PRE-SERIES-OLT-2021-T1-FT-I-KVPY-CLASS-XII
FULL TEST – I

PART – I
MATHEMATICS

1. Number of solution of the equation $\log_2\left(x+3^{\log_6 x}\right)=\log_6 x$ is

(A) 1 (B) 2
(C) 3 (D) 4

Ans. A

Sol. $\log_2\left(x+3^{\log_6 x}\right)=\log_6 x$

Let $u=\log_6 x \Rightarrow x=6^u$

$log_2(6^u+3^u)=u$

$6^u+3^u=2^u$

$1+2^u=\left(\frac{2}{3}\right)^u$

LHS = RHS happens for only one value of u, which is –1

($\because$ LHS is increasing whereas RHS is decreasing)

$\log_6 x=-1 \Rightarrow x=\frac{1}{6}$

2. Give that $S=\left|\sqrt{x^2+4x+5}-\sqrt{x^2+2x+5}\right|$ for all real values of x, then the maximum value of S^4 is

(A) 1 (B) 2
(C) 4 (D) 3

Ans. C

Sol. This problem can be viewed as

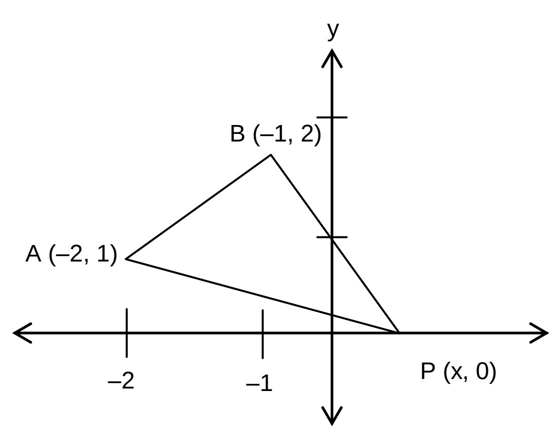
$S=\left|\sqrt{(x+2)^2+1}-\sqrt{(x+1)^2+2^2}\right|$

$S=|PA-PB|$

But $|PA-PB|\le AB$

$\therefore |PA-PB|_{max.}=AB=\sqrt{2}=S$

$\therefore S^4=4$



3. If $J=\int_0^{\infty}\frac{\ln x}{1+x^3}dx$ and $K=\int_0^{\infty}\frac{x\ln x}{1+x^3}dx,$ then

(A) $J+K=0$ (B) $J+K>0$

(C) $J+K<0$ (D) nothing can be said about $J+K$

Ans. A

Sol. $J+K=\int_0^{\infty}\frac{(1+x)\ln x}{1+x^3}dx$

Put $x=\frac{1}{t}$.

$J+K=\int_{\infty}^{0}t^2\frac{t+1}{t^3+1}\ln\left(\frac{1}{t}\right)\left(-\frac{1}{t^2}\right)dt$

$=-\int_0^{\infty}\frac{(t+1)\ln t}{1+t^3}dt=-(J+K)$

4. There is only one way to choose real numbers M and N such that when the polynomial $5x^4+4x^3+3x^2+Mx+N$ is divided by the polynomial x^2+1, the remainder is 0. If M and N assume these unique values, then $M-N$ is:

(A) –6 (B) –2

(C) 6 (D) 2

Ans. C

Sol. Let $P(x)=5x^4+4x^3+3x^2+Mx+N$

Let $Q(x)=x^2+1$

If the quotient is Q

Then $P(x)=Q(x^2+1)$

If $x=i,$ then $P(i)=0$

If $x=-i,$ then $P(-i)=0$

Hence, $5-4i-3+Mi+N=0$

Hence, $N+Mi=-2+4i$

$\therefore$ $N=-2;\ M=4$

$\therefore$ $M-N=6$

5. If A and B are two square matrixes such that $A^2B=BA$ and if $(AB)^{10}=A^kB^{10}$ then k is

(A) 1022 (B) 1023

(C) 1024 (D) 1025

Ans. B

Sol. $(AB).(AB)=A^3B^2$

$(AB)^3=A^7B^3$

$(AB)^n=A^{2^n-1}B^n$

6. Let $\binom{n}{r} = {}^{n}C_{r}$, then the value of $\binom{2000}{2} + \binom{2000}{5} + \binom{2000}{8} + \ldots\ldots\ldots\ldots\ldots + \binom{2000}{2000}$ equals

(A) $\frac{2^{2001}+1}{3}$ (B) $\frac{2^{2000}-1}{3}$

(C) $\frac{2^{2000}+1}{3}$ (D) $\frac{2^{2001}-1}{3}$

Ans. B

Sol. Let $f(x) = (1+x)^{2000} = \sum_{k=0}^{2000} \binom{2000}{k} x^k$

$f(1) + \omega f(\omega) + \omega^2 f(\omega^2) = 3\left(\binom{2000}{2} + \binom{2000}{5} + \ldots\ldots + \binom{2000}{2000}\right)$

$\therefore 2^{2000} + \omega(1+\omega)^{2000} + \omega^2(1+\omega^2)^{2000}$

$so, 3\left(\binom{2000}{2} + \ldots\ldots\ldots + \binom{2000}{2000}\right)\binom{2000}{2} \ldots\ldots\ldots\ldots\ldots \binom{2000}{2000} = \frac{2^{2000}-1}{3}$

7. The minimum distance between the circle $x^2 + y^2 = 9$ and the curve $2x^2 + 10y^2 + 6xy = 1$ is:

(A) $2\sqrt{2}$ (B) 2

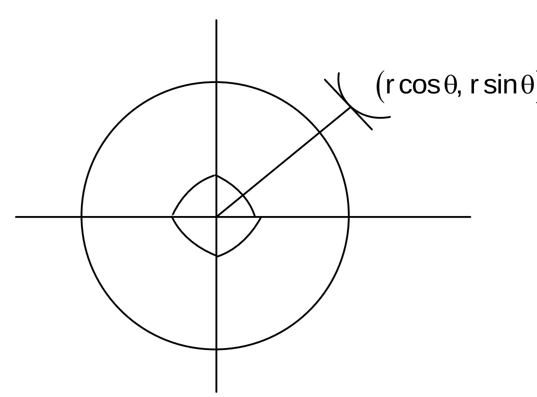
(C) $3-\sqrt{2}$ (D) $3-\frac{1}{\sqrt{11}}$

Ans. B

Sol. $r^2\left(2\cos^2\theta + 10\sin^2\theta + 6\sin\theta\cos\theta = 1\right)$

$r^2 = \frac{1}{3\sin 2\theta - 4\cos 2\theta + 6} \le \frac{1}{6-5} = 1$

Minimum distance between curves
$= 3 - 1 = 2$



8. Let ABC be a triangle. Let A be the point (1, 2), y = x is the perpendicular bisector of AB and $x - 2y + 1 = 0$ is the angle bisector of angle C. If the equation of BC is given by $ax + by - 5 = 0$, then the value of $a + b$ is:

(A) 1 (B) 2

(C) 3 (D) 4

Ans. B

Sol. Image of A say A' w.r.t $x-2y+1=0$ lies on BC.

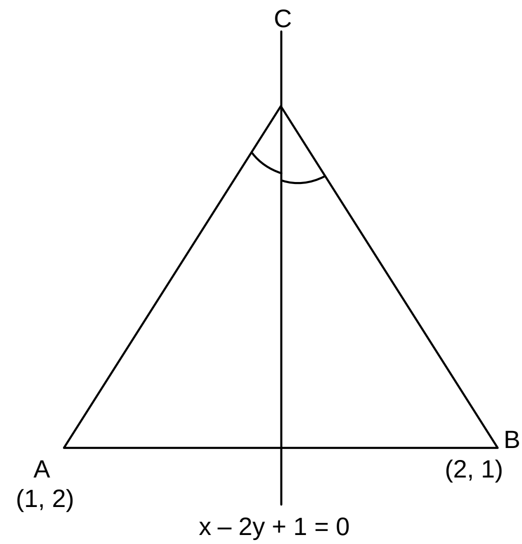
$\frac{x-1}{1}=\frac{y-2}{-2}=-2\frac{(1-4+1)}{1+2^2}=\frac{4}{5}$

$A'=\left(\frac{9}{5},\frac{2}{5}\right)$

Equation of BC joining $A'\left(\frac{9}{5},\frac{2}{5}\right)$ and B

(2, 1) is $y-1=\frac{1-\frac{2}{5}}{2-\frac{9}{5}}(x-2)=\frac{3}{1}(x-2)$

$3x-y-5=0 \Rightarrow a+b=3-1=2$



9. Two tangents drawn to parabola at points (3, 9) and (9, 7) intersect at (2, 2) then slope of directrix is

(A) $\frac{3}{2}$ (B) $-\frac{2}{3}$

(C) $-\frac{5}{2}$ (D) $\frac{2}{5}$

Ans. B

Sol. Use the property : "The line segment joining the point of intersection tangents drawn at the points A and B on the parabola to the midpoint of the corresponding chord of contact is always parallel to the axis of the parabola".

Here, If $A=(3,9)$, $B=(9,7)$ and $C=(2,2)$ and also let M $(6,8)$ is the mid point of AB.

