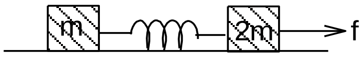
OLTS – 1920 – JEEM 2020

FULL TEST – 1

Part –I : Physics

Section – A

1. Two blocks of mass m and 2m are kept on a smooth horizontal surface. They are connected by an ideal spring of force constant R. Initially the spring is upstretched. A constant force F is applied to the heavier block in the direction shown figure. Suppose at time t displacement of smaller block is x. Then displacement of heavier block at this moment would be



(A) x/2 (B) $\frac{ft^2}{6m}+x/3$ (C) x/3 (D) $\frac{ft^2}{4m}-x/2$

Ans. **D**

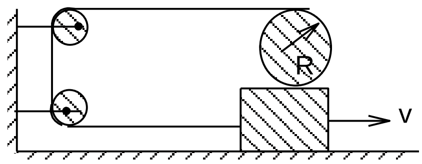
Sol. $\alpha_{com}=\frac{F}{m+2m}=\frac{F}{3m}$

$\therefore x_{com}=\frac{1}{2}\alpha_{com}t^2=\frac{Ft^2}{6m}$

Further $x_{com}=\frac{m(x)+2m(x')}{2+2m}$

Or $\frac{Ft^2}{6m}=\frac{x+2x'}{3}\therefore x'=\frac{Ft^2}{4m}-\frac{x}{2}$

2. In the figure shown, the plank is being pulled to the right with constant speed v. If the cylinder does not slip then.

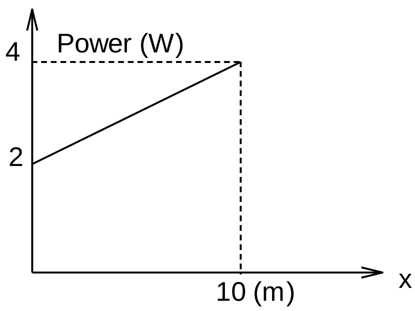


(A) The speed of the centre of mass of the cylinder is 2v.
(B) The speed of the centre of mass of the cylinder is v.
(C) The angular velocity of the cylinder is v/R
(D) The angular velocity of the cylinder is zero.

Ans. **C**

Sol. As per concept v = Rω

3. A particle A of mass $\frac{10}{7}$ kg is moving in position direction of x – axis. At initial position x = o, its velocity is 1 m/s, then velocity of particle at x = 10 m is (use the graph given)



(A) 4 m/s
(B) 2 m/s
(C) $3\sqrt{2}$ m/s
(D) $\frac{100}{3}$ m/s

Ans. **A**

Sol. Area under P – x graph

$=\int Pdx=\int\left(m\frac{dv}{dt}\right)vdx=\int_1^v mv^2\ dV$

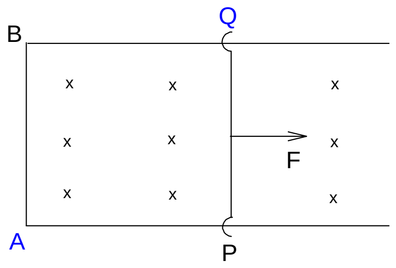
$= \left[\frac{mv^3}{3}\right]_1^v = \frac{10}{7\times 3}\left(v^3 - 1\right)$

From the graph, area = $\frac{1}{2}(2+4)\times 10 = 30$

$= \frac{10}{7\times 3}\left(v^3 - 1\right) = 30$

$\therefore v = 4ms^{-1}$

4. In the shown figure connector PQ can slide on the two frictionless conducting rails. Resistance of the rails as well as fixed connector AB is negligible. Separation between the rails is ℓ and the resistance of the connector is R. A constant force F parallel to the rails is applied to the connector. There exists a uniform magnetic field of induction B perpendicular to the plane of loop. The current in the connector when it achieves the terminal velocity :



(A) $\frac{B\ell}{F}$ (B) $\frac{2B\ell}{F}$

(C) $\frac{F}{B\ell}$ (D) $\frac{2F}{B\ell}$

Ans. **C**

Sol. $\frac{dv}{dt} = \frac{F}{m} - \frac{B^2\ell^2 v}{mR}$

at terminal velocity $\frac{dv}{dt} = 0$

$v_t = \frac{FR}{B^2\ell^2}$

$I = \frac{Bv\ell}{R}$

5. A thermodynamics process obeys the following relation

$2dQ = dU + 2dW$

where dQ, dU and dW has usual meaning. [Given di-atomic gas: R = gas constant] then heat capacity for the process is :

(A) $\frac{5R}{2}$ (B) $\frac{7R}{2}$ (C) $\frac{3R}{5}$ (D) Infinite

Ans. **D**

Sol. $dQ = \frac{dU}{2} + dW$ (1)

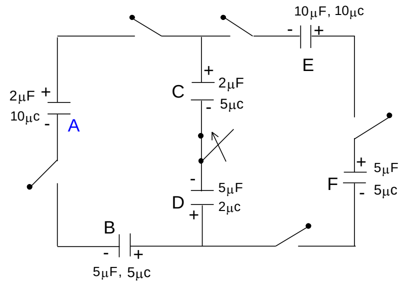
But from 1st law of thermodynamics

$dQ = dU + dW$ (2)

from (1) and (2)

$dU = 0 \quad \Rightarrow C = \infty$

6. Six capacitors A, B, C, D, E & F are charged initially and connected in a circuit as shown in the figure their capacitances, initial charges and polarities are also shown in the figure. If all the keys are switched on simultaneously. The final charge on capacitor F is



(A) $\frac{56}{13}\mu c$ (B) $\frac{75}{13}\mu c$ (C) $\frac{31}{13}\mu c$ (D) $\frac{96}{13}\mu c$

Ans. **D**

Sol. LOOP PQRSP

$$\frac{10+q_1}{2}-\frac{5-q_1}{5}+\frac{2+q}{5}-\frac{5-q}{2}=0$$

$$7(q+q_1)+19=0 \quad (1)$$

LOOP PUTSP

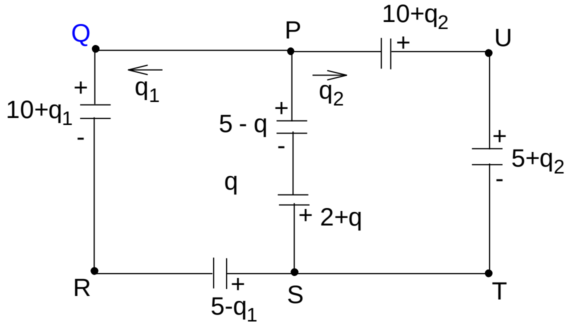
$$-\frac{10-q_2}{10}+\frac{5+q_2}{5}+\frac{2+q}{5}-\frac{5-q}{2}=0$$

$$3q_2+7q=6 \quad (2)$$

$q = q_1 + q_2$ (3)

solving (1), (2) & (3)

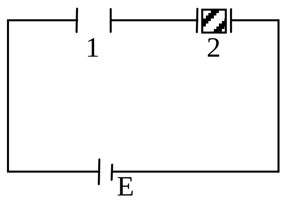
$q_2=\frac{31}{13}\mu c$ $q_1=-\frac{232}{91}/\mu c$



