FIITJEE

CBSE TERM - I

ALL XIITH STUDYING BATCHES

Part Test – II

MATHEMATICS

(9th November 2021)

Time: 1:30 Hours **Maximum Marks: 40**

General Instructions:

1. The question paper contains three sections A, B and C
2. Section A consists of 20 questions MCQ Single Option Correct, out of which students will attempt any 16 questions only. Each question carries +1 Mark.
3. Section B consists of 20 questions MCQ Single Option Correct, out of which students will attempt any 16 questions only. Each question carries +1 Mark.
4. Section C consists of 10 questions MCQ Single Option Correct out of which 5 questions are based on case studies. Students will attempt any 8 questions only. Each question carries +1 Mark.
5. There is no negative marking.

Name of the Candidate : .....

Enroll Number : .....

Date of Examination : .....

MATHEMATICS

SECTION – A

*This section contains **20 Multiple Choice Questions** number **1 to 20**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

1. Let N denote the set of all natural numbers and R be the relation on N×N defined by (a, b) R (c, d) if ad(b + c) = bc(a + d), then R is-
   (A) Symmetric only (B) Reflexive only
   (C) Transitive only (D) An equivalence relation

2. If A = {1, 2, 3}, B = {1, 4, 6, 9} and R is a relation from A to B defined by 'x is greater than y'. The range of R is -
   (A) {1, 4, 6, 9} (B) {4, 6, 9}
   (C) {1} (D) None of these

3. Which of the following is a function?
   (A) {(2,1), (2,2), (2,3), (2,4)} (B) {(1,4), (2,5), (1,6) , (3,9)}
   (C) { (1,2), (3,3), (2,3), (1,4)} (D) { (1,2), (2,2), (3,2), (4,2)}

4. The domain of $f(x)=\frac{1}{x^3-x}$ is
   (A) R – {–1,0,1} (B) R
   (C) R – {0,1} (D) None of these

5. The slope of tangent to the curve $x = a\sin^3 t,\ y = a\cos^3 t$ at $t=\frac{\pi}{3}$ is
   (A) $\sqrt{3}$ (B) $-\sqrt{3}$
   (C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

6. The equation of tangent at the point 't' to the curve $x = a\cos^3 t, y = a\sin^3 t$ is
   (A) $x(\sec t)-y(\operatorname{cosec} t)=a$ (B) $x(\sec t)+y(\operatorname{cosec} t)=a$
   (C) $x(\operatorname{cosec} t)-y(\sec t)=a$ (D) $x(\operatorname{cosec} t)+y(\sec t)=a$

7. Function $f(x)=2x^3-9x^2+12x+29$ is decreasing when
   (A) $x<2$ (B) $x>2$
   (C) $x>3$ (D) $1<x<2$

8. The minimum value of the function $x^x\ (x>0)$ is at
   (A) x = 1 (B) x = e
   (C) $x = e^{-1}$ (D) None of these

9. The maximum value of function $\sin x(1+\cos x)$ occurs at

(A) $x=\frac{\pi}{4}$ (B) $x=\frac{\pi}{2}$

(C) $x=\frac{\pi}{3}$ (D) $x=\frac{\pi}{6}$

10. Which of the following functions defined from R to R is one-one.

(A) $f(x)=|x|$ (B) $f(x)=\cos x$

(C) $f(x)=e^x$ (D) $f(x)=x^2$

11. The equation of the normal to the curve $y=x(2-x)$ at the point (2, 0) is

(A) $x-2y=2$ (B) $x-2y+2=0$

(C) $2x+y=4$ (D) $2x+y-4=0$

12. A wholesale merchant wants to start the business of cereal with Rs.24,000. Wheat is Rs.400 per quintal and rice is Rs.600 per quintal. He has capacity for store 200 quintal cereal. He earns the profit Rs.25 per quintal on wheat and Rs.40 per quintal on rice. If he stores x quintal rice and y quintal wheat, then for maximum profit the objective function is -

(A) $25x+40y$ (B) $40x+25y$

(C) $400x+600y$ (D) $\frac{400}{40}x+\frac{600}{25}y$

13. A function whose graph is symmetrical about the y-axis is given by-

(A) $f(x)=\log_e\left(x+\sqrt{x^2+1}\right)$ (B) $f(x+y)=f(x)+f(y)$ for all $x,y\in R$

