

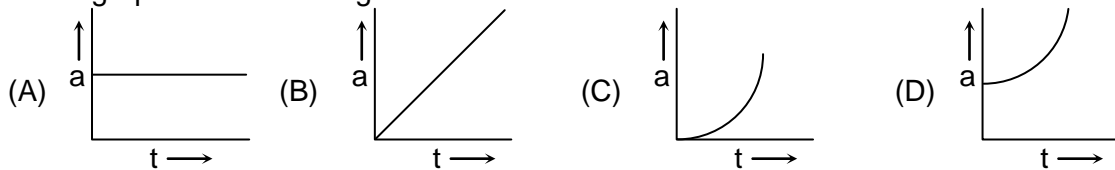
OLTS-1920-JEEM 2020

FULL TEST - 3

PART – A : PHYSICS SECTION-A : Single Correct Answer Type

This section contains **20 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

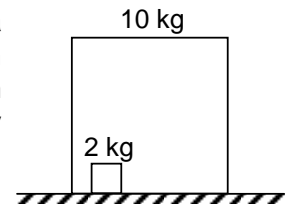
- 1 A small bead slides from rest along a wire that is shaped like a vertical uniform helix (spring). Which graph shows the magnitude of the acceleration 'a' as a function of time?



Ans.: D

Sol. $a_t = g \sin \theta$, $a_r = \frac{u^2}{r} = \frac{a_t^2}{r} t^2$, $a = a_t \sqrt{a_t^2 \frac{t^4}{r^2} + 1}$

- 2 A cubical box of mass 10 kg with edge 5m is free to move on a frictionless horizontal surface. Inside is a small block of mass 2 kg which moves without friction inside the box. At $t = 0$, the block is moving with 5 m/s directly towards one of the faces of the box, while the box is initially at rest. The coefficient of restitution is $\frac{9}{10}$.



After 1 minute, the block is at a position x from original position. Which of the following is closest to x?

- (A) 50m (B) 100m (C) 150m (D) 0m

Ans.: A

Sol. $x_{cm} = 50m$

3. A light bulb has a solid cylindrical filament of length L and radius a and consumes power P. We have to design a new bulb, using cylindrical filament of same material, operating at same voltage and emitting the same spectrum of light, which will consume power nP. The new length and radius of filament are (loss due to convection is minimal, temperature of filament is uniform).

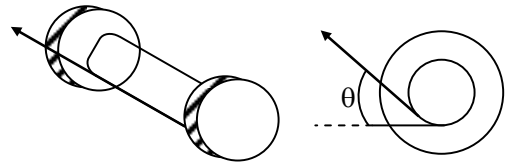
- (A) $n^{1/3}L$, $n^{2/3}a$ (B) $n^{2/3}L$, $n^{1/3}a$ (C) $n^{2/3}L$, $n^{2/3}a$ (D) $n^{1/3}L$, $n^{1/3}a$

Ans.: A

Sol. $P = \frac{U^2}{R}$ and $P \propto a$

4. Kepler's law state that
 (i) The orbits of planets are elliptical with one focus at the Sun
 (ii) A line connecting the sun and a planet sweeps out equal areas in equal times
 (iii) The square of the time period of a planet's orbit is proportional to the cube of its semi major axis.

10. A solid cylinder of radius 1cm attached at the center of two disks of radius 2cm. It is placed on a surface where it can roll but not slip. A thread is wound around the central cylinder when the thread is pulled at $\theta = 90^\circ$ to horizontal, the apparatus rolls to right. Which below is the largest value of θ far which it will not roll to right when pulling on thread?

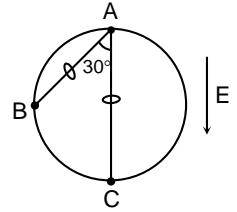


- (A) $\theta = 15^\circ$ (B) 30° (C) $\theta = 45^\circ$ (D) $\theta = 60^\circ$

Ans.: D

Sol. $Fr - fR = I\alpha$, $f - F \cos \theta = ma$, $a = R\alpha \Rightarrow a \geq 0 \Rightarrow r - R \cos \theta \geq 0$

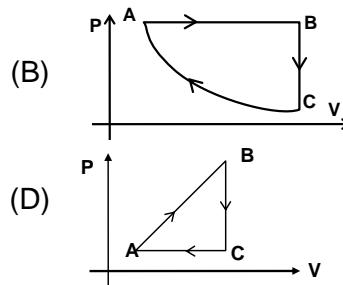
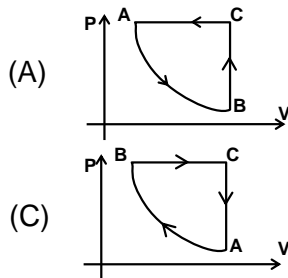
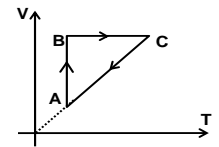
11. A circular ring is fixed in the horizontal plane. AB and AC are frictionless wires stretched across the ring as shown. Beads P and Q of same mass carrying charge +q each are free to slide on the wires. The ratio of the time taken by P to travel from A to B to the time taken by Q to travel from A to C is
- (A) 1 (B) $1/\sqrt{3}$ (C) $\sqrt{3}$ (D) 2



Ans.: A

Sol. The time will be same.

