$ is equal to
(A) 1
(B) 2
(C) 3
(D) 4

3. Consider the points $P = (-\sin(\beta-\alpha), -\cos\beta), Q = (\cos(\beta-\alpha), \sin\beta)$ and $R = (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$, where $0 < \alpha, \beta < \frac{\pi}{4}$ then
(A) P lies on the line segment RQ
(B) Q lies on the line segment PR
(C) R lies on the line segment QP
(D) P, Q, R are non – collinear

4. For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then
(A) $a + b - c > 0$
(B) $a - b + c < 0$
(C) $a - b + c > 0$
(D) $a + b - c < 0$

PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. The number of triangles that can be formed with the sides of lengths a, b and c where a, b, c are integers such that $a \le b \le c$ is
(A) $\frac{1}{4}(c+1)^2$ when c is odd
(B) $\frac{1}{2}c(c+1)$ when c is odd
(C) $\frac{1}{4}c(c+2)$ when c is even
(D) $\frac{1}{4}c^2$ when c is even

Space For Rough Work

6. If $x \in R$, and $S = 1 - C_1 \frac{1+x}{1+nx} + C_2 \frac{1+2x}{(1+nx)^2} - C_3 \frac{1+3x}{(1+nx)^3} + ... + ..$upto $(n+1)$ terms then S

(A) equals x^2
(B) equals 1
(C) equals 0
(D) is independent of x

7. A and B are two points on the x – axis and y – axis respectively. Two circles are drawn passing through the origin and having centre at A and B.
(A) Equation of the common chord is $ax - by = 0$
(B) mid – point of the common chord is $\left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$
(C) AB bisect the common chord.
(D) AB is perpendicular to the common chord.

8. The equation $\frac{b\cos x}{2\cos 2x - 1} = \frac{b + \sin x}{(\cos^2 x - 3\sin^2 x)\tan x}$ posses a solution if

(A) $b < \frac{1}{2}$
(B) $0 < b < \frac{1}{2}$
(C) $b < 0$
(D) $b = \frac{1}{2}$

9. Normals are drawn from the point P (15, 12) to the parabola $y^2 = 4x$. If the feet of the normal form a ΔABC, then which of the following is/are true.
(A) Centroid of ΔABC is $\left(\frac{26}{3}, 0\right)$
(B) Centroid of ΔABC is $\left(\frac{22}{3}, 0\right)$
(C) Area of triangle formed by tangents to the parabola at A, B and C is 35 sq. units
(D) Area of triangle formed by tangents to the parabola at A, B and C is 140 sq. units

10. Let z_1 lies on $|z| = 1$ and z_2 lies on $|z| = 2$, then which of the following is/are true?
(A) maximum value of $|z_1 + z_2|$ is 3
(B) minimum value of $|2z_1 - z_2|$ is 0
(C) maximum value of $|2z_1 + z_2|$ is 4
(D) minimum value of $|2z_1 - 3z_2|$ is 5

11. If pair of tangents are drawn from centre C of the circle $x^2 + y^2 - 5x = 0$ to the nearer branch of the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ to touch it at point A and B, then
(A) product of the slopes of lines CA and CB is $\frac{-64}{75}$
(B) product of the slopes of lines CA and CB is $\frac{-32}{25}$
(C) circumcentre of ΔABC lies inside the ΔABC.
(D) circumcentre of ΔABC lies outside the ΔABC.

Space For Rough Work

12. If $a+b+c=4$ and $a^2+b^2+c^2+3(ab+bc+ac)=21$ where $a,b,c\in\mathbb{R}$, then which of the following is/are true
(A) minimum value of $(a+b)(b+c)(a+c)$ is 18
(B) minimum value of $(a+b)(b+c)(a+c)$ is 24
(C) maximum value of $(a+b)(b+c)(a+c)$ is $\frac{490}{27}$
(D) maximum value of $(a+b)(b+c)(a+c)$ is $\frac{471}{27}$

PART – B (Maximum Marks: 9)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

1. Consider the number $N=10!$. If the number of positive divisors of N is λ, then $\frac{\lambda}{30}$ equals

2. Let L_1, L_2 and L_3 be the length of tangents drawn from a point P to the circles $x^2+y^2=4$, $x^2+y^2-4x=0$ and $x^2+y^2-4y=0$ respectively. If $L_1^4=L_2^2L_3^2+16$, then locus of P are the curves C_1 (a straight line) and C_2 (a circle). Let the circumcentre of the triangle formed by C_1 and other two lines which are inclined at an angle of 45^o with C_1 and is tangent to C_2 is (α,β), then $(\alpha+2\beta)$ equals

3. Let $S_n=\sum_{r=1}^{n} r.2^{\frac{r}{2}\left(1+(-1)^r\right)}.3^{\frac{r}{2}\left(1-(-1)r\right)}$. If $S_{20}=\alpha.3^{21}+\beta.2^{22}+\frac{391}{288}$, then the value of $\left[\frac{32\alpha-9\beta}{4}\right]$ is (where [.] denotes the greatest integer function)

PART – B (Maximum Marks: 15)

(Numerical **Type)**

This section contains ***Five (05)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

4. If $[16\cos x+12\sin x]=4k+10$, then the maximum value of k is _______________

Space For Rough Work

5. The length of the semi – major axis of an ellipse is 8 and the eccentricity is $\frac{1}{2}$. If Δ denotes the area of the rectangle formed by joining the vertices of the latera recta of the ellipse then the value of $\frac{\Delta}{120}$ is equal to

6. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangents to C_2 and C_1, then the one fifth of the radius of the circle C is

