FIITJEE RBT-4 for (JEE-Advanced)

BATCHES – Two Yr CRP2123(All)

# PHYSICS, CHEMISTRY & MATHEMATICS

**QP Code: 100183** **Paper-2**

**Time Allotted: 3 Hours** **Maximum Marks: 180**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

## INSTRUCTIONS

**Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.**

### A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections**.
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & B** in the OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

### B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with HB pencil for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

### C. Marking Scheme For All Three Parts.

**(i)** **PART – A *(01 –06)***: This section contains SIX **(06)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
*Full Marks*: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
*Partial Marks*: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
*Zero Marks*: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

**PART – A *(07 –10)***: This section contains TWO **(02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer. Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **PART – B *(1 – 3)*:** This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.** Each question carries **+4 marks** for correct answer and **there will be no negative marking.**

**(iii)** **PART – B (4 – 9):** This section contains **SIX (06)** questions stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places. Each question carries **+2 marks** for correct answer and **there will be no negative marking.**

**Name of the Candidate :**________________________________________

**Batch :**__________________ **Date of Examination :**__________________

**Enrolment Number :**__________________________________________

# SECTION – I : PHYSICS

## PART – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

1. Two stars $S_1$ and $S_2$ of mass M each are moving in circular orbits about their common centre of mass 'O' due to mutual gravitational force of attraction. The separation between them is 4R. An asteroid of mass m(m <<M) is in a coplaner circular orbit about same centre 'O'. It is observed that the asteroid is also moving in a circular orbit with same angular speed. Neglect size of stars and asteroid compared with R. Then choose the correct option(s).

(A) Time period of revolution of stars and asteroid is $8\pi\sqrt{\frac{2R^3}{GM}}$

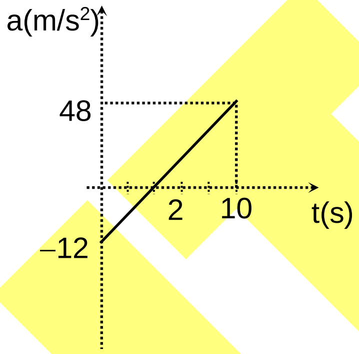
(B) Radius of the circle traced by asteroid is $2\sqrt{3}\,R$

(C) The total energy of the asteroid is $\frac{-5GMm}{16R}$

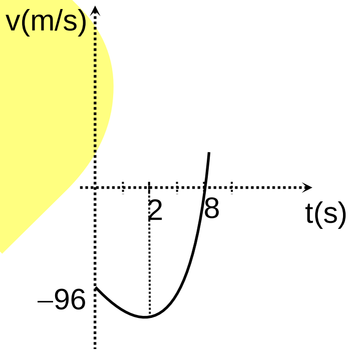
(D) Radius of the circle traced by the star $S_1$ is 2R.

2. The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 96t + 37$, where x is expressed in meters and t is in seconds. Which of the graph(s) represent(s) the motion of the particle?

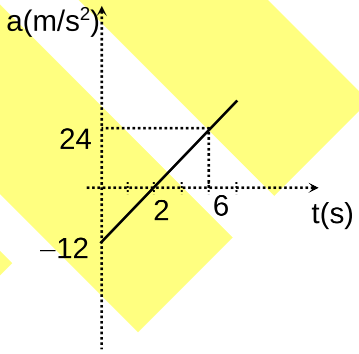
(A)



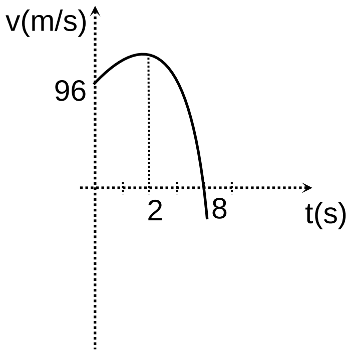

(B)




(C)




(D)




*Space For Rough Work*

3. A flat circular annular disc of mass M, outer radius a and inner radius b. The measured value of M, a and b are, M = (0.30 kg ± 0.0003 kg), a = (4 cm ± 0.001 cm) and b = (2 cm ± 0.001 cm). Then,

(A) Percentage error in the calculation of a and b are in ratio 1 : 2.

(B) Percentage error in the calculation M is 0.1%

(C) Percentage error in the calculation of moment of inertia of the disc about an axis passing through the centre and perpendicular to the plane of disc is 0.16%

(D) Percentage error in the calculation of moment of inertia of the disc about an axis passing through the centre and perpendicular to the plane of disc is 1.6%

4. When a yellow light of wavelength 6000 Å from a sodium lamp falls on a certain photocell, a negative potential of 0.30 volt is needed to stop all the electrons from reaching the collector. Then choose correct option(s).

(A) Work function of the given photocell is 2.5 eV (approximately).

(B) Work function of the given photocell is 1.78 eV (approximately).

(C) A negative potential of 0.6 Volt will be needed to stop all the electrons if light of wavelength $\lambda$ = 4000 Å is used

(D) A negative potential of 1.32 Volt (approximately) will be needed to stop all the electrons if light of wavelength $\lambda$ = 4000 Å is used

5. 10g of ice at 0°C is mixed with 5g of steam at 100°C. If latent heat of fusion of ice is 80 cal-g$^{-1}$ and latent heat of vaporization of 540 cal-g$^{-1}$. When thermal equilibrium is reached then which of the following is/are correct?

(A) Temperature of the mixture is 0°C

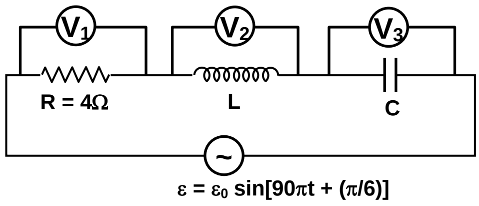
(B) Temperature of mixture is 100°C

(C) Mixture contains 13.3 g of water and 1.67 g of steam

(D) Mixture contains 5.3 g of ice and 9.7 g of water

***Space For Rough Work***

6. In the figure shown the reading of voltmeter are $V_1$ = 80 V, $V_2$ = 90 V and $V_3$ = 30 V. Then choose the correct option(s).



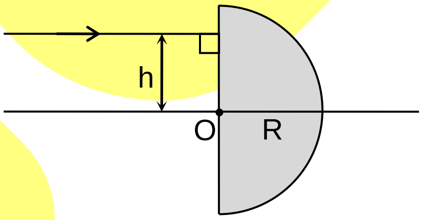

(A) The peak value of current is $20\sqrt{2}$ ampere.
(B) The peak value of emf is $100\sqrt{2}$ volts.
(C) The value of L, is $\frac{1}{20\pi}$H
(D) The value of C for which the circuit will produce maximum heat is $\frac{1}{405\pi}$F

## PART – A (Maximum Marks: 12)

*This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

### Paragraph for Question Nos. 07 and 08

A semi-cylinder made of a transparent plastic has a refractive index of $\mu = 5/3$ and radius of R = 30 cm lies in vacuum. There is a narrow incident light ray perpendicular to the flat side of the semi-cylinder at height 'h' from the axis of symmetry. (take tan37° = 3/4)
**Now answers the following questions.**




7. What is the maximum value of 'h' so that the light ray can still leave the other side of the semi-cylinder?

(A) 12 cm
(B) 8 cm
(C) 18 cm
(D) 24 cm

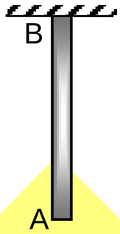
8. When the refractive index of transparent plastic is $\mu = \sqrt{2}$, the value of 'h' is chosen such that total internal reflection just takes place, then time for which light remains inside the cylinder, is

(A) $10^{-9}$ s
(B) $2 \times 10^{-9}$ s
(C) $4 \times 10^{-9}$ s
(D) $8 \times 10^{-9}$ s

***Space For Rough Work***

### Paragraph for Question Nos. 09 and 10

Consider a rope AB of non-uniform density as shown in the figure. If we consider direction A to B as the direction of increasing y, with y = 0, at A. The linear mass density of rope is given by $\mu = \left(\frac{y}{\ell}\right)^3 \mu_0$, where $\mu_0$ is a positive constant and $\ell$ is length of the rope. Now one pulse is produced at the lower end A. Then answer the following questions