Charge on capacitor F is

$$5+q_2=5+\frac{31}{13}=\frac{96}{13}\mu c$$

7. Two identical capacitors 1 and 2 are connected in series to a battery as shown in figure. Capacitor 2 contains a dielectric slab of dielectric constant k as shown. Q_1 and Q_2 are the charges stored in the capacitors. Now the dielectric slab is removed and the corresponding charges are Q'_1 and Q'_2. Then –



(A) $\frac{Q'_1}{Q_1}=\frac{k+1}{k}$ (B) $\frac{Q'_2}{Q_2}=\frac{k+1}{2}$ (C) $\frac{Q'_2}{Q_2}=\frac{k+1}{2k}$ (D) $\frac{Q'_1}{Q_1}=\frac{k}{2}$

Ans. **C**

Sol. Before removing the slab

$\therefore Q_1 = C.V_1$ and $Q_2 = KC.V_2$

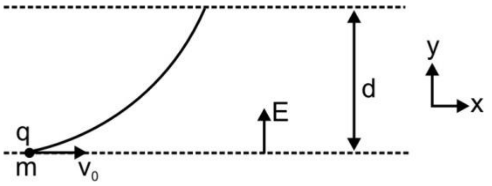
$Q_1=\frac{KCE}{K+1}$ $Q_2=\frac{KCE}{K+1}$

After removing the slap

$$Q_1' = Q_2' = \frac{CE}{2}$$

$$\therefore \frac{Q_2'}{Q_2}=\frac{K+1}{2K}$$

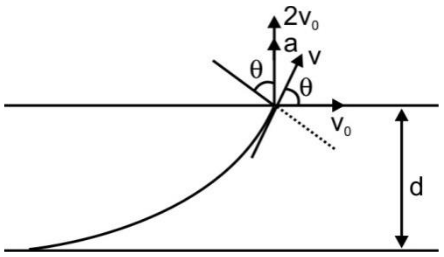
8. Charge q of mass m is projected with velocity v_0 along x-axis in uniform electric field E along y-axis. Radius of curvature of charge when it has travelled distance d along y-axis is ? Neglect gravity and it is given that $\frac{qEd}{m} = 2v_0^2$



(A) $\frac{v_0^2 m}{qE}\sqrt{5}$ (B) $\frac{v_0^2 m}{2qE}\sqrt{5}$ (C) $\frac{5}{2}\frac{v_0^2 m}{qE}\sqrt{5}$ (D) $\frac{5v_0^2 m}{qE}\sqrt{5}$

Ans. D

Sol.



$a\cos\theta = \frac{v^2}{R}$

(Where $v = \sqrt{(2v_0)^2 + v_0^{2-}}$ $a = \frac{qE}{m}$ $\tan\theta = \frac{2v_0}{v_0}$)

9. A rigid circular loop of radius r and mass m lies in the x-y plane on a flat table and has a current i flowing in it. At this particular place. The earth's magnetic field is $\vec{B} = B_x\hat{i} + B_y\hat{j}$. The minimum value of i for which one end of the loop will lift from the table is

(A) $\frac{mg}{\pi r B x}$ (B) $\frac{mg}{\pi r B_y}$ (C) $\frac{mg}{2\pi r\sqrt{B_x^2 + B_y^2}}$ (D) $\frac{mg}{\pi r\sqrt{B_x^2 + B_y^2}}$

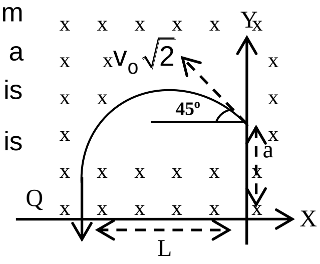
Ans. D

Sol. $\vec{\gamma} = \vec{\mu} \times \vec{\beta}$

$mgr = i\pi r^2\sqrt{\beta x^2 + \beta y^2}$

$\therefore i = \frac{mg}{\pi r\sqrt{\beta x^2 + \beta y^2}}$

10. A particle of charge (+q) and mass m moving under the influence of uniform electric field $\vec{E} = E_o\hat{j}$ and uniform magnetic field $\vec{B} = -B_o\hat{k}$ follows a trajectory from P to Q as shown in figure. The velocity at P is $\vec{v} = -v_o\hat{i} + v_o\hat{j}$ and velocity at Q is - $\frac{v_o}{\sqrt{2}}\hat{j}$. The magnitude of electric field is given by



(A) $\frac{mv_0^2}{8q_o a}$ (B) $\frac{3mv_0^2}{4qa}$

(C) $\frac{7mv_0^2}{8qa}$ (D) $\frac{5mv_0^2}{8qa}$

Ans. **B**

Sol. A work energy theorem,

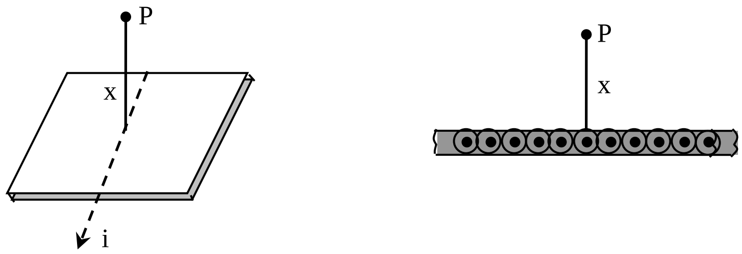
Work done by electric field = Change in kinetic energy

$$q(E_0 a) = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}m\left(v_0\sqrt{2}\right)^2$$

$$E = \frac{-3mv_0^2}{4qa}$$

magnitude of E $= \dfrac{3mv_0^2}{4qa}$

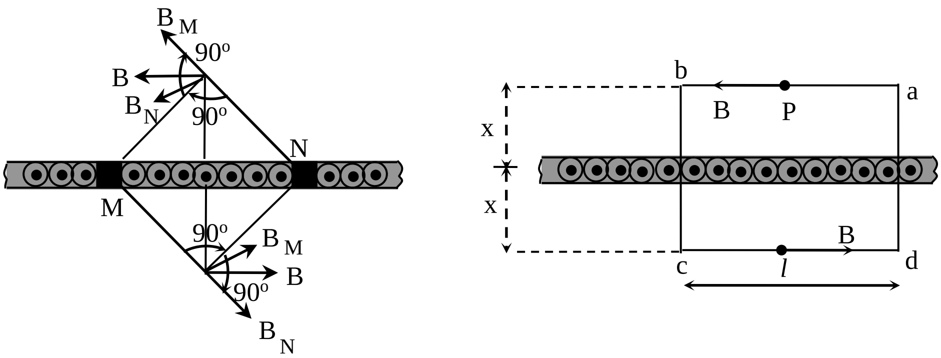
11. Figure shows a cross-section of a large metal sheet carrying an electric current along its axis. The current in a strip of width dl is kdl where k is constant. Find the magnetic field at a point P at a distance x from the metal sheet.



(A) $\dfrac{\mu_0 k}{2}$ (B) $\mu_0 k$ (C) $2\mu_0 k$ (D) $\dfrac{\mu_0 k}{4}$

Ans. **A**

Sol.