7. Value of x if x satisfies the equation $\log_{1-x}(3) - \log_{1-x}(2) = \frac{1}{2}$ is

8. The sum of all the integral roots of $(\log_5 x)^2 + \log_{5x}\left(\frac{5}{x}\right) = 1$ is ________________

Space For Rough Work

QP Code: 100187
Answers
SECTION – I : PHYSICS
PART – A

1.	**B**	2.	**D**	3.	**A**	4.	**C**
5.	**ABD**	6.	**AD**	7.	**AC**	8.	**ACD**
9.	**AC**	10.	**ABC**	11.	**ABCD**	12.	**AB**

PART – B

1.	**2**	2.	**5**	3.	**6**	4.	**5.25**
5.	**5**	6.	**1.25**	7.	**1.73**	8.	**3**

SECTION – II : CHEMISTRY
PART – A

1.	B	2.	B	3.	B	4.	C
5.	**ACD**	6.	**ABD**	7.	**CD**	8.	AB
9.	AC	10.	ABC	11.	ACD	12.	ABC

PART – B

1. 7
2. 4
3. 5
4. 375.34 (Range 375.2 to 375.4)
5. 12.5 (Range12.4 to 12.6)
6. 55.8(Range 55.7 to 55.9)
7. 1.5
8. 40.62(Range 40.6 – 40.7)

SECTION – III : MATHEMATICS
PART – A

1.	A	2.	A	3.	D	4.	A
5.	AC	6.	CD	7.	ABCD	8.	ABC
9.	AC	10.	ABC	11.	AC	12.	AC

PART – B

1.	9	2.	0	3.	4	4.	2.50
5.	0.8	6.	1.6	7.	–1.25	8.	6

Answers & Solutions

SECTION – I : PHYSICS

PART – A

1. **B**

Sol. From range:

$\frac{2u_x u_y}{g} = \frac{6\sqrt{3}}{5}$ $\qquad u_x u_y = 6\sqrt{3}$ ...(1)

Resultant of 2 and u_x $\Rightarrow$ $\sqrt{4+u_x^2}$

Whose resultant with u_y makes an angle 37°.

So, $\frac{u_y^2}{4+u_x^2} = \frac{9}{16}$ ...(2)

Solving (1) + (2), we will get $u_x = 2\sqrt{3}, u_y = 3$ & $u_z = 2\,m/s$

So net $\Rightarrow$ $\sqrt{u_x^2+u_y^2+u_z^2}$ = 5 m/s.

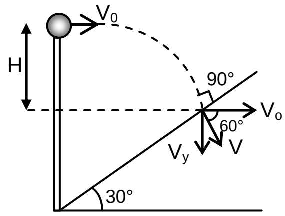
2. **D**

Sol. $\phi_B = 0$

$\because\ \vec{B} \perp \vec{S}$; $\therefore\ \frac{d\phi_B}{dt} = 0$

3. **A**

Sol. $v_y = \sqrt{2gH}$



$\frac{v_y}{v_x} = \frac{\sqrt{2gH}}{v_o} = \tan 60^o = \sqrt{3}$

$v_0 = \sqrt{\frac{2gH}{3}}$

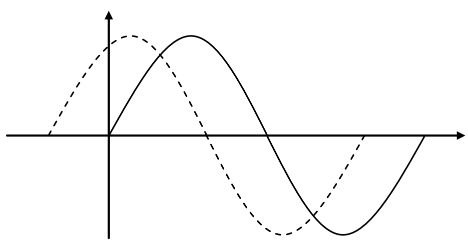
4. **C**

Sol. $i = \frac{2I_o}{T}t$ [0 < t < t/2]

$i_{rms} = \frac{\frac{2I_o}{T}\sqrt{\int_0^{T/2} t^2 dt}}{\sqrt{T/2}} = \frac{I_o}{\sqrt{3}}$.

5. **ABD**

Sol. Results can be obtained by looking at shape of string after a short time (shown dotted). Also, each particle of string executes SHM about mean position which is on x-axis.



6. **AD**

Sol. Use ampere`s law to find magnetic field due to solid cylinder current = $\mu_0 Jd/2$

Due to cylinder on its own axis is 0

Applying superposition principle we can get the required answers.

7. **AC**

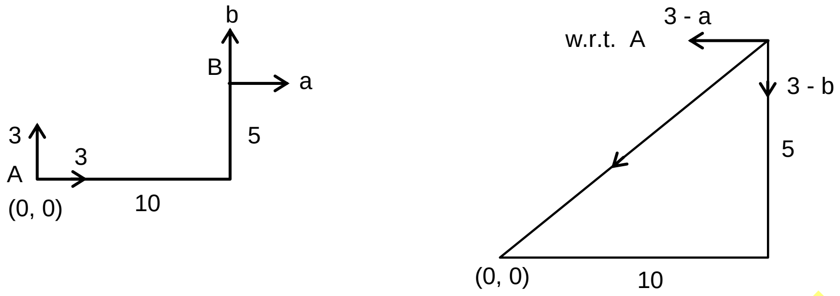
Sol. Force on branch BC is coming out of the plane of paper and force on AD is going into the plane of paper and net force on the loop is zero.

8. **ACD**

Sol. $\vec{F}_m = q\left(\vec{v}\times\vec{B}\right)$ and apply Newton's second law of motion.

9. **AC**

Sol.



In vertical direction $\left(s = \frac{d}{t}\right)$

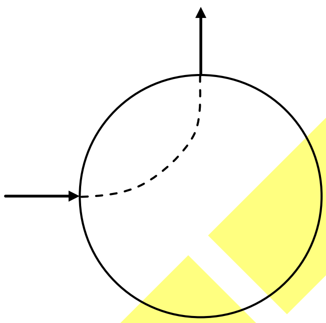
$3 - b = \frac{5}{2} \rightarrow b = \frac{1}{2}$

In horizontal direction

$3 - a = \frac{10}{2} \rightarrow a = -2.$

10. **ABC**

Sol.