Then slope of CM = $\frac{8-2}{6-2}=\frac{6}{4}=\frac{3}{2}$

$\therefore$ Slope of directrix = $\frac{-2}{3}$

10. A pair of dice is rolled till a sum of either 5 or 7 is obtained. Then the probability that 5 come before 7 is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

Ans. B

Sol. A = getting five P(A) = 1/4
B = getting 7 P(B) = 1/6

n (AUB) = 10

Prob. that sum of numbers is 5 or 7 $= \frac{10}{36} = \frac{5}{18}$

$\therefore$ Prob. That sum is neither 5 or 7 $= \frac{13}{18}$

$\therefore$ Required prob. $\frac{1}{9} + \frac{13}{18} \times \frac{1}{9} + \frac{13}{18} \times \frac{13}{18} \times \frac{7}{9} + ...\infty = \frac{2}{5}$

11. Let $f:[0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is

(A) 1 (B) 2

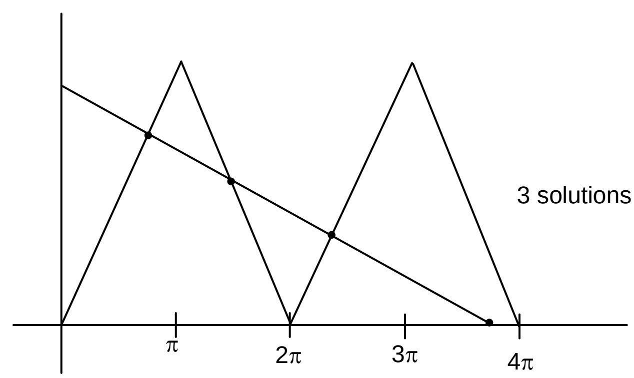
(C) 3 (D) 4

Ans. C

Sol. $[0, 4\pi] \to [0, \pi]$

$f(x) = \cos^{-1} \cos x$

$f(x) = \frac{10-x}{10} = 1 - \frac{x}{10}$



12. If y_1 and y_2 are two different solutions of the equation $\frac{dy}{dx} + P(x)y = Q(x)$ and y = $3\alpha y_1 + 2\beta y_2$ is also the solution of the equation, (where $\alpha - \beta = 4$), then which of the following is not correct. ($\alpha, \beta \in R$)

(A) $4\alpha + \beta = 5$ (B) $\alpha + 4\beta = 7$

(C) $5\alpha + 5\beta + 2 = 0$ (D) $5\alpha + 10\beta + 13 = 0$

Ans. B

Sol. $\frac{dy_1}{dx} + P(x)y_1 = Q(x)$, $\frac{dy_2}{dx} + P(x)y_2 = Q(x)$

$\frac{d(3\alpha y_1 + \beta y_2)}{dx} + P(x)(3\alpha y_1 + 2\beta y_2) = Qx$

$3\alpha Q(x) + 2\beta(Q(x)) = Q(x) \Rightarrow 3\alpha + 2\beta = 1$ and $\alpha - \beta = 4$

$\Rightarrow \alpha = \frac{9}{5}$, $\beta = \frac{-11}{5}$

13. If roots of quadratic equation $(x-3)(x-p)=7$ have integral root then sum of all possible integral values of p is

(A) 6 (B) 9

(C) 7 (D) none of these

Ans. A

Sol. Consider the following case

$x-3$	$x-p$
1	7
7	1
-1	-7
-7	-1

14. If in a triangle ABC, $\cos A\cos B+\sin A\sin B\sin C=1$, then the sides are proportional to

(A) $1:1:\sqrt{2}$ (B) $1:\sqrt{2}:1$

(C) $\sqrt{2}:1:1$ (D) none of these

Ans. A

Sol. $\cos A\cos B+\sin A\sin B\sin C=1$

$\sin C=\frac{1-\cos A\cos B}{\sin A\sin B}$ .....(i)

$\Rightarrow$ $1\le\cos A\cos B+\sin A\sin B$

or $\cos(A-B)\ge 1$

$\therefore$ $\cos(A-B)=1$

$\Rightarrow$ $A=B$

From Eq. (i)

$\sin C=\frac{1-\cos^2}{\sin^2 A}$

= 1

$\therefore$ $\angle C=90^\circ$

$\therefore$ $\angle A=\angle B=45^\circ$

$a:b:c=\sin A:\sin B:\sin C$

$=\frac{1}{\sqrt{2}}:\frac{1}{\sqrt{2}}:1$

or $a:b:c=1:1:\sqrt{2}$

15. Let r, s and t be the roots of the equation, $8x^3+1001x+2008=0$. The value of $(r+s)^3+(s+t)^3+(t+r)^3$ is:

(A) 251 (B) 751

(C) 735 (D) 753

Ans. D

Sol. $8x^3 + 1001x + 2008 = 0$

$r + s + t = 0,\ rst = -\frac{2008}{8} = -251$

So, $(r+s)^3 + (s+t)^3 + (t+r)^3$

$= -\left(t^3 + s^3 + r^3\right) = -3rst = -3(-251)$

= 753

As $r + s + t = 0$, so $r^3 + s^3 + t^3 = 3rst$

16. If $y = \tan^{-1}\left(\frac{\log_e\left(\frac{e}{x^2}\right)}{\log_e\left(ex^2\right)}\right) + \tan^{-1}\left(\frac{3 + 2\log_e x}{1 - 6\log_e x}\right)$, then $\frac{d^2y}{dx^2}$ is

(A) 2 (B) 1

(C) 0 (D) –1

Ans. C

Sol. $y = \tan^{-1}\left(\frac{\log_e\left(\frac{e}{x^2}\right)}{\log_e\left(ex^2\right)}\right) + \tan^{-1}\left(\frac{3 + 2\log_e x}{1 - 6\log_e x}\right)$

$= \tan^{-1}\left(\frac{1 - 2\ \log_e x}{1 + 2\ \log_e x}\right) + \tan^{-1}\left(\frac{3 + 2\ \log_e x}{1 - 3.2\log_e x}\right)$

$= \tan^{-1}(1) - \tan^{-1}(2\ \log_e x) + \tan^{-1}(3) + \tan^{-1}(2\log_e x)$

$= \tan^{-1}(1) + \tan^{-1}(3)$

$\therefore \quad \frac{dy}{dx} = 0$

17. The number of distinct solutions of the equation

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0,\ 2\pi]$ is

(A) 5 (B) 2

(C) 6 (D) 8

Ans. D

Sol. On simplification, w get $\cos 4x = 0 \Rightarrow 4x = 2n\pi \pm \frac{\pi}{2} \Rightarrow x = \frac{n\pi}{2} \pm \frac{\pi}{8}$

Number of distinct solution on $[0,\ 2\pi]$ is 8.

18. Let a, b, c are complex numbers and roots of $z^3 + az^2 + bz + c = 0$ are unimodular then $|a| - |b|$ equal to

(A) 0
(B) 1
(C) –1
(D) 2

Ans. A

Sol. $|\alpha + \beta + \gamma| = |a|$

$|\alpha\beta + \beta\gamma + \gamma \propto| = |b|$

$|\alpha\beta\gamma|\left|\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right| = |b|$

$|\bar{\alpha} + \bar{\beta} + \bar{\gamma}| = |b| = |a|$

19. Let $k = \lim_{n\to\infty} n^2 \int_{-\frac{1}{n}}^{\frac{1}{n}} (2019 \sin x + 2020 \cos x)|x| dx$. The value of k is equal to

(A) 2021
(B) 2022
(C) 2020
(D) 2019

Ans. C

Sol. $\int_{-\frac{1}{n}}^{\frac{1}{n}} (2019 \sin x + 2020 \cos x)|x| dx = 4040\left(\frac{1}{n}\sin\frac{1}{n} + \cos\frac{1}{n} - 1\right)$

$= 4040\left(\frac{1}{n}\sin\frac{1}{n} - 2\sin^2\frac{1}{2n}\right)$

Hence, $k = 4040 \lim_{n\to\infty}\left(\frac{\sin\frac{1}{n}}{\frac{1}{n}} - \frac{\sin^2\frac{1}{2n}}{2\left(\frac{1}{2n}\right)}\right) = 2020$

20. If $x = \sum_{r=1}^{90} r \sin(2r)^\circ$, then the value of x is equal to

(A) $90 \cot 1^\circ \operatorname{cosec} 1^\circ$
(B) $90 \sec 1^\circ$
(C) $90 \cot 1^\circ$
(D) none of these

Ans. C

Sol. $S = 1\sin 2^\circ + 2\sin 4^\circ + 3\sin 6^\circ + ... + 89\sin 178^\circ$

$S = 89\sin 178^\circ + 88\sin 176^\circ + 87\sin 174^\circ + ... + 1\sin 2^\circ$

Adding the two, we get

$S = 90(\sin 2^\circ + \sin 4^\circ + ... + \sin 178^\circ) = 90\left(\frac{\sin 89^\circ}{\sin 1^\circ}\right)\sin 90^\circ = 90\cot 1^\circ$

PHYSICS

21. A thin prism P_1 with angle 4^o and made from glass (μ = 1.54) is combined with another prism P_2 made of another glass of μ = 1.72 to produce dispersion without deviation. The angle of prism P_2 is

(A) 53.3^o (B) 4^o
(C) 3^o (D) 2.6^o

Ans. C

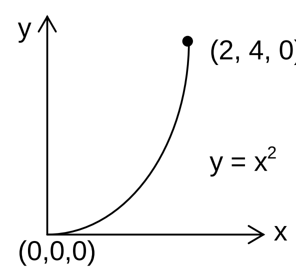
Sol. For no deviation

$(\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$

$4^o(1.54 - 1) = (1.72 - 1)A_2$

$A_2 = \frac{4 \times 0.54}{0.72} = 3°.$

22. By applying a force $\vec{F} = (3xy - 5z)\hat{j} + 4z\hat{k}$ a particle is moved along the path $y = x^2$ from point (0, 0, 0) to (2, 4, 0). The work done by the force F on the particle is



(A) $\frac{280}{5}$ unit (B) $\frac{140}{5}$ unit
(C) $\frac{232}{5}$ unit (D) $\frac{192}{5}$ unit

Ans. D

Sol. $w = \int \vec{F}.d\vec{r}$

$= \int (3xy - 5z)dy$; $\int_0^4 3\sqrt{y}\ ydy = \frac{192}{5}$

23. A boat which has a speed of 6 km/h in still water crosses a river of width 1 km along the shortest possible path in 20 min. The velocity of the river water in km/h is

(A) 1 (B) 3
(C) 4 (D) $3\sqrt{3}$

Ans. D

Sol. t = 20 min = $\frac{1}{3}$hr

1 km = $\sqrt{(6)^2 - V^2} \times \left(\frac{1}{3}\text{hr}\right)$

V = $3\sqrt{3}$ km/hr.

24. The Young's modulus of a wire of length L and radius r is Y newton per square metre. If the length is reduced to L/2 and radius to r/2, its Young's modulus will be

(A) Y/2 (B) Y

(C) 2Y (D) 4Y

Ans. B

Sol. Young's modulus does not depend on the geometry of the wire but its materials.

25. A copper wire of diameter 1.02 mm carries a current of 1.7 amp. The drift velocity (v_d) of electrons in the wire. Given n, number density of electrons in copper = 8.5×10^{28} /m^3.

(A) 3.0 mm/s (B) 2.0 mm/s

(C) 1.5 mm/s (D) 1 mm/s

Ans. C

Sol. I = 1.7 A

J = current density

$$= \frac{I}{\pi r^2} = \frac{1.7}{\pi \times (0.51 \times 10^{-3})^2}$$

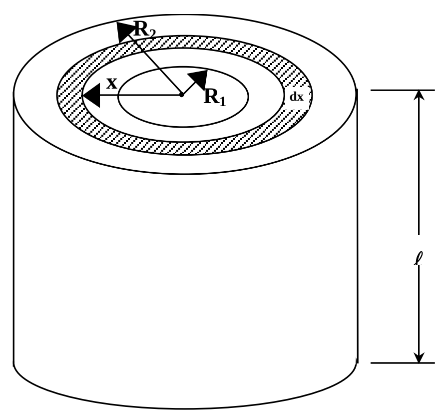
$= nev_d$

$= 8.5 \times 10^{28} \times (1.6 \times 10^{-19}) \times v_d$

$$\therefore v_d = \frac{1.7}{\pi \times (0.51 \times 10^{-3})^2 \times 8.5 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$= 1.5 \times 10^{-3}$ m/sec. = 1.5 mm/sec.