Consider two strips R and S of the sheet situated symmetrically on the two sides of P. The magnetic field at P above sheet and below sheet is parallel to sheet as shown in figure. There is no field perpendicular to the sheet. Now applying ampere's law to the close path a-b-c-d-a as shown in figure.

$$\oint \vec{B}.\vec{dl} = \mu_0 i_m$$

$$\Rightarrow \oint \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.dl + \int_d^a \vec{B}.dl = \mu_0 (kl)$$

or $Bl + 0 + Bl + 0 = \mu_0 kl$

or $B = \dfrac{\mu_0 k}{2}$ Clearly, magnetic field in this case is independent of x.

12. A transverse wave is represented by $y = y_0 \sin\left(\frac{2\pi}{\lambda}(vt - x)\right)$. The maximum particle velocity is twice the wave velocity for λ =

(A) $2\pi y_0$ (B) $\dfrac{2\pi y_0}{3}$ (C) $\dfrac{\pi y_0}{2}$ (D) πy_0

Ans. D

Sol.

Given, $v_{P_{max.}} = 2v$

$\Rightarrow y_0\left(\frac{2\pi v}{\lambda}\right) = 2v \quad \Rightarrow \lambda = \pi y_0$

13. A circular disc is rolling down an inclined plane without slipping. If the angle of inclination is 30°, the acceleration of the disc down the inclined plane is

(A) g (B) $\frac{g}{2}$ (C) $\frac{g}{3}$ (D) $\frac{\sqrt{2}}{3}g$

Ans. C

Sol. $a = \frac{g\sin\theta}{\left(1+\frac{I}{MR^2}\right)} = \frac{g}{3}\left[\because \theta = 30^o, I = \frac{MR^2}{2}\right]$

14. For a particle executing SHM, the potential energy is given by $U = U_0\left(1+\sin^2\omega t\right)$. The maximum kinetic energy of the particle is

(A) U_0 (B) $\frac{U_0}{2}$ (C) $2U_0$ (D) $\frac{3U_0}{2}$

Ans. A

Sol. $KE_{max.} = PE_{max.} - PE_{min.} = 2U_0 - U_0 = U_0$

15. A small square loop of wire of side *l* is placed inside a large square loop of wire of side L (L>>*l*). The loops are co-planar and their centers coincide. The mutual inductance of the system is proportional to

(A) *l*/L (B) l^2/L (C) L/*l* (D) L^2/*l*.

Ans. B

Sol. Magnetic field produced by a current in a large square loop of wire at its center

$B = \frac{2\sqrt{2}\mu_0 i}{\pi L}$

The magnetic flux ϕ_{12} that links big loop with the small square loop of side $\ell\left(\ell << L\right)$ is

$\phi_{12} = B\left(1^2\right) = \frac{2\sqrt{2}\mu_0 i}{\pi}\left(\frac{\ell^2}{L}\right)$,

$\therefore$ The mutual inductance

$M_{12} = \frac{\phi_{12}}{i} = \frac{2\sqrt{2}\mu_0 i}{\pi}\left(\frac{\ell^2}{L}\right)$

i.e., $M_{12} \propto \left(\ell^2 / L\right)$.

16. There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially, both have equal number of nuclei. After n half lives of A, rate of disintegration of both are equal. The value of n is

(A) 4 (B) 2 (C) 1 (D) 5

Ans. C

Sol. Let $\lambda_A = \lambda \;\; \therefore \;\; \lambda_B = 2\lambda$

If N_0 is total no. of atoms in A and B at $t=0$, then initial rate of disintegration of $A=\lambda N_0$, and initial rate of disintegration of $B=2\lambda N_0$

As $\lambda_B = 2\lambda_a$

$\therefore \quad T_B = \frac{1}{2}T_A$

i.e. half life of B is half the half life of A.

After one half life of A

$$\left(-\frac{dN}{dt}\right)_A = \frac{\lambda N_0}{2}$$

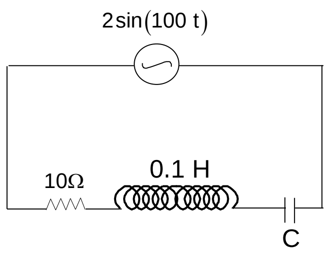
Equivalently, after two half lives of B

$$\left(-\frac{dN}{dt}\right)_B = \frac{2\lambda N_0}{4} = \frac{\lambda N_0}{2}$$

Clearly, $\left(-\frac{dN}{dt}\right)_A = -\left(\frac{dN}{dt}\right)_B$,

after $n=1$, i.e., one half life of A.

17. The power factor of the circuit is $1/\sqrt{2}$. The capacitance of the circuit is equal to



(A) 400 μF (B) 300 μF
(C) 500 μF (D) 200 μF

Ans. **C**

Sol. $\cos\phi = \frac{R}{Z}$

18. The plates of a parallel plate capaitor with no dielectric are connected to a voltage source. Now a dielectric of dielectric constant K is inserted to fill the whole space between the plates with voltage source remaining connected to the capacitor. Which of the following is **incorrect**.

(A) the energy stored in the capacitor will become K–fold
(B) the electric field inside the capacitor will decrease to K–times
(C) the force of attractionbetween the plates will increase to K^2 – times
(D) the charge on the capacitor will increase to K–times

Ans. **B**

Sol. Capacitance of the capacitor becomes K time and potential difference between the plates of capacitor remains constant.

19. An inductance of negligible resistance whose reactance is 22 Ω at 200 Hz is connected to 200 V, 50 Hz power line. The value of inductance is

(A) 0.0175 H (B) 0.175 H
(C) 1.75 H (D) 17.5 H

Ans. **A**

Sol. $22 = L \times 2\pi \times 200;\ L = \frac{22 \times 7}{2 \times 22 \times 200} = 0.0175H$

Value of inductance is the same whatever be the frequency.

20. A cavity of radius R/2 is made inside a solid sphere of radius R. The centre of the cavity is located at a distance R/2 from the centre of the sphere. Find the gravitational force on a particle of mass 'm' at a distance R/2 from the centre of the sphere on the line joining both the centres of sphere and cavity (opposite to the centre of cavity). [Here g = GM/R^2, where M is the mass of the sphere]

(A) $\frac{mg}{2}$ (B) $\frac{3mg}{8}$ (C) $\frac{mg}{16}$ (D) none of these

Ans. **B**

Sol. Inside the cavity gravitational field is uniform.

Section - B

21. A container filled with air under pressure P_0 contains a soap bubble of radius R. When the air pressure is reduced to half isothermally, the bubble radius becomes (5R/4). If the surface tension of the soap water solution is S, then find $\left(\frac{RP_o}{12S}\right)$.

Ans. **8**

Sol. $$\left(P_o + \frac{4S}{R}\right)R^3 = \left(0.5P_o + 0.8 \times \frac{4S}{R}\right)\left(\frac{5R}{4}\right)^3$$

$$P = \frac{96S}{R}$$

22. A glass tube of 1.0 m length is filled with water. The water is drained out very slowly from the tube through a hole in the bottom while a vibrating 500 HZ tuning fork is held near the open upper end of the tube. If the speed of sound is 320 ms^{-1}, find the number of resonance that can be obtained.

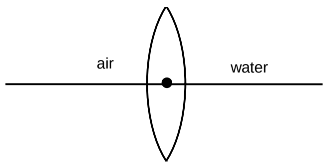
Ans. **4**

Sol. $$L_{min} = \frac{C}{4f} = 16 \text{ cm}$$

so resonance for length 16 cm, 48 cm, 80 cm i.e 3 cases.