So, its radius is also R.

So, $R = \frac{mv}{qB} \rightarrow B = \frac{mv}{qR}$

$T = \frac{2\pi M}{4qB} = \frac{\pi m}{2qB}$

So, A, B & C are correct.

11. **ABCD**

Sol. Let elongation in spring A, B and C be x_1, x_2 and x_3 respectively.

Considering spring forces and constraint relations

$x_2 = 4x_3$ ...(i)

$x_2 = 2x_1$ ...(ii)

and $x_1 + 2x_2 + x_3 = x$ ...(iii)

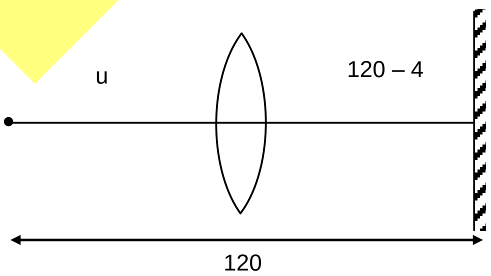
$\Rightarrow x_1 = \left(\frac{2}{11}\right)x \;;\; x_2 = \left(\frac{4}{11}\right)x \;;\; x_3 = \left(\frac{1}{11}\right)x$

Also, F = $2K\left(\frac{x}{11}\right)$

$\Rightarrow T = 2\pi\sqrt{\frac{11m}{2k}}$

12. **AB**

Sol.



For two different positions of lens

$1 = \frac{u}{120 - u} \;;\; 9 = \frac{120 - u}{u}$

$\frac{1}{9} = \frac{u^2}{(120 - u)^2} \rightarrow \frac{1}{3} = \frac{u}{120 - u}$

u = 30 ; v = 90

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{90} + \frac{1}{30}$$

P = 22.5

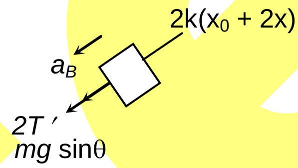
PART – B

1. **2**

Sol. Let elongation of spring be x_0 in equilibrium. Then,

$$2T + mg\sin\theta = 2kx_0 \qquad \ldots(i)$$

and $T = mg \qquad \ldots(ii)$

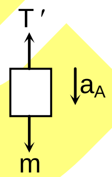


Let Block B is displaced by x down the inclination

F.B.D. of B

$$-ma_B = 2k(x_0 + 2x) - 2T' - mg\sin\theta \qquad \ldots(iii)$$

F.B.D. of *A*



mg – T ′ = ma_A

Also, $a_A = 2a_B$

$$T' = mg - 2ma_B$$

$$-ma_B = 2kx_0 + 4kx - 2mg + 4ma_B - mg\sin\theta$$

$$-ma_B = 4kx + 4ma_B$$

$$a_B = -\frac{4k}{5m}x$$

$$\therefore \qquad T = 2\pi\sqrt{\frac{5m}{4k}}$$

T = 6.28 s.

2. **5**

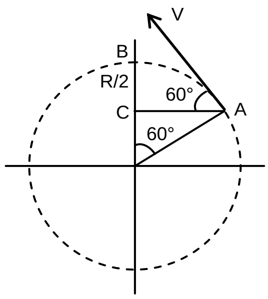
Sol. Conservation of mechanical energy

$$mgH = mgh + \frac{1}{2}mv^2$$

$$0.45\ mg = 3mg\frac{R}{2} + \frac{1}{2}mv^2$$

$$9 - 30R = v^2 \qquad \ldots(i)$$

$$AC = \frac{R\sqrt{3}}{2}$$



. For reaching at B let particle takes t sec.

$$\frac{v}{2} \times t = \frac{R\sqrt{3}}{2} \qquad \ldots(ii)$$

$$\frac{v\sqrt{3}}{2}t - \frac{1}{2} \times g \times t^2 = \frac{R}{2} \qquad \ldots(iii)$$

On solving, $v^2 = 15R$ by eq. (i)

$$9 - 30R = 15R \qquad \Rightarrow \qquad R = \frac{9}{45} \times 100 = 20\text{ cm}$$

= 1/5 meter.

3. **6**

Sol. $dF = \left[\eta(2\pi r dr)\frac{\omega r}{h}\right]2$

dP = dF. V

$= \frac{4\eta\pi}{h}(\omega r)^2 r\,dr$

$\int_o^P dP = \frac{4\eta\pi\omega^2}{h}\int_o^R r^3 dr = \frac{\eta\pi\omega^2 R^4}{h}$

x + y = 6

4. **5.25**

Sol. K.E. = $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}\frac{m(2r)^2}{12}w^2 + \frac{1}{2}mr^2w^2$ = 5.25.

5. **5**

Sol. At stable equilibrium, U is minimum.

$\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} > 0$

$= \frac{1}{dx}\left(\frac{x^3}{3} - \frac{ax^2}{2} + 20x\right) = 0.$

$\Rightarrow$ $x^2 - 9x + 20 = 0$. $\Rightarrow$ (x – 5) (x – 4) = 0.

x = 5 and x = 4 are points of equilibrium.

And U minimum when $\frac{d^2U}{dx^2} > 0$. i.e. at x = 5.

6. **1.25**

Sol. For pulse $\frac{dx}{dt} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{3-10kt}{\mu}}$

$\Rightarrow \int_o^L dx = \int_o^{t_o}\sqrt{\frac{3-10\,kt}{\mu}}\,dt$

And $3 - 10\,k\,t_o = 0$

7. **1.73**

Sol. τ = NIAB = Cθ

$\theta = \frac{NABQ}{C}$

So, $K = \sqrt{3}$

K = 1.73

8. **3**

Sol. 2000 × 15 × 0.5 + m × 80 = 2500 × 25 × 1

m = 500 gm

Water = 2.5 kg + 0.5 kg = 3 kg.