12. A cyclic process ABCA is shown in a $V - T$ diagram. The corresponding PV diagram will be



Ans.: A

Sol. $AB \rightarrow$ Isothermal, $BC \rightarrow$ Isochoric, $CA \rightarrow$ Isobaric

13. A cylinder of radius r and length ℓ is placed in an uniform electric field E such that the axis of cylinder makes an angle θ with the direction of electric field. Total electric flux through curved surface of cylinder is

- (A) $2 \pi r \ell E \cos \theta$ (B) $2 \pi r \ell E \sin \theta$ (C) zero (D) None of these

Ans.: C

Sol. $\phi = \int \vec{E} \cdot d\vec{s}$

14. A point source S of light is emitting a power P. A sphere of radius r is situated at a distance R from source S ($r \ll R$), has a mass M and specific heat capacity C, assuming that the sphere absorbs all the radiation incident on it. The time in which temperature of sphere rise by $\theta^\circ\text{C}$ is:

- (A) $\frac{R^2 MC \theta}{Pr^2}$ (B) $\frac{2R^2 MC \theta}{Pr^2}$ (C) $\frac{3R^2 MC \theta}{Pr^2}$ (D) $\frac{4R^2 MC \theta}{Pr^2}$

Ans.: D

Sol. $\Delta \theta = MS \Delta T$

15. In Young's double slit experiment, the 7th maximum with wavelength λ_1 is at a distance d_1 and that with wavelength λ_2 is at a distance d_2 . Then d_1/d_2 is:

- (A) λ_1/λ_2 (B) λ_2/λ_1 (C) λ_1^2/λ_2^2 (D) λ_2^2/λ_1^2

Ans.: A

Sol. $x = \frac{nD\lambda}{d}$

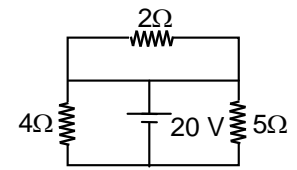
16. When the distance between the object and the screen is more than $4f$, we can obtain the image of the object on the screen for the two positions of a convex lens of focal length f . It is called displacement method. In one case the image is magnified. If l_1 and l_2 be the sizes of the two images, then the size of the object is:

- (A) $(l_1 + l_2)/2$ (B) $l_1 - l_2$ (C) $\sqrt{l_1 l_2}$ (D) $\sqrt{l_1/l_2}$

Ans.: C

Sol. From displacement method for finding

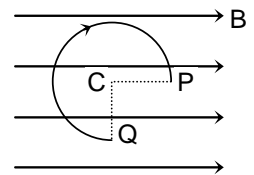
17. In the circuit shown in the figure,
 (A) current passing through 2Ω resistance is 9A
 (B) current passing through 4Ω resistance is 10A
 (C) current passing through 5Ω resistance is 4A
 (D) none of the above



Ans.: C

Sol. $\sum i = 0 \Rightarrow \sum v = 0$

18. A wire carrying current 'i' is bent as shown and placed in the plane of paper in a uniform magnetic field \vec{B} . Magnitude of force on the wire is: (Assuming $CP = CQ = r$).



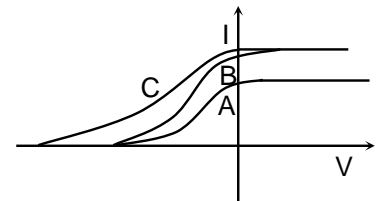
- (A) Bir (B) $Bi2r$
 (C) $\sqrt{2} Bir$ (D) zero

Ans.: A

Sol. $\vec{F} = i(\vec{\ell} \times \vec{B})$

19. In a photoelectric experiment anode potential is plotted against plate current. Then

- (A) A and B will have different intensities while B and C will have different frequencies.
 (B) B and C will have different intensities while A and C will have different frequencies.
 (C) A and B will have different intensities while A and C will have equal frequencies.
 (D) A and B will have equal intensities while B and C will have different frequencies

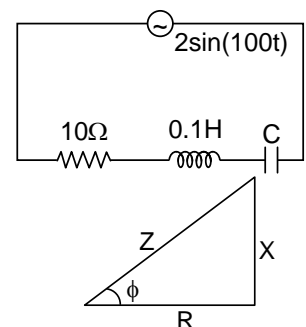


Ans.: A

Sol. From theory of Photo-electric effect.

20. The power factor of the circuit is $1/\sqrt{2}$. The capacitance of the circuit is equal to :

- (A) $400 \mu\text{F}$ (B) $300 \mu\text{F}$
 (C) $500 \mu\text{F}$ (D) $200 \mu\text{F}$



Ans.: C

Sol. $\left| \omega L - \frac{1}{\omega C} \right| = R$
 $\Rightarrow C = 500 \mu\text{F}$.

SECTION – B : Single digit integer

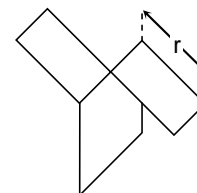
This section contains **2 questions**. The answer to each question is a **single digit integer** ranging from 0 to 9 (both inclusive).

21. If a particle at rest explodes into N_1 or fewer particles with known masses, and the total kinetic energy of the new particles is known the kinetic energy of each of the new particles is completely known.

Ans.: 2

Sol. Conservation of momentum

22. A paper helicopter with rotor radius r and weight w is dropped from a height h in air with a density ρ . Assuming that the helicopter quickly reaches terminal velocity, a function for the time of flight T can be found in the forms $T = kh^{\alpha}r^{\beta}\rho^{\delta}w^{\omega}$ where k is a constant (dimensionless). Determine α



Ans.: 1

Sol. Dimensional analysis

SECTION – C : Numerical Answer Type

This section contains **3 questions**. The answer to each question is a **Numerical Answer Type XXXXX.XX** (for example if answer is **99999.99** then write the same answer without rounding off any digit).

23. Two streams of water flow through the U shaped tubes as shown. The tube on left has cross-sectional area A and speed of water flowing through it is v , the tube on right has area of cross-section $\frac{A}{2}$.



If the net force on the tube assembly is zero, what must be the speed v' of water flowing through the tube on right? (Neglect gravity, speed of water on entry & exit is same).

Ans.: 00001.41

Sol. Force on tube = ρAv^2

24. When a block of wood with a weight 30N is completely submerged under water the buoyant force on the block of wood from water is 500N when the block is released it floats at the surface what fraction of the block will then be visible above the surface of water.

