PRE-SERIES-OLT-2021-T2-FT-II-KVPY-CLASS-XII
FULL TEST – II

PART – I
MATHEMATICS

1. If $y = \lambda x - 3, y = \mu x + 1, y = x + 4$ are three normals from a fixed point P to parabola whose axis is along x-axis then $2\lambda - 3\mu$ is equal to

(A) 5 (B) $\frac{5}{2}$

(C) $-\frac{5}{2}$ (D) none of these

Ans. C

Sol. Sum of slopes=0

$\Rightarrow \lambda + \mu + 1 = 0$ …(1)

also all three lines are concurrent at P

hence $\begin{vmatrix} \lambda & -1 & -3 \\ \mu & -1 & 1 \\ 1 & -1 & 4 \end{vmatrix} = 0$ ...(2)

from (1) & (2) find λ and μ.

2. Suppose that the minimum of $f(x) = \cos 2x - 2a(1 + \cos x)$ is $-\frac{1}{2}$. Then a = ______

(A) $2 - \sqrt{3}$ (B) $-2 + \sqrt{3}$

(C) $2 + \sqrt{3}$ (D) $-2 - \sqrt{3}$

Ans. B

Sol. We have $f(x) = 2\cos x^2 - 1 - 2a - 2a\cos x$

$= 2\left(\cos x - \frac{a}{2}\right)^2 - \frac{1}{2}a^2 - 2a - 1$

For $a > 2$, $f(x)$ takes the minimum value of 1 – 4a when $\cos x = 1$; for $a < -2$, $f(x)$ takes the minimum 1 when $\cos x = -1$; for $-2 \le a \le 2$, $f(x)$ takes the minimum $-\frac{1}{2}a^2 - 2a - 1$ when $\cos x = \frac{a}{2}$. It is easy to see that $f(x)$ will never be $-\frac{1}{2}$ for $a > 2$ or $a < -2$. So it is only possible that $-2 \le A \le 2$. Then from $-\frac{1}{2}a^2 - 2a - 1 = \frac{1}{2}$, we get $a = -2 + \sqrt{3}$ or $a = -2 - \sqrt{3}$ (discarded). Therefore, the correct answer is $a = -2 + \sqrt{3}$.

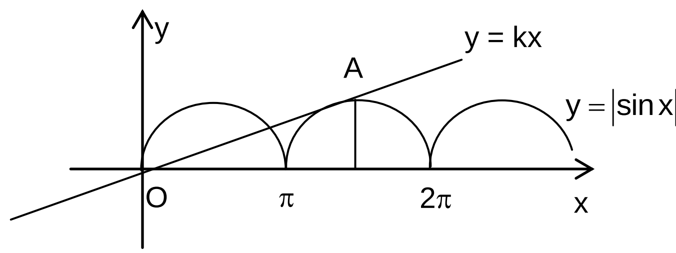
3. It is known that the curve $f(x)=|\sin x|$ intercepts the line $y=kx(k>0)$ at exactly three points, the maximum x coordinate of these points being α. Then $\frac{\cos\alpha}{\sin\alpha+\sin 3\alpha}=$

(A) $\frac{2+\alpha^2}{\alpha}$ (B) $\frac{1+\alpha^2}{\alpha}$

(C) $\frac{1+\alpha^2}{4\alpha}$ (D) α

Ans. C

Sol. The image of the three intercepting point of $f(x)$ and $y=kx$ is shown in the figure. It is easy to see that the curve and the line are tangent to each other at point $A(\alpha,-\sin\alpha)$, and $\alpha\in\left(\pi,\frac{3\pi}{2}\right)$.



As $f'(x)=-\cos x$ for $x\in\left(\pi,\frac{3\pi}{2}\right)$, we have $-\cos\alpha=-\frac{\sin\alpha}{\alpha}$, i.e. $\alpha=\tan\alpha$. Then

$$\frac{\cos\alpha}{\sin\alpha+\sin 3\alpha}=\frac{\cos\alpha}{2\sin 2\alpha\cos\alpha}=\frac{1}{4\sin\alpha\cos\alpha}$$

$$=\frac{\cos^2\alpha+\sin^2\alpha}{4\sin\alpha\cos\alpha}=\frac{1+\tan^2\alpha}{4\tan\alpha}$$

$$=\frac{1+\alpha^2}{4\alpha}.$$

4. On a coordinate plane there are two region, M and N: M is confined by $\begin{cases} y\geq 0, \\ y\leq x, \\ y\leq 2-x; \end{cases}$ and N is determined by the inequalities $t\leq x\leq t+1, 0\leq t\leq 1$. Then the size of the common area of M and N is given by $f(t)=$ _______ .

(A) $-t^2+t+\frac{1}{2}$ for $0\leq t\leq 1$ (B) $-t^2+t+\frac{1}{4}$ for $0\leq t\leq 1$

(C) $t^2+t+\frac{1}{2}$ for $0\leq t\leq 1$ (D) $t+\frac{1}{2}$ for $0\leq t\leq 1$

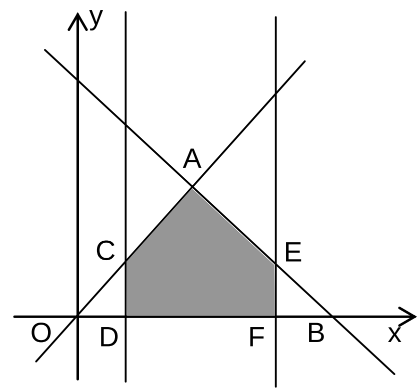
Ans. A

Sol. As shown in the figure, we have $f(t) = S_{\text{shaded area}}$

$= S_{\Delta AOB} - S_{\Delta OCD} - S_{\Delta BEF}$

$= 1 - \frac{1}{2}t^2 - \frac{1}{2}(1-t)^2$

$= -t^2 + t + \frac{1}{2},\ 0 \le t \le 1$



5. Given points P, Q on an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b > 0)$, satisfying $OP \perp OQ$, the minimum of $|OP| \times |OQ|$ is __________.

(A) $\frac{a^2b^2}{a^2+b^2}$ (B) $\frac{2a^2b^2}{a^2+b^2}$

(C) $\frac{a^2+b^2}{2ab}$ (D) $\frac{2a^2b^2}{a+b}$

Ans. B

Sol. Define $P(|OP|\cos\theta, |OP|\sin\theta)$, $Q\left(|OQ|\cos\left(\theta \pm \frac{\pi}{2}\right), |OQ|\sin\left(\theta \pm \frac{\pi}{2}\right)\right)$.

We have $\frac{1}{|OP|^2} = \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}$, …(i)

$\frac{1}{|OQ|^2} = \frac{\sin^2\theta}{a^2} + \frac{\cos^2\theta}{b^2}$. …(ii)

Then

$$\frac{1}{|OP|^2} + \frac{1}{|OQ|^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$

Therefore, $|OP| \times |OQ|$ reaches the minimum $\frac{2a^2b^2}{a^2+b^2}$ when $|OP| = |OQ| = \sqrt{\frac{2a^2b^2}{a^2+b^2}}$.

6. If p(x) = ax² + bx and q(x) = lx² + mx + n with p(1) = q(1), p(2) – q(2) = 1 and p(3) – q(3) = 4, then p(4) – q(4) is

(A) 0 (B) 5

(C) 6 (D) 9

Ans. D

Sol. We have, p(1) – q(1) = 0 ⇒ (a + b) – (l + m + n) = 0 … (i)

p(2) – q(2) = 1 ⇒ (4a + 2b) – (4l + 2m + n) = 1 … (ii)

p(3) – q(3) = 4 ⇒ (9a + 3b) – (9l + 3m + n) = 4 … (iii)

now use the operation 3 × (3) + 1 × (1) – 3 × (2),

we get (16a + 4b) – (16l + 4m + n) = 9 ⇒ p(4) – q(4) = 9.

7. For the mapping f: A $\rightarrow$ B where A = {1, 2, 3, 4} and B = {a, b, c} if X and Y are the total number of into and onto functions then |X – Y| is equal to

(A) 10 (B) 8

(C) 9 (D) none of these

Ans. C

Sol. If n(A) = n, n(B) = r then total number of functions = r^n.

Total number of into function = ${}^rC_1 (r-1)^n - {}^rC_2 (r-2)^n + \ldots$

Here r = 3, n = 4

$r^n = 3^4 = 81$

$X = {}^3C_1\, 2^4 - {}^3C_2\, 1^4 = 45$

Y = 81 – 45 = 36

|X – Y| = 9.

8. The locus of the point of intersection of the lines x sin θ + (1 – cos θ) y = a sin θ and x sin θ - (1 + cos θ) y + a sin θ = 0 is

(A) $x^2 - y^2 = a^2$ (B) $x^2 + y^2 = a^2$

(C) $y^2 = ax$ (D) none of these

Ans. B

Sol. From the given equations we have

$\frac{1-\cos\theta}{\sin\theta} = \frac{a-x}{y}$ and $\frac{1+\cos\theta}{\sin\theta} = \frac{a+x}{y}$

Multiplying we get $\frac{1-\cos^2\theta}{\sin^2\theta} = \frac{a^2-x^2}{y^2} \Rightarrow x^2 + y^2 = a^2$

9 The function $f : R \rightarrow R,\ f(x) = \begin{cases} x|x| - 4, & x \in Q \\ x|x| - \sqrt{3}, & x \notin Q \end{cases}$ is

(A) one-one and onto (B) many-one and onto

(C) one-one and into (D) many-one and into

Ans. D

Sol. For $x = 2$ and $x = 3^{1/4}$, $f(x) = 0$ and hence many-one.

For no value of x, $f(x) = -\sqrt{3}$ and hence into.

10. Let there be three sketch pens of different colors. A regular pentagon with unit side length is to be drawn using these pens. Exactly one of the colors is used to draw a side of the pentagon and at least two colors are used to draw the pentagon. The total number of ways in which this can be done, is equal to

(A) 30 (B) 39

(C) 48 (D) 51

Ans. C

Sol. Number of ways $=\frac{1}{5}\left(3^5-3\right)$.

11. If y_1 and y_2 are two different solutions of the equation $\frac{dy}{dx}+P(x)y=Q(x)$ and $y = 3\alpha y_1 + 2\beta y_2$ is also the solution of the equation, (where $\alpha - \beta = 4$), then which of the following is not correct. ($\alpha, \beta \in R$)

(A) $4\alpha + \beta = 5$
(B) $\alpha + 4\beta = 7$
(C) $5\alpha + 5\beta + 2 = 0$
(D) $5\alpha + 10\beta + 13 = 0$

Ans. B

Sol. $\frac{dy_1}{dx}+P(x)y_1=Q(x)$, $\frac{dy_2}{dx}+P(x)y_2=Q(x)$

$\frac{d(3\alpha y_1+\beta y_2)}{dx}+P(x)\left(3\alpha y_1+2\beta y_2\right)=Qx$

$3\alpha Q(x) + 2\beta(Q(x)) = Q(x) \Rightarrow 3\alpha + 2\beta = 1$ and $\alpha - \beta = 4$

$\Rightarrow \alpha=\frac{9}{5}$, $\beta=\frac{-11}{5}$

12. Suppose A,B,C are defined as $A=a^2b+ab^2-a^2c-ac^2, B=b^2c+bc^2-a^2b-ab^2$, and $C=a^2c+c^2a-cb^2-c^2b$ where a>b>c>0 and the equation $Ax^2+Bx+C=0$ has equal roots, then a,b,c are in

(A) A.P.
(B) G.P.
(C) A.G.P
(D) H.P.