SECTION – II : CHEMISTRY
PART – A

1. B

Sol. Back bonding in BCl_3 is easier as it is electron deficient compound. Between OCl_2 and NCl_3 the electronegative difference between atoms in OCl_2 is greater than between atoms in NCl_3. $\therefore OCl_2$ forms stronger back bond than NCl_3.

2. B

Sol. R or R′ can't be CHO, CO, COOH as they are reduced by $LiAlH_4$.

3. B

Sol. The O $\rightarrow$ S back bond in SOF_2 is stronger than O $\rightarrow$ S in $SOCl_2$. Since F is more electronegative than Cl, the S atom in SOF_2 strongly attract the lone pair of oxygen to form $p\pi$ - $d\pi$ back bond.

4. C

Sol. No alkene can produce the alcohol in (C) because loss of H_2O can't form any alkene.

5. **ACD**

Sol. This is Hoffmann's bromamide reaction.

6. **ABD**

Sol. The alkene part of the carboxylic acid derivatives reacts with Br_2/H_2O.

7. **CD**

Sol. $d_{x^2-y^2}$ lie along X- and Y- axis, which are perpendicular to Z-axis.

8. AB

Sol. Due to $t_{2g}^3e_g^2$ crystal field electronic configuration of Mn^{2+} and Fe^{3+} ions

9. AC

Sol. Cleavage of C = C bond takes place during ozonolysis.

10. ABC

Sol. Racemic mixture is formed from compounds containing chiral carbon atom.

11. ACD

Sol. $t_{½}$ only decreases by increasing temperature.

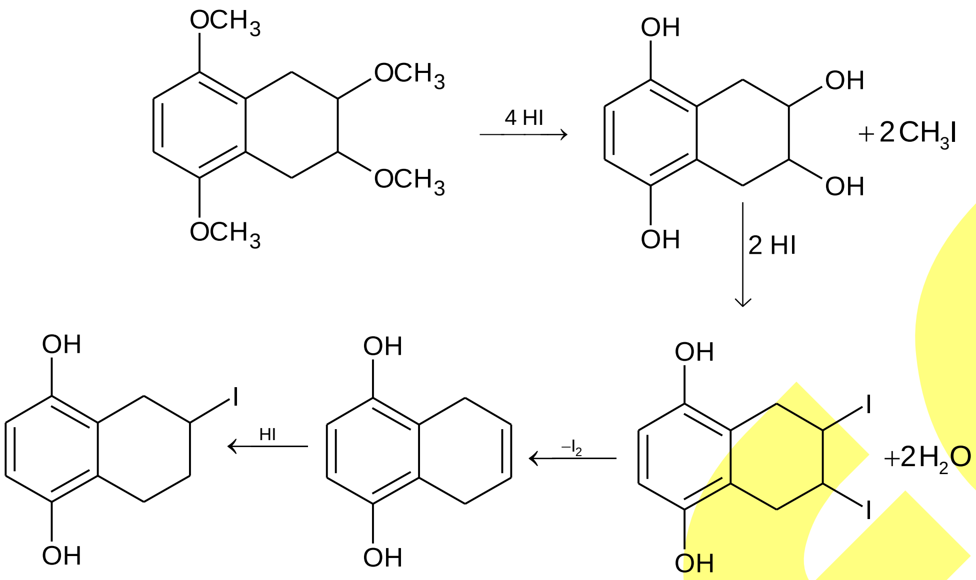
12. ABC

Sol. In A, B and C the products are mesitylene, cyclohexane and benzene respectively.

PART – B

1. 7

Sol.



2. 4

Sol. $4Fe^{3+} + 12SCN^{-} \longrightarrow 4[Fe(SCN)_3]$

One mole Fe^{3+} consumes three moles of SCN^- ion. Four moles of Fe^{3+} will consume 4 × 3 = 12 moles $Fe(SCN)_3$.

∴ x = 4

3. 5

Sol. $pH = \frac{1}{2}\left[p^{k_w} - p^{k_b} - \log C\right] = \frac{1}{2}\left[14 - 5 - \log 10^{-1}\right] = \frac{1}{2} \times 10 = 5$

4. 375.34 (Range 375.2 to 375.4)

Sol. $\Delta T_b = iK_b m = 1.8 \times 0.52 \times 2.5 = 2.34$

∴ $T_b - T_b^0 = 2.34$

or $T_b - 100^oC = 2.34$

∴ $T_b = 102.34^oC = 375.34$ K

5. 12.5 (Range12.4 to 12.6)

Sol. M_{eq} of HCl added = 500 × 1.25 = 625

M_{eq} of NaOH used = 250 × 0.5 = 125

M_{eq} of HCl reacted with MO = 625 – 125 = 500

500 = M_{eq} of MO = $\frac{W}{E} \times 1000 = \frac{14}{E} \times 1000 \Rightarrow E = 28, \text{molar mass} = 28 \times 2 = 56$

∴At weight of MO = 56 – 16 = 40

∴ 2x + 15 = 40

∴ x = 12.5

6. 55.8(Range 55.7 to 55.9)

Sol. The atom contains two unpaired electrons.

∴ Formula of oxide = MO

Molar mass = 39.8 + 16 = 55.8

7. 1.5

Sol.

$$ROH + CH_3COCl \xrightarrow{(CH_3CO)_2O} R - O - \overset{\overset{\displaystyle O}{\|}}{C} - CH_3 + HCl$$

For one OH group (OH → $OCOCH_3$), loss of one H and gain of $COCH_3$ takes place.