(C) $f(x)=\cos x+\sin x$ (D) None of these

14. If $f(x)=\frac{x-3}{x+1}$, then $f\left[f\{f(x)\}\right]$ equals

(A) x (B) $\frac{1}{x}$

(C) –x (D) $\frac{-1}{x}$

15. If $f(x)=2|x-2|-3|x-3|$, then the value of $f(x)$ when $2<x<3$ is

(A) 5 – x (B) x – 5

(C) 5x – 13 (D) None of these

16. A vertex of the linear inequalities $2x+3y\le 6$, $x+4y\le 4$ and $x,y\ge 0$ is

(A) (1, 0) (B) (1, 1)

(C) $\left(\frac{12}{5},\frac{2}{5}\right)$ (D) $\left(\frac{2}{5},\frac{12}{5}\right)$

17. If $f:R-\{3\}\to R-\{1\}$, $f(x)=\dfrac{x-2}{x-3}$ then function $f(x)$ is

(A) Only one – one
(B) one – one and into
(C) Many one and onto
(D) one – one and onto

18. A shopkeeper wants to purchase two articles A and B of cost price Rs. 4 and 3 respectively. He thought that he may earns 30 paise by selling article A and 10 paise by selling article B. He has not to purchase total article more than Rs. 24. If he purchases the number of articles of A and B, x and y respectively, then linear constraints are -

(A) $x\geq 0, y\geq 0, 4x+3y\leq 24$
(B) $x\geq 0, y\geq 0, 30x+10y\leq 24$
(C) $x\geq 0, y\geq 0, 4x+3y\geq 24$
(D) $x\geq 0, y\geq 0, 30x+40y\geq 24$

19. Find the minimum value of $z=2x_1-10x_2$ subjected to constraints $x_1-x_2\geq 0, x_1-5x_2\leq -5$ and $x_1,x_2\geq 0$

(A) 10
(B) –10
(C) 5
(D) None of these

20. The solution of a problem to maximize the objective function $z=x+2y$ under the constraints $x-y\leq 2, x+y\leq 4$ and $x,y\geq 0$, is

(A) $x=0, y=4, z=8$
(B) $x=1, y=2, z=5$
(C) $x=1, y=4, z=9$
(D) $x=0, y=3, z=6$

SECTION – B

*This section contains **20 Multiple Choice Questions** number **21 to 40**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

21. The point at which the maximum value of x + y, subject to the constraints $x+2y\le 70,\ 2x+y\le 95,\ x,y\ge 0$ is obtained, is
(A) (30, 25) (B) (20, 35)
(C) (35, 20) (D) (40, 15)

22. The domain of the function
$f(x)=\sin^{-1}\left(\log_2\frac{x^2}{2}\right)$ is
(A) $[-2,2]-(-1,1)$ (B) $[-1,2]-\{0\}$
(C) $[1,2]$ (D) $[-2,2]-\{0\}$

23. Let R be the real line. Consider the following subsets of the plane R × R :
S = {(x, y): y = x + 1 and 0 < x < 2}
T = {(x, y) : x – y is an integer}.
Which one of the following is true ?
(A) Both S and T are equivalence relations on R
(B) S is an equivalence relation on R but T is not
(C) T is an equivalence relation on R but S is not
(D) Neither S nor T is an equivalence relation on R

24. The feasible solution of a L.P.P. belongs to-
(A) First and second quadrant (B) First and third quadrant
(C) Second quadrant (D) Only first quadrant

25. The range of function $f(x)=\frac{x^2}{1+x^2}$ is
(A) $R-\{1\}$ (B) $R^+\cup\{0\}$
(C) $[0,1]$ (D) None of these

26. If $f(x)=\cos(\log x)$, then $f(x)f(y)-\frac{1}{2}\left(f\left(\frac{x}{y}\right)+f(xy)\right)$ is equal to
(A) –1 (B) $\frac{1}{2}$
(C) –2 (D) 0

27. Find the maximum value of $x=6x_1+10x_2$ subject to $3x_1+5x_2\le 13$, $5x_1+3x_2\le 15$ and $x_1,x_2\ge 0$
(A) 10 (B) 20
(C) 40 (D) Infinite solution