Section - C

23. Find the focal length of the glass (μ = 3/2) lens (bi convex lens) of radius of curvature 20 cm, with air as medium on one of its side and water on the other.
(μ_{air} = 1 μ_{water} = 1.3)



Ans. **00037.14**

Sol. For an object placed at infinite image will be formed at focus.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{20}$$

$$\Rightarrow \quad \frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5 - 1}{20}$$

$\Rightarrow$ v = 60 cm

Now this will act as an object for second surface, we have

$$\frac{\mu_3}{v} - \frac{\mu_2}{60} = \frac{\mu_3 - \mu_2}{-20}$$

$$\Rightarrow \quad \frac{1.3}{v} = \frac{3}{2 \times 60} + \frac{-0.2}{-20} = \frac{1}{40} + \frac{2}{200} = \frac{5+2}{200}$$

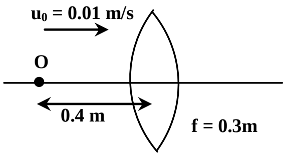
$$\Rightarrow \quad \frac{1.3}{v} = \frac{7}{200} \quad \Rightarrow \quad 7v = 260$$

v = 37.14 cm

24. A point object is moving with velocity 0.01 m/s on principal axis towards a convex lens of focal length 0.3 m. When object is at a distance of 0.4 m from the lens, find
(a) rate of change of position of the image, and
(b) rate of change of lateral magnification of image.

Ans. **00001.80**

Sol. $\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow -\frac{1}{v^2}\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0$



$\Rightarrow \frac{dv}{dt}=\frac{v^2}{u^2}\frac{du}{dt}$ .. . . (i)

$\frac{1}{30}=\frac{1}{v}-\frac{1}{-40} \Rightarrow$ v =120 cm

$\Rightarrow \frac{dv}{dt}=0.09\ m/sec$

$\Rightarrow$ m = $\frac{dv}{du}=\frac{v^2}{u^2}=\left(1-\frac{v}{f}\right)^2$

$\frac{dm}{dt}=-\frac{2}{f}\left(1-\frac{v}{f}\right)\frac{dv}{dt} \quad =\frac{-2}{0.3}\left(1-\frac{120}{30}\right)\times 0.09=1.8\ s^{-1}$

25. Electromagnetic waves travel in a medium at speed of $2\times10^8 m/s$. The relative permeability of medium is $\perp$. Find relative permittivity.

Ans. **00002.25**

Sol. $c^1=\frac{1}{\sqrt{\mu c}}=\frac{c}{\sqrt{\mu_r \in_r}}$

Part – II : Chemistry

Section – A

26. The enthalpies of combustion of $C_{(graphite)}$ and $C_{(diamond)}$ are –393.5 and –395.4 kJ/mol respectively. The enthalpy of conversion of $C_{(graphite)}$ to $C_{(diamond)}$ in kJ/mol is

(A) –1.9
(B) –788.9
(C) 1.9
(D) 788.9

Ans. C

Sol.

$C_{graphite} + O_2 \longrightarrow CO_2$; $\Delta H = -393.5$

$C_{diamond} + O_2 \longrightarrow CO_2$; $\Delta H = -395.4$

$\therefore C_{graphite} \longrightarrow C_{diamond} = -393.5 + 395.4 = 1.9$

27. Which one of the following statement is false?

(A) work is a state function.
(B) temperature is a state function.
(C) work appears at the boundary of the system.
(D) change in the state is completely defined when the initial and final states are specified.

Ans. A

Sol. Work can't be a state function, because it is proportional to distance i.e. depend on path.

28. A reaction for which ΔH = –11.7 kJ mol^{-1} and ΔS = –105 JK^{-1} mol^{-1} would be spontaneous when temperature is

(A) equal to 111.4°C
(B) equal to 111.4 K
(C) > 111.4 K
(D) < 111.4 K

Ans. D

Sol. ΔG = (–)ve for spontaneous reaction

$\Delta G = \Delta H - T\Delta S$

$= -11.69 \times 10^3 + 105 \times T < 0$

If T < 111.4 K then ΔG is (–)ve and reaction is spontaneous.

29. A reaction, $A(g) + 2B(g) \rightleftharpoons 2C(g) + D(g)$ was studied using an initial concentration of B which was 1.5 times that of A. But the equilibrium concentrations of A and B were found to be equal. The value of K_p for the equilibrium is

(A) 4
(B) 6
(C) 8
(D) 12

Ans. A

Sol.

$A(g) + 2B(g) \rightleftharpoons 2C(g) + D(g)$

	A	B	C	D
Initial conc.	x	$1.5x$	0	0
At equilibrium	$x - y$	$1.5x - 2y$	2y	y

Given: $x - y = 1.5x - 2y$; $y = 0.5x$. Thus, at equilibrium [A] = $0.5x$ [B] = $0.5x$ [C] = x [D] = $0.5x$

$$\therefore K_c = K_p = \frac{(x^2)\,(0.5x)}{(0.5x)\,(0.5x)^2} = 4.$$

30. In a system,

$$A(s) \rightleftharpoons 2B(g) + 3C(g)$$

if the concentration of C at equilibrium is increased by a factor of 2, it will cause the equilibrium concentration of B to change to

(A) two times the original value
(B) one half of its original value
(C) $2\sqrt{2}$ times its original value
(D) $\frac{1}{2\sqrt{2}}$ times the original value

Ans. **D**

Sol. $A(s) \rightleftharpoons 2B(g) + 3C(g)$

$K_p = p_B^2 \cdot p_C^3$ ...(i)

$K_p = p_B'^2 \cdot (2p_C)^3$ ...(ii)

$\therefore \quad \frac{p_B'}{p_B} = \sqrt{\frac{p_C^3}{8p_C^3}} = \frac{\mathbf{1}}{\mathbf{2\sqrt{2}}}$.

31. One mole of N_2O_4(g) at 300 K is kept in a closed container under one atmosphere. It is heated to 600 K when 20% by mass of N_2O_4(g) decomposes to NO_2(g). The resultant pressure is

(A) 1.2 atm
(B) 2.4 atm
(C) 2.0 atm
(D)1.0 atm

Ans. **B**

Sol. $N_2O_4 \rightleftharpoons 2NO_2$

At equilibrium, $1 - 0.2$ moles $\quad 0.4$ moles

At 300 K, $1 \times V = 1 \times R \times 300$ ...(1)

At 600 K, $P \times V = (1.2) \times R \times 600$ ...(2)

Dividing (2) by (1) we get P = 2.4 atm.

32. When $NaNO_3$(s) is heated in a closed vessel, oxygen is liberated and $NaNO_2$(s) is left behind. At equilibrium,

(A) addition of $NaNO_2$ favours reverse reaction.
(B) addition of $NaNO_3$ favours forward reaction.
(C) increasing temperature favours forward reaction.
(D) decreasing pressure favours reverse reaction.

Ans. **C**

Sol. $NaNO_3(s) \rightleftharpoons NaNO_2(s) + \frac{1}{2}O_2(g)$

Since the reaction is endothermic, increase in temperature favours forward reaction.