Ans. D

Sol. $A+B+C=0 : B^2=4AC \Rightarrow A=C$

13. The greatest value of $\left(3x+2y\right)$ subject to the condition that $x>0,\ y>0$ and $x^2+xy+y^2=\frac{27}{7}$ is __________

(A) 2
(B) 4
(C) does not exist
(D) 6

Ans. A

Sol. For the maximum value, $\frac{d}{dx}\left(3x+2y\right)=0$. So, $\frac{dy}{dx}=\frac{3}{2}$

Now, differentiating the given relation, we get, $\frac{dy}{dx}=\frac{2x+y}{x+2y}=-\frac{3}{2}$

$\Rightarrow 4x+2y=3x+6y \Rightarrow x=4y$

So, $16y^2+4y^2+y^2=\frac{27}{7}\Rightarrow y=\frac{3}{7}$

The maximum value of $3x+2y=14y=6$

14 The domain of the function $f(x)=\ln\left(\ln\frac{x}{\{x\}}\right)$ (where {.} denotes the fractional part function) is

(A) $R-Z$

(B) $(0,\infty)-Z$

(C) $(2,\infty)-Z$

(D) $(1,\infty)-Z$

Ans. D

Sol. $\ln\frac{x}{\{x\}}>0\Rightarrow\frac{x}{\{x\}}>1\Rightarrow x>\{x\}\Rightarrow x>1$

Obviously, x cannot be an integer.

15. Let $P(x)=x^2+bx+c$, where *b* and *c* are integers. If $P(x)$ is a factor of both x^4+6x^2+25 and $3x^4+4x^2+28x+5$, then

(A) the value of $P(1)$ is 5

(B) the value of $P(-1)$ is 7

(C) the minimum value of $P(x),\ x\in R$ is 4

(D) the maximum value of $P(x),\ x\in R$ is 14

Ans. C

Sol. Since $P(x)$ divides both of them, $P(x)$ also divides

$(3x^4+4x^2+28x+5)-3(x^4+6x^2+25)$

$=-14x^2+28x-70=-14(x^2-2x+5)$.

Hence, $P(x)=x^2-2x+5$

Alternatively:

$x^4+6x^2+25=(x^4+10x^2+25)-4x^2$

$=(x^2-2x+5)(x^2+2x+5)$

Hence, $P(x)$ can be x^2-2x+5 or x^2+2x+5.

By using long division, we find that only x^2-2x+5 is a factor of $3x^4+4x^2+28x+5$, which is equal to $(x^2-2x+5)(3x^2+6x+1)$.

$\therefore\quad P(x)=x^2-2x+5$

16. $(\vec{a}.\hat{i})\ (\vec{a}\times\hat{i})+(\vec{a}.\hat{j})\ (\vec{a}\times\hat{j})\ +(\vec{a}.\hat{k})\ (\vec{a}\times\hat{k})$ is equal to;

(A) $3\vec{a}$

(B) $\vec{r}$

(C) $2\vec{r}$

(D) none of these

Ans. D

Sol. $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \Rightarrow \vec{a}.\hat{i} = a_1 \Rightarrow (\vec{a}.\hat{i})(\vec{a}\times\hat{i}) = a_1(-a_2\hat{k} + a_3\hat{j})$

Hence D

17. If $f(r) = r(r^2 - 1)$, then $\sum_{r=2}^{n} \frac{1}{f(r)}$ is equal to

(A) $\frac{1}{4}\left(1 - \frac{1}{n(n+1)}\right)$ (B) $\frac{1}{4}\left(2 - \frac{1}{n(n+1)}\right)$

(C) $\frac{1}{4}\left(1 - \frac{2}{n(n+1)}\right)$ (D) none of these

Ans. C

Sol. $\sum_{r=2}^{n} \frac{1}{2}\left(\frac{(r+1)-(r-1)}{r(r+1)(r-1)}\right) = \frac{1}{2}\sum_{r=2}^{n}\left(\frac{1}{r(r-1)} - \frac{1}{r(r+1)}\right) = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{n(n+1)}\right)$

18 A complex number z lying below the real axis in the Argand plane satisfies the relation $|z-1|^2 + 2|z-2|^2 = 3|z-3|^2$. The principal argument of the complex number $4z - 9$ is equal to

(A) $\frac{\pi}{2}$ (B) $-\frac{5\pi}{6}$

(C) $-\frac{\pi}{6}$ (D) $-\frac{\pi}{2}$

Ans. D

Sol. Put $z = x + iy$. We get $z = \frac{9}{4} + iy$, where $y < 0$.

19. If the equation $4x^2 - 4\sqrt{3(ab+bc+ca)}x + (a+b+c)^2 = 0$ has real roots, where a, b, c are the sides of triangle ABC, then the triangle is

(A) isosceles (B) right angled

(C) equilateral (D) none of these

Ans. C

Sol. $D \geq 0$

$3(ab + bc + ca) - (a + b + c)^2 \geq 0$

$ab + bc + ca - (a^2 + b^2 + c^2) \geq 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 \leq 0$

20 The value of a,b,c so that $\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$

(A) $a = 1, b = 2, c = 1$ (B) $a = 2, b = 1, c = 2$

(C) $a = -1, b = -2, c = -1$ (D) none of these

Ans. A

Sol.

$$\frac{a\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.......\right)-b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-........\right)+c\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+........\right)}{x\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-.......\right)}=2$$

$$\frac{\frac{a-b+c}{x^2}+\frac{a-c}{x}+\left(\frac{a}{2!}+\frac{b}{2!}+\frac{c}{2!}\right)+x(.....)+x^3(.....)+.....}{1-\frac{x}{3!}+\frac{x^3}{5!}.......}=2$$

$a+b+c=0$

$a-c=0$

$\frac{a+b+c}{2}=2$

Solving for a, b, c

a = b = 1

PHYSICS

21. A train having 60 wagons each weighing 25 ton is moving with a speed of 72 km/hr. If the frictional force is 10N per ton, the power developed is:

(A) 3×10^5 W
(B) 3×10^6 W
(C) 3×10^7 W
(D) 3×10^4 W

Ans. A

Sol. power = FV

= 60 x 25 x 10 x 20

22. A 1.5 kW (kilo-watt) laser beam of wavelength 6400Å is used to levitate a thin aluminum disk of same area as the cross section of the beam. The laser light is reflected by the aluminum disk without any absorption. The mass of the foil is close to

(A) 10^{-9} kg
(B) 10^{-3} kg
(C) 10^{-4} kg
(D) 10^{-6} kg

Ans. D

Sol. F = mg $\Rightarrow \frac{2.(IA)}{C}=mg$

$\Rightarrow \frac{2.\times1.5\times10^3}{3\times10^8}=m\times10$

$\Rightarrow$ m = 10^{-6} kg.

23. Consider two thermometers T_1 and T_2 of equal length which can be used to measure temperature over the range θ_1 to θ_2. T_1 contains mercury as the thermometric liquid while T_2 contains bromine. The volume of the two liquids are the same at the temperature θ_1. The volumetric coefficients of expansion of mercury and bromine are $18 \times 10^{-5}\ K^{-1}$ and $108 \times 10^{-5}\ K^{-1}$, respectively. The increase in length of each liquid is the same for the same increase in temperature. If the diameters of the capillary tubes of the two thermometers are d_1 and d_2 respectively, then the ratio $d_1 : d_2$ would be closest to

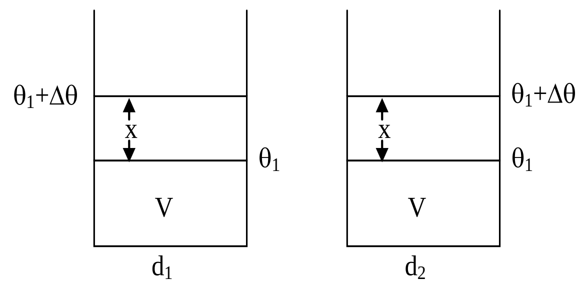
(A) 6.0 (B) 2.5
(C) 0.6 (D) 0.4

Ans. D

Sol. $\Delta V_1 = \pi\left(\frac{d_1}{2}\right)^2 x = 18 \times 10^{-5} V\,\Delta\theta$

$\Delta V_2 = \pi\left(\frac{d_2}{2}\right)^2 x = 108 \times 10^{-5} V\,\Delta\theta$

$\therefore \frac{d_1}{d_2} = \sqrt{\frac{18}{108}} = \frac{1}{\sqrt{6}} = \frac{1}{2.449} \approx 0.4$



24. One mole of a monatomic ideal gas is expanded by a process described by $PV^3 = C$ where C is a constant. The heat capacity of the gas during the process is given by (R is the gas constant)

(A) 2R (B) $\frac{5}{2}R$
(C) $\frac{3}{2}R$ (D) R

Ans. D

Sol. heat capacity = C_V + PdV

$= \frac{3}{2}R + \frac{P_1V_1 - P_2V_2}{3-1}$

$= \frac{3}{2}R - \frac{1 \cdot R}{2} = R$

25. 1 gm of a radioactive substance takes. 50 sec to loose 0.01 gm then the half life of the sample will be

(A) $\frac{50 \ln(2)}{\ln\left(\frac{100}{99}\right)}$ (B) $\frac{50 \ln(2)}{\ln(100)}$
(C) $\frac{50 \ln(2)}{\ln(99)}$ (D) $\frac{50 \ln(2)}{\ln(0.99)}$

Ans. A

Sol. $N = N_0 e^{-\lambda t}$

$0.99 = 1\, e^{-\lambda t} \Rightarrow \lambda t = \ln\left(\frac{100}{99}\right)$

$50 \text{ sec} = t = \frac{\ln\left(\frac{100}{99}\right)}{\lambda}$

$\Rightarrow \; t_{1/2} = \frac{50\ln(2)}{\ln(100/99)}$

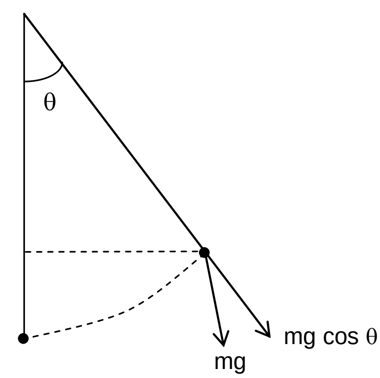
26. The maximum value attained by the tension in the string of a swinging pendulum is four times the minimum value it attains. There is no slack in the string. The angular amplitude of the pendulum is