26. A cylindrical conductor of length ℓ and inner radius R_1 and outer radius R_2 has specific resistance ρ. A cell of emf ε is connected across the two lateral faces of the conductor. Find the current drown from the cell.



(A) $\dfrac{2\pi\ell\varepsilon}{\rho \ln \frac{R_2}{R_1}}$

(B) $\dfrac{2\pi\ell\varepsilon}{\rho (R_2^2 - R_1^2)}$

(C) $\dfrac{2\ell\varepsilon}{\rho\pi(R_2^2 - R_1^2)}$

(D) None of thee

Ans. A

Sol. Consider the differential element of the cylinder as shown in the figure.

$$\therefore \quad R = \int_{R_1}^{R_2} \rho \frac{dx}{2\pi x l} \qquad (\because R = \rho \frac{l}{a})$$

$\Rightarrow R = \frac{\rho}{2\pi l} \ln\left(R_2/R_1\right)$

$I = \frac{\varepsilon}{R}$; $\Rightarrow I = \frac{2\pi l \varepsilon}{\rho \ln\left(\frac{R_2}{R_1}\right)}$

27. Two capillary tubes of radii 0.2 cm and 0.4 cm are dipped in the same liquid. The ratio of heights through which liquid will rise in the tube is

(A) 1:2
(B) 2:1
(C) 1:4
(D) 4:1

Ans. B

Sol. capillary rise

$h = \frac{2T\cos(\theta)}{r\rho g}$

$\therefore \frac{h_1}{h_2} = \frac{r_2}{r_1} = \frac{0.4}{0.2} = 2:1$

28. A particle executes S.H.M. with a time period of 4 s. Find the time taken by the particle to go directly from its mean position to half of its amplitude.

(A) 1 s
(B) $\frac{1}{2}$ s
(C) $\frac{1}{3}$ S
(D) $\frac{1}{4}$ S

Ans. C

Sol. $x = A\sin(\omega t + \phi_o)$

At t = 0, x = 0

$\Rightarrow A\sin\phi_o = 0$ or $\phi_o = 0$

Hence, $x = A\sin(\omega t)$

or $A/2 = A\sin(\omega t)$

or $1/2 = \sin(\omega t)$

$\omega t = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$t = \frac{\pi}{6\omega} = \frac{\pi . T}{6(2\pi)}$

as $\omega = 2\pi / T$

$\Rightarrow$ t T/12 = 1/3 s

29. In hydrogen like atoms the ratio of difference of energies $E_{4n} - E_{2n}$ and $E_{2n} - E_n$ varies with atomic number z and principle quantum number n as

(A) $\frac{z^2}{n^2}$ (B) $\frac{z^4}{n^4}$

(C) $\frac{z}{n}$ (D) none of these

Ans. D

Sol. $$\frac{E_{4n} - E_{2n}}{E_{2n} - E_n} = \frac{\frac{E_1}{16n^2} - \frac{E_1}{4n^2}}{\frac{E_1}{4n^2} - \frac{E_1}{n^2}} = \frac{1}{4} = \text{constant}$$

30. A small sphere with radius r and density ρ falls at a speed v through a fluid of density σ and coefficient of viscosity η. The dimensions of 'η' are $ML^{-1}T^{-1}$. If the friction force F opposing the motion is proportional to V, it could also be proportional to

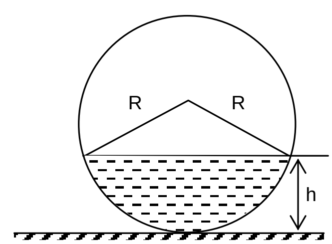
(A) $\frac{\eta}{\rho}$ but independent of r (B) ηr but independent of ρ

(C) $\eta\rho^2$ but independent of ρ (D) ρr but independent of η

Ans. B

Sol. $\vec{F} = -6x\eta r\vec{v}$

31. A liquid is filled in a spherical container of radius R till a height h. At this position the liquid surface at the edges is also horizontal. The contact angle is

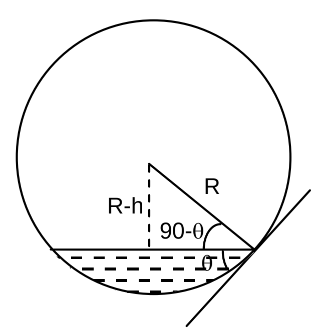


(A) 0 (B) $\cos^{-1}\left(\frac{R-h}{R}\right)$

(C) $\cos^{-1}\left(\frac{h-R}{R}\right)$ (D) $\sin^{-1}\left(\frac{R-h}{R}\right)$

Ans. B

Sol. $\sin(90-\theta) = \frac{R-h}{R}$



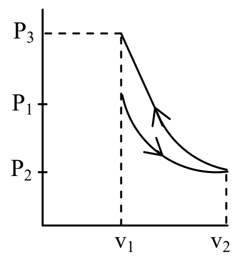
32. An ideal gas initially at P_1, V_1 is expanded isothermally to P_2, V_2 and then compressed adiabatically to the same volume V_1 and pressure P_3. If W is the net work done by the gas in complete process, which of the following is true

(A) $W > 0$; $P_3 > P_1$ (B) $W < 0$; $P_3 > P_1$

(C) $W > 0$; $P_3 < P_1$ (D) $W < 0$; $P_3 < P_1$

Ans. B

Sol.



Area under curve is work done

33. A train of mass M is moving on a circular track of radius R with constant speed v. The length of train is half the perimeter of track. The magnitude of linear momentum of the train will be

(A) 0 (B) $2Mv/\pi$

(C) MvR (D) Mv

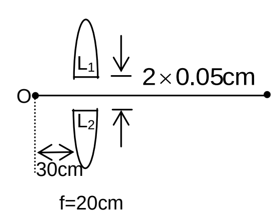
Ans. B

Sol. Total momentum = $\int_{\theta=-90}^{+90} (dm)v\cos\theta$

$dm = \frac{M}{\pi R} \times Rd\theta$

Total momentum = $\int \frac{M}{\pi R} vRd\theta \cos\theta = \frac{2Mv}{\pi}$

34. A point object O is placed at a distance of 0.3 m from a convex lens (focal length 0.2 m) cut into two halves each of which is displaced by 0.0005 m as shown in figure. Find the distance between the images (in mm)



(A) 1 (B) 2

(C) 3 (D) 4

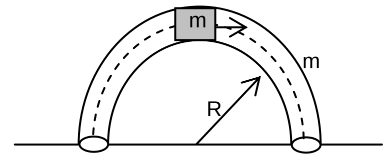
Ans. C

Sol. V = 0.6 m

$\Rightarrow \frac{h_I}{h_o} = \frac{h_I}{0.05} = 2 \Rightarrow h_I = 0.1$ cm.

$\Rightarrow$ Distance between them = 0.3 cm = 3 mm.

35. In a vertical plane inside a smooth hollow thin tube, a block of same mass as that of tube is released as shown. When it is slightly disturbed it moves towards right. By the time the block reaches the right end of the tube, the displacement of the tube will be (where 'R' is the mean radius of tube the assume that the tube remains in vertical plane) towards left



(A) $\frac{2R}{\pi}$ (B) $\frac{4R}{\pi}$

(C) $\frac{R}{2}$ (D) R

Ans. C

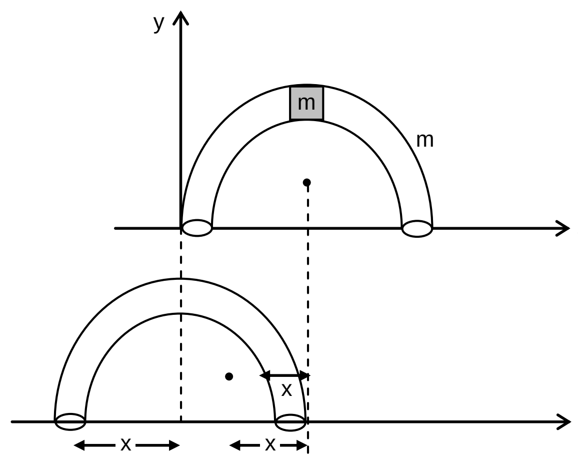
Sol. Initial position of block

$x_{cm} = R$

Final position of block

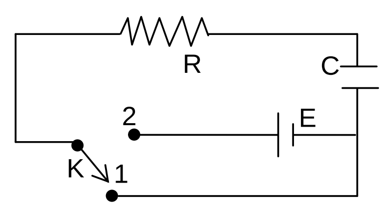
$$x_{cm} = \frac{m(R-x)+m(2R-x)}{2m}$$

Now, $R = \frac{m(R-x)+m(2R-x)}{2m}$



36. In the shown circuit involving a resistor of resistance R Ω, capacitor of capacitance C farad and an ideal cell of emf E volt, the capacitor is initially uncharged and the key is in position 1. At t = 0 second the key is pushed to position 2 for t_0 = RC seconds and then key is pushed back to position 1 for t_0 = RC seconds. This process is repeated again and again. Assume the time taken to push key from position 1 to 2 and vice versa to be negligible.

The charge on capacitor at t = 2RC second is



(A) CE (B) $CE\left(1-\frac{1}{e}\right)$

(C) $CE\left(\frac{1}{e}-\frac{1}{e^2}\right)$ (D) $CE\left(1-\frac{1}{e}+\frac{1}{e^2}\right)$

Ans. C

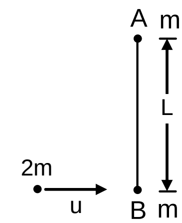
Sol. For charging $q = CE(1-e^{-t/RC})$

Charge at t = RC $\Rightarrow$ $q_o = CE(1 - e^{-1})$

At t = RC discharging starts

$\Rightarrow$ $q = q_o(e^{-t/RC}) = CE(1 - e^{-1}) \times \frac{1}{e} = CE\left(\frac{1}{e}-\frac{1}{e^2}\right)$

37. Two small balls A and B, each of mass m, are joined rigidly at the ends of a light rod of length L. They are placed on a frictionless horizontal surface. Another ball of mass 2 m moving with speed u towards one of the ball and perpendicular to the length of the rod on the horizontal frictionless surface as shown in the figure. If the coefficient of restitution is 1/2 then the angular speed of the rod after the collision will be



(A) $\frac{4}{3}\frac{u}{\ell}$ (B) $\frac{u}{\ell}$

(C) $\frac{2}{3}\frac{u}{\ell}$ (D) None of these

Ans. C

Sol. Velocity of ball ‘A’ just after the collision $V=\left(1+\frac{1}{2}\right)\cdot\frac{2}{3}u=u$

Velocity of ball ‘B’ just after the collision = 0.