33. For the equilibrium $H_2(g) + CO_2(g) \rightleftharpoons H_2O(g) + CO(g)$, $K_c = 16$ at 1000 K. If 1.0 mole of CO_2 and 1.0 mole of H_2 are taken in a 1 L flask, the final equilibrium concentration of CO at 1000 K will be

(A) 0.8 M
(B) 0.08 M
(C) 1.6 M
(D) 1.8 M

Ans. **A**

Sol. $H_2(g) + CO_2(g) \rightleftharpoons H_2O(g) + CO(g)$

$\quad 1-x \quad 1-x \quad\quad x \quad\quad x$

$$K_C = 16 = \frac{x^2}{(1-x)^2} \quad x = \frac{4}{5} = 0.8 \text{ M}.$$

34. Which of the following solution will have pH close to 1.0?
(A) 100 ml of M/10 HCl + 100 ml of M/10 NaOH
(B) 55 ml of M/10 HCl + 45 ml of M/10 NaOH
(C) 10 ml of M/10 HCl + 90 ml of M/10 NaOH
(D) 75 ml of M/5 HCl + 25 ml of M/5 NaOH

Ans. **D**

Sol. In (a)

	HCl	+ NaOH	$\longrightarrow$ NaCl	+ H_2O
m. mol	10	10	0	0
	0	0	10	10

pH = 7

In (b)

	HCl	+ NaOH	$\longrightarrow$ NaCl	+ H_2O
m. mole	5.5	4.5	0	0
	1.0	0	4.5	4.5

$[H^+] = \frac{1}{100} = 10^{-2}$

pH = 2

In (c)

	HCl	+ NaOH	$\longrightarrow$ NaCl	+ H_2O
m. mole	1	9	0	0
	0	8	1	1

$[OH^-] = \frac{8}{100}$

pOH = 1.096 ; pH = 12.9

In (d)

	HCl	+ NaOH	$\longrightarrow$ NaCl	+ H_2O
m. mole	15	5	0	0
	10	0	5	5

$[H^+] = \frac{10}{100} = 10^{-1}$

pH = 1

35. An acid of pH 6 is diluted hundred times. The pH of the solution becomes
(A) 4 (B) 8
(C) 6.95 (D) 6

Ans. **C**

Sol. After dilution, pH = 8, which is not possible for the acid.
So, pH should be nearly less than 7.

36. Three sparingly soluble salts A_2X, AX and AX_3 have the same solubility product. Their solubilities will be in the order
(A) $AX_3 > AX > A_2X$ (B) $AX_3 > A_2X > AX$
(C) $AX > AX_3 > A_2X$ (D) $AX > A_2X > AX_3$

Ans. **D**

Sol.

$$A_2X \rightleftharpoons \underset{(2s)}{2A^+} + \underset{(s)}{X^{2-}} \quad ; \quad K_{sp} = 4s^3 \quad ; \quad s = \left(\frac{K_{sp}}{4}\right)^{1/3}$$

$$AX \rightleftharpoons \underset{(s')}{A^+} + \underset{(s')}{X^-} \quad ; \quad K_{sp} = s'^2 \quad ; \quad s' = \sqrt{K_{sp}}$$

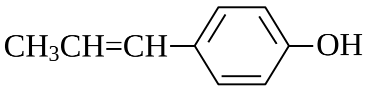
$$AX_3 \rightleftharpoons \underset{(s'')}{A^{+3}} + \underset{(3s'')}{3X^-} \quad ; \quad K_{sp} = 27(s'')^4 \quad ; \quad s'' = \left(\frac{K_{sp}}{27}\right)^{1/4}$$

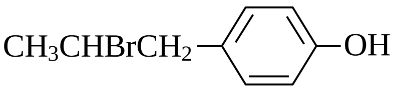
37. The pH values of 1 M solutions of CH_3COOH (I), CH_3COONa (II), CH_3COONH_4 (III) and KOH (IV) will be in the order

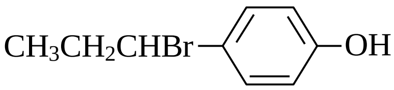
(A) IV > III > II > I
(B) IV > II > III > I
(C) I > III > II > IV
(D) II > I > III > IV

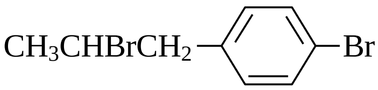
Ans. **B**

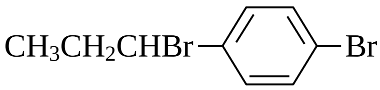
Sol. CH_3COO^- when hydrolysed generates OH^- ions making the solution basic.
CH_3COONH_4 results in a neutral solution since $K_a(CH_3COOH) = K_b(NH_4OH)$

38. The reaction of  with HBr gives

(A) 

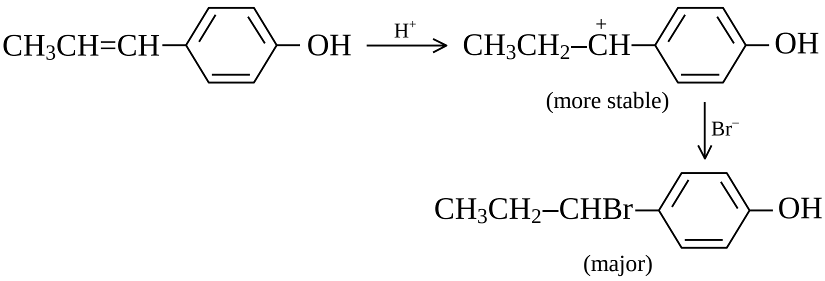
(B) 

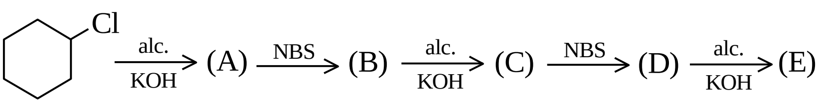
(C) 

(D) 

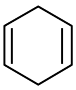
Ans. **B**

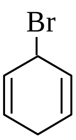
Sol.

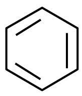


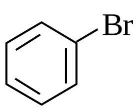
39. 

The product E is

(A) 

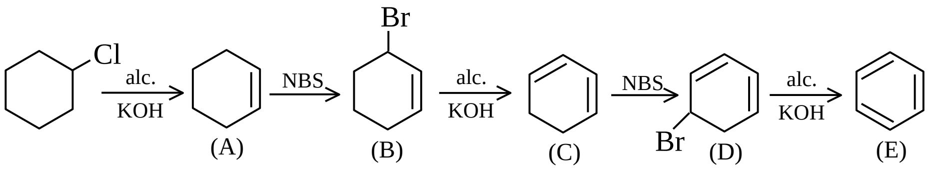
(B) 

(C) 

(D) 

Ans. **C**

Sol.