(A) 90° (B) 60°

(C) 45° (D) 30°

Ans. B

Sol. $\frac{1}{2}mv^2 = mgR(1-\cos\theta)$



$\frac{mv^2}{R} = 2mg(1-\cos\theta)$

$T_{max} = mg + \frac{mv^2}{R} = mg(3-2\cos\theta)$

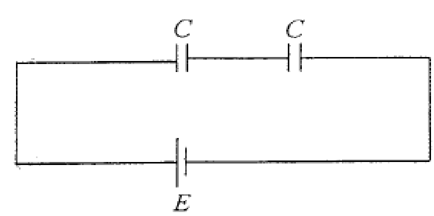
$T_{min} = mg\cos\theta$

$\therefore \frac{4}{1} = \frac{mg(3-2\cos\theta)}{mg\cos\theta}$

$\therefore \cos\theta = \frac{1}{2}$

$\theta = 60°$

27. Two identical parallel pate capacitors of capacitance C each are connected in series with a battery of emf, E as shown. If one of the capacitors is now filled with a dielectric of dielectric constant k, the amount of charge which will flow through the battery is (neglect internal resistance of the battery)



(A) $\frac{k+1}{2(k-1)}CE$ (B) $\frac{k-1}{2(k+1)}CE$

(C) $\frac{k-2}{k+2}CE$ (D) $\frac{k+2}{k-2}CE$

Ans. B

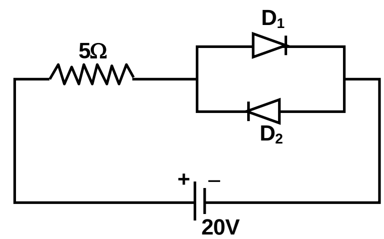
Sol. $C_{eq} = \frac{c}{2}$; $q_{initial} = \frac{CE}{2}$

C_{eq} (find) = $\frac{KC \times c}{KC + c} = \frac{KC}{K+1}$, $q_{final} = \frac{KCE}{(K+1)}$

$\therefore$ extra charge supplied by battery = $\frac{KCE}{K+1} - \frac{CE}{2}$

$= \frac{(K-1)CE}{2(K+1)}$

28. In the figure shown below, chose the correct answer.
(A) Voltage across D_2 is 20 Volt.
(B) Voltage across D_1 is 20 Volt.
(C) Current through diode D_1 is 4A.
(D) Current through diode D_1 is 0 A.



Ans. C

Sol. Since, D_1 will be acting as short circuit, D_2 will act as open circuit so,
Current through 5Ω resistance and diode D_1 is $\frac{20}{5}$ = 4 A

29. The flat face of a plano-convex lens of focal length 10 cm is silvered. A point source placed 30 cm in front of the curved surface will produce a
(A) real image 15 cm away from the lens
(B) real image 6 cm away from the lens
(C) virtual image 15 cm away from the lens
(D) virtual image 6 cm away from the lens

Ans. B

Sol. $\because$ $P_{net} = 2P_L + P_M = \frac{2}{10} - \frac{1}{\infty} = \frac{2}{10} = \frac{1}{5}$

$\therefore$ $\frac{1}{f}$ of mirror $= \frac{1}{V} + \frac{1}{-30} = -\frac{1}{5}$

$\frac{1}{V} = \frac{1}{30} - \frac{1}{5} = \frac{1-6}{30} = \frac{-5}{30} = \frac{-1}{6}$

$\therefore$ V = –6

30. The river 'A' and river 'B' merge to form a river 'C' with their speeds in the ratio 1 : 1.5. The cross-sectional areas of the river 'A', river 'B' and the river 'C' are in the ratio 1 : 2 : 3. Assuming streamline flow, the ratio of the speed of river 'C' to that of the river 'B'.
(A) 7 : 9 (B) 4 : 3
(C) 8 : 9 (D) 5 : 3

Ans. C

Sol. Let

Area of river 'A' $\Rightarrow$ A

Area of river 'B' $\Rightarrow$ 2A

Area of river 'C' $\Rightarrow$ 3A

$V_A : V_B : V_C \quad \Rightarrow \quad V : \frac{3}{2}V : V_1$

By equation of continuity

$AV + \frac{3}{2}A \cdot 2V \quad \Rightarrow \quad 3A\,V_1$

$V_{river\ 'C'} = \frac{4}{3}V$

$\frac{V_{river\ 'B'}}{V_{river\ 'C'}} \Rightarrow \frac{\frac{3}{2}V}{\frac{4}{3}V} = \frac{9}{8}$

$V_{river\ 'C'} : V_{river\ 'B'} = 8 : 9.$

31. A ball dropped from a high altitude acquires a terminal velocity before hitting the ground, where it bounces off elastically. If air resistance depends on the speed of the ball, what will it acceleration be just after the first bounce?

(A) zero
(B) g downwards
(C) 2g downwards
(D) 3g downwards

Ans. C

Sol. Let F_{air} = kv then mg = kv

$S_Q \quad v = \frac{mg}{k}$

But just after the bounce F_{net} = mg + kv

$$a = g + \frac{kv}{m} = 2g$$

32. The sun orbits the centre of the galaxy (Milky-way) in almost circular path of radius 'R' in a period 'T' and earth also orbits the sun in an almost circular path of radius 'r' in a period 't'. Assume whole mass of the galaxy concentrated at its centre and find an expression for the ratio of mass of galaxy to that of sun.

(A) $\left(\frac{R}{r}\right)^2 \left(\frac{t}{T}\right)^2$
(B) $\left(\frac{R}{r}\right)^3 \left(\frac{T}{t}\right)^2$
(C) $\left(\frac{R}{r}\right)^2 \left(\frac{t}{T}\right)^3$
(D) $\left(\frac{R}{r}\right)^2 \left(\frac{T}{t}\right)^3$

Ans. A

Sol. $T = 2\pi\sqrt{\frac{R^3}{GM}}$, $M = \frac{4\pi^2 R^3}{GT^2}$

33. In Young's Double slit Experiment, the wavelength of the red light is 7800 Å and that of blue light is 5200 Å. The value of n for which n^{th} bright band due to red light coincides with $(n + 1)^{th}$ bright band due to blue light, is

(A) 1
(B) 2
(C) 3
(D) 4

Ans. B

Sol. $y_{n(red)} = \frac{n\lambda_1 D}{d}$

$y_{nt(blue)} = \frac{(n+1)\lambda_2 D}{d}$

Apply $y_{n\ (red)} = y_{n+1\ (blue)}$

$n(\lambda_r) = (n + 1)\lambda_b$

$n(7800) = (n + 1)(5200)$

$\Rightarrow \quad n = 2$

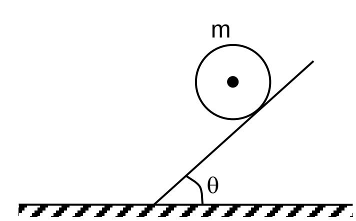
34. A point source is emitting sound in all directions. The ratio of distance of two points from the point source where the difference in loudness levels is 3 dB is: ($\log_e 2 = 0.3$)

(A) $\frac{1}{2}$
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{4}$
(D) $\frac{2}{3}$

Ans. B

Sol. $3 = 10 \log\left(\frac{r_2}{r_1}\right)^2$

35. A sphere of mass m has to purely roll on a rough inclined plane of coefficient plane of coefficient of friction 'μ'. The friction force acting on the sphere is



(A) $\mu mg\cos\theta$
(B) $\frac{2mg\sin\theta}{7}$ downward
(C) $\frac{2mg\sin\theta}{7}$ upward
(D) $\frac{5mg\sin\theta}{7}$ downward

Ans. C

Sol. $f = \frac{mg\sin\theta}{1+\frac{R^2}{K^2}}$

36. Two particles move parallel to x axis about the origin with same amplitude a and frequency ω. At a certain instant they are found at a distance a/3 from the origin on opposite sides but their velocities are in the same direction. What is the phase difference between the two.

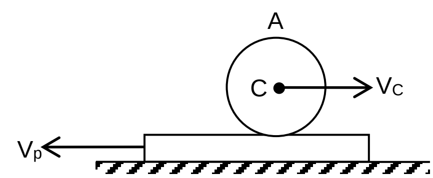
(A) $\cos^{-1}\left(\frac{7}{9}\right)$ (B) $\cos^{-1}\left(\frac{5}{9}\right)$

(C) $\cos^{-1}\left(\frac{4}{9}\right)$ (D) $\cos^{-1}\left(\frac{1}{9}\right)$

Ans. A

Sol. $\cos\phi = 1 - 2\times\left(\frac{1}{3}\right)^2 = \frac{7}{9}$

37. The velocities are in ground frame and the cylinder is performing pure rolling on the plank. Velocity of point 'A' would be

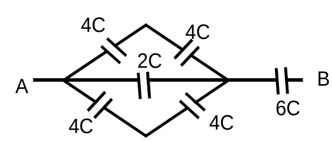


(A) $2V_c$
(B) $2V_c + V_p$
(C) $2V_c - V_p$
(D) $2(V_c + V_p)$

Ans. C

Sol. $V_{Ap} = 2V_c$
$V_{Ag} = V_{Ap} + V_{Pg} = 2V_c - V_p$

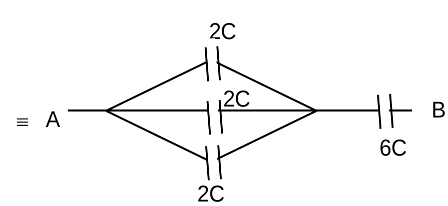
38. The equivalent capacitance between points A and B of the circuit will be

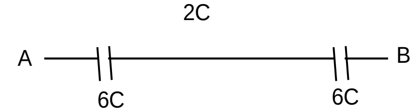


(A) 12C (B) 6 C
(C) 3 C (D) 24 C

Ans. C

Sol. Equivalent circuit of the above figure can be drawn as





$C_{AB} = 3C$

39. When the electron in a hydrogen atom jumps from the second orbit to the first orbit, the wavelength of the radiation emitted is λ. When the electron jumps from the third to the first orbit, the wavelength of the radiation emitted as

(A) $\frac{9}{4}\lambda$ (B) $\frac{4}{9}\lambda$

(C) $\frac{27}{32}\lambda$ (D) $\frac{32}{27}\lambda$

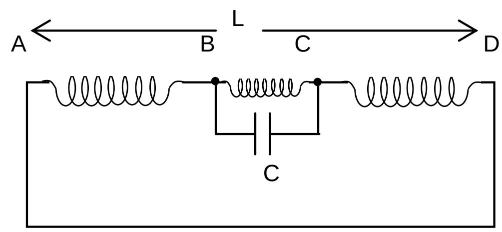
Ans. C

Sol. $\frac{hc}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R$ ...(i)