$\therefore \quad \omega=\frac{u-0}{\ell}=\frac{u}{\ell}$

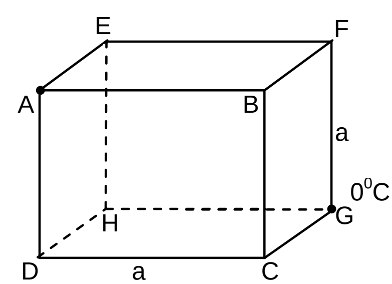
38. A cubical frame is made by connecting 12 identical uniform conducting rods as shown in the figure. In the steady state the temperature of junction A is 100^0 C while that of the G is 0^0C. Then,

(A) H will be Hotter than B

(B) Temperature of F is 40^0C

(C) Temperature of D is 66.67^0C

(D) Temperature of E is 50^0C



Ans. B

Sol. $100°C - Ir - \frac{I}{2}r - Ir = 0°C$

$\Rightarrow \quad Ir = 40°C \quad ; \quad \therefore \; t_F = 0° + Ir = 40°C$

39. An open organ pipe of length L vibrates in second harmonic mode. The pressure vibration is maximum

(A) at the two ends

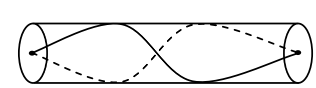
(B) at a distance L/4 from either end inside the tube

(C) at the mid-point of the tube

(D) none of these

Ans. B

Sol. Pressure node is formed at the both the ends and in the middle.



40. In order to shift a body of mass m from a circular orbit of radius 2R to a higher orbit of radius 4R around the earth, (Where R is radius and M is mass of earth) the work done on the body is

(A) $\frac{GMm}{8R}$ (B) $\frac{GMm}{2R}$

(C) $\frac{GMm}{4R}$ (D) $\frac{GMm}{R}$

Ans. A

Sol. $W = -\frac{GMm}{2(4R)} - \left[-\frac{GMm}{2(2R)}\right] = \frac{GMm}{8R}$

CHEMISTRY

41. Molecule AB has a bond length if 1.61 Å and a dipole moment of 0.380 D. The fractional charge on each atom (absolute magnitude) is ($e_0 = 4.802 \times 10^{-10}$ esu)

(A) 0.5 (B) 0.05

(C) 0 (D) 1.0

Ans. B

Sol. $\mu = q \times d$

$0.380 = q \times 1.61 \times 10^{-10} \times 3.33 \times 10^{-30}$

$q = 0.786 \times 10^{-20}$

Fraction charge $\Rightarrow \frac{0.786 \times 10^{-20}}{1.6 \times 10^{-19}}$

$\Rightarrow 0.5 \times 10^{-1}$

$\Rightarrow 0.05$

42. Electromagnetic radiations having $\lambda = 310\text{Å}$ are subjected to a metal sheet having work function = 12.8 eV. What will be the velocity of photo electrons with maximum kinetic energy

(A) 0, no emission will occurs (B) 2.18×10^6 m/s

(C) $2.18\sqrt{2} \times 10^6$ m/s (D) 8.72×10^6 m/s

Ans. C

Sol. $E = \frac{12500}{310} \Rightarrow 40.32$

$27.3 \times 1.6 \times 10^{-19} = \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2$

$v^2 = 6 \times 1.6 \times 10^{10}$

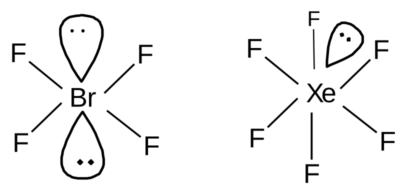
$\Rightarrow 9.6 \times 10^{12} \Rightarrow 2.18\sqrt{2} \times 10^6$ m/s

43. The number of lone pair on central atom in $[BrF_4]^-$, XeF_6 and SF_4 are:

(A) 2, 0, 1 (B) 1, 0, 0
(C) 2, 1, 1 (D) 2, 1, 0

Ans. C

Sol.

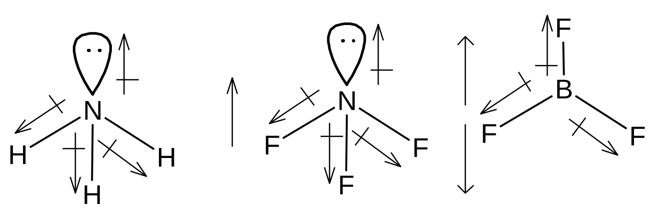


44. Which of the following arrangements of molecules is correct on the basis of their dipole moments?

(A) $BF_3 > NF_3 > NH_3$ (B) $NF_3 > BF_3 > NH_3$
(C) $NH_3 > BF_3 > NF_3$ (D) $NH_3 > NF_3 > BF_3$

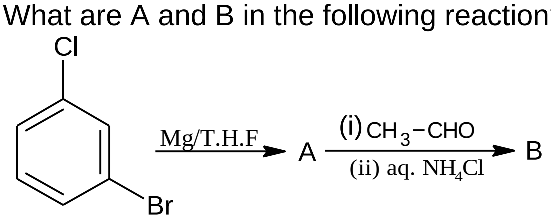
Ans. D

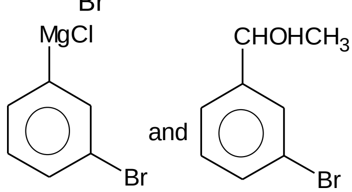
Sol.

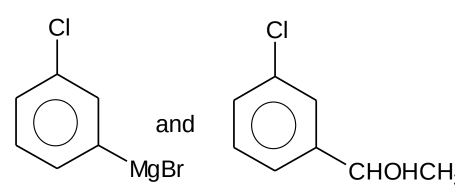


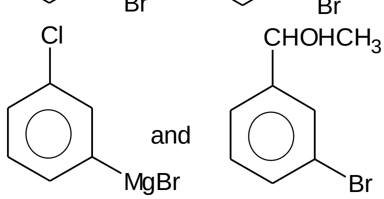
$\mu = 0$

45. What are A and B in the following reaction?



(A) 

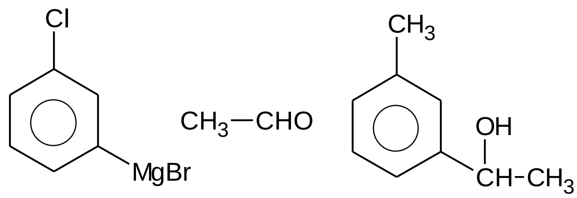
(B) 

(C) 

(D) None of these

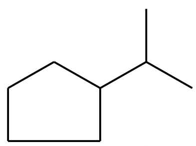
Ans. B

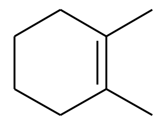
Sol.

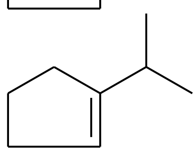


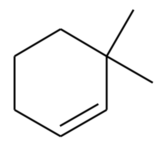
46. $C_5H_{10} \xrightarrow{Br_2/hv} C_5H_9Br \xrightarrow{Mg/Ether} \xrightarrow[(ii)\ H^+]{(i)\ Acetone} \xrightarrow[\Delta]{H_2SO_4}$ P . P is

(only one isomer)

(A) 

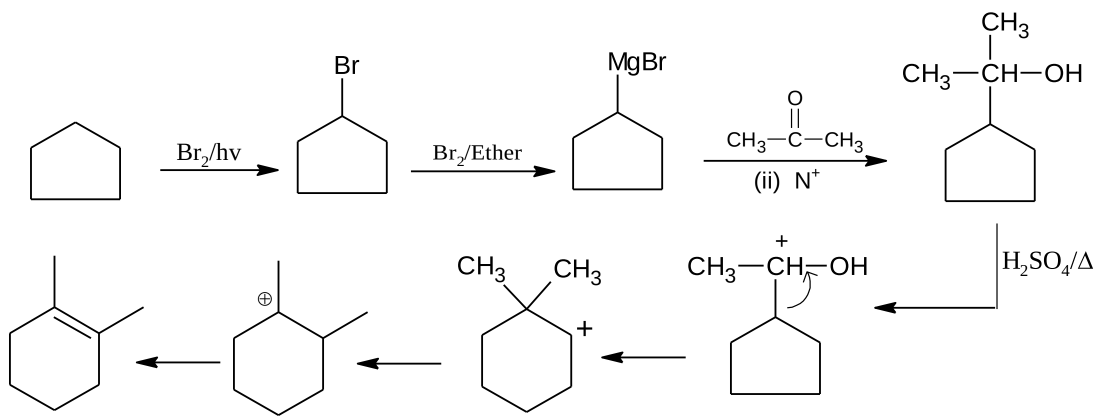
(B) 

(C) 

(D) 

Ans. B

Sol.



47. Which of the following compound is not formed at all?

(A) IF_3 (B) $BrCl_3$

(C) FCl_3 (D) ClF_3

Ans. C

Sol. Fluorine combines through one bond only.

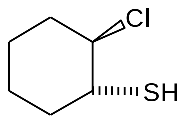
48. Which of the following species exhibit the diamagnetic behaviour?

(A) NO (B) O_2^{2-}

(C) O_2^+ (D) O_2

Ans. B

Sol. Refer MOT

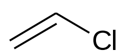
49.  $\xrightarrow{\text{Acetone/NaSH}}$ Reaction Mechanism

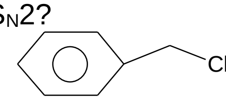
(A) Only one product will form
(B) Two products are formed and they will be enantiomer of each other
(C) Mechanism of reaction is SN^2
(D) Mechanism of the reaction is SN^1

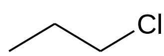
Ans. B

Sol. Based on SNi mechanism.

50. Which of the following is most reactive towards S_N2?

(A) 

(B) 

(C) 

(D) $Ph-\overset{O}{\overset{\|}{C}}-CH_2-Cl$

Ans. D

Sol. e^- w.g. stabilize T.S. so answer D.