$\therefore$ For one OH group, the mass increases by 42 units.

$\therefore$ Number of OH group = $\frac{252}{42} = 6 = x$

$\therefore \frac{x}{4} = \frac{6}{4} = 1.5$

8. 40.62(Range 40.6 – 40.7)

Sol. $\Delta H_2 = \Delta H_2 + n\Delta C_P dT$

$= 40.5 + n\Delta C_P \int_{320}^{330} dT$

$= 40.5 + 0.1(1.5\ R)\ (330-320)$

$= 40.5 + 1.5R \times 10 = 40.5 + 15R$

$= 40.5 + 15 \times 8.3 = 40.62$

SECTION – III : MATHEMATICS

PART – A

1. A

Sol. Let $\alpha \in \mathbb{R} - \mathbb{Q}$ be a common root of $f(x)=0$ and $g(x)=0$. Then $\alpha^2 = -a\alpha - b$. Substituting this in $\alpha^3 + p\alpha + q = 0$, we get $(a^2 - b + p)\alpha + ab + q = 0$

As α is irrational and $a, b, p, q \in \mathbb{Q}$, $p = b - a^2$, $q = -ab$. This gives, $g(x) = (x-a)f(x)$.

2. A

Sol. We have $7 - 4\sqrt{3} = \dfrac{1}{7+4\sqrt{3}}$

$\therefore 0 < 7 - 4\sqrt{3} < 1 \Rightarrow 0 < \left(7-4\sqrt{3}\right)^{2m} < 1$

Let $F = \left(7-4\sqrt{3}\right)^{2n}$.

Then $x + F = \left(7+4\sqrt{3}\right)^{2n} + \left(7-4\sqrt{3}\right)^{2n}$

$= 2\left[{}^{2n}C_0 7^{2n} + {}^{2n}C_2 7^{2n-2}\left(4\sqrt{3}\right)^2 + {}^{2n}C_4 7^{2n-4}\left(4\sqrt{3}\right)^4 + \ldots + {}^{2n}C_{2n}\left(4\sqrt{3}\right)^{2n}\right]$

$= 2m$, where m is some positive integer.

$\Rightarrow [x] + f + F = 2m \Rightarrow f + F = 2m - [x]$

Since $0 \le f < 1$ and $0 < F < 1$, we get $0 < f + F < 2$. Also, since $f + F$ is an integer, we must have $f + F = 1$. Thus, $x(1-f) = xF = \left(7+4\sqrt{3}\right)^{2n}\left(7-4\sqrt{3}\right)^{2n}$

$= (49-48)^{2n} = 1^{2n} = 1.$

3. D

Sol. Put $\beta - \alpha - \phi$ and consider the determinant $\Delta = \begin{vmatrix} -\sin\phi & -\cos\beta & 1 \\ \cos\phi & \sin\beta & 1 \\ \cos(\phi+\theta) & \sin(\beta-\theta) & 1 \end{vmatrix}$

Using $R_3 \to R_3 - \cos\theta R_2 - \sin\theta R_1$

$\Delta = \begin{vmatrix} -\sin\phi & -\cos\beta & 1 \\ \cos\phi & \sin\beta & 1 \\ 0 & 0 & 1-\cos\theta-\sin\theta \end{vmatrix}$

$= (1-\cos\theta-\sin\theta)\cos(\phi+\beta)$

$= (1-\cos\theta-\sin\theta)\cos(2\beta-\alpha)$

$= \left[1-\sqrt{2}\sin\left(\theta+\dfrac{\pi}{4}\right)\right]\cos(2\beta-\alpha)$

As $0 < \theta < \dfrac{\pi}{4} \Rightarrow \dfrac{\pi}{4} < \theta + \dfrac{\pi}{4} < \dfrac{\pi}{2}$

$\Rightarrow \dfrac{1}{\sqrt{2}} < \sin\left(\theta+\dfrac{\pi}{4}\right) < 1$ and $0 < \alpha, \beta < \dfrac{\pi}{4} \Rightarrow \dfrac{-\pi}{4} < 2\beta - \alpha < \dfrac{\pi}{2}$

$\Rightarrow \cos(2\beta-\alpha) \ne 0$

Thus $\Delta \ne 0$ and the points *P*, *Q*, *R* are non – collinear.

4. A

Sol. The lines $ax + by + c = 0$ and $bx + ay + c = 0$ intersect in $\left(\dfrac{-c}{a+b}, \dfrac{-c}{a+b}\right)$.

Distance between (1, 1) and $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$

$= \sqrt{2}\left|1+\frac{c}{a+b}\right| = \sqrt{2}\frac{a+b+c}{a+b}$.

As $\sqrt{2}\frac{a+b+c}{a+b} < 2\sqrt{2}$, we get $a+b+c < 2(a+b)$

$\Rightarrow a+b-c > 0$.

5. AC

Sol. Let $c = 2m+1$ where m is a positive integer. Since $c < a+b$ and $a \le b$, we get $c < 2b \Rightarrow b > \frac{1}{2}c = m+\frac{1}{2}$. Thus, b can take values from $m+1$ to $2m+1$.

If $b = 2m+1$, a can be $1, 2, ...(2m+1)$. Thus, there are $(2m+1)$ values.

If $b = 2m$, a can be 2, 3,….2m. In this case a can take $(2m-1)$ values. And so on. When $b = m+1$, a can take just one value viz. $(m+1)$. Thus, there are $(2m+1)+(2m-1)+....+1 = (m+1)^2$.