$\frac{hc}{\lambda'} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8}{9}R$ ...(ii)

$\frac{\lambda'}{\lambda} = \frac{3}{4} \times \frac{9}{8}, \quad \lambda' = \frac{27}{32}\lambda$

40. An inductance L is split into three equal parts AB, BC and CD and a capacitor C is connected across its two centre terminals (figure). The outer terminals are short circuited. The frequency of oscillation is



(A) $\frac{3}{2\sqrt{2}\pi} \times \frac{1}{\sqrt{LC}}$

(B) $\frac{1}{2\pi} \times \frac{1}{\sqrt{LC}}$

(C) $\frac{\sqrt{3}}{2\pi} \times \frac{1}{\sqrt{LC}}$

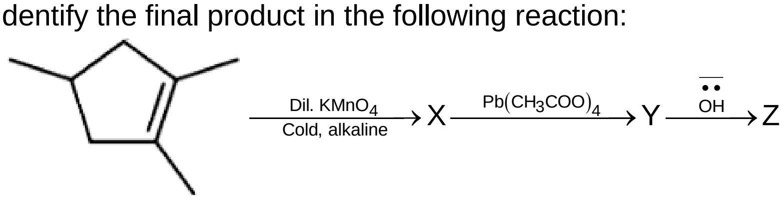
(D) $\frac{3}{2\pi} \times \frac{1}{\sqrt{LC}}$

Ans. A

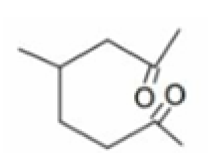
Sol. $L' = \frac{2/3\ L.\ 1/3\ L}{2/3\ L + 1/3\ L} = \frac{2}{9}L, \omega = \frac{1}{\sqrt{2/9\ L \times C}}$

<u>CHEMISTRY</u>

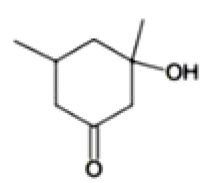
41. Identify the final product in the following reaction:



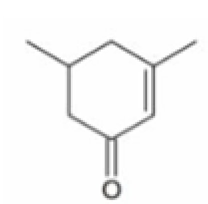
(A)



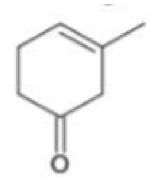
(B)



(C)

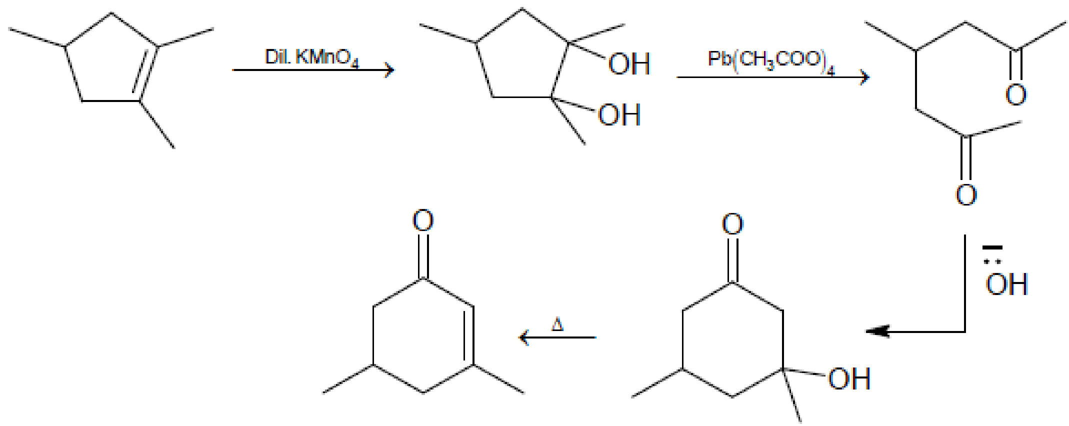


(D)

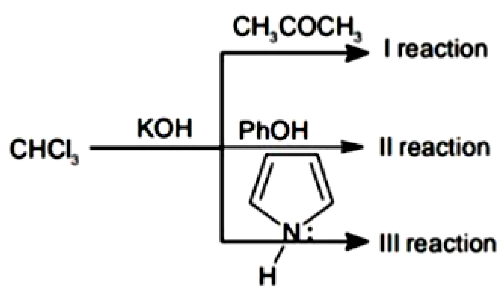


Ans. C

Sol.



42.



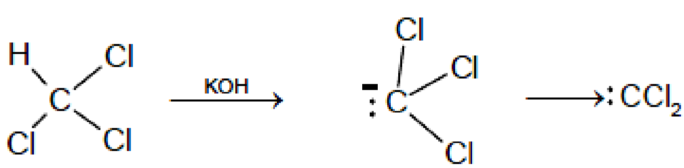
In which of the following reaction carbine is intermediate

(A) I, II (B) II, III

(C) III, I (D) I, II, III

Ans. B

Sol.



43. The normality of a solution of sodium hydroxide 100 mL of which contains 4 grams of NaOH is

(A) 0.1 (B) 40

(C) 1.0 (D) 0.4

Ans. C

Sol. Use: $\text{Normality} = \dfrac{\text{No. of gm equivalents of solute}}{\text{Volume of solution in litres}}$

44. Which of the following combination of solutions does not make a buffer?

(A) $NaH_2PO_4 + Na_2HPO_4$ (B) $NaHCO_3 + H_2CO_3$

(C) $NaH_2PO_4 + Na_3PO_4$ (D) $KHCO_3 + K_2CO_3$

Ans. C

Sol. For acidic buffer we need weak acid and its conjugate base.

45. Cyclopropane rearranges to form propene 

This follows first order kinetics the half life is 13.86 $\times 10^{-3}$ min. The initial concentration of cyclopropane is 16 M. What will be the concentration of cyclopropane after 27.72 $\times 10^{-3}$ min.

(A) 0.035 M
(B) 4M
(C) 3.5 M
(D) .35 M

Ans. B

Sol. $R = \frac{2.303}{t}\log_{10}\frac{C_o}{C_t}$

Where $R = \frac{0.693}{t_{1/2}}$ & $C_o = 16$

46. Select the methanides from compound given below:

Al_4C_3	Be_2C	MgC_2	BaC_2
I	II	III	IV

(A) I only
(B) I and IV
(C) I and II
(D) I II III IV

Ans. C

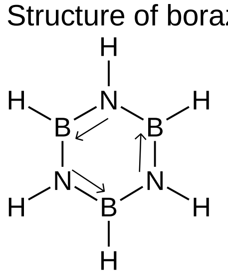
Sol. Methanides(C^{4-}) release methane gas on hydrolysis.

47. The bonds present in borazole are:

(A) 12σ, 3π
(B) 9σ, 6π
(C) 6σ, 6π
(D) 9σ, 9π

Ans. A

Sol. Structure of borazole is



48. Which gives blood red colour with ammonium thiocyanate?

(A) Fe^{3+}
(B) Fe^{2+}
(C) Cu^{2+}
(D) Cd^{2+}

Ans. A

Sol. Fe^{3+} forms red coloured complex with SCN^-.

49. When H_2O_2 is added to an acidified solution of $K_2Cr_2O_7$
(A) Solution turns green due to formation of Cr_2O_3
(B) A deep blue-violet colored compound CrO_5
(C) Solution turns yellow due to formation of K_2CrO_4
(D) Solution gives green ppt. of $Cr(OH)_3$ is formed

Ans. B

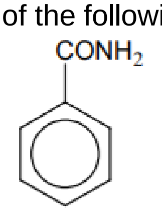
Sol. Fact based.

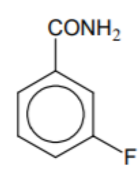
50. The degree of dissociation of HI at a particular temperature is 0.8. The volume of 2 M hypo solution required to neutralize the iodine present in an equilibrium mixture of reaction which is started by 2 mole each of H_2 and I_2 in a vessel of two litre capacity is
(A) 0.4 litre (B) 0.8 litre
(C) 1.6 litre (D) 3.2 litre

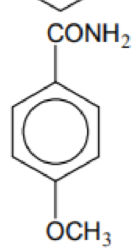
Ans. C

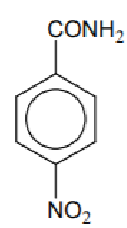
Sol. $2Na_2S_2O_3 + I_2 \longrightarrow 2NaI + Na_2S_4O_6$

51. Which of the following can undergo Hofmann Bromamide reaction most easily?

(A) 

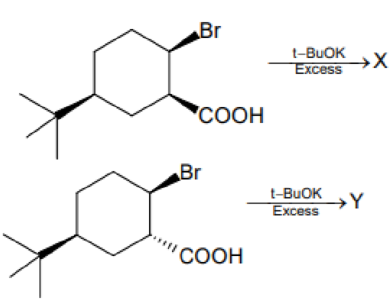
(B) 

(C) 

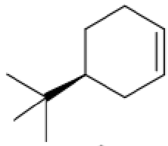
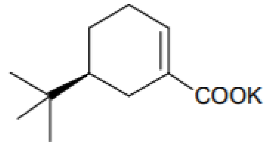
(D) 

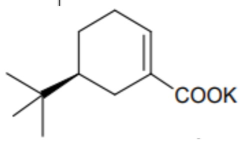
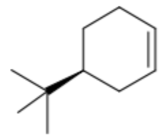
Ans. C

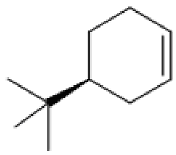
Sol. Rate of reaction depends upon migration aptitude.

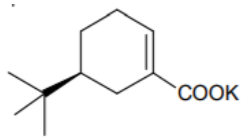
52. 

Which of the following option is correct?