40. Identify (C) in the following reactions:

$CH_2{=}CH_2 \xrightarrow{Cl_2,\ H_2O} (A) \xrightarrow[\text{Heat}]{H_2SO_4} (B) \xrightarrow{\text{alcoholic KOH}} (C)$

(A) $CH_2{=}CH_2$ (B) $CH_2{=}CH{-}Cl$

(C) $Cl{-}CH{=}CH{-}OH$ (D) $CH_2{=}CH{-}O{-}CH{=}CH_2$

Ans. **D**

Sol.

$$CH_2{=}CH_2 \xrightarrow{Cl_2,\ H_2O} CH_2Cl{-}CH_2OH \xrightarrow[\text{Heat}]{H_2SO_4} \underset{(B)}{ClCH_2CH_2OCH_2CH_2Cl} \xrightarrow{\text{alc. KOH}} \underset{(C)}{CH_2{=}CH{-}O{-}CH{=}CH_2}$$

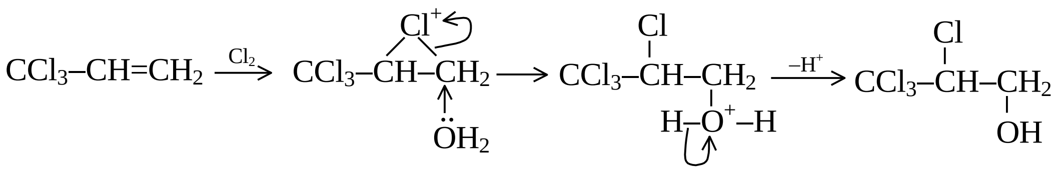
41. $CCl_3CH{=}CH_2 \xrightarrow{Cl_2 + H_2O}$ (A). (A) is

(A) $CCl_3CH(OH){-}CH_2Cl$ (B) $CCl_3{-}CH(Cl){-}CH_2OH$

(C) $CCl_3{-}CH(Cl){-}CH_2Cl$ (D) $CCl_3CH(OH){-}CH_2OH$

Ans. B

Sol.



42. In a solid AB_2 co-ordination number of A is 8. It has a cubic close packed lattice. Half of the B atoms are however ejected from the solid. Now number of tetrahedral hole (voids) remain filled are

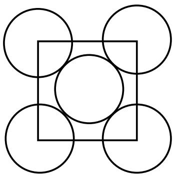
(A) 2A atoms (B) 4A atoms

(C) 9A atoms (D) equal to A atoms

Ans. **D**

Sol. There are in total 8 tetrahedral voids. A will form ccp and B are present in all tetrahedral voids. When half of the B atoms are ejected from the solid, half remains filled i.e. equal to A atoms.

43. An element has unit cell made up of planes as shown below:



Co-ordination number of a lattice point in the above solid is

(A) 4 (B) 8
(C) 6 (D) 12

Ans. **D**

Sol. The given planes will form an fcc lattice where coordination number of a lattice point = 12.

44. If a is the edge length of unit cell of sodium chloride, the distance between nearest Na^+ and Cl^- ions will be

(A) a (B) $\sqrt{2}$ a
(C) a/2 (D) $\sqrt{3}$ a

Ans. **C**

Sol. As $r_{Na^+} + r_{Cl^-} = \frac{\mathbf{a}}{\mathbf{2}}$

45. The cubic unit cell of aluminium has an edge length of 400 pm. Its density is 2.8 g cm^{-3}. The unit cell is

(A) primitive (B) face–centered
(C) body–centered (D) end–centered

Ans. **B**

Sol. $2.8 = \frac{Z \times 27}{(400 \times 10^{-10}\,cm)^3 \times (6.023 \times 10^{23})}$

$Z \approx 4.0$

$\therefore$ The unit cell of aluminium is face–centred.

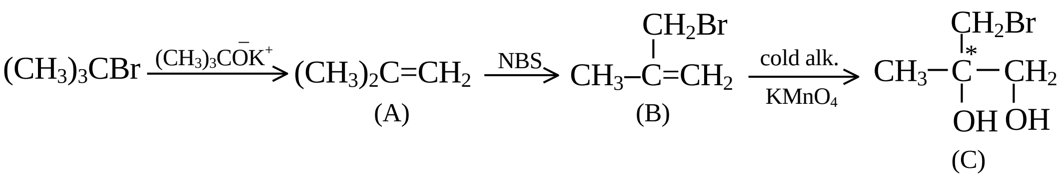
Section – B

46. $(CH_3)_3CBr \xrightarrow{(CH_3)_3C\bar{O}K^+} (A) \xrightarrow{NBS} (B) \xrightarrow[KMnO_4]{Cold\,alk.} (C)$.

The total number of stereoisomers possible for the compound (C) is

Ans. **2**

Sol.



Compound (C) has one chiral C–atom. Therefore, it has two stereoisomers.

47. In a face centred cubic arrangement of A and B atoms whose A atoms are at the corner of the unit cell and B atoms at the face centres. One of the B atoms is missing from one of the faces in the unit cell. The simplest formula of compound is A_xB_y. (x + y) = ?

Ans. **7**

Sol. $Z_A = \frac{1}{8}\times 8 = 1$; $Z_B = \frac{1}{2}\times 5 = \frac{5}{2}$ $AB_{5/2}$ i.e. A_2B_5, (2 + 5) = 7

Section – C

48. One mole of ice is converted into water at 273 K. The entropies of $H_2O(s)$ and $H_2O(l)$ are 38.20 and 60.01 J mole^{-1} K^{-1} respectively. The enthalpy change for the conversion is

Ans. 05954.13

Sol. $\Delta G = \Delta H - T\Delta S$. At equilibrium $\Delta G = 0$, so $\Delta H = T\Delta S$
$\Delta H = 273 \times (60.01 - 38.20) = 5954.13$ J mole^{-1}.

49. The pH of a solution obtained by mixing equal volume of solutions having pH = 3 and pH = 4. [log 5.5 = 0.7404]

Ans. 00003.26

Sol. At pH = 3, $[H^+] = 10^{-3}$ M
At pH = 4, $[H^+] = 10^{-4}$ M
When in equal volume of the two solutions are mixed, the $[H^+]$

$$= \frac{10^{-3} + 10^{-4}}{2}$$

$$= \frac{10^{-3}[1+0.1]}{2} = \frac{1.1}{2}\times 10^{-3}$$

$[H^+] = 5.5 \times 10^{-4}$
$-\log [H^+] = -\log (5.5) - \log 10^{-4}$
pH = –0.7404 + 4 = 3.26

50. 50 mL of 0.1M of H_3CCOOH is titrated against 0.1M NaOH solution. What will be the pH of the solution when 25 mL of NaOH is added? [Given: K_a of $H_3C–COOH = 2 \times 10^{-5}$; log 2 = 0.3]

Ans. 00004.70

Sol. $CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$

Millimoles	t = 0	5	2.5	0	0
Millimoles	t = t´	2.5	0	2.5	

Here, millimoles of CH_3COOH and CH_3COONa are same. Together they constitute acidic buffer and for acidic buffer

$$pH = pKa + \log \frac{[salt]}{[acid]}$$

because [salt] = [acid], so
pH = pKa
$pH = -\log 2 \times 10^{-5}$
pH = **4.7**

Part – III : Mathematics

Section – A

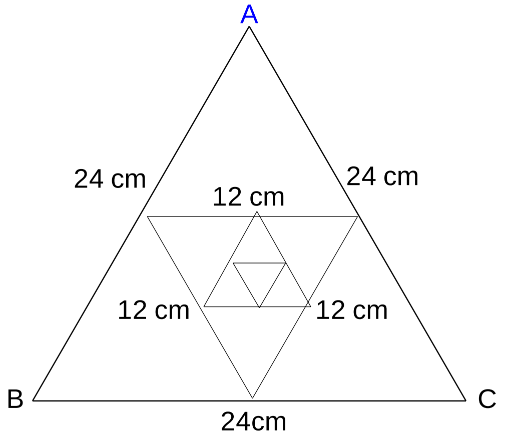
51. If ω is a complex cube root of unity, then $\cos\left[\left\{(1-\omega)(1-\omega^2)+---+(10-\omega)(10-\omega^2)\right\}\frac{\pi}{900}\right]$ is equal to