51. To neutralise completely 20 mL of 0.1 M aqueous solution of phosphorous acid (H_3PO_3), the volume of 0.1 M aqueous KOH solution required is
(A) 60 mL
(B) 20 mL
(C) 40 mL
(D) 10 mL

Ans. C

Sol. milli equivalent of H_3PO_3 = milli equivalent of KOH

$2 \times 20 \times 0.1 = V \times 0.1$

$V = 40\,ml$

52. A mixture of N_2 and H_2 is caused to react in a closed container to form NH_3. The reaction ceases before reactant has totally consumed. At this stage, 2.0 moles each of N_2, H_2 and NH_3 are present. The moles of N_2 and H_2 present originally were respectively.
(A) 4 and 4 moles
(B) 3 and 5 moles
(C) 3 and 4 moles
(D) 4 and 5 moles

Ans. B

Sol.
$$\underset{x}{N_2} + \underset{y}{3H_2} \longrightarrow 2NH_3$$
$$x-z \quad y-3z \quad 2z$$
$$2 \quad 2 \quad 2$$

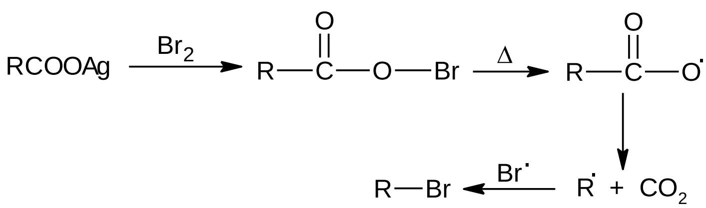
$2Z = 2, \quad x - Z = 2, \quad y - 3Z = 2$

$Z = 1 \quad x = 3 \quad y = 5$

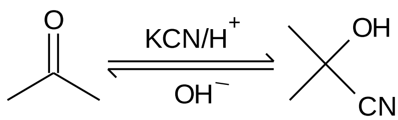
53. Which is incorrect about Hunsdiecker reaction?
(A) Cl_2 can give alkyl halide
(B) I_2 will give ester as major product when treated with RCOOAg
(C) The reaction proceeds through free radical Intermediate
(D) The R-COO$^{\oplus}$ is formed during reaction

Ans. D

Sol.



54.

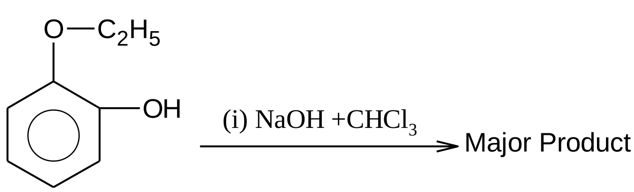


(A) Forward reaction is nucleophilic addition and backward reaction is E_1
(B) Forward reaction is nucleophilic addition and backward reaction is E_1CB
(C) Forward reaction is nucleophilic addition and backward reaction is E_2
(D) Forward reaction is nucleophilic addition and backward reaction is S_N1

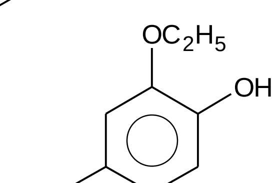
Ans. B

Sol. Anion is formed so answer B.

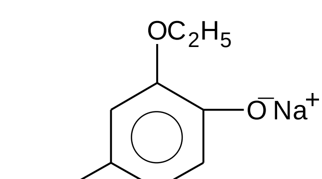
55.



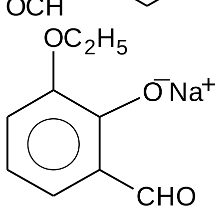
(A)



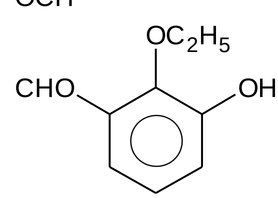
(B)



(C)

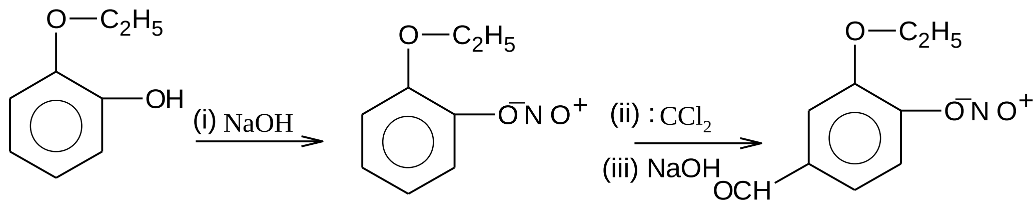


(D)



Ans. B

Sol.



O^- having more +m effect more – O – C_2H_5 .

56. 100 ml of tap water containing $Ca(HCO_3)_2$ was titrated with $N/50\,HCl$ with methyl orange as indicator. It 30 ml of HCl were required, what is the degree of temporary hardness as ppm.

(A) 300 ppm
(B) 500 pm
(C) 30 ppm
(D) 50 ppm

Ans. A

Sol. mili equivalent of HCl = $Ca(HCO_3)_2 = CaCO_3 = \frac{30}{50}\times 10^{-3}$

Mass of $CaCO_3 = \frac{\text{mili eq.}\times \text{molar mass}}{nf}$

$= \frac{30}{50}\times\frac{10^{-3}\times 100}{2} = 0.03$

Hardness in ppm = $\frac{0.03\times 10^{6}}{100} = 300\,ppm$

57. 2g of aluminium is treated separately with excess of dilute H_2SO_4 and excess of NaOH solution. The ratio of volumes of hydrogen evolved under similar conditions is

(A) 1: 2
(B) 1: 1
(C) 2: 1
(D) 2: 3

Ans. B

Sol. $2A\ell + 3H_2SO_4 \longrightarrow A\ell_2(SO_4)_3 + 3H_2$

$2A\ell + 2NaOH + 2H_2O \longrightarrow 2NaA\ell O_2 + 3H_2$

58. The difference in the wavelength of the 1st line of Lyman series and 2nd line of Balmer series in a hydrogen atom is

(A) $\frac{9}{2R}$
(B) $\frac{4}{R}$
(C) $\frac{88}{15R}$
(D) None of these

Ans. B

Sol. 1st line in Lyman, $2 \longrightarrow 1$

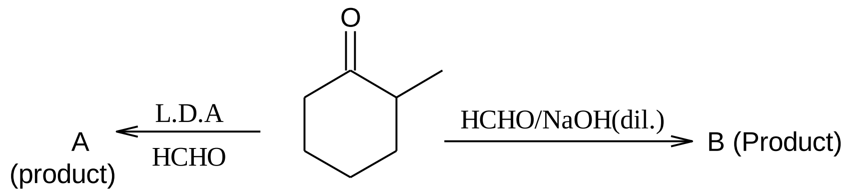
$\frac{1}{\lambda_1} = \bar{\upsilon} = R\left(\frac{1}{12} - \frac{1}{22}\right) = \frac{3R}{4}$

IInd line in Balmer $4 \longrightarrow R$

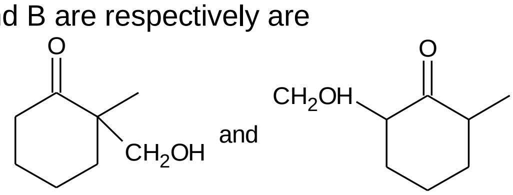
$$\frac{1}{\lambda_2} = \bar{\upsilon} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3}{16}R$$

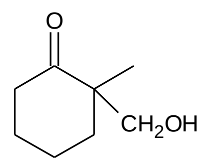
Difference $\lambda_1 - \lambda_2 = \frac{4}{3R} - \frac{16}{3R} = \frac{4}{R}$

59.

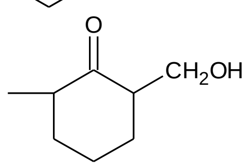


A and B are respectively are

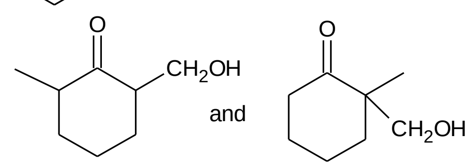
(A) 

(B) 

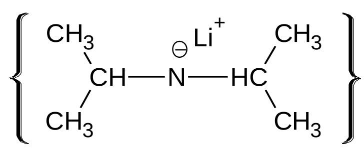
Both

(C) 

Both

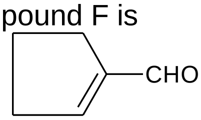
(D) 

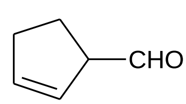
Ans. A

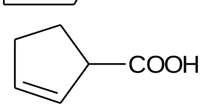
Sol. L.D.A 

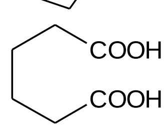
is Buky Base then NaOH.

60. Cyclohexene on ozonolysis followed by reaction will zinc dust and water gives compound E. Compound E on further treatment with aqueous KOH yield compound F. Compound F is

(A) 

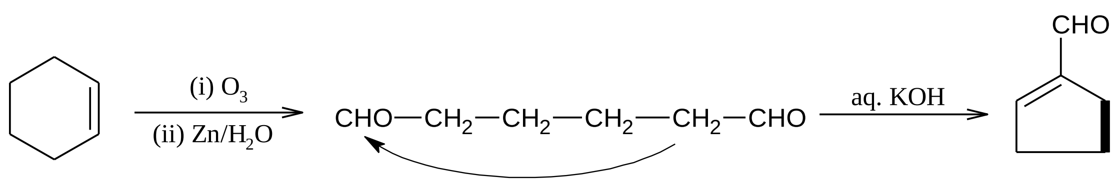
(B) 

(C) 

(D) 

Ans. A

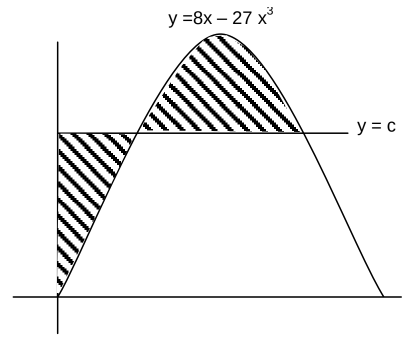
Sol.