6. CD

Sol. Putting $\frac{1}{1+nx} = y$, we can write $S = \left[C_0 - C_1 y + C_2 y^2 - ... + (-1)^n C_n y^n\right]$

$-xy\left[C_1 - 2C_2 y + 3C_2 y^2 - ... + (-1)^{n-1} nC_n y^{n-1}\right]$

$= (1-y)^n + xy\frac{d}{dt}\left\{(1-t)^n\right\}_{t=y}$

$= (1-y)^n - nxy(1-y)^{n-1}$

$= (1-y)^{n-1}[1-y-nxy]$

$= (1-y)^{n-1}\left[1-(1+nx)y\right] = 0$.

7. ABCD

Sol. Let OA = a and OB = b (figure), so that the equation of the line AB is $\frac{x}{a}+\frac{y}{b} = 1$. Equations of the circles passing through the origin and having centres at A (a, 0) and B (0, b) are, respectively.

$x^2 - 2ax + y^2 = 0$ (1)

and $x^2 - 2by + y^2 = 0$ (2)

so that the equation of the common chord OP is

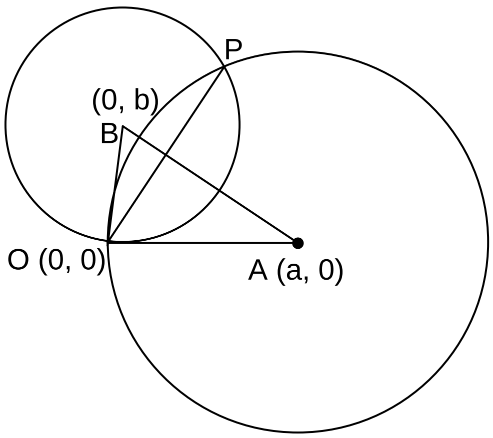
$ax - by = 0$ (3)

which is perpendicular to AB

This has one end at the origin O and the other end P is given by solving (2) and (3).

$\left(\frac{b}{a}y\right)^2 - 2by + y^2 = 0$

$\Rightarrow y = \frac{2a^2 b}{a^2+b^2} \Rightarrow x = \frac{2ab^2}{a^2+b^2}$



Therefore, the mid – point of the common chord OP is $\left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$ which lies on AB

8. ABC

Sol. The conditions for the existence of a solution are

1. $2\cos 2x - 1 \neq 0 \Rightarrow \cos 2x \neq \frac{1}{2} \Rightarrow 2x \neq \frac{\pi}{6}$

2. $\tan x \neq 0 \Rightarrow x \neq \pm\frac{\pi}{2}$

3. $\cos^2 x - 3\sin^2 x \neq 0 \Rightarrow \tan^2 x \neq \frac{1}{3} \Rightarrow x \neq \pm\frac{\pi}{6}$

As $2\cos 2x - 1 = 2\left(\cos^2 x - \sin^2 x\right) - 1$

$= 2\left(\cos^2 x - \sin^2 x\right) - \left(\cos^2 x + \sin^2 x\right)$

$= \cos^2 x - 3\sin^2 x$

So the given equation can be written as $b\sin x = b + \sin x$

$\Rightarrow \quad \sin x = \frac{b}{b-1}$

Which is possible if $-1 \leq \frac{b}{b-1} \leq 1$

$\Rightarrow \quad \frac{2b-1}{b-1} \geq 0$ and $\frac{1}{b-1} \leq 0$

$\Rightarrow \quad 2b - 1 \leq 0 \Rightarrow b \leq \frac{1}{2}$

But for $b = \frac{1}{2}, \sin x = -1$ which is not possible as $x \neq \frac{-\pi}{2}$.

Hence equation has a solution if $b < \frac{1}{2}$.

9. AC

Sol. $tx + y = 2t + t^3$ passes through (13, 12) $t^3 - 13t - 12 = 0$

$\Rightarrow t^2(t+1) - t(t+1) - 12(t+1) = 0$

$(t+1)(t-4)(t+3) = 0$

A, B, C $= \left(t^2, 2t\right)$

$A(1,-2); B(16,8); C(9,-6)$

Centroid $= \left(\frac{26}{3}, 0\right)$, Area $= \frac{1}{2}\begin{vmatrix} 1 & -2 & 1 \\ 16 & 8 & 1 \\ 9 & -6 & 1 \end{vmatrix}$

$A = 70$

Area of triangle formed by tangents $= \frac{70}{2} = 35$.

10. ABC

Sol. $|z_1| = 1$ and $|z_2| = 2$

(A) $|z_1 + z_2| \le |z_1| + |z_2| \le 3$

(B) $|2z_1 - z_2| \ge |2|z_1| - |z_2|| \ge 0$

(C) $|2z_1 + z_2| \le 2|z_1| + |z_2| \le 4$

(D) $|2z_1 - 3z_2| \ge |2|z_1| - 3|z_2|| \ge 4$

11. AC

Sol. centre: $\left(\frac{5}{2}, 0\right)$; $y = mx \pm \sqrt{25m^2 - 16}$

$\frac{5}{2}m = \pm\sqrt{25m^2 - 16} \Rightarrow \frac{25m^2}{4} = 25m^2 - 16$

$\frac{75m^2}{4} = 16 \Rightarrow m^2 - \frac{64}{65} = 0$, product of slopes of CA . CB is $-\frac{64}{75}$ circumcentre of ΔABC lies inside the ΔABC

12. AC

Sol. $(a+b) + (b+c) + (a+c) = 8$

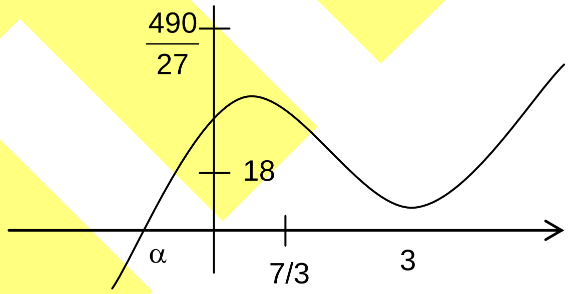
Let $a + b = \alpha, b + c = \beta, a + c = \gamma$

$\Rightarrow \alpha + \beta + \gamma = 8$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = a^2 + b^2 + c^2 + 3(ab + bc + ac)$

= 21

$\Rightarrow x^3 - 8x^2 + 21x - \lambda = 0$ where $\lambda = (a+b)(b+c)(a+c)$ these equations have 3 real roots.