(A) $\frac{1}{5}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

Ans.: 00000.04

Sol. $V_i = \frac{\rho_B}{\rho_w} = \frac{3}{50}$, $V_o = \frac{2}{50}$

25. A train moves towards a stationary observer with speed 34 m/s. The train sound a whistle and its frequency registered by the observer is f_1 . If the train's speed is reduced to 17 m/s, the frequency registered is f_2 . If the speed of sound is 340 m/s then ratio f_1/f_2 is:

Ans.: 00001.06

Sol. From Doppler's effect

PART-B : CHEMISTRY

SECTION-A : Single Correct Answer Type

This section contains **20 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

26. In which case, the order acidic strength is not correct?

- (A) $\text{HI} > \text{HBr} > \text{HCl}$ (B) $\text{HIO}_4 > \text{HBrO}_4 > \text{HClO}_4$
(C) $\text{HClO}_4 > \text{HClO}_3 > \text{HClO}_2$ (D) $\text{HF} > \text{H}_2\text{O} > \text{NH}_3$

Ans.: B

Sol. $\text{HClO}_4 > \text{HBrO}_4 > \text{HIO}_4 \Rightarrow$ acidic strength has been decided on the basis of electronegativity or charge density on central atom.

27. Stability of the species $\text{Li}_2, \text{Li}_2^-$ and Li_2^+ has the order:

- (A) $\text{Li}_2 > \text{Li}_2^- > \text{Li}_2^+$ (B) $\text{Li}_2 > \text{Li}_2^+ > \text{Li}_2^-$ (C) $\text{Li}_2^- > \text{Li}_2 > \text{Li}_2^+$ (D) $\text{Li}_2^- > \text{Li}_2^+ > \text{Li}_2$

Ans.: B

Sol. From Bond order calculation, it is found $\text{Li}_2 = 1, \text{Li}_2^+ = \text{Li}_2^- = 0.5$

But due to more antibonding electrons $\text{Li}_2^- < \text{Li}_2^+$

28. Pick out an incorrect statement from the following:

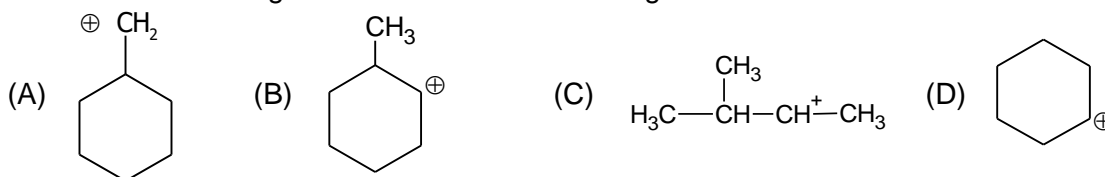
- (A) Benzene cannot be iodinated with I_2 directly
(B) The product of the reaction between benzene and $(\text{CH}_3\text{CH}_2\text{CH}_2\text{CO})_2\text{O}$ in presence of anhydrous AlCl_3 is $\text{PhCOCH}_2\text{CH}_2\text{CH}_3$
(C) The electrophile formed with HNO_3 & H_2SO_4 is SO_3
(D) Iodination of Arene may occur with ICl also

Ans.: C

Sol. Iodination of benzene with I_2 cannot be done directly because, it is reversible and it can be done with the help of HNO_3 .

The electrophile formed with HNO_3 and H_2SO_4 is not SO_3 it is NO_2^+ .

29. Which of the following carbocation will not rearrange :



Ans.: D

Sol.

Because all carbon of Option (D) is 2°

30. Which test can distinguish 1,1 dichloroethane and 1,2 dichloroethane.

- (A) (i) aq KOH, (ii) 2,4 - DNP (B) NaHSO_3
(C) Na_2CO_3 (D) FeCl_3 solution

Ans.: A

Sol. Gem-dihalides on substitution by OH^- produces carbonyl compound, which will reacts with 2,4-DNP but 1,2-dichloroethane will give vic-diol.

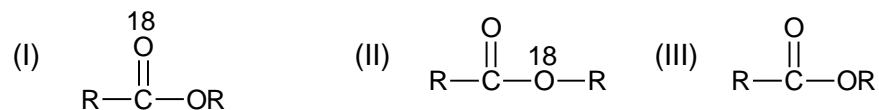
31. In an amino acid the carboxylic acid group has $K_a=10^{-4}$ and amino group has $K_b= 10^{-5}$. The isoelectric point of that amino acid is

- (A) 4 (B) 5 (C) 6.5 (D) 4.5

Ans.: C

Sol. pH at isoelectric point = $\frac{pK_{a1} + pK_{a2}}{2} = \frac{4 + 9}{2} = 6.5$

32. The reaction of $R-\overset{\overset{18}{O}}{\parallel}{C}-OH$ with H^+/ROH will give:



(A) I, II only (B) I, III only (C) I, II and III only (D) I only

Ans.: B

Sol. It is Acyl Cleavage

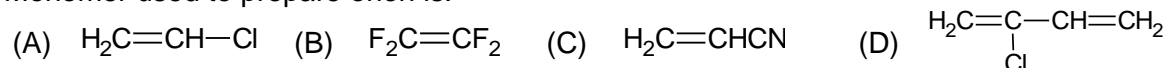
33. Benzenediazonium chloride forms coloured compound when treated with:

(A) Phenol (B) Cresol (C) Resorcinol (D) All of the above

Ans.: D

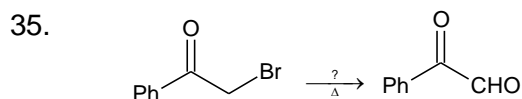
Sol. All have electron donating group attached to the ring which makes dye formation favourable.