PART – II

MATHEMATICS

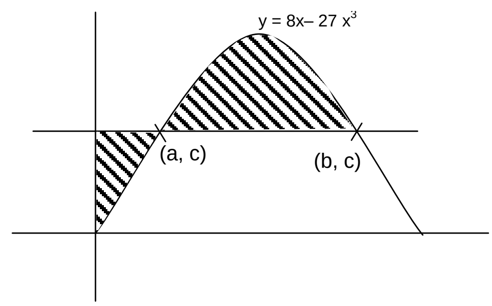
61. The value of c such that areas of shaded region are equal is



(A) $\frac{32}{27}$ (B) $\frac{23}{27}$

(C) $\frac{34}{27}$ (D) $\frac{25}{274}$

Ans. A



Sol. $\int_0^a \left(c-\left(8x-27x^3\right)\right)dx = \int_a^b \left(\left(8x-27x^3\right)-c\right)dx$

$O = 4b^2 - 27b^4 - bc$

$O = 4b^2 - \frac{27}{4} - b\left(8b-27b^3\right)$

$b^4\left(\frac{81}{4}b^2 - 4\right) = 0$

$b > 0 \;\; b^2 = \frac{4^2}{81} \qquad b = \frac{4}{81}$

$C = 8b - 27b^3$

$= \frac{32}{27}$

62. Value of Definite integration $\int_0^1 \frac{2x-\left(1+x^2\right)^2\cot^{-1}x}{\left(1+x^2\right)\left(1-\left(1+x^2\right)\cot^{-1}x\right)}dx$ is

(A) 1 +ln 2 (B) 3+ln 2

(C) 2+ ln 2 (D) 4+ln 2

Ans. A

Sol. $I = \int_0^1 \frac{\frac{2x}{\left(1+x^2\right)^2} - \cot^{-1}x}{\frac{1}{1+x^2} - \cot^{-1}x}dx$

$= \int_0^1 \frac{\frac{2x}{\left(1+x^2\right)^2} - \frac{1}{1+x^2} + \frac{1}{1+x^2} - \cot^{-1}x}{\frac{1}{1+x^2} - \cot^{-1}x}dx$

$$= -\int_0^1 \frac{\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2}}{\frac{1}{1+x^2} - \cot^{-1}x} dx + \int_0^1 1dx$$

$$= -\log_e \left(\frac{1}{1+x^2} - \cot^{-1}x \right) \Big|_0^1 + 1$$

$$= -\log_e \left(\frac{1}{2} - \frac{\pi}{4} \right) + \log_e \left(1 - \frac{\pi}{2} \right) + 1$$

$$= \log_e \left(\frac{1 - \frac{\pi}{2}}{\frac{1}{2} - \frac{\pi}{4}} \right) + 1$$

$$= 1 + ln\,2$$

63. In a ΔABC the maximum value of $\frac{\sum a\cos^2\left(\frac{A}{2}\right)}{a+b+c}$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{3}{4}$ (D) 1

Ans. C

Sol. $\frac{a(1+\cos A) + b(1+\cos B) + c(1+\cos C)}{2(a+b+c)}$

$$= \frac{1}{2} + \frac{R}{4s}(\sin 2A + \sin 2B + \sin 2C)$$

$$= \frac{1}{2} + \frac{R}{4s}(4\,sin\ A\ sin\ B\ sin\ C)$$

$$= \frac{1}{2} + \frac{R}{4s}\left(4\frac{abc}{8R^3}\right) = \frac{1}{2} + \frac{abc}{8R^2s}$$

$$= \frac{1}{2} + \frac{4R\Delta}{8R^2s} = \frac{1}{2} + \frac{r}{2R} = \frac{1}{2}\left(1+\frac{r}{R}\right) \le \frac{1}{2}\left(1+\frac{1}{2}\right) \le \frac{3}{4}$$

64. If the line $x + y - 1 = 0$ is a tangent to a parabola with focus (1, 2) at A and intersects the directrix at B and tangent at vertex at C respectively, then AC. BC is equal to:

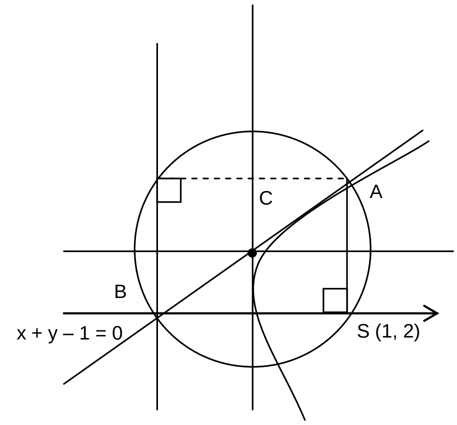
(A) 2 (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. A

Sol. Using power of C

$(BC)(AC) = (CS)^2 = \left(\frac{1+2-1}{\sqrt{2}}\right)^2 = 2$



65. For $x > 0$, let $A = \begin{bmatrix} \frac{x^2+1}{x} & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 16 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{5x}{x^2+1} & 0 & 0 \\ 0 & \frac{3}{x} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ be two matrices.

Three other matrices X, Y, and Z are defined as

$X = (AB)^{-1} + (AB)^{-2} + (AB)^{-3} + ...(AB)^{-n}$, $Y = \lim_{n\to\infty} X$ and $Z = Y^{-1} - 2I$,

Then the value of $\det.\left(\text{adj}\sqrt{5}\,Y^{-1}\right)$ is equal to

(A) $(5!)^2$

(B) $5^3(5!)^2$

(C) $5(5!)^2$

(D) $5^2(5!)^2$

Ans. C

Sol. $AB = \begin{bmatrix} \frac{x^2+1}{x} & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 16 \end{bmatrix} \begin{bmatrix} \frac{5x}{x^2+1} & 0 & 0 \\ 0 & \frac{3}{x} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$(AB)^{-1} \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}, (AB)^{-2} = \begin{bmatrix} \frac{1}{5^2} & 0 & 0 \\ 0 & \frac{1}{3^2} & 0 \\ 0 & 0 & \frac{1}{4^2} \end{bmatrix}$ and so on

$$x=\begin{bmatrix}\frac{1}{5}+\frac{1}{5^2}+.....\frac{1}{5^n} & 0 & 0\\ 0 & \frac{1}{3}+\frac{1}{3^2}+.....\frac{1}{3^n} & 0\\ 0 & 0 & \frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+.....+\frac{1}{4^n}\end{bmatrix}$$

$$Y=\lim_{n\to\infty}\begin{bmatrix}\frac{\frac{1}{5}}{1-\frac{1}{5}} & 0 & 0\\ 0 & \frac{\frac{1}{3}}{1-\frac{1}{3}} & 0\\ 0 & 0 & \frac{\frac{1}{4}}{1-\frac{1}{4}}\end{bmatrix}\Rightarrow Y=\begin{bmatrix}\frac{1}{4} & 0 & 0\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{3}\end{bmatrix}$$

$$\therefore\ Y^{-1}=\begin{bmatrix}4 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$$

66. Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in R$. If the roots of the equation $x^2+2(a+b+c)x+3\lambda(ab+bc+ca)=0$ are real and distinct, then

(A) $\lambda<\frac{4}{3}$

(B) $\lambda>\frac{5}{3}$

(C) $\lambda\in\left(\frac{1}{3},\frac{1}{5}\right)$

(D) $\lambda\in\left(\frac{4}{3},\frac{5}{3}\right)$

Ans. A

Sol. $\because$ Roots are real and distinct.

$\therefore \quad \Delta>0$

$\Rightarrow \quad 4(a+b+c)^2-12\lambda(ab+bc+ca)>0$

$\Rightarrow \quad \left(a^2+b^2+c^2\right)+2(ab+bc+ca)-3\lambda(ab+bc+ca)>0$

$\Rightarrow \quad \frac{\sum a^2}{\sum ab}>(3\lambda-2) \quad ....(i)$

Now, in a triangle

Difference of two sides < third side

i.e. $\quad |a-b|<c,\ |b-c|<a$ and $|c-a|<b$

$\Rightarrow \quad (a-b)^2+(b-c)^2+(c-a)^2<a^2+b^2+c^2$

$\Rightarrow \quad a^2+b^2+c^2<2(ab+bc+ca)$

or $\dfrac{\sum a^2}{\sum ab} < 2$ ....(ii)

From Eqs. (i) and (ii), we get

$$3\lambda - 2 < \frac{\sum a^2}{\sum ab} < 2$$

$\Rightarrow$ $3\lambda - 2 < 2 \Rightarrow \lambda < \dfrac{4}{3}$

67. f is an odd function. It is also known that $f(x)$ is continuous for all values of x and is periodic with period 2. If $g(x) = \int_0^x f(t)\,dt,$ then:

(A) $g(x)$ is odd
(B) $g(n) = 0,\ n \in N$
(C) $g(2n) = 0,\ n \in N$
(D) $g(x)$ is non – periodic

Ans. C

Sol. $g(x) = \int_0^x f(t)\,dt$, so, $g(-x) = \int_0^{-x} f(t)\,dt = -\int_0^x f(-t)\,dt$

$g(-x) = \int_0^x f(t)\,dt$, as $f(-t) = -f(t)$, so, $g(-x) = g(x)$

Also $g(x+2) = \int_0^{x+2} f(t)\,dt = \int_0^2 f(t)\,dt + \int_2^{2+x} f(t)\,dt$

$g(x+2) = g(2) + \int_0^x f(t+2)\,dt$

$= g(2) + \int_0^x f(t)\,dt = g(2) + g(x)$

Now, $g(2) = \int_0^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_{-1}^0 f(t+2)\,dt$

$g(2) = \int_0^1 f(t)\,dt + \int_{-1}^0 f(t)\,dt = \int_{-1}^1 f(t)\,dt = 0$ as $f(t)$ is odd

$\Rightarrow$ $g(2) = 0 \Rightarrow g(x+2) = g(x)$

$\Rightarrow g(x)$ is periodic with period 2.

$\Rightarrow$ $g(4) = 0$

$\Rightarrow$ $f(6) = 0,\ g(2n) = 0,\ n \in N$

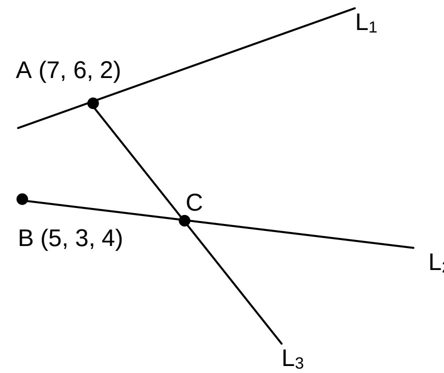
68. A line L_1 parallel to $3\hat{i} + 2\hat{j} + 4\hat{k}$ passes through $A(\vec{a}) = 7\hat{i} + 6\hat{j} + 2\hat{k}$ and another line L_2 parallel to $2\hat{i} + \hat{j} + 3\hat{k}$ passes through $B(\vec{b}) = 5\hat{i} + 3\hat{j} + 4\hat{k}$. If line L_3 perpendicular to $2\hat{i} - 2\hat{j} - \hat{k}$ intersects L_1 and L_2 at $A(\vec{a})$ and C respectively, then find the value of $\left[\left|\overrightarrow{AC}\right|\right]$

[Note : [y] denotes greatest integer less than or equal to y]