9. The time to move the pulse from A to B.

(A) $\sqrt{\frac{\ell}{g}}$ (B) $2\sqrt{\frac{\ell}{g}}$

(C) $3\sqrt{\frac{\ell}{g}}$ (D) $4\sqrt{\frac{\ell}{g}}$

10. Acceleration of the pulse when y = $\ell/4$.

(A) g/4 (B) g/2

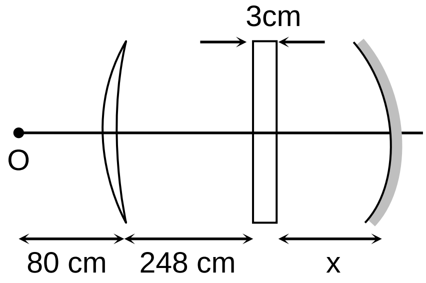
(C) g (D) g/8

## PART – B (Maximum Marks: 12)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

1. Two rods each of length $L_1$ and one rod of length $L_2$ form isosceles triangle having base of length $L_2$. Co-efficient of linear expansion of base is $4 \times 10^{-6}$ /°C while for rod of length $L_1$ is $25 \times 10^{-6}$ /°C. Find the ratio $\left(\frac{L_2}{L_1}\right)$, if the length of latitude remains same when the temperature of the system is increased by 10°C.

2. A concavo-convex lens made of glass of refractive index 1.5 has surfaces of radii 20 cm and 60 cm. A concave mirror of radius of curvature 20 cm is placed co-axially to the lens. A glass slab of thickness 3 cm and refractive index 1.5 is placed close to the mirror in the space between the mirror and lens as shown in the figure. The distance of the nearest surface of the slab from lens is 248 cm. An object is placed 80 cm to the left of the lens. If the final position of the image formed after the refraction from lens, refraction from slab, reflection from mirror, refraction from the slab and lens is at the object O, find the distance x (in cm) of the mirror from nearer surface of slab.




*Space For Rough Work*

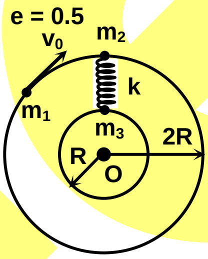
3. A train moving towards a hill at a speed 144 km/hr, sounds whistle of frequency 300 Hz. Wind is blowing from the hill with a velocity 360 km/hr at an angle of 53° with the railway track. If the speed of sound in air is $\frac{400}{3}$ m/s, then find the frequency (in Hz) heard by a man on the hill.

## PART – B (Maximum Marks: 12)

*This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

### Question Stem for Question Nos. 4 and 5

Three particles each of mass ‘m’ can slide on fixed frictionless circular track in the same horizontal plane as shown in the figure. Particle $m_1$ moves with velocity $v_0$ and hits particle $m_2$, the coefficient of restitution being e = 1/2. Assume $m_2$ and $m_3$ are initially in rest and lie along a radial line before impact and the spring is initially unstretched. The velocity of $m_3$ when extension in the spring is maximum, is X m/s and the maximum extension in the spring is Y meter. (Take $v_0$ = 20 m/s and $\frac{m}{k} = 5$ kg/(N/m))

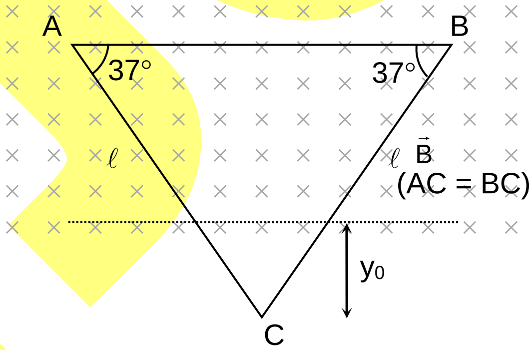



4. The value of X is ………..

5. The value of Y is ………..

### Question Stem for Question Nos. 6 and 7

A current carrying loop is in the shape of isosceles triangle of mass m lies in the vertical plane. There exists uniform horizontal magnetic field B in the region as shown in the figure. The current in the loop is i. The value of $y_0$ is X meter when loop is in equilibrium. If the loop is displaced slightly in its plane perpendicular to its side AB from its equilibrium position and released, then time period of its oscillation is Y sec. (neglect the induced emf in the loop, take $\frac{mg}{iB} = \frac{80}{3}$, $\pi$ = 3.14 and g = 10 m/s²)

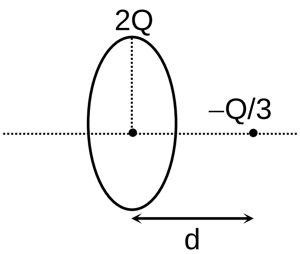



6. The value of X is……………….

7. The value of Y is……………….

### Question Stem for Question Nos. 8 and 9

A ring of mass 2M, radius R and charge 2Q is placed as shown in the figure. A point charge $-\frac{Q}{3}$ and mass M is placed on the axis of ring at a distance d from the centre of the ring as shown in the figure. (d<<R). When system is released it performs oscillations. The energy of oscillation of the system is $\frac{Q^2}{m\pi\varepsilon_0 R^n}$ and the time period of oscillation is $T = \sqrt{\frac{X\pi^3\varepsilon_0 MR^3}{YQ^2}}$.




8. The value of |m – n| is ………..

9. The value of |X – Y| is ………..

*Space For Rough Work*

# SECTION – II : CHEMISTRY

## PART – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

1. Among the following intensive properties are

(A) $E^{o}_{cell}$

(B) $E_{cell}$

(C) Normality

(D) Entropy

2. Which of the following solution is/are hypertonic at constant T with 0.4 M NaCl showing 80% dissociation?

(A) 0.7 M glucose

(B) 0.3 M $Na_3PO_4$ showing 90% dissociation

(C) 1 M $H-\overset{\overset{O}{||}}{C}-OH$ showing 30% association

(D) 0.5 M $MgCl_2$ showing 20% dissociation

3. In isothermal compression of ideal gas

(A) W is +ve

(B) $\Delta H$ is zero

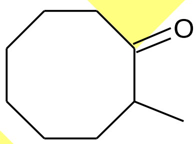
(C) $\Delta S_{gas}$ is – ve

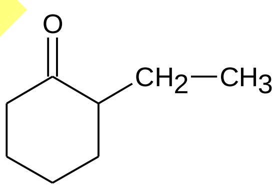
(D) $\Delta E$ is zero

4. Tautomer of which of the following can show geometrical isomerism

(A) $CH_3-\overset{\overset{O}{||}}{C}-H$

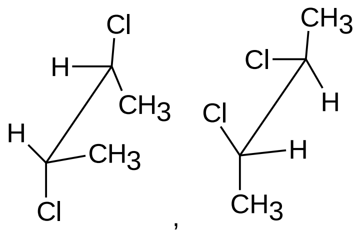
(B) $CH_3-CH_2-CH_2-\overset{\overset{O}{||}}{C}-H$

(C) 

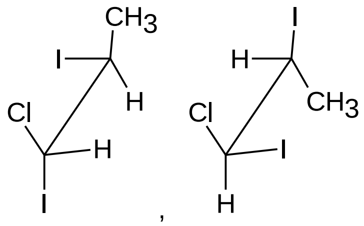

(D) 

5. Which of the following pairs of compounds is/are identical?

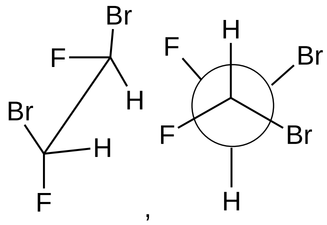
(A)



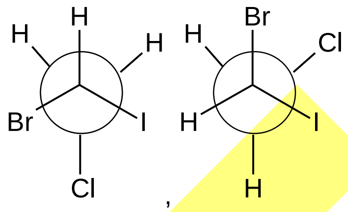
(B)



(C)



(D)



6. Which set of quantum numbers is/are not consistent with the theory?

(A) $n = 2, \ell = 0, m = -1, s = +\frac{1}{2}$

(B) $n = 2, \ell = 0, m = 0, s = -\frac{1}{2}$

(C) $n = 1, \ell = -1, m = 0, s = -\frac{1}{2}$

(D) $n = 3, \ell = 3, m = -3, s = +\frac{1}{2}$

***Space For Rough Work***

## PART – A (Maximum Marks: 12)

*This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

### Paragraph for Question Nos. 07 and 08

An orange solid (A) on heating gives a green residue (B), a colourless gas (C) and water vapours. The dry gas (C) on passing over heated Mg gave a white solid (D). (D) on reaction with water gave a gas (E) which formed black precipitate with mercurous nitrate solution.