34. Monomer used to prepare orlon is:



Ans.: C

Sol. Orlon is polyvinyl cyanide.

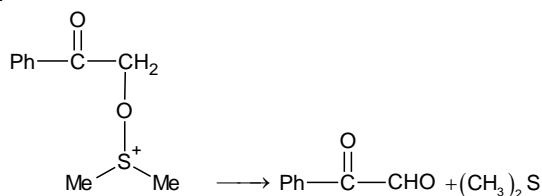


The reagent used in the above transformation is:

(A) $(CH_3)_2SO$ (B) $H-\overset{\overset{O}{\parallel}}{C}-N(CH_3)_2$ (C) Acetone (D) P_4O_{10}

Ans.: A

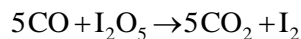
Sol.



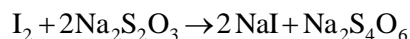
36. Which of the following halogen oxide is used for estimation of carbon monoxide in automobile exhaust gases?

(A) Cl_2O_7 (B) I_2O_5 (C) ClO_2 (D) BrO_3

Ans.: B



Sol.



37. The vapour density of PCl_5 is 104.25 but when heated to $230^\circ C$ its vapour density is reduced to 62. the degree of dissociation of PCl_5 at this temperature will be

(A) 6.8% (B) 68% (C) 46% (D) 64%

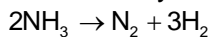
Ans.: B

Sol. $\alpha = \frac{D-d}{(n-1)d}$

38. A mixture of $\text{NH}_3(\text{g})$ and $\text{N}_2\text{H}_4(\text{g})$ is placed in a sealed container at 27°C , 0.5 atm. After complete decomposition of NH_3 and N_2H_4 at 927°C according to following reaction,
 $2\text{NH}_3 \rightarrow \text{N}_2 + 3\text{H}_2$, $\text{N}_2\text{H}_4 \rightarrow \text{N}_2 + 2\text{H}_2$
 the total pressure reached to 4.5 atm. The percentage of NH_3 (by mole) is
 (A) 75% (B) 25% (C) 40% (D) 60%

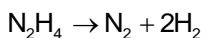
Ans.: A

Sol. Let, $\text{NH}_3 = x$ mole
 $\text{N}_2\text{H}_4 = y$ mole



$$x \quad 0 \quad 0$$

$$0 \quad \frac{x}{2} \quad \frac{3x}{2}$$



$$y \quad 0 \quad 0$$

$$0 \quad y \quad 2y$$

Initial total mole = $x + y$

Final total mole = $2x + 3y$

$$\frac{2x + 3y}{x + y} = \frac{4.5}{(0.5) \times (4)}$$

$$\Rightarrow x = 3y \Rightarrow \%x = 75$$

39. A solution is 0.09 M in HCl and 0.09 M in CH_2ClCOOH . The pH of the solution is 1. What is the K_a for CH_2ClCOOH ?

- (A) 1.8×10^{-5} (B) 1.25×10^{-2} (C) 3.6×10^{-3} (D) 2.8×10^{-4}

Ans.: B

Sol. $\text{CH}_2\text{ClCOOH} \rightleftharpoons \text{CH}_2\text{ClCOO}^- + \text{H}^+$

$$0.09$$

$$(0.09 - x) \quad x \quad (0.09 + x)$$

$[\text{H}^+]$ in the solution = 10^{-1}

$$0.09 + x = 10^{-1}$$

$$x = 0.01$$

$$K_a \text{ for } \text{CH}_2\text{ClCOOH} = \frac{0.01 \times 0.01}{(0.09 - 0.01)} = 1.25 \times 10^{-2}$$

40. Borazine is called Inorganic Benzene. Which of the following statement is incorrect about borazine?

- (A) Each B and N atom is sp^2 hybridized
 (B) Borazine satisfies the $(4n+2)$ Huckel's rule
 (C) Like organic benzene, borazine does not give addition product with HCl
 (D) Borazine contains dative $p\pi-p\pi$ bond

Ans.: C

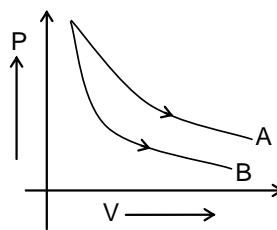
Sol. Borazine give addition product due to polar π bond.

41. In the nuclear reaction, ${}_{92}^{235}\text{U} \longrightarrow {}_{82}^{207}\text{Pb}$, the number of α - and β -particles lost would be
 (A) 8, 4 (B) 6, 2 (C) 7, 4 (D) 4, 3

Ans.: C

Sol. ${}_{92}^{235}\text{U} \longrightarrow {}_{82}^{207}\text{Pb} + 7{}_2\text{He}^4 + 4{}_{-1}\beta^0$

42. P-V plot for two gases (assuming ideal) during adiabatic processes are given in the figure. Plot A and plot B should correspond respectively to:



- (A) He and H₂
 (B) O₂ and He
 (C) Ar and Ne
 (D) H₂ and Cl₂

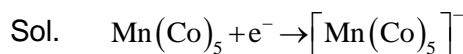
Ans.: B

Sol. $PV^\gamma = \text{constant}$

43. Which of the following is an oxidizing agent?

- (A) Mn(CO)₅ (B) Fe(CO)₅ (C) Mn₂(CO)₁₀ (D) Fe₂(CO)₉

Ans.: A



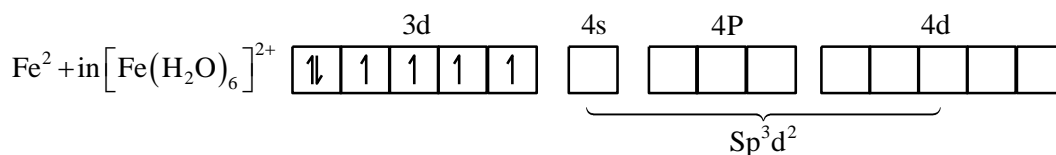
Less stable more stable, EAN of Mn = 36 (kr)