(A) 3
(B) 4
(C) 5
(D) 6

Ans. B

Sol. $L_1 = \frac{x-7}{3} = \frac{y-6}{2} = \frac{z-2}{4} = r_1$



$L_2 : \frac{x-5}{2} = \frac{y-3}{1} = \frac{z-4}{3} = r_2$

Any point C on L_2 is $(5+2r_2, 3+r_2, 4+3r_2)$

$\therefore$ Direction ratio of L_3 line is

$(5+2r_2-7, 3+r_2-6, 4+3r_2-2)$

$\Rightarrow (2r_2-2, r_2-3, 3r_2+2)$ and it is perpendicular to vector $2\hat{i}-2\hat{j}-\hat{k}$

$\therefore (2r_2-2)\times 2 + (r_2-3)\times(-2) + (3r_2+2)\times(-1)$

$= 0 \Rightarrow r_2 = 0$

$\therefore$ C is (5, 3, 4)

$\therefore |AC| = \sqrt{(7-5)^2 + (6-3)^2 + (2-4)^2} = \sqrt{17}$

$\therefore \left[\left|\overrightarrow{AC}\right|\right] = 4$

69. If p is the product of the sines of angles of a triangle and q the product of their cosines, the tangents of the angle are roots of the equation

(A) $qx^3 - px^2 + (1+q)x - p = 0$

(B) $px^3 - qx^2 + (1+p)x - q = 0$

(C) $(1+q)x^3 - px^2 + qx - q = 0$

(D) none of the above

Ans. A

Sol. Here, $p = \sin A \sin B \sin C$

and $q = \cos A \cos B \cos C$

$\therefore \quad \tan A \tan B \tan C = \frac{p}{q}$

And $\tan A \tan B + \tan B \tan C + \tan C \tan A$

$\sin A \sin B \cos C + \sin B \sin C \cos A$

$= \frac{+\sin C \sin A \cos B}{\cos A \cos B \cos C}$

$= \frac{\sin B(\sin A \cos C + \cos A \sin C) + \sin C(\sin A \cos B)}{\cos A \cos B \cos C}$

$= \frac{\sin B \sin(A+C) + \sin C \sin A \cos B}{\cos A \cos B \cos C}$

$= \frac{\sin^2 B + \sin C \sin A \cos B}{\cos A \cos B \cos C}$

$= \frac{1 - \cos B(\cos B - \sin C(\sin A))}{q}$

$$=\frac{1-\cos B\left(-\cos(A+C)-\sin A\sin C\right)}{q}$$

$=\left(\frac{1+q}{q}\right)$ and $\tan A+\tan B+\tan C$

$=\tan A\tan B\tan C(\text{in a }\Delta ABC)$

$=\frac{p}{q}$

$\therefore$ Required equation is $x^3-\frac{p}{q}x^2+\left(\frac{1+q}{q}\right)x-\frac{p}{q}=0$

$\Rightarrow \quad qx^3-px^2+(1+q)x-p=0$

70. If the domain of f(x) is $x\in R-(-1,1)$, then the domain of the function $f\left([\sin x]\cos\left(\frac{\pi}{|x-1|}\right)\right)$ (where [.] denotes greatest function) is

(A) $x\in R$

(B) $x\in R-(-1,1)$

(C) $x\in\phi$

(D) $x\in(-1,1)$

Ans. C

Sol $1\le[\sin x]\cos\left(\frac{\pi}{[x-1]}\right)\le 1$

And domain of function is $x\in R-(-1,1)$

$\Rightarrow[\sin x]\cos\left(\frac{\pi}{[x-1]}\right)=\pm1$

$\Rightarrow[\sin x]\cos\left(\frac{\pi}{[x-1]}\right)=\pm1$

$\Rightarrow$ either $[\sin x]=1,\cos\left(\frac{\pi}{[x-1]}\right)=-1$

$[\sin x]=1\Rightarrow x=(4n+1)\frac{\pi}{2}$

$\cos\left(\frac{\pi}{[x-1]}\right)=-1\Rightarrow[x-1]=\pm1$

$\Rightarrow x\in[0,1]\cup[2,3]$

$\Rightarrow x\in\phi$

Or [sinx]=-1, $\cos\left(\frac{\pi}{[x-1]}\right)=1$ but $\cos\left(\frac{\pi}{[x-1]}\right)\ne1$

hence $x\in\phi$

PHYSICS

71. If the kinetic energy of a body is directly proportional to time 't', the magnitude of the force acting on the body is (Body is moving on straight line path)
(A) directly proportional to $\sqrt{t}$
(B) inversely proportional to $\sqrt{t}$
(C) directly proportional to the speed of the body
(D) inversely proportional to the speed of the body

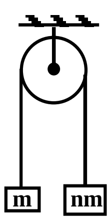
Ans. D

Sol. $\frac{1}{2}mV^2 \propto t \quad \Rightarrow \quad V^2 \propto t$

$V \propto \sqrt{t} \quad \Rightarrow \quad \frac{dV}{dt} \propto \frac{1}{\sqrt{t}}$

$\Rightarrow F \propto \frac{1}{\sqrt{t}} \quad \Rightarrow \quad F \propto \frac{1}{V}$

72. Two blocks of masses m and nm are connected by a massless string passing over a frictionless pulley. The value of n for which both the blocks moves with an acceleration of g/10 is



(A) 9/11 (B) 11/9
(C) Both (A) and (B) (D) None

Ans. C

Sol. Case (1) Assume that mass m is accelerating upward.

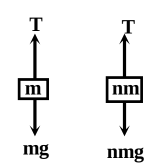


Figure 1

$\therefore \quad T - mg = \frac{mg}{10} \quad \Rightarrow \quad T = \frac{11mg}{10}$

$\therefore \quad nmg - T = \frac{nmg}{10} \quad \Rightarrow \quad nmg - \frac{11mg}{10} = \frac{nmg}{10}$

$\frac{9}{10}n = \frac{11}{10}$ and $n = \frac{11}{9}$

Case (2) Assume
If m mass is moving downward

$mg - T = \frac{mg}{10} \quad \Rightarrow \quad T = \frac{9mg}{10}$

$\therefore \quad T - nmg = nmg/10$

$\Rightarrow \quad \frac{9}{10}mg - nmg = \frac{nmg}{10}$

$\frac{9}{10} = \frac{n}{10} + n = \frac{11n}{10}$

$\therefore \quad n = 9/11$

73. A particle of mass 2 kg is performing SHM, given by equation $x = 10 \sin 2\pi t$ (x is in m and t in s). The work done on the particle in time 0.25 sec to 0.75 sec, will be

(A) $200\pi^2$J
(B) $100\pi^2$J
(C) 50πJ
(D) zero

Ans. D

Sol. $F = m\frac{d^2x}{dt^2} = 2x - (10\times 4\pi^2 \sin 2\pi t)$

$= -80\pi^2 \sin 2\pi t$

$dx = 20\pi \cos 2\pi t\, dt$

$W = \int_{0.25}^{0.75} F\, dx = -80\pi^2 \times 20 \quad \pi \int_{0.25}^{0.75} \sin 2\pi t \cos 2\pi t\, dt$

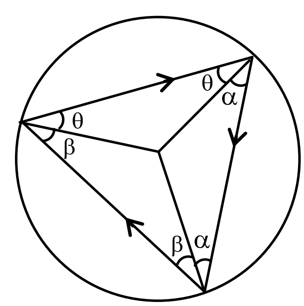
$= -40\pi^2 \times 20\pi \int_{0.25}^{0.75} \sin 4\pi t\, dt = \frac{800 \pi^3}{4\pi}\left[\cos 4\pi t\right]_{0.25}^{0.75} = 0$

74. A cylindrical hall has a horizontal smooth floor. A ball is projected along the floor from A point on the wall in a direction making an angle θ with the radius through the point the ball returns back to the initial point after two impacts with the wall. If the coefficient of restitution is e then $\tan^2\theta$ will be

(A) $\frac{1+e+e^2}{e^3}$
(B) $\frac{1+e}{e^2}$
(C) $\frac{e^2}{1+e}$
(D) $\frac{e^3}{1+e+e^2}$

Ans. D

Sol. $\tan\alpha = \frac{u\sin\theta}{eu\cos\theta} = \frac{\tan\theta}{e}$



$\therefore\ \tan\alpha = \frac{1}{e}\tan\theta$

$\therefore\ \tan\beta = \frac{1}{e}\tan\theta$

$2\theta + 2\alpha + 2\beta = 180°$

$\alpha + \beta = (90 - \theta)$

$\tan(\alpha + \beta) = \cot\theta$

$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{1}{\tan\theta}$

$\Rightarrow \frac{\frac{1}{e}\tan\theta + \frac{1}{e^2}\tan\theta}{1 - \frac{1}{e^3}\tan^2\theta} = \frac{1}{\tan\theta}$

$\Rightarrow \tan^2\theta\left(\frac{1}{e} + \frac{1}{e^2}\right) = 1 - \frac{1}{e^3}\tan^2\theta$

$$\Rightarrow \tan^2\theta\left(\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}\right)=1$$

$$\tan^2\theta\left(\frac{1}{\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}}\right)=\frac{e^3}{1+e+e^2}$$

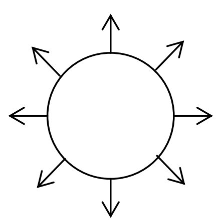
75. There is a horizontal film of soap solution. On it a thread is placed in the form of a loop. The film is pierced inside the loop and thread becomes a circular loop of radius R. If the surface tension of the solution be T, then what will be the tension in the thread?

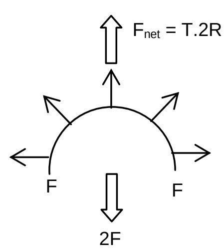
(A) $\frac{\pi R^2}{T}$ (B) $\pi R^2 T$

(C) $2\pi RT$ (D) RT

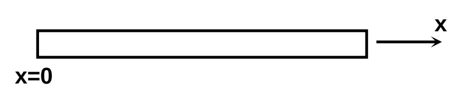
Ans. D

Sol.