7. Select the incorrect statement

(A) The central atom (s) of the anion of solid (A) has $sp^3$ hybridization.

(B) The orange solid (A) is diamagnetic in nature.

(C) The anion of orange solid (A) is oxidizing in nature.

(D) All metal oxygen bond lengths are equal in anion of solid (A).

8. Which of the following is false for the gas (E)?

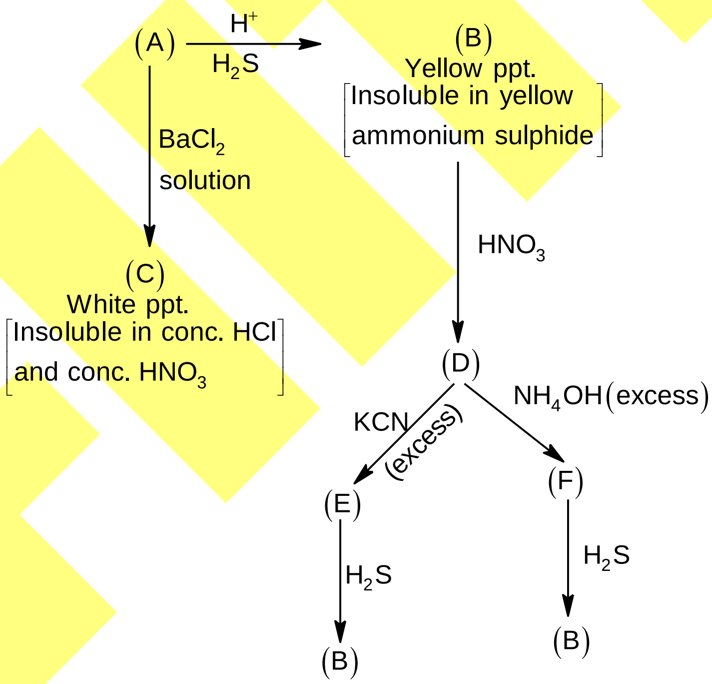
(A) It gives a deep blue colouration with $CuSO_4$ solution.

(B) It is oxidized to a colourless gas (neutral oxide) at 1200 K in presence of a catalyst Pt/Rh in air.

(C) It gives the same gas (C) with potassium permanganate solution.

(D) It gives black precipitate with $HgCl_2$ .

### Paragraph for Question Nos. 09 and 10




9. What is "C"?

(A) $PbCl_2$ (B) $BaSO_4$

(C) $BaSO_3$ (D) $PbSO_4$

10. What is “B”?

(A) CdS
(B) $PbCrO_4$
(C) HgO
(D) $K_3\left[Co(NO_2)_6\right]$

## PART – B (Maximum Marks: 12)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

1. Number of co-ordination isomerism for the complex $\left[\overset{II}{Pt}(NH_3)_4\right]\left[\overset{II}{Pt}Cl_4\right]$ is (including itself)

2. Number of stereo isomers in the complex $\left[Ma_2b_2c_2\right]$ [where a, b, c are monodentate ligand]

3. The following equilibrium are established in mixing of two gases $x_2$ and y in sealed container at constant temperature T.
$3x_2(g) \rightleftharpoons x_6(g), \quad K_p = 1.6\ atm^{-2}$
$x_2(g) + y(g) \rightleftharpoons x_2y(g), \quad K_P = z\ atm^{-1}$
When $x_2(g)$ and $y(g)$ are mixed in 2 : 1 molar ratio, the total pressure of gases at equilibrium is found to be 1.4 atm and partial pressure of $x_6(g)$ is 0.2 atm. The value of 6W is.

## PART – B (Maximum Marks: 12)

*This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

### Question Stem for Question Nos. 4 and 5

A reaction is said to be first order if it’s rate is proportional to the concentration of reactant. Let us consider a reaction

| | $A(g) \longrightarrow$ | $B(g)$ | $+ C(g)$ |
|---|---|---|---|
| At $t = 0$ | a | 0 | 0 |
| At time t | a – x | x | x |

The rate of reaction is given by the expression $\frac{dx}{dt} = k(a - x)$ and integrated rate equation for given reaction is represented as $k = \frac{1}{t}\ell n\left(\frac{a}{a-x}\right)$ where a = initial concentration and (a – x) = concentration of A after time t.

4. Thermal decomposition of compound A is a first order reaction. If 75% of A is decomposed in 100 min. How long will it take for 90% of the compound to decompose?
[Given: log2 = 0.30]

5. Consider a reaction $A(g) \longrightarrow 3B(g) + 2C(g)$ with rate constant $1.386 \times 10^{-2}\ min^{-1}$. Starting with 2 moles of A in 12.5 litre vessel initially, if reaction is allowed to takes place at constant pressure and at 298 K then find the concentration of B after 100 min? $\left[e^{-1.386} = 0.25\right]$

***Space For Rough Work***

**Question Stem for Question Nos. 6 and 7**

0.257 g of an organic compound is analysed for nitrogen by Kjeldahl's method. The ammonia evolved was absorbed in 50 mL of 0.1 M HCl which required 23.2 ml of $\frac{N}{10}$ NaOH for the complete neutralization.

6. What is percentage of 'N' in the compound?

7. In Carius method of estimation of halogen, 0.15 g of an organic compound gave 0.12 g of AgBr. What is % of Bromine in the compound?
[Take : $M_{AgBr} = 188\ g/mol;\ M_{Br} = 80\ g/mol$]

**Question Stem for Question Nos. 8 and 9**

EMF of a cell corresponding to the reaction:

$$Zn(s) + 2H^{+}(aq) \longrightarrow Zn^{2+}(aq)[0.10\ M] + H_2(g)\,|\,[1.0\ atm]$$

is 0.61 V at $25^{o}C$. Also, at $25^{o}C$, $E^{o}_{Zn^{2+}/Zn} = -0.76\ V$. $\left[\frac{2.303RT}{F} = 0.06\right]$

8. The pH of solution in cathode chamber is………

9. $\Delta G$ of the above cell reaction is y,' then the value of |y| in kJ? (Given 1F = 96500)

***Space For Rough Work***

# SECTION – III : MATHEMATICS

## PART – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

1. If the value of limit ; $\lim_{n\to\infty}\left(n+1-\sum_{i=2}^{n}\sum_{k=2}^{i}\frac{k-1}{k!}\right)$ is L, then (where [.] denotes greatest integer function)

(A) $\left[\frac{[L]+2L}{[L]}\right]=3$

(B) $\left[\frac{[L]+2L}{[L]}\right]=4$

(C) $\left[\frac{[L]+[L-1]}{2[L]}\right]=0$

(D) $\left[\frac{[L]+[L-1]}{2[L]}\right]=1$

2. Suppose that the function f : R → R and g : R → R satisfy the relation f(x + g(y)) = 2x + y + 5, then if function g(x + f(y)) = $k_1(x) + k_2(y) + k_3$, then

(A) $2k_1 + k_2 = 1$

(B) $2k_1 + k_2 = 2$

(C) $k_3 - k_1 = 0$

(D) $k_3 - k_1 = 2$

3. ${}^{m}C_1 \cdot {}^{n}C_m - {}^{m}C_2 \cdot {}^{2n}C_m + {}^{m}C_3 \cdot {}^{3n}C_m$ ..... is equal to