44. [Fe(H₂O)₆]²⁺ and [Fe(CN)₆]⁴⁻ differ in

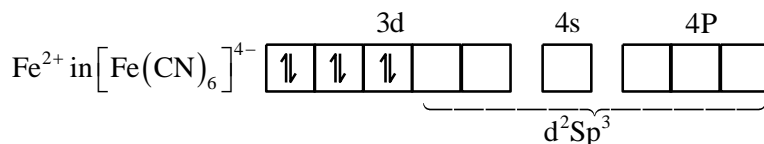
- (A) geometry, magnetic moment (B) geometry, hybridization
 (C) Magnetic moment, colour (D) hybridization, number of d-electrons

Ans.: C

Sol.



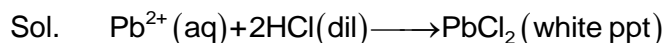
Colour: Pale green, $\mu = 4.9 \text{ B.M}$ octahedral



45. An aqueous solution of a substance gives a white precipitate on treatment with dil. HCl which dissolves on heating, When H₂S is passed through the hot acidic solution, black precipitate is obtained. The substance is

- (A) Hg₂²⁺ salt (B) Hg²⁺ salt (C) Ag⁺ salt (D) Pb²⁺ salt

Ans.: D



White ppt of PbCl₂ is soluble in hot water Pb²⁺ ions give black ppt of Pbs with H₂S

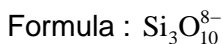
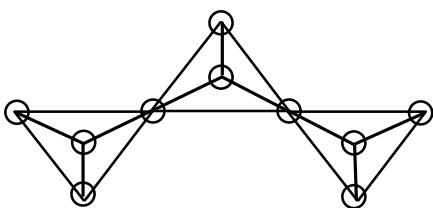
SECTION-B: Single Digit Integer

This section contains **2 questions**. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive).

46. The silicate anion in the mineral kinoite is a chain of three SiO₄ tetrahedra, that share corners with adjacent tetrahedra. The change of the silicate anion is

Ans.: 8

Sol.



47. The density of gas X is twice that of Y at the same temperature, the molecular weight of gas Y is twice that of X. The pressure of gas X w.r.t. pressure of gas 'Y' will be:

Ans.: 4

Sol. $PV = RT$

$$\text{or } \frac{PM}{d} = RT \Rightarrow \frac{P_1}{P_2} \times \frac{M_1}{M_2} = \frac{d_1}{d_2} \quad \left(\frac{d_x}{d_y} = 2 \text{ and } \frac{M_x}{M_y} = \frac{1}{2} \right)$$

$$\frac{P_x}{P_y} \cdot \frac{1}{2} = 2 \Rightarrow \frac{P_x}{P_y} = \frac{4}{1}$$

SECTION – C : Numerical Answer Type

This section contains **3 questions**. The answer to each question is a **Numerical Answer Type XXXXX.XX** (for example if answer is **99999.99** then write the same answer without rounding off any digit).

48. What is the melting point of benzene if $\Delta H_{\text{fusion}} = 9.95 \text{ kJ / mol}$ and $\Delta S_{\text{fusion}} = 35.7 \text{ J/K-mol}$?

Ans.: 00278.70

Sol. $T = \frac{\Delta H}{\Delta S} = \frac{9.95 \times 1000}{35.7} = 278.7 \text{ K}$

49. The pH of a solution containing 0.4 M HCO_3^- and 0.2 M CO_3^{2-} is: ($K_{a1}(\text{H}_2\text{CO}_3) = 4 \times 10^{-7}$, $K_{a2}(\text{HCO}_3^-) = 4 \times 10^{-11}$) ($\log 2 = 0.3010$)

Ans.: 00010.10

Sol. $\text{pH} = \text{p}K_{a2} + \log \frac{[\text{CO}_3^{2-}]}{[\text{HCO}_3^-]} = (11 - \log 4) + \log \left(\frac{0.2}{0.4} \right) = 10.1$

50. Heat of neutralization between HCl and NaOH is $-13.7 \text{ kcal eq}^{-1}$. Heat of neutralization of $\text{H}_2\text{C}_2\text{O}_4$ (oxalic acid) with NaOH is $-26 \text{ kcal mole}^{-1}$. Hence, heat of dissociation of $\text{H}_2\text{C}_2\text{O}_4$ as $\text{H}_2\text{C}_2\text{O}_4 \rightleftharpoons 2\text{H}^+ + \text{C}_2\text{O}_4^{2-}$, is

Ans.: 00001.40

Sol. $\Delta H_{(\text{neu.})\text{obs.}} = \Delta H_{(\text{neu.})\text{exp.}} + \Delta H_{(\text{I.E.})\text{acid}} + \Delta H_{(\text{I.E.})\text{base}}$

PART – C: MATHEMATICS
SECTION-A : Single Correct Answer Type

This part contains **20 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

51. The mirror image of the directrix of the parabola $y^2 = 4(x + 1)$ in the line mirror $x + 2y = 3$ is
(A) $x = -2$ (B) $4y - 3x = 16$ (C) $x - 3y = 0$ (D) $x + y = 0$

Ans. B

Sol. directrix of $y^2 = 4(x + 1)$ is $x = -2$
any point on $x = -2$ is $(-2, k)$
mirror image of $(-2, k)$ in the line
 $x + 2y = 3$ is given by $x = -\frac{4k}{5}$ (1) and $y = \frac{20 - 3k}{5}$ (2)
from equation (1) and (2) $y = 4 + \frac{3x}{4}$

52. 5 Indian and 5 American couples meet at a party and shake hands. If no wife shakes hands with her own husband and no Indian wife shakes hands with a male, then the number of hands shakes that takes place in the party is
(A) 95 (B) 110 (C) 135 (D) 150