(A) X is  : Y is  (Major products)

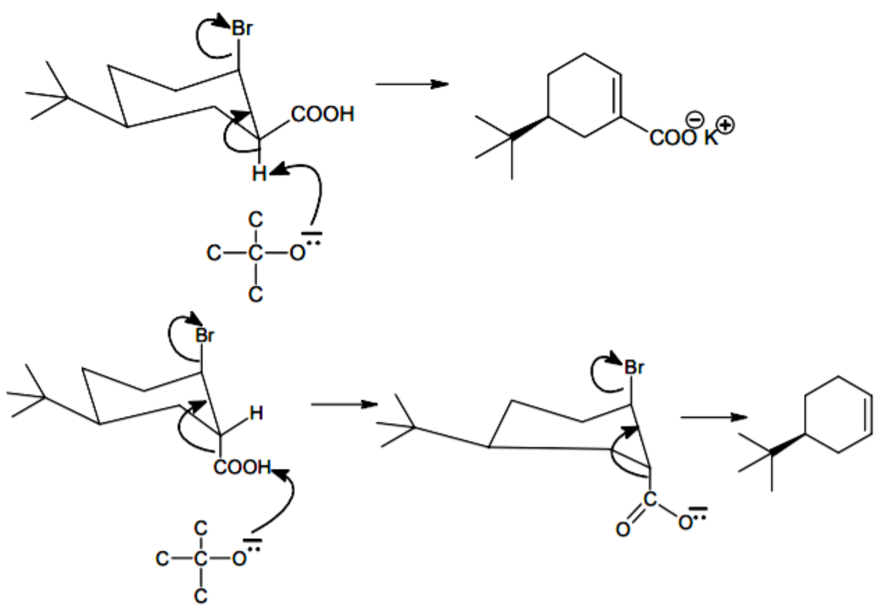
(B) X is  : Y is  (Major products)

(C) Both X & Y are  (Only)

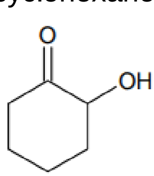
(D) Both X & Y are  (Only)

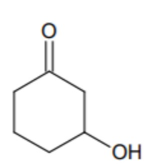
Ans. B

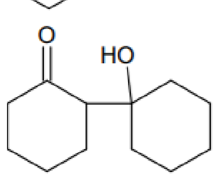
Sol.

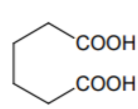


53. When cyclohexanone is treated with Na_2CO_3 soln, we get

(A) 

(B) 

(C) 

(D) 

Ans. C

Sol. Intermolecular aldol reaction.

54. The chief ore of Zn is the sulphide, ZnS. The ore is concentrated by froth floatation process and then heated in air to convert ZnS to ZnO.

$$2ZnS + 3O_2 \xrightarrow{80\%} 2ZnO + 2SO_2$$

$$ZnO + H_2SO_4 \xrightarrow{100\%} ZnSO_4 + H_2O$$

$$2ZnSO_4 + 2H_2O \xrightarrow{80\%} 2Zn + 2H_2SO_4 + O_2$$

The number of moles of ZnS required for producing 2 moles of Zn will be:

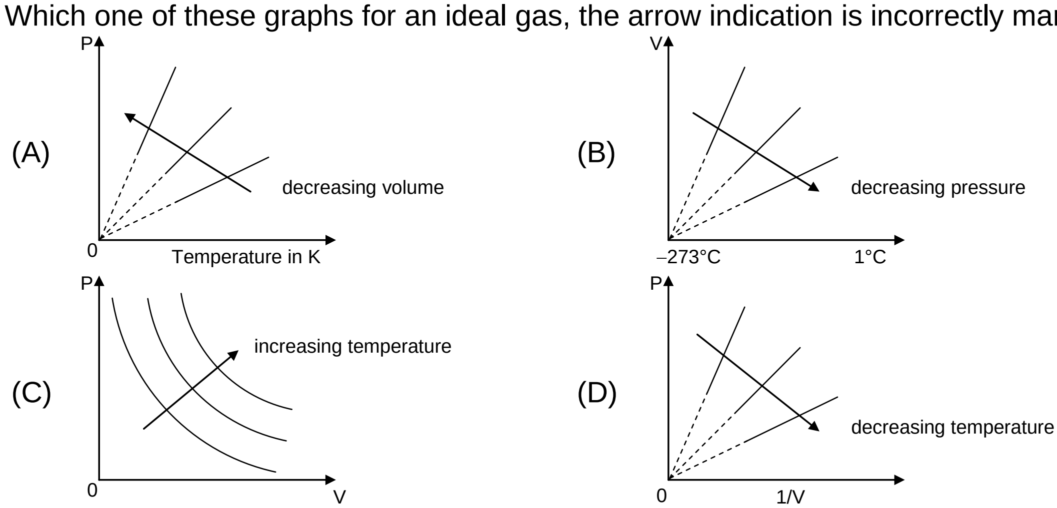
(A) 3.125 (B) 2
(C) 2.125 (D) 4

Ans. A

Sol. Let moles Of ZnS be ‘a’
Moles of ZnO produced = 0.8(a)
Moles of $ZnSO_4$ produced = 0.8 a
Moles of Zn produced = 0.8(0.8)a = 2 moles(given)

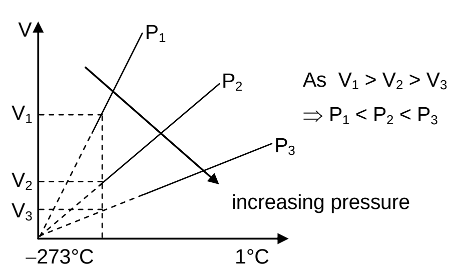
$$\text{So } a = \frac{2}{(0.8)^2} = 3.125$$

55. Which one of these graphs for an ideal gas, the arrow indication is incorrectly marked?



Ans. B

Sol.



56. At 273 K and 1 atm, 10 litre of N_2O_4 decomposes to NO_2 according to equation

$$N_2O_4(g) \rightleftharpoons 2NO_2(g)$$

What is degree of dissociation (α) when the original volume is 25% less than that of existing volume?

(A) 0.25 (B) 0.33
(C) 0.66 (D) 0.5

Ans. B

Sol. For ideal gas mole % = volume %

	$N_2O_4(g) \rightleftharpoons$	$2NO_2(g)$
Initial moles	a	0
at eqm	$a(1-\alpha)$	$2a\alpha$

As per given original volume

$$= \frac{75}{100} \times \text{Volume at eqm}$$

At constant T and P : $V \propto n$

$\therefore$ $a = 0.75 \times a(1 + \alpha)$

$\Rightarrow$ $\alpha = 0.33$

57. The pH of the resultant solution of 20 mL of 0.1 M H_3PO_4 and 20 mL of 0.1 M Na_3PO_4 is:

(A) $pK_{a_1} + \log 2$ (B) pK_{a_1}
(C) pK_{a_2} (D) $\frac{pK_{a_1} + pK_{a_2}}{2}$

Ans. C

Sol.

	H_3PO_4 +	PO_4^{3-} $\longrightarrow$	$H_2PO_4^-$ +	HPO_4^{2-}
initial milli moles	20×0.1	20×0.1		
after reaction	–	–	2	2

Buffer solution of $H_2PO_4^-$ (acid) and HPO_4^{2-} (conjugate base) is formed;

$$pH = pK_{a_2} + \log\frac{[HPO_4^{2-}]}{[H_2PO_4^-]} = pK_{a_2}$$

58. An ideal solution has two components A and B. A is more volatile than B, i.e., $P_A^o > P_B^o$ and also $P_A^o > P_{total}$. If X_A and Y_A are mole fractions of components A in liquid and vapour phases, then:

(A) $X_A = Y_A$ (B) $X_A > Y_A$
(C) $X_A < Y_A$ (D) Data insufficient

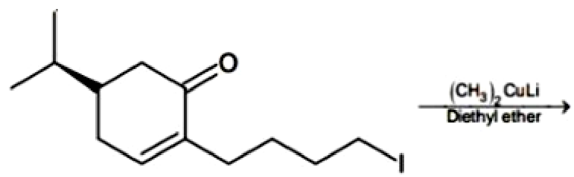
Ans. C

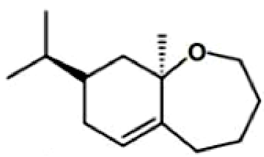
Sol. We know that

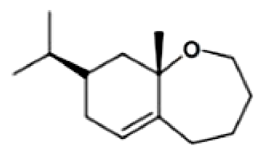
$$Y_A = \frac{P_A^o X_A}{P_{total}} \quad \text{or} \quad \frac{Y_A}{X_A} = \frac{P_A^o}{P_{total}}$$

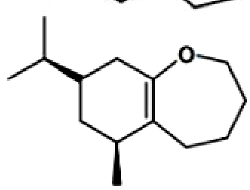
$$\because P_A^o > P_{total} \quad \text{so} \quad \frac{Y_A}{X_A} > 1 \quad \text{or} \quad Y_A > X_A$$

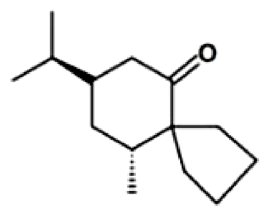
59. Major product formed in the following reaction is



(A) 

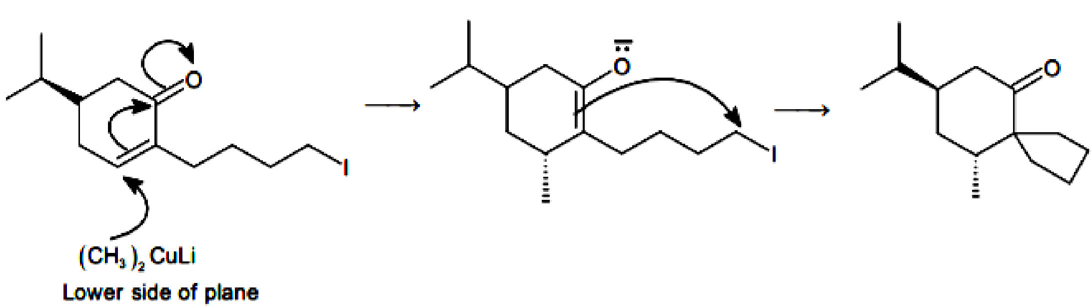
(B) 

(C) 

(D) 

Ans. D

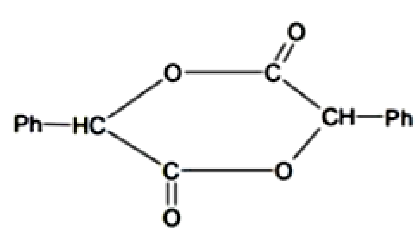
Sol.



60. Identify the final product in the given reaction:

$$Ph-CH_2COOH \xrightarrow[\text{(ii) OH}]{\text{(i) } P_{red}+Br_2} X \xrightarrow{H^+,\Delta} Y$$

(A) $Ph-CH_2-\underset{\overset{\|}{O}}{C}-O-CH_2-Ph$

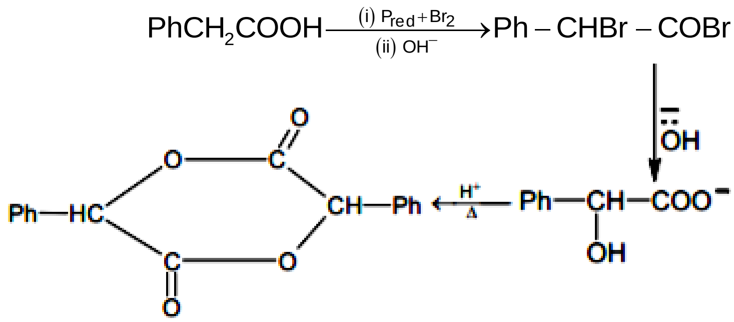
(B) 

(C) $Ph-CH_2-\underset{\overset{\|}{O}}{C}-O-\underset{\overset{\|}{O}}{C}-CH_2Ph$

(D) $Ph-\underset{\underset{COOH}{|}}{CH}-O-\underset{\underset{COOH}{|}}{CH}-Ph$

Ans. B

Sol.