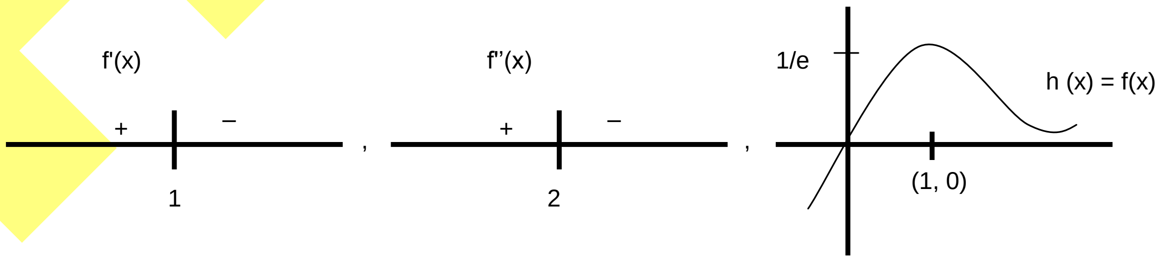


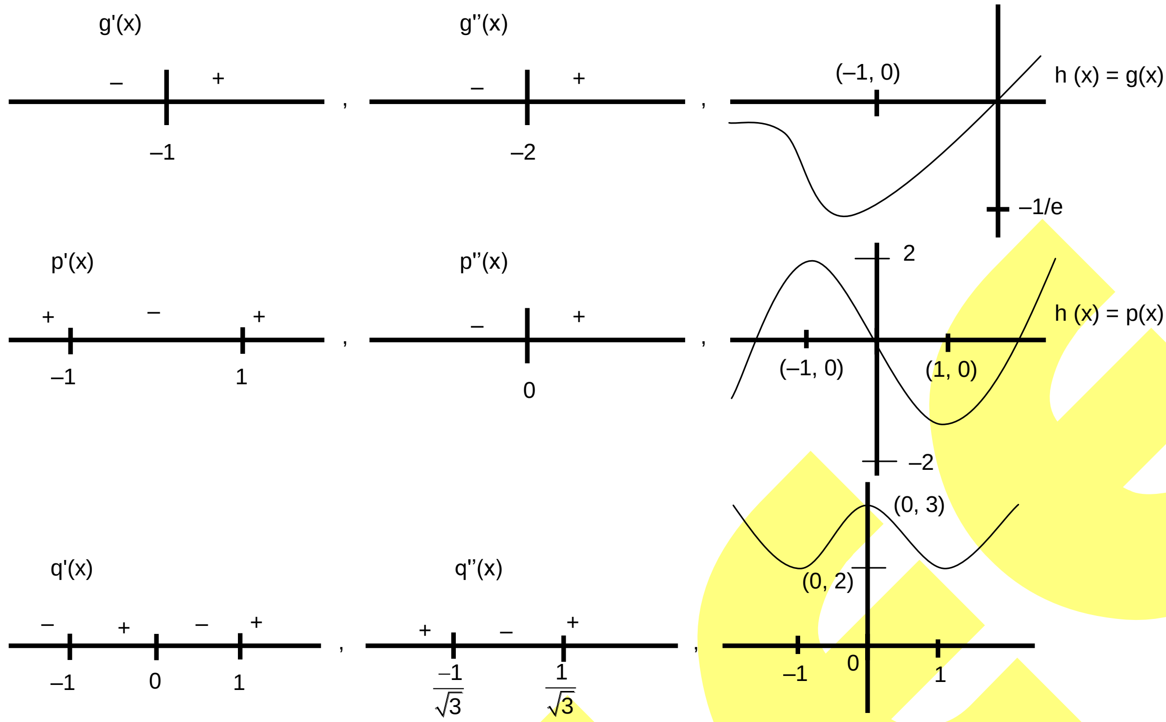
$f(x) = xe^{-x}, f'(x) = -(x-1)e^{-x}, f''(x) = e^{-x}(x-2)$

$g(x) = xe^{x}, g'(x) = (x+1)e^{x}, g''(x) = (x+2)e^{x}$

$p(x) = x^3 - 3x, p'(x) = 3x^2 - 3, p''(x) = 6x$

$q(x) = x^4 - 2x^2 + 3, q'(x) = 4x(x-1)(x+1), q''(x) = 12x^2 - 4$





(I), (i) (iv), (R)
(II), (i) (iv), (R, S)
(III), (ii) (iv), (P, Q, R)
(IV), (ii) (iii), (P, S)