Ans. C

Sol. 10 male can hand shake $m \text{ } ^{10}C_2$ ways
10 women can hand shake $m \text{ } ^{10}C_2$ ways
10 women can hand shake with 10 men = 5×9 ways
Total hand shake = $^{10}C_2 + ^{10}C_2 + 45 = 135$ ways

53. The number of points with integral coordinates $(2a, a - 1)$ for integral values of a that fall in the interior of the larger segment of the circle $x^2 + y^2 = 25$ cut off by the parabola $x^2 + 4y = 0$ is
(A) 4 (B) 2 (C) 0 (D) none of these

Ans. B

Sol. Coordinates are $(2, 0), (4, 1)$

54. In the truth table for the statement
 $p \rightarrow (p \vee q)$, the last column has the truth value in the following order is
(A) FFFF (B) TFFT (C) FFTT (D) TTTT

Ans. D

Sol.

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

55. The value of $\lambda \in \mathbb{R}$ for which the equation
 $(x + 6)^4 - \lambda(x + 6)^2 \cdot (x^2 + 12x + 37) + (\lambda - 1) \cdot (x^2 + 12x + 37)^2 = 0$ has no real root.
(A) $\lambda \in (1, 2)$ (B) $\lambda \in (-\infty, 1) \cup (2, \infty)$
(C) $\lambda = 3/2$ (D) none of these

Ans. B

Sol. Put $(x + 6)^2 = t$

$$t^2 - \lambda t(t+1) + (\lambda - 1)(t+1)^2 = 0$$

$$\Rightarrow t = \frac{1-\lambda}{\lambda-2} = (x+6)^2$$

For no real root $\frac{1-\lambda}{\lambda-2} < 0 \Rightarrow \lambda \in (-\infty, 1) \cup (2, \infty)$

56. If each of n numbers $x_i = i, i=1, 2, 3, \dots, n$ is replaced by $(i+1)x_i$, then the new mean is

- (A) $\frac{(n+1)(n+2)}{n}$ (B) $n+1$ (C) $\frac{(n+1)(n+2)}{3}$ (D) none of these

Ans. C

Sol. $AM = \frac{1}{n} \sum_{i=1}^n (i+1)x_i = \frac{1}{n} \sum_{i=1}^n (i+1)i = \frac{1}{n} [\sum i^2 + \sum i] = \frac{(n+1)(n+2)}{3}$

57. Three normals are drawn from point $(5, 0)$ to parabola $y^2 = 4x$. The centroid of the triangle formed by feet of these three normals is

- (A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (B) $(5, 0)$ (C) $(2, 0)$ (D) $(0, 2)$

Ans. C

Sol. Hint : Sum of the ordinates of the feet of the three normals drawn from a point to the parabola is zero.

Solution : Since normals are drawn from $(5, 0)$, it cannot be centroid

point $(am^2, -2am)$

$$y = mx - 2am - am^3$$

It passes through $(5, 0)$

$$0 = 5m - 2m - m^3$$

$$m = 0 \text{ or}$$

$$m^2 = 3 \therefore m = \pm\sqrt{3}$$

$$\text{centroid} = \frac{3+3+0}{3}, 0 = (2, 0)$$

58. The coefficient of $\frac{1}{n^4}$ in the expression $\frac{2}{3n+2} + \frac{2}{(3n+2)^2} + \frac{8}{3(3n+2)^3} + \dots, n \in \mathbb{N}$, is

- (A) $-\frac{4}{81}$ (B) $\frac{8}{81}$ (C) $-\frac{2}{9}$ (D) Cannot be determined

Ans. A

Sol. $\frac{2}{3n+2} + \frac{4}{2(3n+2)^2} + \frac{8}{3(3n+2)^3} + \dots, \infty = -\log\left(1 - \frac{2}{3n+2}\right) = \log\left(1 + \frac{2}{3n}\right)$

59. A packet of 10 CD's contains 4 defected. The CD's are selected at random, one by one, examined and are not replaced. The probability that 7th CD is the last defective is

- (A) $\frac{2}{21}$ (B) $\frac{4}{9}$ (C) $\frac{7}{27}$ (D) none of these

Ans. A

Sol. A = 3 defective in first six CD's

B = 7th CD is defective

$$P(A \cap B) = P(A).P(B/A)$$

$$P(A) = \frac{{}^4C_3 \times {}^6C_3}{{}^{10}C_6} = \frac{8}{21}$$

$$P(B/A) = \frac{1}{4} \quad \therefore \text{Required prob.} = \frac{8}{21} \times \frac{1}{4} = \frac{2}{21}$$

60. Let $f: \left[\frac{-\pi}{3}, \frac{k\pi}{3} \right] \rightarrow [-1, 2]$ when $f(x) = \sqrt{3} \sin x - \cos x + 1$. The value of k such that $f(x)$ becomes invertible function

- (A) 1 (B) 2 (C) 3/2 (D) 1/2

Ans. A

Sol.
$$f(x) = 2 \left(\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x \right) + 1$$

$$= 2 \sin \left(x - \frac{\pi}{6} \right) + 1$$

to be onto function $-1 \leq \sin \left(x - \frac{\pi}{6} \right) \leq \frac{1}{2}$

$$-\frac{\pi}{2} \leq x - \frac{\pi}{6} \leq \frac{\pi}{6} \Rightarrow \frac{-\pi}{3} \leq x \leq \frac{\pi}{3}$$

$$\frac{k\pi}{3} = \frac{\pi}{3} \Rightarrow k = 1$$

61. The value of ${}^{11}C_2 + {}^{11}C_4 + {}^{11}C_6 + {}^{11}C_8$ is equal to:

- (A) $2^{10}-1$ (B) $2^{10}-11$ (C) $2^{10}-12$ (D) none of these

Ans. C

Sol. Coefficient of even term = 2^{n-1}

$$\Rightarrow 2^{10} - {}^{11}C_0 - {}^{11}C_{10}$$

62. Let $z_k; k = 1, 2, 3, 4$ be four complex numbers such that $|z_k| = \sqrt{k+1}$ and $|30z_1 + 20z_2 + 15z_3 + 12z_4| = \lambda |z_2z_3z_4 + z_3z_4z_1 + z_1z_2z_4 + z_1z_2z_3|$, then $\lambda =$