(A) coefficient of $x^n$ in $(1 - (1 + (1 - x)^n)^m)$

(B) coefficient of $x^n$ in $(1 - (1 - (1 + x)^n)^m)$

(C) coefficient of $x^m$ in $(1 - (1 - (1 + x)^n)^m)$

(D) coefficient of $x^m$ in $(1 - (1 + (1 - x)^n)^m)$

4. Given an alphabet with three letters a, b, c, then the number of words of n letter which contain even number of a's?

(A) $2^n\left(\left(1+\frac{1}{2}\right)^n+\left(1-\frac{1}{2}\right)^n\right)$

(B) $2^{n-1}\left(\left(1+\frac{1}{2}\right)^n+\left(1-\frac{1}{2}\right)^n\right)$

(C) $\frac{1}{4}(3^n+1)$

(D) $\frac{1}{2}(3^n+1)$

***Space For Rough Work***

5. For each integer $n \geq 1$. Let $a_n = 1 + \frac{1}{2} + \frac{1}{3} ..... \frac{1}{n} - \log n$ and $b_n = a_n - \frac{1}{n}$, then

(A) $a_{n+1} > a_n$
(B) $a_{n+1} < a_n$
(C) $b_{n+1} > b_n$
(D) $b_{n+1} < b_n$

6. Let point A is taken on line x + y – 9 = 0 and B and C are taken on circle $2x^2 + 2y^2 - 8x - 8y - 1 = 0$ such that line AB passes through centre of circle and ∠BAC = 45°, then range of x coordinate of point A is superset of which of the following?

(A) [3, 4]
(B) [2, 4]
(C) $\left[\pi, \frac{3\pi}{2}\right]$
(D) $[\pi , 5]$

## PART – A (Maximum Marks: 12)

*This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

### Paragraph for Question Nos. 7 and 8

N dice are rolled once, then the probability of getting sum t is expressed as $P_d(t, N)$. Where dice is a d sided dice with face marked 1, 2, 3, 4, ...... d.

7. Which of the following is an **incorrect** combination?

(A) $P_6(14, 3) = P_6(14, 5)$
(B) $P_3(5, 2) = P_3(5, 3)$
(C) $P_3(10, 4) = P_3(10, 5)$
(D) $P_4(9, 3) = P_4(9, 4)$

8. The value of $P_9(15, 2)$ is equal to

(A) $\frac{4}{71}$
(B) $\frac{4}{81}$
(C) $\frac{8}{81}$
(D) $\frac{8}{71}$

***Space For Rough Work***

**Paragraph for Question Nos. 09 and 10**

Let consider lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $2x + 5y – z + 47 = 0 = 2x + y + z + 7$

9. The shortest distance between both given line is

(A) $2\sqrt{30}$

(B) $3\sqrt{30}$

(C) $\sqrt{30}$

(D) $\sqrt{75}$

10. Equation of plane parallel to both of lines and which lies midway between the lines

(A) $2x + 5y – z + 2 = 0$

(B) $2x – 5y + z – 2 = 0$

(C) $2x + 5y – z – 2 = 0$

(D) $2x – 5y – z – 2 = 0$

## PART – B (Maximum Marks: 12)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

1. Let R be the circum-radius of triangle ABC and let G and H be its centroid and orthocentre respectively. Let F be the midpoint of GH, then if $AF^2 + BF^2 + CF^2 = kR^2$, then k is equal to

2. In the coordinate plane, the area of the region bounded by the line passing through (1, 2) with the slope $\alpha$ and the parabola $y = x^2$ is denoted by $S(\alpha)$ where $\alpha$ varies in the range of $0 \le \alpha \le 6$, then find the value of $\alpha$ such that $S(\alpha)$ is minimized

3. Given a triangle ABC, let a, b, c denotes the length of its sides and m, n, p the length of its medians. For every positive real $\alpha$, let $\lambda(\alpha)$ be the real number satisfying
$a^\alpha + b^\alpha + c^\alpha = (\lambda(\alpha))^\alpha \{m^\alpha + n^\alpha + p^\alpha\}$, then of value of $\lambda(2)$ is k, then [k] is
(where [.] denotes the greatest integer function)

***Space For Rough Work***

## PART – B (Maximum Marks: 12)

*This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

### Question Stem for Question Nos. 4 and 5

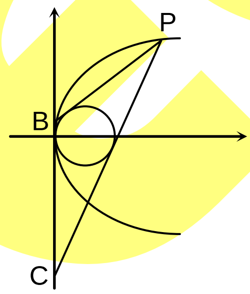
If the abscissa and the ordinate of on equilateral triangle are the roots of $a_1x^3 + b_1x + c_1 = 0$ and $a_2x^3 + b_2x + c_2 = 0$, then if $k_1\left[\frac{b_1}{a_1}+\frac{b_2}{a_2}\right]+k_2R^3=0$ (where R is the circumradius of triangle ABC); $k_1$ and $k_2$ are coprime to each other), then

4. Value of $k_1$ is

5. Value of $k_2$ is

### Question Stem for Question Nos. 6 and 7

As is shown in figure, $P(x_0, y_0)$ is a moving point on the parabola $y^2 = 2x$, point B and C are on y-axis, and the sides of triangle PBC touches the circle $(x - 1)^2 + y^2 = 1$.




6. The minimum value of area of triangle PBC is _____ (in sq. unit)

7. The value of $x_0$ when the area of $\Delta$PBC is minimum

### Question Stem for Question Nos. 8 and 9

The largest area of rectangle whose

8. Base is on x-axis and upper two vertices lies on parabola $y = 12 - x^2$

9. Base is along with x-axis and inscribed in equilateral triangle having base as x-axis and side length 20 unit

*Space For Rough Work*

# QP Code: 100183

# ANSWERS

## SECTION-1 : PHYSICS

### PART – A

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | A, B, C, D | 2. | A, B, C | 3. | A, B, C | 4. | B, D |
| 5. | B, C | 6. | A, B, C, D | 7. | C | 8. | C |
| 9. | D | 10. | D | | | | |

### PART – B

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | 5 | 2. | 10 | 3. | 2100 | 4. | 06.00 |
| 5. | 15.00 | 6. | 10.00 | 7. | 06.28 | 8. | 09.00 |
| 9. | 15.00 | | | | | | |

## SECTION – 2 : CHEMISTRY

### PART – A

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | A, B, C | 2. | B, C | 3. | A, B, C, D | 4. | B, C |
| 5. | A | 6. | A, C, D | 7. | D | 8. | D |
| 9. | B | 10. | A | | | | |

### PART – B

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | 2 | 2. | 6 | 3. | 9 | 4. | 166.67 |
| 5. | 00.07 | 6. | 17.72 | 7. | 34.04 | 8. | 03.00 |
| 9. | 117.73 | | | | | | |

## SECTION – 3 : MATHEMATICS

### PART – A

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | A, C | 2. | B, D | 3. | C | 4. | B, D |
| 5. | B, C | 6. | A, C, D | 7. | C | 8. | B |
| 9. | B | 10. | A | | | | |

### PART – B

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | 3 | 2. | 2 | 3. | 1 | 4. | 02.00 |
| 5. | 03.00 | 6. | 08.00 | 7. | 04.00 | 8. | 32.00 |
| 9. | 86.50 (Range 86.50 to 86.60) | | | | | | |

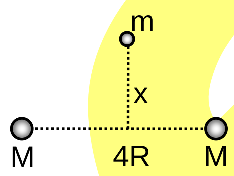
# Answers & Solutions

## SECTION-1 : PHYSICS

### PART – A

1. A, B, C, D

Sol. For the stars




$$\frac{GM^2}{(4R)^2} = M(2R)\omega^2$$

$$\omega = \sqrt{\frac{GM}{32R^3}} \quad \ldots(i)$$

The asteroid should be equidistant from both the stars.

$$\frac{2GMmX}{\left[(X^2 + (2R)^2\right]^{3/2}} = mX\omega^2 \quad \ldots(ii)$$

From (i) and (ii)

$$X = 2\sqrt{3}R$$

Total energy = $-\frac{2GMm}{4R} + \frac{1}{2}m\left(2\sqrt{3}R\sqrt{\frac{GM}{32R^3}}\right)^2 = -\frac{5GMm}{16R}$

2. A, B, C

Sol. $x = t^3 - 6t^2 - 96t + 37$ …(i)
$v = 3t^2 - 12t - 96$ …(ii)
$a = 6t - 12$ …(iii)
at t = 0, v = – 96 m/s
when v = 0, t = 8 sec
and when v is maximum t = 2 sec.