PhCH2COOH
(i) Pred+Br2
(ii) OH−
Ph – CHBr – COBr
OH
Ph—CH—COO−
OH
H+
Δ
Ph—HC
O
C
O
CH—Ph
C
O
O

PART – II

MATHEMATICS

61. Find the maximum of the function $y=\sqrt{x+27}+\sqrt{13-x}+\sqrt{x}$.

(A) 11 (B) 12
(C) 13 (D) 14

Ans. A

Sol. The domain of y is $x\in[0,13]$. We have $y=\sqrt{x+27}+\sqrt{13-x}+\sqrt{x}$

$=\sqrt{x+27}+\sqrt{13+2\sqrt{x(13-x)}}\geq\sqrt{27}+\sqrt{13}=3\sqrt{3}+\sqrt{13}$.

The equality holds when $x=0$. Therefore, the minimum of y is $3\sqrt{3}+\sqrt{13}$.
On the other hand, by the Cauchy inequality we have

$y^2=\left(\sqrt{x}+\sqrt{x+27}+\sqrt{13-x}\right)^2\leq\left(\frac{1}{2}+1+\frac{1}{3}\right)\left[2x+(x+27)+3(13-x)\right]$

= 121.

62. Suppose that the sides a, b, c of ΔABC, corresponding to the angles A, B, C respectively, constitute a geometric sequence. Then the range of $\frac{\sin A\cot C+\cos A}{\sin B\cot C+\cos B}$ is

(A) $(0,+\infty)$ (B) $\left(0,\frac{\sqrt{5}+1}{2}\right)$

(C) $\left(\frac{\sqrt{5}-1}{2},\frac{\sqrt{5}+1}{2}\right)$ (D) $\left(\frac{\sqrt{5}-1}{2},+\infty\right)$

Ans. C

Sol. Suppose that the common ratio of a, b, c is q. Then $b=aq, c=aq^2$. We have

$\frac{\sin A\cot C+\cos A}{\sin B\cot C+\cos B}=\frac{\sin A\cos C+\cos A\sin C}{\sin B\cos C+\cos B\sin C}$

$=\frac{\sin(A+C)}{\sin(B+C)}=\frac{\sin(\pi-B)}{\sin(\pi-A)}$

$=\frac{\sin B}{\sin A}=\frac{b}{a}=q$

So we only need to determine the range of q. As a, b, c are the sides of a triangle, they satisfy $a+b>c$ and $b+c>a$.

That is to say, $\begin{cases}a+aq>aq^2\\aq+aq^2>a\end{cases}$.

It follows that $\begin{cases}q^2-q-1<0\\q^1+q-1>0\end{cases}$.

Their solutions are $\begin{cases} \dfrac{\sqrt{5}-1}{2} < q < \dfrac{\sqrt{5}+1}{2}, \\ q > \dfrac{\sqrt{5}-1}{2} \text{ or } q < -\dfrac{\sqrt{5}+1}{2} \end{cases}$

It is only possible that $\dfrac{\sqrt{5}-1}{2} < q < \dfrac{\sqrt{5}+1}{2}$.

The equality holds when $4x = 9(13-x) = x+27$. It is so for $x = 9$. Therefore, the maximum of y is 11.

63. The set of real value of 'a' for which at least one tangent to the parabola $y^2 = 4ax$ becomes normal to the circle $x^2 + y^2 - 2ax - 4ay + 3a^2 = 0$ is

(A) [1, 2]
(B) [$\sqrt{2}$, 3]
(C) R
(D) ϕ (no set)

Ans. C

Sol. Any tangent of parabola will be of the form $ty = x + at^2$ at the point $(at^2, 2at)$. If this is normal to circle, then this will passes through centre of the circle which is (a, 2a)

$2at = a + at^2 \Rightarrow t^2 - 2t + 1 = 0$

$\Rightarrow t = 1, 1$, for any value of a

so the condition is satisfies for all real values of 'a'

64. If sin A = 3 sin (A+2B), angle B is acute and A is obtuse; then

(A) tan b = $1/\sqrt{2}$
(B) tan B > $1/\sqrt{2}$
(C) tan B < $1/\sqrt{2}$
(D) $0 < \tan B < 1/\sqrt{2}$

Ans. B

Sol. $\dfrac{\sin(A+2B)}{\sin A} = \dfrac{1}{3}$

As in Ex. 13, by componendo the dividendo, tan (A + B) = – 2 tan B

$\Rightarrow \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -2\tan B$

$\Rightarrow \tan A = \dfrac{3\tan B}{2\tan^2 B - 1} < 0$ (as A is obtuse)

$\Rightarrow$ 2 tan B – 1 < 0 (as tan B > 0)

$\Rightarrow 0 < \tan B < 1/\sqrt{2}$

65. Three distinct dice are thrown and the sum of the numbers appearing on the top faces is 12. The number of ways in which this is possible is

(A) 55
(B) 30
(C) 25
(D) 91

Ans. C

Sol. $x_1 + x_2 + x_3 = 12$

$t_1 + t_2 + t_3 = 9$ Total solutions

$y_1 + t_2 + t_3 = 3$ Extra solutions

66. If A is skew symmetric matrix, then I – A (where I is Identity matrix of the order equal to that of A)
(A) skew symmetric (B) symmetric
(C) singular (D) non – singular

Ans. D

Sol. Consider a homogeneous of equation (1 – A) X = 0 $\Rightarrow$ AX = IX = X ……..(1)
$\Rightarrow (AX)^T = X^T = X^T A^T = X^T = X^T \Rightarrow -X^T A = X^T$ ($\because$ A is skew symmetric)
$\Rightarrow -X^T AX = X^T X \Rightarrow -X^T X = X^T X$ (using (1))
$\Rightarrow X^T X = 0 \Rightarrow X = 0$
$\therefore$ (I – A) X = 0 has only trivial solution
$\therefore$ det (I – A) $\neq$ 0 or I – A is non – singular.

67. A coplanar beam of light emerging from a point source have equation $\lambda x - y + 2(1+\lambda) = 0,\ \lambda \in R$. The rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having equation $\mu x - y + 2(1-\mu) = 0,\ \mu \in R$. Foot of the perpendicular from the point (2, 2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0$. The eccentricity of the ellipse is equal to
(A) 1/3 (B) $1/\sqrt{3}$
(C) 2/3 (D) 1/2

Ans. C

Sol. The given lines are family of lines passing through (–2, 2) and (2, 2) which will be focii of the elliptical surface. i.e. 2ae = 4.
Foot of perpendicular from focus to tangent lies on auxilliary circle whose radius = 3
$\Rightarrow a = 3 \Rightarrow e = \frac{2}{3}$

68. The numbers 1, 2, 3,……..,n are arrange in random order . The probability that the digits 1, 2,3………k(k<n) appear as neighbors in that order is
(A) $\frac{1}{n\,!}$ (B) $\frac{k\,!}{n\,!}$
(C) $\frac{(n-k)\,!}{n\,!}$ (D) None of these

Ans. D

Sol. n(s) = n!, n(E) = (n – r + 1)!
$\therefore$ P(E) = $\frac{(n-r+1)!}{n!}$

69. The number of triplets of integers for which $2a^2 + b^2 - 8c = 7$ is equal to

(A) 0 (B) 1
(C) 2 (D) infinite

Ans. A

Sol. $2a^2 + b^2 = 8c + 7$ $\Rightarrow$ b must be odd.
Let $b = 2m + 1$.
$\therefore\ a^2 + 2m^2 + 2m = 4c + 3$ $\Rightarrow$ a must be odd.
Let $a = 2n + 1$.
$\therefore\ 2n^2 + 2n + m(m+1) = 2c + 1$, which is not possible.

70. Let $f(x) = x^3 - \frac{3x^2}{2} + x + \frac{1}{4}$. The value of $\int_{1/4}^{3/4} f(f(x))dx$

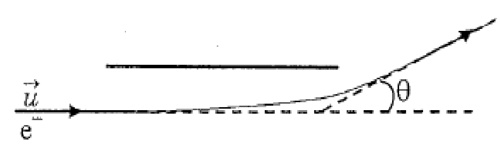
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$
(C) 0 (D) $\frac{3}{4}$

Ans. A

Sol. $4f(x) = x^4 - (1-x)^4 + 2$
$\Rightarrow f(x) + f(1-x) = 1$

PHYSICS

71. An electron enters a parallel plate capacitor with horizontal speed u and is found to deflect by angle θ on leaving the capacitor as shown. It is found that tan θ = 0.4 and gravity is negligible. If the initial horizontal speed is doubled, then tan θ will be



(A) 0.1 (B) 0.2
(C) 0.8 (D) 1.6

Ans. A

Sol. Let length of plate = l

then, time taken to cross the plate = $\frac{l}{v}$ & perpendicular velocity gained in this interval =

$\frac{eE}{m}t = \frac{eE}{m} \times \frac{l}{v}$

$\therefore$ 0.4 = tanθ = $\frac{\left(\frac{eE}{m}\frac{l}{v}\right)}{v} = \frac{eE}{m}\frac{l}{v^2}$ ...(1)

$\therefore\ \tan\theta^1 = \frac{eE}{m}\frac{l}{(2v)^2}$ ...(2)

Solving (1) & (2), $\tan\theta^1 = 0.1$

72. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to

(A) $b\left(\frac{u-f}{f}\right)^{1/2}$ (B) $b\left(\frac{u-f}{f}\right)$

(C) $b\left(\frac{f}{u-f}\right)^{1/2}$ (D) $b\left(\frac{f}{u-f}\right)^{2}$

Ans. D

Sol. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

$\Rightarrow\ \frac{1}{v}=-\frac{1}{f}+\frac{1}{u}$

$\Rightarrow\ v = \frac{uf}{(f-u)}$

Differentiating $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

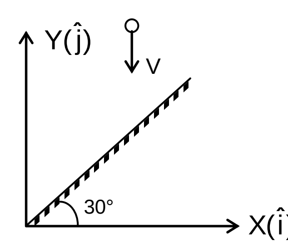
$\Rightarrow\ -\frac{1}{u^2}du=-\frac{1}{v^2}dv$

$\because$ f is constant

$\therefore\ \frac{dv}{du}=-\frac{v^2}{u^2}=-\frac{f^2}{(u-f)^2}$ $m = \frac{I}{O}=-\frac{dv}{du}=\frac{f^2}{(u-f)^2}$

$\therefore\ I = \frac{bf^2}{(u-f)^2}$

73. An object moves with velocity $V(-\hat{j})$ towards the inclined plane mirror as shown in figure. The velocity of the image is



(A) $V\hat{j}$

(B) $V\frac{\sqrt{3}}{2}(-\hat{i})+\frac{V}{2}\hat{j}$

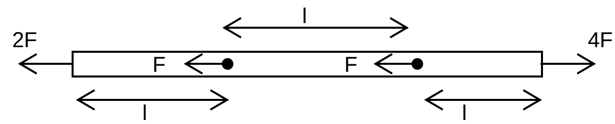
(C) $V\frac{\sqrt{3}}{2}\hat{i}+\frac{V}{2}\hat{j}$

(D) $\frac{V}{2}(-\hat{i})+V\frac{\sqrt{3}}{2}(\hat{j})$

Ans. B

Sol. $\vec{v} = 2V\cos 30°\sin 30°(-\hat{i}) + V(\cos^2 30° - \sin^2 30°)\hat{j}$

74. Force acting on a uniform rod having length 3l & area of crossection A and young's modulus Y are shown in figure. Find elongation in the rod (take $\frac{Fl}{YA} = 1$)



(A) 3
(B) 6
(C) 9
(D) None of these

Ans. C

Sol. $\Delta\ell = \frac{\ell}{AY}(2F + 3F + 4F)$

75. What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil decreases down to zero uniformly during a time interval Δt?