(A) – 1 (B) 0 (C) 1 (d) $\sqrt{3}/2$

Ans. **B**

Sol. $\cos\left[\left\{\sum_{r=1}^{10}(r^2+r+1)\frac{\pi}{900}\right]=\cos\frac{\pi}{2}=0$

52. One side of an equilateral triangle is 24 cm. The mid points of its sides are joined to form another triangle whose mid points are in turn joined to form still another triangle. This process continues infinitely. The sum of the perimeters of all the triangles is

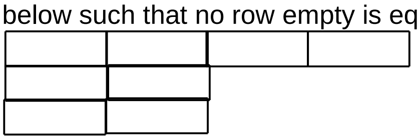


(A) 144 cm (B) 169 cm (C) 400 cm (D) 625 cm

Ans. **A**

Sol. sum of perimeter = $3\{24+12+6+3+........\infty)$

53. The number of different ways the letters of the word VECTOR can be placed in the 8 boxes of the given below such that no row empty is equal to



(A) 26 (B) 26 × 6! (C) 6! (D) 2! × 6!

Ans. **B**

Sol. Total ways = $(8c_2-2)\times 6!$

54. Matrix A such that $A^2=2A-I$, where I is the identity matrix. Then for n ≥ 2, A^n is equal to
(A) nA – (n – 1) I (B) nA – I (C) $2^{n-1}A-(n-1)I$ (D) $2^{n-1}A - I$

Ans. **A**

Sol. $\because A^2=2A-I$

$\therefore A^3 = A^2.A = 2A^2 - IA = 2A^2 - A = 2(2A - I) - A$

$= 3A - 2I = 3A - (3-1)I$

$- - - - - - - - - - - - - -$

$- - - - - - - - - - - - - -$

$- - - - - - - - - - - - - -$

$\therefore A^2 = nA - (n-1)I$

55. The solution of the inequality $(\cot^{-1} x)^2 - 5\cot^{-1} x + 6 > 0$ is

(A) $(\cot 3, \cot 2)$ (B) $(-\infty, \cot 3) U (\cot 2, \infty)$

(C) $(\cot 2, \infty)$ (D) none of these

Ans. **B**

Sol. $(\cot^{-1} x - 3)(\cot^{-1} x - 2) > 0$

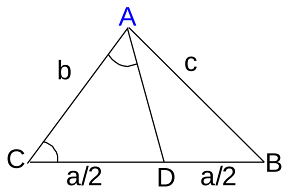
$\therefore x > \cot 2 \, and \, x < \cot 3$

56. If D is the mid point of the side BC of ΔABC and AD is perpendicular to AC, then

(A) $3a^2 = b^2 - 3c^2$ (B) $3b^2 = a^2 - c^2$ (C) $b^2 = a^2 - c^2$ (D) $a^2 + b^2 = 5c^2$

Ans. **B**

Sol.



In ΔACD,

$$\cos c = \frac{1}{\left(\frac{a}{2}\right)}$$

$$\therefore \frac{a^2 + b^2 - c^2}{2ab} = \frac{2b}{a}$$

$\therefore 3b^2 = a^2 - c^2$

57. Consider a tetrahedral with faces F_1, F_2, F_3, F_4. Let $\vec{V}_1, \vec{V}_2, \vec{V}_3, \vec{V}_4$ be the vectors whose magnitude are respectively equal to areas of F_1, F_2, F_3, F_4 and whose directions are perpendicular to their faces in outward direction. Then $|\vec{V}_1 + \vec{V}_2 + \vec{V}_3 + \vec{V}_4|$ equal to

(A) 1 (B) 4 (C) 0 (D) none

Ans. **C**

Sol. $\because \vec{V}_1 = \frac{1}{2}(\vec{a} \times \vec{b})$

$\vec{V}_2 = \frac{1}{2}(\vec{b} \times \vec{c})$

$\vec{V}_3 = \frac{1}{3}(\vec{c} \times \vec{a})$

$\vec{V}_4 = \frac{1}{2}\{(\vec{c} - \vec{a}) \times (\vec{b} - \vec{a})\}$

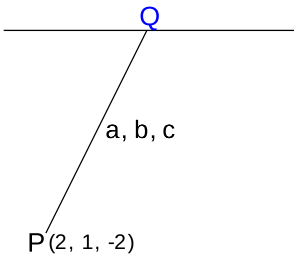
$\therefore \vec{V}_1 + \vec{V}_2 + \vec{V}_3 + \vec{V}_4 = 0$

58. The distance of the point (2, 1, -2) from the line $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-3}$ measured parallel to the plane x + 2y + z = 4 is

(A) $\sqrt{10}$ (B) $\sqrt{20}$ (C) $\sqrt{5}$ (D) $\sqrt{30}$

Ans. **D**

Sol.



Let d.r. of PQ be a, b, c

$\therefore \frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-3}$

$then\ \frac{x-2}{a}=\frac{y-1}{b}=\frac{z+2}{c}=v$

$\therefore Q=(ar+2, br+1, cr-2)$

Which will lie on $\frac{x-1}{2}=\frac{y+1}{2}=\frac{z-3}{3}$

$\therefore \frac{ar+1}{2}=\frac{br+1}{2}=\frac{cr-5}{-3}=\lambda(say)$

$\therefore a=\frac{2\lambda-1}{r}, b=\frac{2\lambda-2}{r}, c=\frac{-3\lambda+5}{r}$

$\because PQ\ is \parallel to\ x+2y+z=4, then$

$a+2b+c=0$

$\therefore \frac{2\lambda-1}{r}+\frac{4\lambda-4}{r}+\frac{-3\lambda+5}{r}=0$

$\therefore \lambda=0$

$\therefore a=\frac{-1}{r}, b=\frac{-2}{r}, c=\frac{5}{r}$

$\therefore Q=(1,-1,3)$

$\therefore PQ=\sqrt{30}$

59. For the parabola $x^2+y^2+2xy-6x-2y+3=0$, the focus is

(A) (1, - 1) (B) (- 1, 1) (C) (3, 1) (D) none

Ans. **D**

Sol. Do yourself

60. A man running round a race course notes that the sum of the distances of two flag points is from him is always 10 m and the distance between the flag points is 8 m. The area of the path he encloses in square meter is

(A) 15π (B) 12π (C) 18π (D) 8π

Ans. **A**

Sol. Do yourself

61. The negotient of $\sim s \vee (\sim r \wedge s)$ is equivalent to

(A) $s \wedge \sim r$ (B) $s \wedge (r \wedge \sim s)$ (C) $s \vee (r \vee \sim s)$ (D) $s \wedge r$

Ans. **D**

Sol. Do yourself

62. If the angles of elevation of the top of a tower from three collinear points A, B and C on a line leading to the fort of the tower are 30^0, 45^0 and 60^0 respectively, then the ratio AB : BC is

(A) $\sqrt{3}:1$ (B) $\sqrt{3}:\sqrt{2}$ (C) $1:\sqrt{3}$ (D) 2 : 3

Ans. **A**

Sol. Do yourself

63. The median of 100 observations grouped in classes of equal width is 25. If the median class interval is 20-30 and the number of observations less than 20 is 45, then the frequency of median class is