3. A, B, C

Sol. Percentage error in a, b and M are $\frac{\Delta a}{a}\times 100 = \frac{0.001}{4}\times 100 = 0.025\%$

$$\frac{\Delta b}{b}\times 100 = \frac{0.001}{2}\times 100 = 0.05\%$$

$$\frac{\Delta M}{M}\times 100 = \frac{0.0003}{0.3}\times 100 = 0.1\%$$

Moment of Inertia

$$I = \frac{M}{2}(a^2 + b^2)$$

$$\frac{\Delta I}{I} = \frac{\Delta M}{M} + \frac{2a\cdot\Delta a + 2b\cdot\Delta b}{a^2 + b^2}$$

$$\frac{\Delta I}{I} = 0.001 + \frac{2(4)(0.001) + 2(2)(0.001)}{20}$$

$$\frac{\Delta I}{I} = 0.001 + \frac{0.008 + 0.004}{20} = 0.0016$$

$$\frac{\Delta I}{I}\times 100 = 0.16\%$$

4. B, D

Sol. As we know that,

$\frac{hC}{\lambda} = \phi + K \Rightarrow \frac{12400}{\lambda} = \phi + v_s$, where $\phi$ is in electron Volt and $\lambda$ is in Å

When light of wavelength $\lambda$ = 6000 Å is used

$\frac{12400}{6000} = \phi + 0.3 \Rightarrow \phi = 1.77 \approx 1.78$ eV (approximately)

When light with wavelength $\lambda$ = 4000 Å is used

$\frac{12400}{4000} = \phi + eV$

$\Rightarrow$ eV = (3.1 – 1.78) e volt = 1.32 eV

$\Rightarrow$ V = 1.32 Volt

5. B, C

Sol. Heat gain by ice to melt = $mL_f = 10 \times 80 = 800$ cal

Heat gain by 10 g water to raise its temperature from 0°C to 100°C = $10 \times 1 \times 100 = 1000$ cal

Total heat gain = 1800 cal

Mass of stem converted into water

$\therefore$ $1800 = m \times 540 \Rightarrow m = 3.33$ g

$\therefore$ equilibrium temperature 100°C

Amount of water = 10 + 3.33 = 13.33 g

Amount of steam = 5 – 3.33 = 1.67 g

6. A, B, C, D

Sol. (A) $i_{rms} = \frac{80}{4} = 20$ amp, $i_{peak} = 20\sqrt{2}$ amp

(B) $\varepsilon_{rms} = \sqrt{(40)^2 + (40-10)^2} = 100$V, $\varepsilon_0 = 100\sqrt{2}$V

(C) $\omega L i_{rms} = 90 \Rightarrow L = \frac{90}{10 \times 90\pi} = \frac{1}{20\pi}$H

(D) For maximum heat, current through the circuit will be maximum $\omega L = \frac{1}{\omega C}$

$C = \frac{1}{405\pi}$F

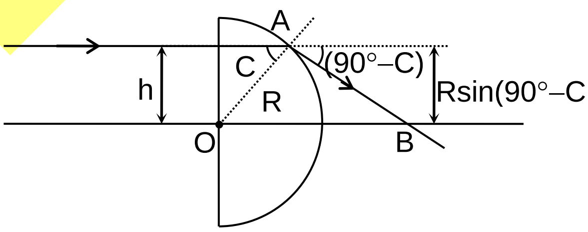
7. C

Sol. $\frac{5}{3}\sin C = 1\sin 90°$

sin C = 3/5 (C $\rightarrow$ critical angle)

C = 37°

h = R sin 37° = $\frac{3}{5}R = \frac{3}{5}(30) = 18$ cm




8. C

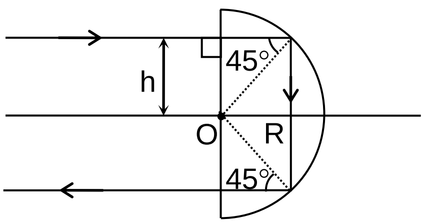
Sol. when $\mu = \sqrt{2}$

$\sqrt{2}\sin C = 1$ (C $\rightarrow$ critical angle)

C = 45°

Total distance travel in semi-cylinder

$= \left(\frac{R}{\sqrt{2}}\right)4 = 2\sqrt{2}R = (2\sqrt{2})(30) = 60\sqrt{2}$ cm

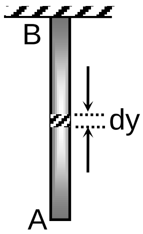



Time $= \frac{60\sqrt{2}}{C_m} \times 10^{-2} = \frac{60\sqrt{2} \times 10^{-2}}{\left(\frac{3 \times 10^8}{\sqrt{2}}\right)} = \frac{120 \times 10^{-2}}{3 \times 10^8} = 4 \times 10^{-9}$ s ($C_m \rightarrow$ speed of light in the medium)

9. D

10. D

Sol. **(for Q. 9-10):**




dm = $\mu$dy

$$m = \int_0^y \mu_0\left(\frac{y}{\ell}\right)^3 dy = \left(\frac{\mu_0}{\ell^3}\right)\left(\frac{y^4}{4}\right)$$

Velocity of wave

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{gy}{4}}$$

$$\frac{dy}{dt} = \frac{\sqrt{gy}}{2}$$

$$\int_0^{\ell} \frac{dy}{\sqrt{y}} = \frac{\sqrt{g}}{2}\int_0^t dt$$

$$t = \left(4\sqrt{\frac{\ell}{g}}\right)$$

$$v = \frac{\sqrt{gy}}{2}$$

$$v^2 = \frac{gy}{4}$$

$$2v\frac{dv}{dy} = \frac{g}{4}$$

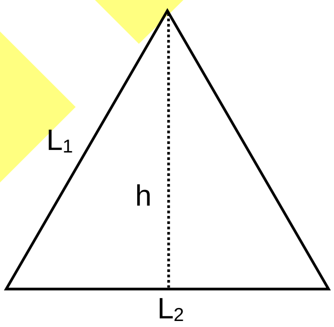
$$v\frac{dv}{dy} = \left(\frac{g}{8}\right)$$

a = g/8

## PART – B

1. 5

Sol.




$$h^2 = L_1^2 - \frac{L_2^2}{4}$$

Differentiating and using dL=L $\alpha$ $\Delta$t

$$0 = 2L_1\alpha_1L_1\Delta t - \frac{1}{4}2L_2\alpha_2L_2\Delta t$$

$$L_1L_1\alpha_1\Delta t = \frac{1}{4}L_2L_2\alpha_2\Delta t$$

$$\Rightarrow \frac{L_2}{L_1} = 2\sqrt{\frac{\alpha_1}{\alpha_2}} = 2\sqrt{\frac{25\times10^{-6}}{4\times10^{-6}}} = 5$$

2. 10

Sol. For lens, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Here, $R_2$ = +60 cm and $R_1$ = 20 cm

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{60}\right)$$

$$\Rightarrow \frac{1}{60} = \frac{1}{v} + \frac{1}{80}$$

v = 240

Shift due to slab, $s = 2\left[1 - \frac{1}{1.5}\right] = 1\,\text{cm}$

For final image to form at O, ray should retrace its path after reflection from mirror. So, $I_2$ should be at centre of curvature of mirror i.e.