(A) $\frac{4}{3}\frac{q^2R}{\Delta t}$
(B) $\frac{2}{3}\frac{q^2R}{\Delta t}$
(C) $\frac{3}{4}\frac{q^2R}{\Delta t}$
(D) $\frac{3}{2}\frac{q^2R}{\Delta t}$

Ans. A

Sol. Suppose initial current is i_0, then

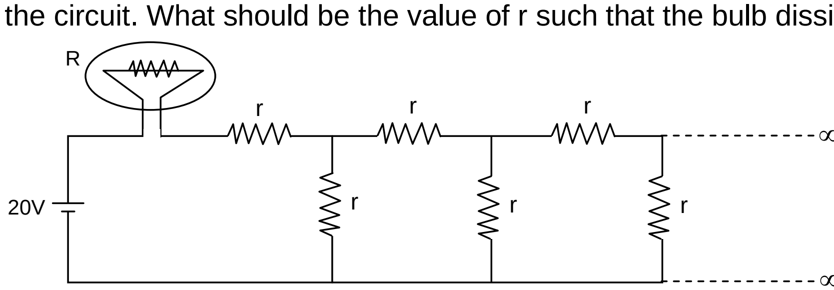
$i(t) = i_0\left(1 - \frac{t}{\Delta t}\right)$

$q = i_0\int_0^{\Delta t}\left(1 - \frac{t}{\Delta t}\right)dt$

So, $i_0 = \frac{2q}{\Delta t}$

$H = \int_0^{\Delta t}\left\{\frac{2q}{\Delta t}\left(1 - \frac{t}{\Delta t}\right)\right\}^2 Rdt$

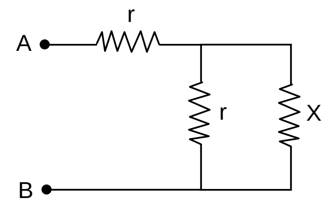
76. A light bulb of resistance $R = 8(\sqrt{5} + 1)\Omega$ is attached in series with an infinite resistor network with identical resistances 'r' as shown in figure. A 20V battery drives current in the circuit. What should be the value of r such that the bulb dissipates maximum power?



(A) 16 Ω
(B) 8 Ω
(C) 4 Ω
(D) $8(\sqrt{5} + 1)\Omega$

Ans. A

Sol. Let 'x' be the equivalent resistance of infinite network.



$$R_{AB} = \frac{rx}{r+x} + r = x$$

$rx + r^2 + rx = rx + x^2$

$x^2 - rx - r^2 = 0$

$$x = \frac{r + \sqrt{r^2 + 4r^2}}{2} = \left(\frac{\sqrt{5}+1}{2}\right)r$$

According to maximum power transfer theorem, for maximum power in R.

$x = R$

$$\left(\frac{\sqrt{5}+1}{2}\right)r = 8\left(\sqrt{5}+1\right)$$

$r = 16\ \Omega$

77. On doping germanium with donor atoms of density 10^{21} cm^{-3}, its conductivity in $mhocm^{-1}$ will be _________ if μ = 3800 cm^2/V-s and $n_i = 2.5 \times 10^{13}\, cm^{-3}$

(A) 240×10^4 (B) 180×10^4

(C) 120×10^4 (D) 60.8×10^4

Ans. D

Sol. $n_e n_h = n_i^2$

$$n_h = \frac{n_i^2}{n_e} = \frac{\left(2.5 \times 10^{13}\right)^2}{10^{21}} cm^{-3}$$

$n_h = 6.25 \times 10^5 cm^{-3}$

conductivity

$\sigma = e(n_e + n_h)\mu$

$= 1.6 \times 10^{-19}\left(10^{21} + 6.25 \times 10^5\right)\mu$

$\approx 1.6 \times 38 \times 10^4$

$\approx 608 \times 10^3$

78. A particle moves along x-axis. The position of the particle at time t is given as

$x = t^3 - 9t^2 + 24t + 1$

The distance traveled in first 5 seconds is

(A) 20 m (B) 10 m

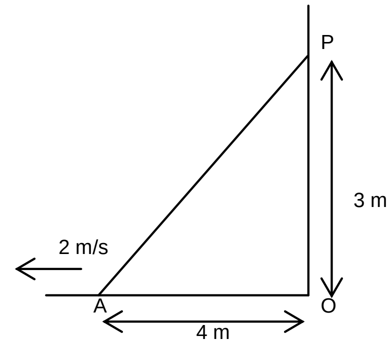
(C) 18 m (D) 28 m

Ans. D

Sol. Distance Travelled = $\int_0^5 |\vec{v}|\, dt = \int_0^5 \left|3t^2 - 18t + 24\right| dt = 28$

79. A ladder AP of length 5m is inclined to a vertical wall is slipping over a horizontal surface with velocity of 2 m/s when P is at a distance 3m from ground what is the velocity of centre of mass at this moment.

(A) 1.25 m/s
(B) 5/3 m/s
(C) 1 m/s
(D) 2 m/s



Ans. B

Sol. $V_{cm} = \frac{2}{3} \times \frac{\sqrt{(3)^3 + (4)^2}}{2}$

80. A particle is projected with initial speed $u = \sqrt{1500}$ m/s such that the radius of the path at highest point is equal to maximum height of the projectile. The horizontal range of the projectile is

(A) 150 m
(B) $100\sqrt{2}$ m
(C) $100\sqrt{3}$ m
(D) $150\sqrt{2}$ m

Ans. B

Sol. Radius at highest point $r = \frac{u^2 \cos^2 \theta}{g} = H$

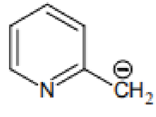
$\frac{u^2 \cos^2 \theta}{g} = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow \tan \theta = \sqrt{2}$

i.e. $\sin \theta = \frac{\sqrt{2}}{\sqrt{3}}$ and $\cos \theta = \frac{1}{\sqrt{3}}$

Range = $\frac{u^2}{g} \times 2 \sin\theta \cos\theta = \frac{1500}{10} \times \frac{2 \times \sqrt{2}}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = 100\sqrt{2}$ m

CHEMISTRY

81. The number of resonating structures possible for

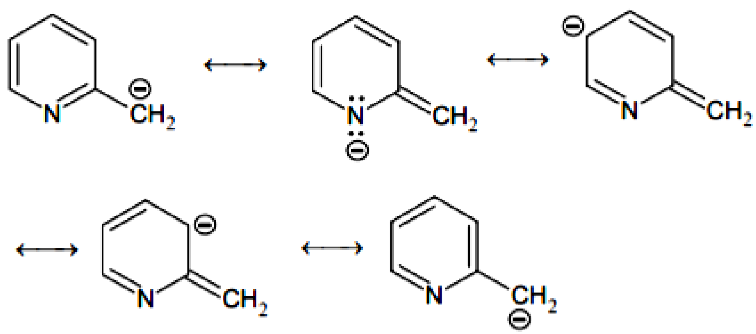


Are?

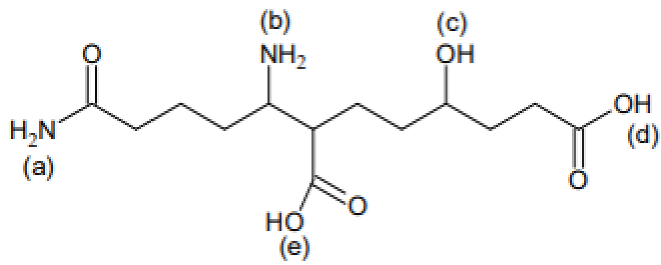
(A) 4
(B) 5
(C) 3
(D) 6

Ans. B

Sol.



82. The most acidic portion among the following is:

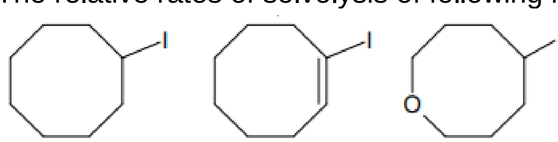


(A) a
(B) c
(C) e
(D) b

Ans. C

Sol. The carboxylic acid group stabilizes the conjugate base and – I of other groups at closest distance.

83. The relative rates of solvolysis of following iodides are

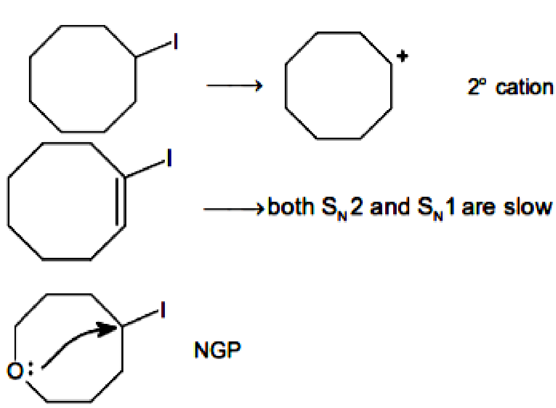


(x) (y) (z)

(A) x > y > z
(B) y > x > z
(C) x > z > y
(D) z > x > y

Ans. D

Sol.



84. Which one is a copolymer?

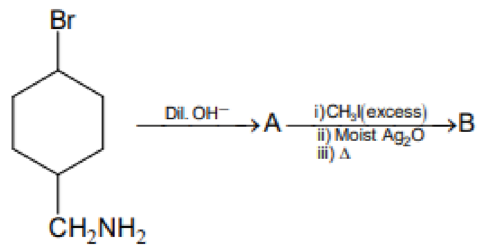
(A) PVC (B) Polypropene

(C) Polystyrene (D) Glyptal

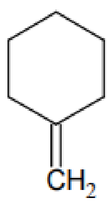
Ans. D

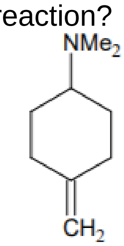
Sol. Glyptal is made from polymerization of ethylene glycol and pthalic acid.