(A) 20 (B) 12 (C) 10 (D) 15

Ans. **C**

Sol. Do yourself

64. The mean of the 100 observations is 50 and their standard deviation is 5. The sum of the squares of all observations is

(A) 50000 (B) 250000 (C) 252500 (D) 255000

Ans. **C**

Sol. Do yourself

65. Let $f : R \to \left[0, \frac{\pi}{2}\right]$ be a function defined by $f(x) = \tan^{-1}(x^2 + x + a)$. If f is into , then a equals

(A) 0 (B) 1 (C) 1/2 (D) 1/4

Ans. **D**

Sol. Do yourself

66. $\lim_{n\to\infty} \sum_{r=1}^{n} \cot^{-1}\left(r^2 + \frac{3}{4}\right)$ is

(A) 0 (B) $\tan^{-1} 1$ (C) $\tan^{-1} 2$ (D) none

Ans. **C**

Sol. Do yourself

67. The function $f(x) = |x^2 - 3x + 2| + \cos|x|$ is not differentiable at x is equal to

(A) – 1 (B) 0 (C) 1 (D) 2

Ans. **C**

Sol. $\because f(x) = |x-1||x-2| + \cos|x|$

$$f(x) = \begin{cases} x^2 - 3x + 2 + \cos x, x < 0 \\ -x^2 + 3x - 2 + \cos x, 0 \le x < 1 \\ -x^2 + 3x - 2 + \cos x, \le x < 2 \\ x^2 - 3x + 2 + \cos x, x \ge 2 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 2x - 3 - \sin x, x < 0 \\ 2x - 3 - \sin x, 0 \le x < 1 \\ -2x + 3 - \sin x, 1 \le x < 2 \\ 2x - 3 - \sin x, x > 2 \end{cases}$$

It is clear that f(x) is not differentiable at x = 1

$\therefore f'(1^-) = -1 - \sin 1$

$f^1(1^+) = 1 - \sin 1$

68. The point of intersection of the tangents drawn to the curve $x^2y = 1 - y$ at the points where it is meet by the curve xy = 1 – y, is given by

(A) (0, - 1) (B) (1, 1) (C) (0, 1) (D) none

Ans. C

Sol. Do yourself

69. If [.] stands for the greatest integer function, then the value of $\int_4^{10} \frac{[x^2]dx}{[x^2 - 28x + 196] + [x^2]}$ is

(A) 0 (B) 1 (C) 3 (D) none

Ans. C

Sol. $Let\ I = \int_4^{10} \frac{[x^2]dx}{[(x-14)^2] + [x^2]} \longrightarrow (i)$

$= \int_4^{10} \frac{[14-x]^2}{[x^2] + [(x-14)^2]} dx \longrightarrow \text{(ii)}$

(i) + (ii)

$2\text{I} = \int_4^{10} 1.dx = 10 - 4 = 6$

$\therefore I = 3$

70. Solution of the differential equation $\sin y.\frac{dy}{dx} = \cos y(1 - x.\cos y)$ is

(A) $\sec y = x - 1 - ce^x$ (B) $\sec y = x + 1 + ce^x$ (C) $\sec y = x + e^x + c$ (D) none of these

(Where c is arbitrary constant)

Ans. B

Sol. $\sin y \frac{dy}{dx} = \cos y(1 - x.\text{cosy})$

$\therefore \tan y\dfrac{dy}{dx} = 1 - x\cos y$

$\therefore \sec y.\tan y\dfrac{dy}{dx} = \sec y - x$

$\therefore \sec y.\tan y\dfrac{dy}{dx} - \sec y = -x \longrightarrow$(i)

$\sec y = V$

$\therefore \sec y.\tan y\dfrac{dy}{dx} = \dfrac{dv}{dx}$

$\therefore \dfrac{dv}{dx} - v = -x$

$\therefore I.f. = e^{\int -1.dx} = e^{-x}$

$\therefore V.e^{-x} = \int(-x).e^{-x}dx$

Section – B

71. If $a^2 - 4a + 1 = 4$, then the value of $\dfrac{a^3 - a^2 + a - 1}{a^2 - 1}\left(a^2 \neq 1\right)$ is equal to ....

Ans. **4**

Sol. $\because a^2 - 4a + 1 = 4 \Rightarrow a^2 + 1 = 4(1 + a)$

$\therefore \dfrac{a^3 - a^2 + a - 1}{a^2 - 1} = \dfrac{(a-1)\left(a^2+1\right)}{a^2 - 1} = \dfrac{a^2 + 1}{a + 1} = 4$

72. Numbers from 1 to 1000 are divisible by 60 but not by 24 is ....

Ans. **8**

Sol. Let n(A) = number divisible by 60 = (60, 120, …….,960 = 16)
n(B) = number divisible by 24 = (24, 48, ……..,984) = 41
n(A n B) = number divisible by both = 120, 240, ……….960 = 8
∴ number divisible by 60 but not 24 = 16 – 8 = 8

Section – C

73. Evaluate

$$\lim_{x\to 0}\frac{8}{x^8}\left\{1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}.\cos\frac{x^2}{4}\right\}$$

Ans. 00000.03

Sol. Let $P = \lim_{x\to 0}\dfrac{8}{x^8}\left\{\left(1 - \cos\dfrac{x^2}{4}\right)\left(1 - \cos\dfrac{x^2}{2}\right)\right\}$

$= \lim_{x\to 0}\dfrac{8}{x^8}.4\sin^2\dfrac{x^2}{8}.\sin^2\dfrac{x^2}{4}$

$= \lim_{x\to 0}\dfrac{32}{64\times 16}.\dfrac{\sin^2\dfrac{x^2}{8}}{\dfrac{x^4}{64}}.\dfrac{\sin^2\dfrac{x^2}{4}}{\dfrac{x^4}{16}} = \dfrac{1}{32}$

74. Let f(x) be a continuous function given by

$f(x)=\begin{cases} 2x, & |x|\le 1 \\ x^2+ax+b, & |x|>1 \end{cases}$. Find the area of the region in the third quadrant bounded by the curves x = -2y^2 and y = f(x) lying on the left of the line 8x + 1 = 0

Ans. **00001.34**

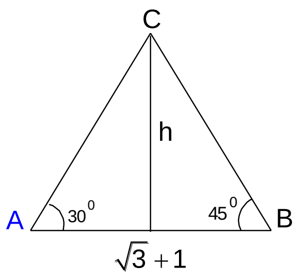
Sol. Required area = $\int_{-2}^{-}\left[-\sqrt{\frac{-x}{2}}-\left(x^2+2x-1\right)dx\right]+\int_{-1}^{-\frac{1}{8}}\left[-\sqrt{\frac{-x}{2}}-2x\right]dx$

$=\frac{257}{192}$

75. If the angles of a triangle are 30^0 and 45^0 and the included side is $\left(\sqrt{3}+1\right)cm$. Then area of the triangle is

Ans. **00001.37**

Sol.



$\angle C=180-(30+45)$

By sine rule in Δ ABC, we have

$\frac{a}{\sin 30^0}=\frac{b}{\sin 45^0}=\frac{\sqrt{3}+1}{\sin 105^0}$

$\therefore a=\sqrt{2}, b=2$

$\therefore \Delta=\frac{1}{2}ab\sin C=\frac{\sqrt{3}+1}{2}sq.units$