$7 + 3 + x = 20$
$x = 10$ cm

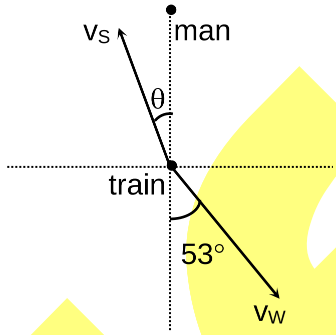
3. 2100

Sol. $v_W \to$ velocity of wind
$v_S \to$ velocity of sound
$v_S.\sin\theta = v_W \sin 53°$

$$\sin\theta = \frac{v_W}{v_S}\left(\frac{4}{5}\right) = \frac{100}{(400/3)} \times \frac{4}{5} = \frac{3}{5}$$

$\theta = 37°$

$$f' = f\left(\frac{v_s\cos\theta - v_W\cos 53°}{v_S\cos\theta - v_W\cos 53° - v_T}\right) = 300 \times 7 = 2100 \text{ Hz}$$




4. 06.00

5. 15.00

Sol. **(for Q. 4-5):**
By the conservation of linear momentum between $m_1$ and $m_2$ and by using coefficient of restitution velocity $m_2$ just after collision $\frac{3v_0}{4}$.
After the collision, the maximum extension in the spring occurs when angular velocity of $m_2$ and $m_3$ about O become same. Now by conservation of angular momentum about 'O' and energy conservation

Velocity of $m_2 = 2R\omega = \frac{3}{5}v_0$

Velocity of $m_3 = R\omega = \frac{3}{10}v_0$ and $\Delta x$ maximum = $\frac{3}{4}v_0\sqrt{\frac{m}{5k}}$

6. 10.00

7. 06.28

Sol. **(for Q. 6-7):**

$$i\left(\frac{8y_0}{3}\right)B = mg$$

$$y_0 = \frac{3mg}{8iB}$$

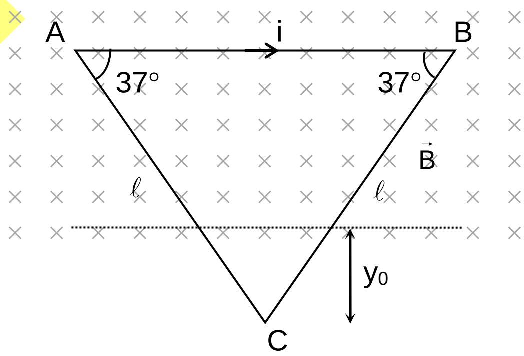
$y_0 = 10$
If loop is displaced by small y downward. Then,

$$ma = mg - iB\left[\frac{8}{3}(y + y_0)\right]$$

$$a = -\left(\frac{8iB}{3m}\right)y$$

$$\omega = \sqrt{\frac{8iB}{3m}}$$

$$T = \frac{2\pi}{\omega} = 2\pi$$




8. 09.00

9. 15.00

Sol. **(for Q. 8-9):**

Difference of potential energy of two configuration when point charge is at rest at a distance d to when it passes through centre.
The energy of oscillation system

$$E = \frac{2KQ^2}{3\sqrt{R^2+d^2}} - \frac{2KQ^2}{3R} = \frac{Q^2d^2}{12\pi\varepsilon_0 R^3}$$

|m – n| = 12 – 3 = 9
Force between the ring and the charge particle

$$F = -\left(\frac{2KQ^2}{3R^3}\right)d$$

$$M_{eq} = \frac{2}{3}M$$

Hence time period of oscillation is $T = \sqrt{\frac{16\pi^3\varepsilon_0 MR^3}{Q^2}}$

# SECTION – 2 : CHEMISTRY

## PART – A

1. A, B, C

Sol. $E^o_{cell}$, $E_{cell}$ and normality are intensive property, where as entropy is extensive property.

2. B, C

Sol. $[\pi_{obs}]_{0.4\text{ M NaCl}} = i \times C \times R \times T$

$= i \times 0.4 \times R \times T$

$= [1+(n-1)\alpha] \times 0.4 \times R \times T$

$= [1+0.8] \times 0.4 \times R \times T$

$= 1.8 \times 0.4 \times R \times T$

= 0.72 RT

$[\pi_{obs}]_{0.3\text{ M }Na_3PO_4} = [1+(4-1)\times 0.9] \times 0.3 \times R \times T$

= 1.11 RT

$[\pi_{obs}]_{0.7\text{ M Glucose}} = 0.7 \times R \times T$

= 0.7 RT

$[\pi_{obs}]_{1M\,H-\overset{O}{\overset{\|}{C}}-OH} = [1+(2-1)0.3] \times R \times T$

$= 1.3 \times R \times T$

$[\pi_{obs}]_{0.5\text{ M }MgCl_2} = 0.5 \times R \times T[1+(2)\times 0.2]$

$= R \times T \times 0.5[1.4]$

= 0.7 RT

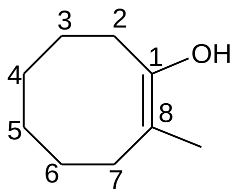
Here only option (B) and (C) have greater osmotic pressure then given solution.

3. A, B, C, D

Sol. In isothermal ideal gas compression as volume decrease. Hence, W is +ve, $\Delta H$ is zero, $\Delta S_{gas}$ is – ve and $\Delta E$ is zero

4. B, C

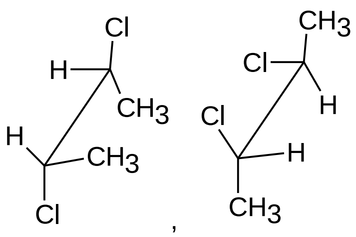
Sol. $CH_3-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-H$ will show geometrical isomerism.




will show geometrical isomerism

5. A

Sol.



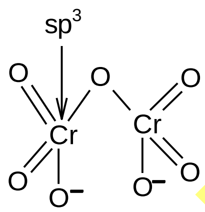

are identical.

6. A, C, D

Sol. (A) $\ell = 0$, then $m \neq 1$. Hence, (A) is incorrect.
(B) It is the correct set.
(C) If n = 1, then $\ell = -1$. Hence, (C) is not correct.
(D) If n = 3, then $m \neq 3$. Hence, (D) is also incorrect.

7. D

Sol.




, all electrons are paired. So, diamagnetic ($3d^0 4s^0$)

$Cr_2O_7^{2-}$ acts as strong oxidizing agent in acidic medium.

8. D

Sol. (E) = $NH_3$

(A) $Cu^{2+} + 4NH_3 \longrightarrow [Cu(NH_3)_4]^{2+}$ (deep blue colouration)

(D) $HgCl_2 + NH_3 + H_2O \longrightarrow HgO.Hg(NH_2)Cl \downarrow$ (white)

9. B

10. A

Sol. **(for Q. 9 to 10):**

$A \xrightarrow{H^+/H_2S} B$ (Yellow ppt.)

Only CdS and $As_2S_3$ are yellow

CdS insoluble in Yellow Ammonium Sulphide

$As_2S_3$ soluble in Yellow Ammonium Sulphide

So, B is CdS

$A \xrightarrow{BaCl_2} C$ (White ppt.)

$BaSO_4$ are white ppt.

$BaSO_4$ Insoluble in mineral acids

$BaSO_3$ soluble in mineral acids

So, C is $BaSO_4$

$\therefore$ A is $CdSO_4$

$$CdS[B] \xrightarrow{HNO_3} \underbrace{Cd^{2+}(aq.) + S\downarrow + NO_3^{1-}(aq)}_{D}$$

$$Cd^{2+}(aq) \xrightarrow{KCN(excess)} K_2\left[Cd(CN)_4\right]$$

$[D]$

## PART – B

1. 2

Sol. $\left[Pt(NH_3)_4\right][PtCl_4]$ and $\left[Pt(NH_3)_3 Cl\right][PtCl_3NH_3]$

2. 6

Sol. Number of stereo isomers for the complex is 6.