- (A) $|z_1 z_2 z_4|$ (B) $|z_2 z_3 z_4|$ (C) $|z_1 z_3 z_4|$ (D) $|z_1 z_2 z_3|$

Ans. A

Sol.
$$\left| \frac{z_1}{2} + \frac{z_2}{3} + \frac{z_3}{4} + \frac{z_4}{5} \right| = \frac{\lambda}{60} |z_1z_2z_3z_4| \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \frac{1}{z_4} \right|$$

Now, $z_1\bar{z}_1 = 2, z_2\bar{z}_2 = 3, z_3\bar{z}_3 = 4$ and $z_4\bar{z}_4 = 5$ ($\because |z_k| = \sqrt{k+1}$)

$$\Rightarrow \lambda = \frac{60}{|z_1z_2z_3z_4|} = \frac{60}{\sqrt{2}\sqrt{3}\sqrt{4}\sqrt{5}} = \sqrt{30} = |z_1z_2z_4|$$

63. The sum of $\frac{3}{1.2} \cdot \frac{1}{2} + \frac{4}{2.3} \cdot \left(\frac{1}{2}\right)^2 + \frac{5}{3.4} \cdot \left(\frac{1}{2}\right)^3 + \dots$ to n terms is equal to

- (A) $1 - \frac{1}{(n+1)2^n}$ (B) $1 - \frac{1}{n.2^{n-1}}$ (C) $1 + \frac{1}{(n+1)2^n}$ (D) none of these

Ans. A

Sol.
$$t_n = \frac{n+2}{n(n+1)} \cdot \left(\frac{1}{2}\right)^n = \frac{2(n+1) - n}{n(n+1)} \cdot \left(\frac{1}{2}\right)^n = \frac{1}{n} \cdot \left(\frac{1}{2}\right)^{n-1} - \frac{1}{n+1} \cdot \left(\frac{1}{2}\right)^n$$

$$S_n = \sum T_n = 1 - \frac{1}{(n+1)} \left(\frac{1}{2}\right)^n$$

64. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x \rightarrow 0} \left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals

- (A) 1 (B) $e^{1/2}$ (C) e^2 (D) e^3

Ans. C

Sol. $\lim_{x \rightarrow 0} \left(\frac{f(1+x)}{f(1)}\right)^{1/x} = e^{\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{f(1+x)-f(1)}{f(1)}\right)} = e^{\frac{f'(1)}{f(1)}} = e^2$.

65. Let a hyperbola has its transverse and conjugate axis coinciding with the major and minor axes of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ respectively, if the hyperbola passes through one of the foci of the ellipse and the product of eccentricities of hyperbola and ellipse is one, then

- (A) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (B) focus of hyperbola is (4, 0)
 (C) focus of hyperbola is $(5\sqrt{2}, 0)$ (D) none of these

Ans. A

Sol. $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{16}{25} = \frac{9}{25} \therefore e = \frac{3}{5}$

Hence e' for hyperbola = $\frac{5}{3}$ as $ee' = 1$

Also foci of ellipse are $(\pm ae, 0) = (\pm 3, 0)$. The equation of hyperbola is $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ and it passes through $(\pm 3, 0)$.

$\therefore A^2 = 9$

$e'^2 = 1 + \frac{B^2}{A^2}$ for hyperbola

$\therefore \frac{25}{9} = 1 + \frac{B^2}{9} \therefore B^2 = 16$

Hence equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Its foci are at $(\pm Ae', 0) = \left(\pm 3 \cdot \frac{5}{3}, 0\right) = (\pm 5, 0)$.

66. The number of solutions of $\tan(5\pi \cos \alpha) = \cot(5\pi \sin \alpha)$ for α in $(0, 2\pi)$ is

- (A) 7 (B) 12 (C) 14 (D) 18

Ans. C

Sol. $\tan(5\pi \cos \alpha) = \cot(5\pi \sin \alpha)$

$\sin \alpha + \cos \alpha = \frac{2n+1}{10}, n \in \mathbb{I}$

$\Rightarrow \sin\left(\frac{\pi}{4} + \alpha\right) = \frac{2n+1}{10\sqrt{2}}$

$-1 \leq \sin\left(\frac{\pi}{4} + \alpha\right) \leq 1 \Rightarrow \frac{-10\sqrt{2}-1}{2} \leq n \leq \frac{10\sqrt{2}-1}{2}$

\therefore No. of values of $n = 14$.

67. $\int \frac{7x^{13} + 5x^{15}}{(x^7 + x^2 + 1)^3} dx$
 (A) $\frac{1}{2} \frac{x^{14}}{(x^7 + x^2 + 1)^2} + C$ (B) $\frac{1}{2} \frac{x^{12}}{(x^7 + x^2 + 1)^2} + C$ (C) $-\frac{1}{2} \frac{x^{12}}{(x^7 + x^2 + 1)^2} + C$ (D) none of these

Ans. A

Sol. $\frac{1}{2} \frac{x^{14}}{(x^7 + x^2 + 1)^2} + C$

68. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$, then $f(x)$ increases in

- (A) (2, 2) (B) no value of x (C) (0, ∞) (D) ($-\infty$, 0)

Ans. D

Sol. $f'(x) = e^{-(x^2+1)^2} \cdot 2x - e^{-(x^2)^2} \cdot 2x = 2xe^{-(x^4+1+2x^2)} (1 - e^{2x^2+1})$
 $\Rightarrow f'(x) > 0 \quad \forall x \in (-\infty, 0).$

69. The sum of two natural numbers n_1 and n_2 is known to be equal to 100. Then what is the probability that their product being greater than 1600?