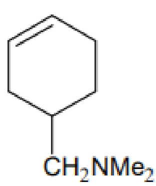
85.

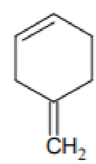


Identify the product in the following sequence of reaction?

(A) 

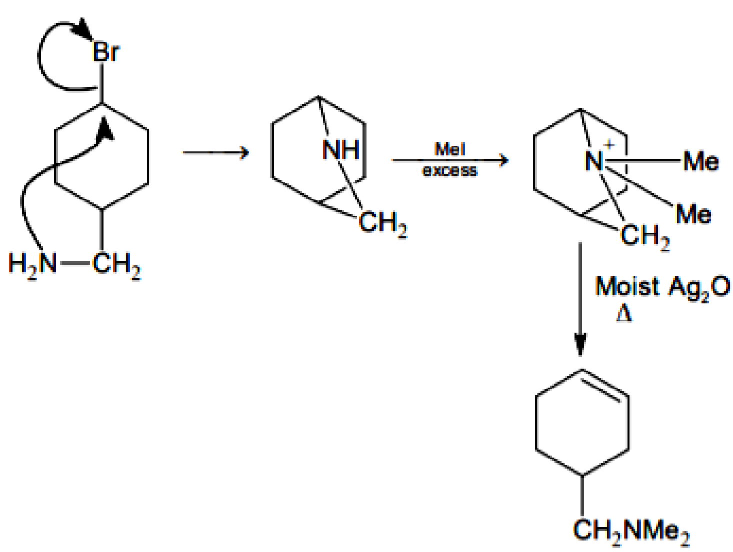
(B) 

(C) 

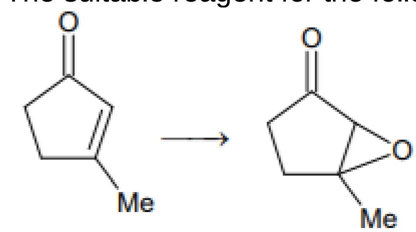
(D) 

Ans. C

Sol.



86. The suitable reagent for the following conversion is

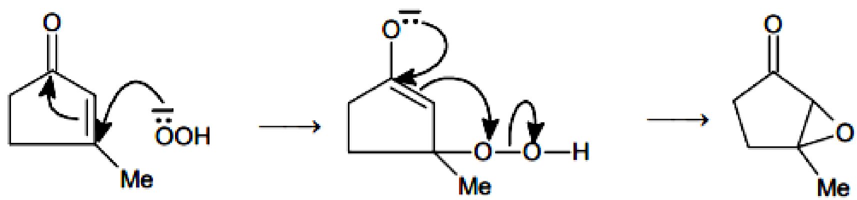


(A) m-CPBA
(B) H_2O_2/AcOH
(C) BuOH/HCl
(D) H_2O_2/NaOH

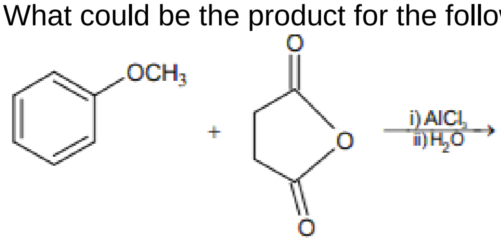
Ans. D

Sol.

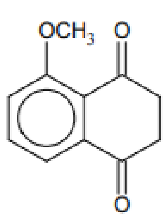
$$H_2O_2 + \ddot{\overline{O}}H \longrightarrow H-O-\ddot{\overline{O}} + H_2O$$



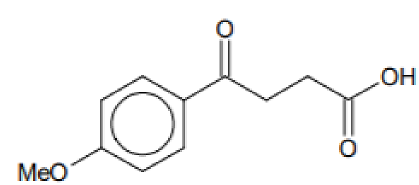
87. What could be the product for the following reaction?



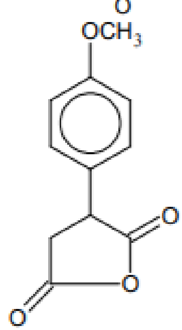
(A)



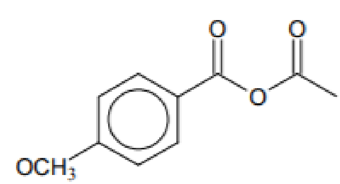
(B)



(C)

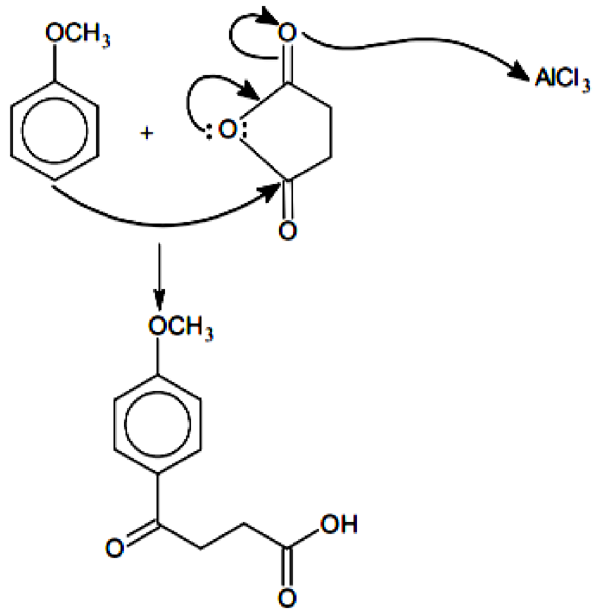


(D)

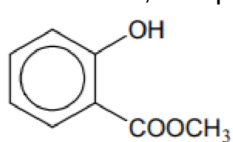


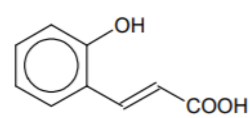
Ans. B

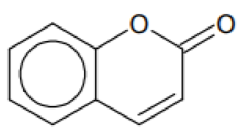
Sol.



88. When o-hydroxybenzaldehyde is heated with ethanoic anhydride in the presence of sodium ethanoate, compound formed during the reaction is?

(A) 

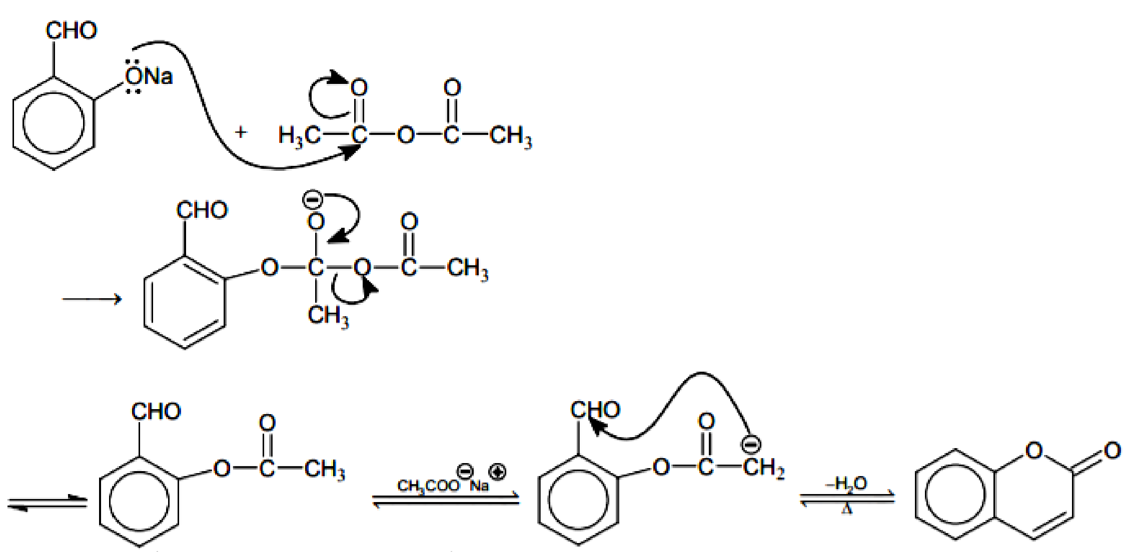
(B) 

(C) 

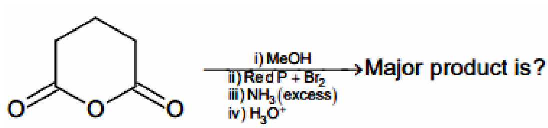
(D) Both B and C

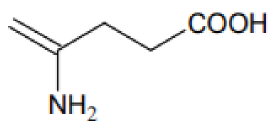
Ans. D

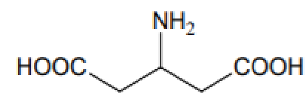
Sol.

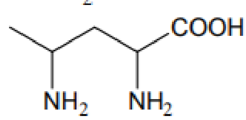


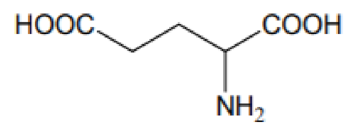
89.



(A) 

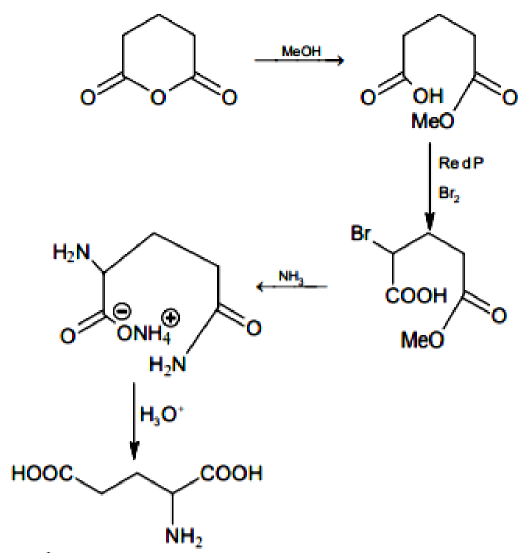
(B) 

(C) 

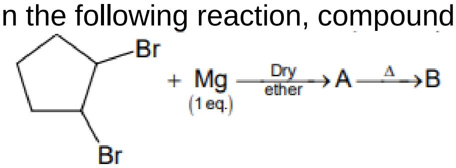
(D) 

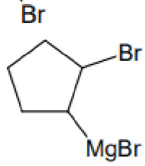
Ans. D

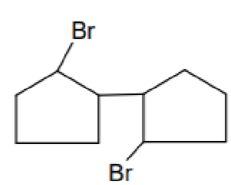
Sol.

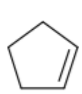


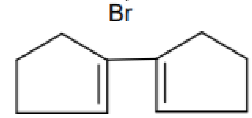
90. In the following reaction, compound (B) is



(A) 

(B) 

(C) 

(D) 

Ans. C

Sol.