3. 9

Sol.
$$3x_2(g) \rightleftharpoons x_6(g), \quad K_p = 1.6\ atm^{-2}$$
$$2a - x - z \qquad \frac{x}{3}$$
$$x_2(g) + y(g) \rightleftharpoons x_2y(g), \quad K_P = W\ atm^{-1}$$
$$2a - x - z \qquad a - z \qquad z$$
$$\frac{x}{3} = 0.2 \Rightarrow x = 0.6$$
$$1.6 = \frac{P_{X_6}}{\left[P_{X_2}\right]^3} \Rightarrow \left[P_{X_2}\right]^3 = \frac{0.2}{1.6}$$
$$P_{X_2} = \frac{1}{2} = 0.5\ atm$$
Now, 0.5 + 0.2 + a – z + z = 1.4
a = 0.7
2a – x – z = 0.5
$\Rightarrow 2\times0.7 - 0.6 - 0.5 = z$
$z = 0.3$
$$K_P = W = \frac{0.3}{0.5\times[0.7-0.3]} = 1.5$$
Hence, 6W = 6 × 1.5 = 9

4. 166.67

Sol. 1st order kinetic
$$t = \frac{2.303}{k}\log\frac{a_o}{a_t}$$
$$\frac{0.6932}{50} = \frac{2.303}{t}\log\frac{100}{10}$$

5. 00.07

Sol.
$$A(g) \longrightarrow 3B(g) + 2C(g)$$
$$t = 0 \qquad \left[\frac{2}{12.5}\right] \qquad 0 \qquad 0$$
$$t = t \qquad \frac{2-x}{V} \qquad \frac{3x}{V} \qquad \frac{2x}{V}$$
Now, $t = \frac{2.303}{K}\log\frac{a_0}{a_t}$

$$\Rightarrow 100 = \frac{2.303}{1.386\times10^{-2}}\log\frac{2/12.5}{a_t}$$

$$\Rightarrow at = 0.04$$

Now, $\frac{2-x}{V} = 0.04$ … (1)

And as the P, T constant

$\frac{2}{12.5} = \frac{2+4x}{V}$ ... (2)

On solving Eq. (1) and (2), we get

$[B_t] = 0.07\text{ M}$

6. 17.72

Sol. Meq. of $NH_3$ = Meq. of HCl – Meq. of NaOH

$$\frac{W\times1000}{17} = 5-2.32 = 2.68$$

$$W = \frac{17\times2.68}{1000} = 0.4556$$

$$\%N = \frac{0.04556}{0.257}\times100$$

= 17.72%

7. 34.04

Sol. $\%Br = \frac{80}{188}\times\frac{0.12}{0.15}\times100 = 34.04\%$

8. 03.00

Sol. $E = 0.61 = 0.76 - \frac{0.06}{2}\log\frac{[Zn^{2+}]}{[H^+]^2}$

$$\Rightarrow \log\frac{(0.1)}{[H^+]^2} = \frac{0.76-0.61}{0.03}$$

$$\Rightarrow pH = 3$$

9. 117.73

Sol. $\Delta G = -nFE_{cell}$

$$|\Delta G| = \frac{2\times0.61\times96500}{1000}\text{ kJ}$$

$$= 117.73\text{ kJ}$$

# SECTION – 3 : MATHEMATICS

## PART – A

1. A, C

Sol. $\lim\limits_{n\to\infty}\left(n+1-\sum\limits_{i=2}^{n}\sum\limits_{k=2}^{i}\frac{k-1}{k!}\right)=\lim\limits_{n\to\infty}\left(n+1-\sum\limits_{i=2}^{n}\sum\limits_{k=2}^{i}\left\{\frac{1}{(k-1)!}-\frac{1}{k!}\right\}\right)$

$L=\lim\limits_{n\to\infty}\left(n+1-\sum\limits_{i=2}^{n}\left(1-\frac{1}{i!}\right)\right)$ ; $L=\lim\limits_{n\to\infty}\left(2+\sum\limits_{i=2}^{n}\frac{1}{i!}\right)$

$\therefore L = e$

2. B, D

Sol. Let g(0) = c then put x = x and y = 0 in functional equation f(x + g(y)) = 2x + y + 5
$\Rightarrow$ f(x + c) = 2x + 5 $\Rightarrow$ f(x) = 2x + 5 – 2c
$\Rightarrow$ f(x + g(y)) = 2x + 2(g(y)) – 2c + 5 compare with give equation
$\Rightarrow g(y)=\frac{y}{2}+c \Rightarrow g(x+f(y))=\frac{x+f(y)}{2}+c \Rightarrow g(x+f(y))=\frac{x+2y-2c+5}{2}+c = \frac{x+2y+5}{2}$

3. C

Sol. ${}^mC_1\cdot{}^nC_m - {}^mC_2\cdot{}^{2n}C_m + {}^mC_3\cdot{}^{3n}C_m \,.....\, (-1)^{m-1}\cdot{}^mC_m\cdot{}^{mn}C_n$
$\Rightarrow$ Coefficient of $x^m$ in expression
${}^mC_1\cdot(1+x)^n - {}^mC_2\cdot(1+x)^{2n} + {}^mC_3\cdot(1+x)^{3n} \,.....\, {}^mC_m\cdot((1+x)^{mn})\cdot(-1)^{m-1}$
$\Rightarrow {}^mC_0 - \left[{}^mC_0 - {}^mC_1\cdot(1+x)^n + {}^mC_2\cdot(1+x)^{2n} - {}^mC_3\cdot(1+x)^{3m} \,.....\, {}^mC_m\cdot(-1)^{mn}\cdot(1+x)^{mn}\right]$
$\Rightarrow$ Coefficient of $x^m$ in ${}^mC_0-(1-(1+x)^n)^m \Rightarrow$ Coefficient of $x^m$ in $\left(1-(1-(1+x)^n)^m\right)$

4. B, D

Sol. If there is 2k occurrence of a then this can be done by ${}^nC_{2k}$ ways and remaining positions can be filled in $2^{n-2k}$ ways. So total number of ways are $\sum\limits_{k=0}^{n}{}^nC_{2k}\,2^{n-2k} = \frac{1}{2}(3^n+1)$

5. B, C

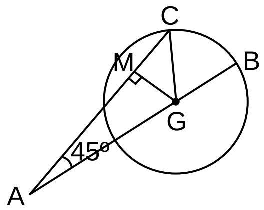
Sol. Let $f(x)=\frac{1}{1+x}$, then $\frac{1}{n}\left(\frac{1}{1+\frac{1}{n}}\right)<\int_0^{\frac{1}{n}}f(x)dx<\frac{1}{n}$, $\frac{1}{1+n}<\ln\left(1+\frac{1}{n}\right)<\frac{1}{n}$

Now, $a_n-a_{n+1}=-\log n-\frac{1}{n+1}+\log(n+1) = \log\left(1+\frac{1}{n}\right)-\frac{1}{n+1}>0$

$b_{n+1}-b_n = a_{n+1}-a_n-\frac{1}{n+1}+\frac{1}{n} = \frac{1}{n+1}-\log\left(1+\frac{1}{n}\right)-\frac{1}{n+1}+\frac{1}{n} = -\log\left(1+\frac{1}{n}\right)+\frac{1}{n}>0$

6. A, C, D

Sol. Let coordinate of A(a, 9 – a), then the distance of the centre of circle
$x^2 + y^2 - 4x - 4y - \frac{1}{2} = 0$ from AC will be
= |AG| sin 45°
$=\sqrt{(a-2)^2+(7-a)^2}\cdot\frac{1}{\sqrt{2}}$



Now, $d \le \sqrt{\frac{17}{2}} \Rightarrow \sqrt{(a-2)^2+(7-a)^2}\cdot\frac{1}{\sqrt{2}} \le \sqrt{\frac{17}{2}}$

$\Rightarrow a^2 - 9a + 18 \le 0 \Rightarrow a \in [3, 6]$

7. C

8. B

Sol. **(for Q. 7 & 8):**

We can determine the value of $P_d(t, N)$ as = $\frac{\text{coefficient of } x^t \text{ in } (x^1+x^2+x^3.....x^d)^N}{d^N}$