PART – B

1. 9

Sol. $10! = 2^8.3^4.5^2.7$

Number of divisors $= 9 \times 5 \times 3 \times 2 = 270 = \lambda$

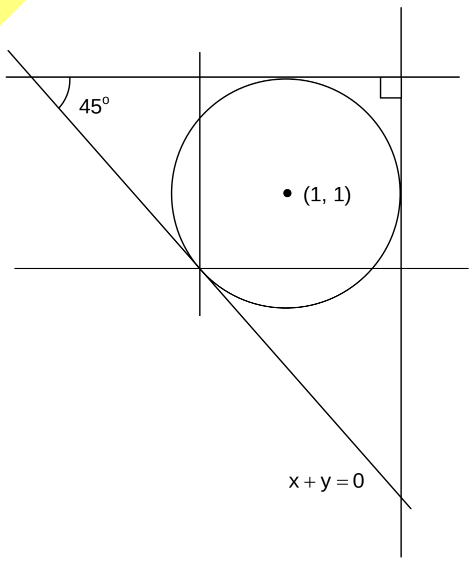
2. 0

Sol. $(h^2 + k^2 - 4)^2 = (h^2 + k^2 - 4h)(h^2 + k^2 - 4k) + 16$

$\Rightarrow (h + k)(h^2 + k^2 - 2h - 2k) = 0$ circle

$C_1 : x^2 + y^2 - 2x - 2y = 0$ and

$C_2 : x + y = 0$ circumcentre will be mid point of hypotenuse in right angle triangle $(\alpha, \beta) = (0, 0)$



3. 4

Sol. $S_n = 1.3 + 2.2^2 + 3.3^3 + 4.2^4 + ... + \text{upto n – terms}$

$S_n = (1.3 + 3.3^3 + 5.3^5 + ...) + (2.2^2 + 4.2^4 + 6.2^6 + ...)$

$S_1 = 1.3 + 3.3^3 + 5.3^5 + ... + \text{ upto 10 – terms}, \; S_1 = \frac{15}{32}(5.3^{21} + 1)$

$S_2 = 2.2^2 + 4.2^4 + 6.2^6 + ... + \text{ upto 10 – terms}, \; S_2 = \frac{58.2^{22} + 8}{9}$

$$S_{20} = S_1 + S_2 = \frac{15}{32}\left(5.3^{21}+1\right) + \left(\frac{58.2^{22}+8}{9}\right)$$
$$= \frac{75}{32}.3^{21} + \frac{58}{9}.2^{22} + \frac{391}{288}$$

4. 2.50

Sol. $16\cos x + 12\sin x = \sqrt{16^2+12^2}\cos(x-\alpha), \alpha = \tan^{-1}\left(\frac{3}{4}\right)$.

5. 0.8

Sol. Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $a^2 = 8^2$ and $b^2 = 8^2\left(1-\frac{1}{4}\right)$

Vertices of the letra recta are $\left(\pm ae, \pm\frac{b^2}{a}\right)$

So $\Delta = 2ae \times \frac{2b^2}{a} = 4b^2e$

$= 4 \times 8^2 \times \frac{3}{4} \times \frac{1}{2} = 12 \times 8$

$\Rightarrow \frac{\Delta}{120} = 0.8$

6. 1.6

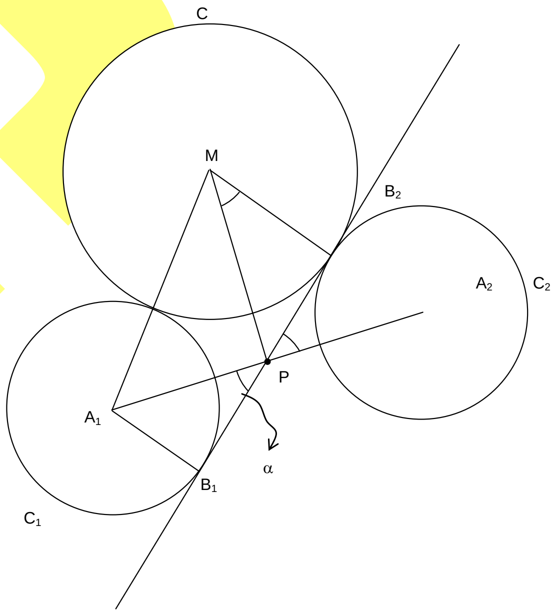
Sol. Let A_1, A_2 and M be the centres of the circles C_1, C_2 and C respectively. Let the common tangent through P to C_1 and C touch C_1 at B_1, C at B_2 and C_2 also at B_2.

From right angled triangle A_1B_1P if $\angle A_1PB_1 = \alpha, \sin\alpha = \frac{A_1B_1}{A_1P} = \frac{1}{3}$

$\Rightarrow \cos\alpha = \frac{2\sqrt{2}}{3} \Rightarrow PB_1 = 2\sqrt{2} = PB_2$

From triangle MPB_2, $\tan\alpha = \frac{PB_2}{MB_2} = \frac{2\sqrt{2}}{r}$

$\Rightarrow \frac{1}{2\sqrt{2}} = \frac{2\sqrt{2}}{r} \Longrightarrow r = 8$.



7. –1.25

Sol. $1-x>0,\ 1-x\neq 1 \Rightarrow x<1,\ x\neq 0$

Also, $\log_{1-x}\left(\frac{3}{2}\right) = \frac{1}{2} \Rightarrow \frac{3}{2} = (1-x)^{1/2}$

$\Rightarrow \frac{9}{4} = 1-x \Rightarrow x = 1-\frac{9}{4} = \frac{-5}{4}$.

$\Rightarrow -4x = 5$

$\Rightarrow x = -1.25$

8. 6

Sol. Clearly $x>0$ and $x \neq \frac{1}{5}$

$$\log_{5x}\left(\frac{5}{x}\right)=\frac{\log_5 5-\log_5 x}{\log_5 5+\log_5 x}$$

Putting $\log_5 x = t,$ then equation (1) becomes

$$t^2+\frac{1-t}{1+t}=1 \Leftrightarrow t^3+t^2-2t=0$$

$$\Leftrightarrow t(t-1)(t+2)=0 \Leftrightarrow t=0, 1, -2$$

So integral roots of (1) are 1 and 5.