76. If along a uniform rod of length ℓ carrying current I, the voltage V changes with position x along the length of the rod such that dV/dx = – k, where k is a positive number, then the resistance of the rod is



(A) $k\ell/2I$ (B) $k\ell/I$

(C) $I/k\ell$ (D) $k\ell I$

Ans. B

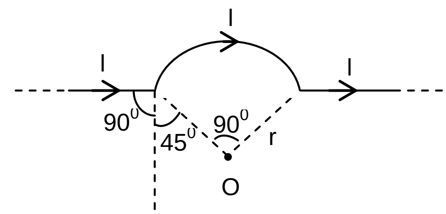
Sol. $dV = -IdR = = -I\rho\frac{dx}{A}$

$$\frac{dV}{dx}=-I\frac{\rho}{A}=-k$$

$\Rightarrow R = k\ell/I$

II Method: $V_2 - V_1 = \int\left(\frac{dV}{d\ell}\right)d\ell = kL = IR$

77. The magnetic field at the centre O of the arc in figure is (r is the radius of circular arc)



(A) $\frac{\mu_0 I}{4\pi \times r}\left[\sqrt{2}+\pi\right]$

(B) $\frac{\mu_0 I}{2\pi r}\left[\frac{\pi}{4}+\left(\sqrt{2}-1\right)\right]$

(C) $\frac{\mu_0}{4\pi}\times\frac{I}{r}\left[\sqrt{2}+r\right]$

(D) $\frac{\mu_0}{4\pi}\times\frac{I}{r}\left[\sqrt{2}+\frac{\pi}{4}\right]$

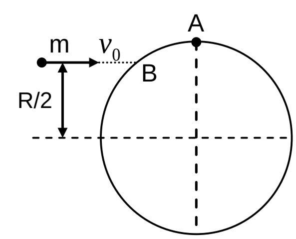
Ans. B

Sol. $B=\frac{\mu_0 I}{2\pi\cdot\frac{r}{\sqrt{2}}}\left[\sin 90^o-\sin 45^o\right]+\frac{\mu_0 I}{4\pi r}\cdot\frac{\pi}{2}$

$=\frac{\mu_0 I}{4\pi r}\left[2\sqrt{2}-2+\frac{\pi}{2}\right]$

$=\frac{\mu_0 I}{2\pi r}\left[\frac{\pi}{4}+(\sqrt{2}-1)\right]$

78. A disc of mass m and radius R is lying on a smooth horizontal surface. A particle of mass m moving horizontally with a velocity v_0, collides with the disc at B and sticks to it. Speed of the point A on the disc just after impact will be



(A) $\frac{\sqrt{31}}{8}v_0$

(B) $\frac{\sqrt{5}}{16}v_0$

(C) $\frac{5v_0}{16}$

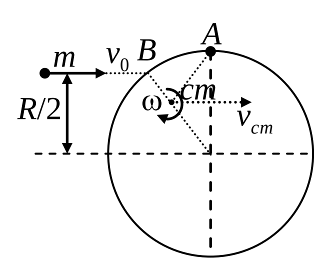
(D) $\frac{v_0}{2}$

Ans. A

Sol. COM, $mv_0\hat{i}=2m\vec{v}_{cm}\Rightarrow\vec{v}_{cm}=\frac{v_0}{2}\hat{i}$

COAM about CM $\left(mv_0\frac{R}{2}\sin 30^o\right)(-\hat{k})=\left(m\frac{R^2}{4}+\frac{1}{2}mR^2+\frac{mR^2}{4}\right)\vec{\omega}$

$\vec{\omega}=\frac{v_0}{4R}(-\hat{k})$, $\vec{v}_A=\vec{v}_{cm}+\vec{\omega}\times\vec{r}$



$=\frac{v_0}{2}\hat{i}+\frac{v_0}{4R}(-\hat{k})\times\left(\frac{R}{2}.\frac{\sqrt{3}}{2}\hat{i}+\frac{3R}{4}\hat{j}\right)$

$=\frac{11v_0}{16}\hat{i}-\frac{\sqrt{3}v_0}{16}\hat{j}$, $|\vec{v}_A|=\frac{\sqrt{31}}{8}v_0$

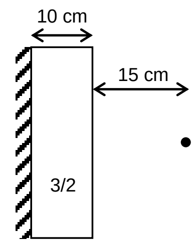
79. A mirror (μ = 3/2) is 10 cm thick. An object is placed 15 cm in front of it. The position of image from the front surface is

(A) 15 cm (B) 21.67 cm
(C) 28.34 cm (D) 35 cm

Ans. C

Sol. $X_{app} = \frac{10}{3/2} + \frac{10}{3/2} + \frac{15}{1}$

$= \frac{40}{3} + 15 = 28.33$



80. In a radioactive reaction an unstable nucleus A dis-integrates into a stable nucleus B. But A is generated at a constant rate of q nucleus per second. Then at steady state number of nucleus of A will be

$q\ (sec^{-1}) \longrightarrow A \xrightarrow{\lambda} B$
(Parent Nucleus) (Daughter Nucleus)

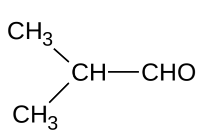
(A) $q\lambda$ (B) $\frac{q}{\lambda}$
(C) $q - \lambda$ (D) $\frac{\lambda}{q}$

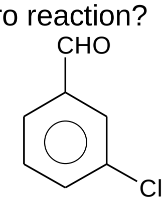
Ans. B

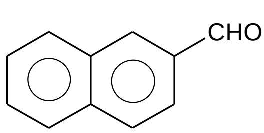
Sol. At steady state.
Rate of generation of A = Rate of decay of A.
$q = \lambda N_A$
$\Rightarrow N_A = \frac{q}{\lambda}$

CHEMISTRY

81. Which of the following will not under go Cannizaro reaction?

(A) 

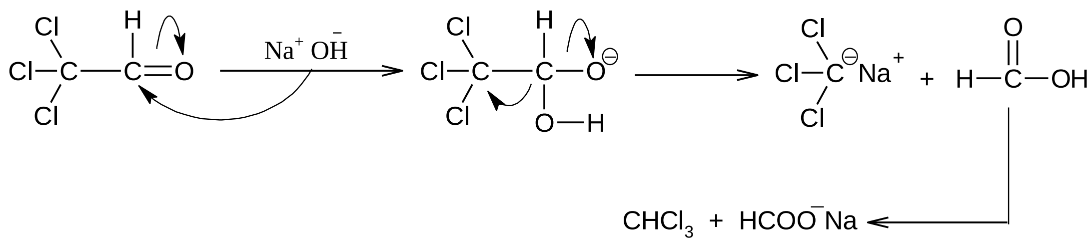
(B) 

(C) 

(D) Cl_3C—CHO

Ans. D

Sol.



82. The de broglie wavelength of neutron at 27ºC is λ. The wavelength at 927ºC will be

(A) $\lambda/9$
(B) $\lambda/4$
(C) $\lambda/2$
(D) $\lambda/3$

Ans. C

Sol. $\lambda = \dfrac{h}{\sqrt{3m}\,KT} \Rightarrow \dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{T_2}{T_1}} = \dfrac{\lambda}{\lambda_2} = \sqrt{\dfrac{1200}{3000}} \Rightarrow 4$

83. If the uncertainity in velocity and position is same, the uncertainity in momentum will be

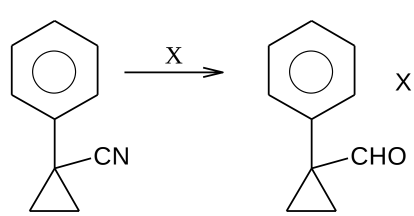
(A) $\sqrt{\dfrac{hm}{4\pi}}$
(B) $m\sqrt{\dfrac{4}{4\pi}}$
(C) $\sqrt{\dfrac{h}{4\pi m}}$
(D) $\dfrac{1}{m}\sqrt{\dfrac{h}{4\pi}}$

Ans. A

Sol. $\Delta x.\Delta V = \dfrac{h}{4\pi m} \quad (\Delta x = \Delta V)$

$\Delta X = \sqrt{\dfrac{h}{4\pi m}}$

$\Delta P = m\Delta V = \sqrt{\dfrac{hm}{4\pi}}$

84. 

X can be

(A) $LiAlH_4$/Ether/H_2O
(B) $NaBH_4/C_2H_5OH$
(C) DIBAl –H
(D) Na/C_2H_5OH

Ans. C

Sol. Information Based.

85. Which of the following represents correct order of decreasing reducing nature in aqueous solution?

(A) $Li > Na > K > Rb$ (B) $Rb > K > Na > Li$
(C) $Rb > Li > Na > K$ (D) $Li > Rb > K > Na$

Ans. D

Sol. On moving down the group, reducing nature increases. Li behave exceptionally due to its small size.

86. Which of the following is correct?

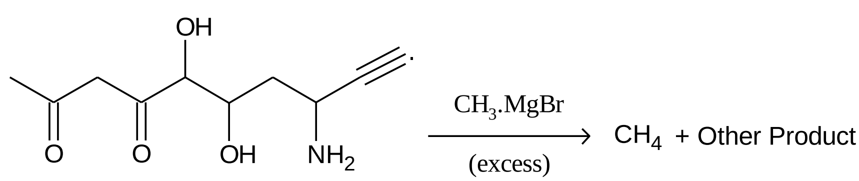
(A) Radius of $Ca^{2+} < Cl^{-} < S^{2-}$ (B) Radius of $Cl^{-} < S^{2-} < Ca^{2+}$
(C) Radius of $S^{2-} = Cl^{-} = Ca^{2+}$ (D) Radius of $S^{2-} < Cl^{-} < Ca^{2+}$

Ans. A

Sol.

	Ca^{2+}	Cl^{-}	S^{2-}
Z/e	20/18	17/18	16/18

Size decrease as z/e increases

87. 

The number of moles of CH_4 liberated is

(A) 3 (B) 4
(C) 5 (D) 6

Ans. C

Sol. 5 Acidic H are present in reactant.

88. The total volume of 0.1 M $KMnO_4$ solution that is needed to oxidize 100 mg each of ferrous oxalate and ferrous sulphate in a mixture in acidic medium

(A) 1.096 ml (B) 1.32 ml
(C) 5.48 ml (D) none of these

Ans. C

Sol. Equivalent of $KMnO_4$ = Equivalent of FeC_2O_4 + Equivalent of $FeSO_4$

$$0.1 \times v \times 5 = \frac{100}{144} \times 3 + \frac{100}{152} \times 1$$

$$v = 5.48 \text{ ml}$$

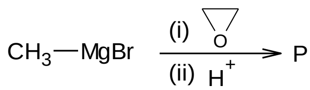
89. The dehydration step in aldol condensation in presence of dilute NaOH follow

(A) E_1 mechanism (B) E_2 mechanism
(C) E_i mechanism (D) E_1 CB mechanism

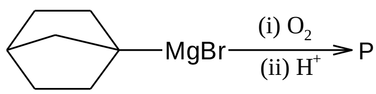
Ans. D

Sol. Anion is formed as intermediate.

90. Which of the following reaction will not produce alcohol?

(A) 

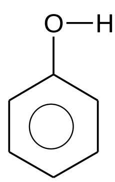
(B) $Ph-MgBr \xrightarrow[\text{(ii) } H^+]{\text{(i) } O_2} P$

(C) 

(D) $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-MgBr \xrightarrow[\text{(ii) } H^+]{\text{(i) } O_2}$

Ans. B

Sol.

In B product is  which is not alcohol