The value of $P_3(5,2) = P_3(5,3) = \frac{2}{9}$; $P_9(15,2) = \frac{4}{81}$

$P_6(14,3) = P_6(14,5) = \frac{5}{72}$ ; $P_3(10,4) = P(10,6) = \frac{10}{81}$

$P_4(9,3) = P_4(9,4) = \frac{5}{32}$

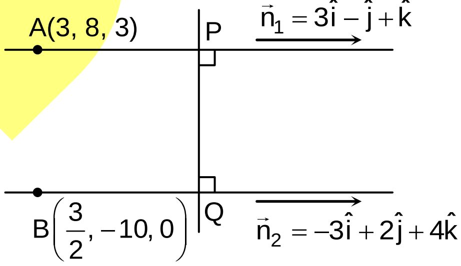
9. B

10. A

Sol. **(for Q. 9 & 10):**

Equation of line-1 $\equiv \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$

Equation of line-2 ≡ 2x + 5y – z + 47 = 2x + y + z + 7 = 0 in symmetric form $\frac{x-\frac{3}{2}}{-3} = \frac{y+10}{2} = \frac{z}{4}$




Shortest distance between both line

$= \left|\frac{\overrightarrow{AB}\cdot(\vec{n}_1 \times \vec{n}_2)}{|\vec{n}_1 \times \vec{n}_2|}\right| = 3\sqrt{30}$

Now, P is point on AP ⇒ P = (3 + 3r, 8 – r, 3 + r)

Q is point of BQ = $Q = \left(\frac{3}{2} - 3r', \; -10 + 2r', 4r'\right)$

Now, $\overrightarrow{PQ} \parallel (\vec{n}_1 \times \vec{n}_2)$

$\Rightarrow \frac{\frac{3}{2}+3r+3r'}{-2} = \frac{19-r-2r'}{-5} = \frac{3+r-4r'}{1} \Rightarrow r = 0 \text{ and } r' = \frac{3}{2}$

∴ P = (3, 8, 3) and Q = (–3, –7, 6) midpoint of PQ = $\left(0, \frac{1}{2}, \frac{9}{2}\right)$,

so required plane 2x + 5y – z + 2 = 0

## PART – B

1. 3

Sol. Let vertices $\vec{A}, \vec{B}, \vec{C}$ and circumcentre as origin, then

$\vec{H}$ (orthocentre) = $\vec{A} + \vec{B} + \vec{C}$

$\vec{G}$ (centroid) = $\frac{\vec{H}}{3}$

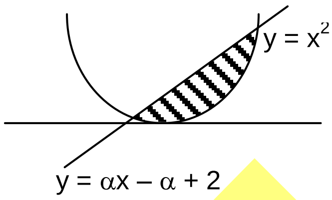
$\vec{F} = \frac{\vec{G}+\vec{H}}{2} = \frac{2\vec{H}}{3}$

Now, $AF^2 + BF^2 + CF^2 = (\vec{A}-\vec{F})(\vec{A}-\vec{F}) + (\vec{B}-\vec{F})(\vec{B}-\vec{F}) + (\vec{C}-\vec{F})(\vec{C}-\vec{F})$

$= \vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{B} + \vec{C}\cdot\vec{C} - 2(\vec{A}+\vec{B}+\vec{C})\cdot\vec{F} + 3\vec{F}\cdot\vec{F} = 3R^2 - \vec{F}\cdot\left(2(\vec{A}+\vec{B}+\vec{C}) - 3\vec{F}\right) = 3R^2$

2. 2

Sol. Let point of intersection of line $y = \alpha x - \alpha + 2$ with $y = x^2$ are $x = x_1$ and $x = x_2$




Then, $x_1 + x_2 = \alpha$ ; $x_1x_2 = \alpha - 2$

$$S(\alpha) = \int_{x_1}^{x_2} \left((\alpha x - \alpha + 2) - x^2\right)dx$$

For area to minimized $\frac{d(S(\alpha))}{d\alpha} = 0$

$$\Rightarrow \frac{d(S(\alpha))}{d(\alpha)} = \frac{d}{d\alpha}\int_{x_1}^{x_2}\left((\alpha x - \alpha + 2) - x^2\right)dx = \int_{x_1}^{x_2}\frac{d}{d\alpha}\left(\alpha x - \alpha + 2 - x^2\right)dx$$

$$= \int_{x_1}^{x_2}(x-1)dx = (x_2 - x_1)\left(\frac{x_1 + x_2}{2} - 1\right) = 0 \Rightarrow x_1 + x_2 = 2 \Rightarrow \alpha = 2$$

3. 1

Sol. $m = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$ ; $n = \frac{1}{2}\sqrt{2a^2 + 2c^2 - a^2}$ ; $p = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$

$\Rightarrow a^2 + b^2 + c^2 = (\lambda(2))^2\{m^2 + n^2 + p^2\}$

$\Rightarrow a^2 + b^2 + c^2 = (\lambda(2))^2\left\{\frac{1}{4}\{(2b^2 + 2c^2 - a^2) + (2(a^2) + 2c^2 - b^2) + (2a^2 + 2b^2 - c^2)\}\right\}$

$\Rightarrow \lambda(2) = \frac{2}{\sqrt{3}}$

4. 02.00

5. 03.00

Sol. **(for Q. 4.-5):**
Let the vertices of triangle ABC are $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$
Then $x_1$, $x_2$, $x_3$ are root of $a_1x^3 + b_1x + c_1 = 0$

$\Rightarrow x_1 + x_2 + x_3 = 0$ and $\sum x_1x_2 = \frac{b_1}{a_1}$.

Similarly $y_1 + y_2 + y_3 = 0$ and $\sum y_1y_2 = \frac{b_2}{a_2}$

$\Rightarrow$ Circumcentre is at origin

$\Rightarrow x_1^2 + y_1^2 = x_2^2 + y_2^2 = x_3^2 + y_3^2 = R^2 \Rightarrow x_1^2 + x_2^2 + x_3^2 + y_1^2 + y_2^2 + y_3^2 = 3R^2$

$\Rightarrow (x_1 + x_2 + x_3)^2 - 2\sum x_1x_2 + (y_2 + y_2 + y_3)^2 - 2\sum y_1y_2 = 3R^2$

$\Rightarrow -2\left[\sum xx_2 + \sum y_1y_2\right] = 3R^2 \Rightarrow -2\left[\frac{b_1}{a_1} + \frac{b_2}{a_2}\right] = 3R^2$

6. 08.00

7. 04.00

Sol. Let point B(0, b) and C(0, c) and assume b > c

The equation of PB is $y - b = \frac{y_0 - b}{x_0}(x)$

$\Rightarrow (y_0 - b)x - x_0y + x_0b = 0$ as its tangent of circle $(x - 1)^2 + y^2 = 1$

$\Rightarrow 1 = \left|\frac{y_0 - b + x_0b}{\sqrt{(y_0 - b)^2 + x_0^2}}\right| \Rightarrow (x_0 - 2)b^2 + 2y_0b - x_0 = 0$ similarly $(x_0 - 2)c^2 + 2y_0c - x_0 = 0$

$\Rightarrow b+c=\dfrac{-2y_0}{x_0-2}$ and $bc=\dfrac{-x_0}{x_0-2}$ $\Rightarrow (b-c)^2=\dfrac{4x_0^2+4y_0^2-8x_0}{(x_0-2)^2}$ $\Rightarrow b-c=\dfrac{2x_0}{x_0-2}$ (as $y_0^2=2x_0$)

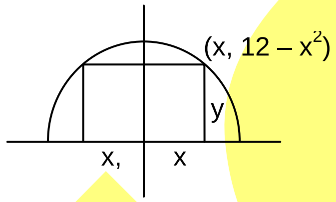
Now, area of $\Delta PBC = \dfrac{1}{2}(b-c)x_0 = \dfrac{x_0^2}{x_0-2} = (x_0-2)+\dfrac{4}{(x_0-2)}+4 \ge 8$

8. 32.00

Sol. $A = 2x(12 - x^2)$

$\dfrac{dA}{dx} = 2(12-x^2+(-2x(2x))) = 24-6x^2$

For maximum x = 2, A = 32 sq units




9. 86.50
(Range 86.50 to 86.60)

Sol. $A = 2x\sqrt{3}(10-x)$

$\dfrac{dA}{dx} = 2\sqrt{3}(10-2x)$

Maximize when x = 5

$A = 50\sqrt{3}$