- (A) $\frac{39}{99}$ (B) $\frac{59}{100}$ (C) $\frac{59}{99}$ (D) none of these

Ans. C

Sol. Total number of ways in which $n_1 + n_2 = 100$ is equal to 99.

Now, $n_1 \cdot n_2 > 1600$
 $\Rightarrow n_1 (100 - n_1) > 1600$
 $\Rightarrow n_1^2 - 100n_1 + 1600 < 0$
 $\Rightarrow (n_1 - 80)(n_1 - 20) < 0$
 $\Rightarrow 20 < n_1 < 80$
 $\Rightarrow 21 \leq n_1 \leq 79.$

Thus number of favourable ways = $79 - 21 + 1 = 59$

Hence required probability = $\frac{59}{99}.$

70. Let $\vec{b} = 4\hat{i} + 3\hat{j}$ and \vec{c} be two vectors perpendicular to each other in the xy -plane. Then one of the vector in the same plane having projections 1 and 2 along \vec{b} and \vec{c} can be

- (A) $\vec{a} = (2\hat{i} - \hat{j})$ (B) $\vec{a} = (2\hat{i} + \hat{j})$ (C) $\vec{a} = (\hat{i} - 2\hat{j})$ (D) None of these

Ans. A

Sol. Since \vec{b}, \vec{c} are perpendicular

$\therefore \vec{b} \cdot \vec{c} = 0$
 $\therefore (4\hat{i} + 3\hat{j}) \cdot (x\hat{i} + y\hat{j}) = 0$
 where $\vec{c} = x\hat{i} + y\hat{j}$

$\Rightarrow 4x + 3y = 0 \Rightarrow y = -\frac{4}{3}x$

$\therefore \vec{c} = x\hat{i} - \frac{4}{3}x\hat{j} = \frac{x}{3}(3\hat{i} - 4\hat{j})$

Now, projection of $\vec{a} = \pm(\vec{a}_1\hat{i} + \vec{a}_2\hat{j})$ and on \vec{b} is 1 and on \vec{c} is 2.

$$\therefore 1 = \left| \frac{4a_1 + 3a_2}{5} \right| \text{ and } 2 = \left| \frac{3a_1 - 4a_2}{5} \right|$$

$$\Rightarrow a_1 = 2, a_2 = -1 \quad \therefore \bar{a} = \pm(2\hat{i} - \hat{j}).$$

SECTION – B

Single digit integer

This section contains **2 questions**. The answer to each question is a **single digit integer** ranging from 0 to 9 (both inclusive).

71. Evaluate $\lim_{x \rightarrow \infty} \log_{\left(\sqrt{x^4 + x^2 + 1} - x^2\right)} \left(\sqrt{x^2 + x + 1} - x\right) =$

Ans. 1

Sol. $\lim_{x \rightarrow \infty} \sqrt{x^2 + x + 1} - x = \frac{1}{2}$

$$\lim_{x \rightarrow \infty} \sqrt{x^4 + x^2 + 1} - x^2 = \frac{1}{2}$$

72. A point representing the complex number z lies on or inside a circle of radius 3 and centre $(-4, 0)$, then the greatest value of $|z + 1|$.

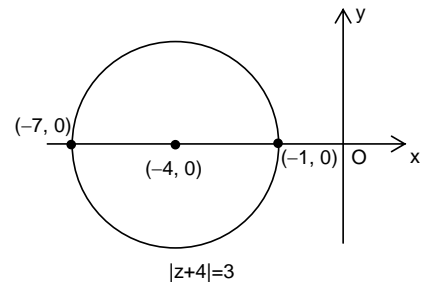
Ans. 6

Sol. Since z satisfies $|z + 4| \leq 3$.

$$\Rightarrow |z + 1| \text{ is the distance of } z \text{ from } (-1, 0)$$

$$\Rightarrow |z + 1|_{\min} = |-1 + 1| = 0$$

$$\Rightarrow |z + 1|_{\max} = |-7 + 1| = 6 \text{ units.}$$



SECTION – C

Numerical Answer Type

This section contains **3 questions**. The answer to each question is a **Numerical Answer Type XXXXX.XX** (for example if answer is **99999.99** then write the same answer without rounding off any digit).

73. $\cos^{-1}\left(\cos\left(\frac{-17\pi}{5}\right)\right)$ is equal to

Ans. 00013.20

Sol. $\cos^{-1}\left(\cos\left(4\pi - \frac{3\pi}{5}\right)\right) = \cos^{-1}\left(\cos\frac{3\pi}{5}\right) = \frac{3\pi}{5}$

74. If A is a diagonal matrix of order 3×3 is commutative with every square matrix of order 3×3 under multiplication and trace $(A) = 12$, then

Ans. 00064.00

Sol. A diagonal matrix is commutative with every square matrix if it is scalar matrix so every diagonal element is 4.

$$\therefore |A| = 64$$

75. The value of $\int_{-1}^{15} \text{sgn}\{\{x\}\} dx$, where $\{.\}$ denotes the fractional part function, is

Ans. 00016.00

Sol.
$$\int_{-1}^{15} \operatorname{sgn}\{x\} dx = \int_0^{16} \operatorname{sgn}\{x-1\} dx = \int_0^{16} \operatorname{sgn}\{x\} dx \text{ (by properties)}$$
$$= 16 \int_0^1 \operatorname{sgn}\{x\} dx = 16 \int_0^1 1 dx = 16$$