28. If $g(x) = x^2 + x - 2$ and $\frac{1}{2}(gof)(x) = 2x^2 - 5x + 2$, then $f(x)$ is equal to

(A) $2x - 3$ (B) $2x + 3$
(C) $2x^2 + 3x + 1$ (D) $2x^2 - 3x - 1$

29. The angle of intersection of the curves, $y = 4 - x^2$ and $y = x^2$ is

(A) $\frac{\pi}{2}$ (B) $\tan^{-1}\left(\frac{4}{3}\right)$

(C) $\tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$ (D) None of these

30. The maximum value of $z = 3x_1 + 2x_2$ subject to $x_1 + x_2 \geq 1$, $x_2 - 5x_1 \leq 0$, $5x_2 - x_1 \geq 0$ $x_1 - x_2 \geq -1$, $x_1 + x_2 \leq 6$, $x_1 \leq 3$ and $x_1 \geq 0$, $x_2 \geq 0$ will be

(A) 15 (B) 20
(C) 25 (D) None of these

31. $f(x) = 2x^2 - \log|x|, (x \neq 0)$ is monotonic increasing in the interval

(A) $\left(\frac{1}{2}, \infty\right)$ (B) $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

(C) $\left(-\infty, \frac{-1}{2}\right) \cup \left(0, \frac{1}{2}\right)$ (D) $\left(\frac{-1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

32. The maximum value of $z = 5x_1 + 7x_2$ subjected to $x_1 + x_2 \leq 4$, $3x_1 + 8x_2 \leq 24$, $10x_1 + 7x_2 \leq 35$ and $x_1 \geq 0$, $x_2 \geq 0$ will be

(A) $z = 24.8$ (B) $z = 25.8$
(C) $z = 23.8$ (D) $z = 21.8$

33. The optimal value of the objective function is attained at the points –
(A) given by intersection of inequations with the axes only.
(B) given by intersection of inequations with x-axis only
(C) given by corner points of the feasible region
(D) None of these

34. If $y = ax^3 + 3x^2 + (2a+1)x + 1000$ is strictly increasing function for all values of x, then

(A) $\frac{-3}{2} < a < 1$ (B) $a > 1$

(C) $a < \frac{-3}{2}$ (D) $a > 1$ or a $a < -32$

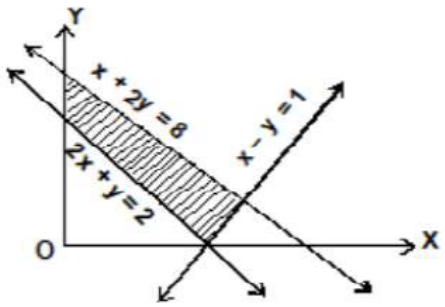
35. Find the maximum value of $z = 3x_1 + 4x_2$ subject to $5x_1 + 4x_2 \leq 200$, $3x_1 + 5x_2 \leq 150$ $5x_1 + 4x_2 \geq 100$, $8x_1 + 5x_2 \geq 80$ and $x_1 . x_2 \geq 0$

(A) 138.1 (B) 38.1
(C) 183.1 (D) None of these

36. Which of the following statements is correct?
(A) Every L.P.P. admits an optimal solution
(B) A L.P.P. admits a unique solution
(C) If a L.P.P. admits two optimal solutions, then it has an infinite number of optimal solutions.
(D) A L.P.P. admits two optimal solutions.

37. If $0 \le c \le 5$, then the minimum distance of the point (0, c) from parabola $y = x^2$ is
(A) $\sqrt{c-4}$ (B) $\sqrt{c-1/4}$
(C) $\sqrt{c+1/4}$ (D) None of these

38. For the following shaded area, the linear constraints except $x \ge 0$ and $y \ge 0$, are



(A) $2x + y \le 2,\ x - y \le 1,\ x + 2y \le 8$ (B) $2x + y \ge 2,\ x - y \le 1,\ x + 2y \le 8$
(C) $2x + y \ge 2,\ x - y \ge 1,\ x + 2y \le 8$ (D) $2x + y \ge 2,\ x - y \ge 1,\ x + 2y \ge 8$

39. A factory uses three different resources for the manufacture of two different products, 20 units of the resources A, 12 units of B and 16 units of C being available. 1 unit of the first product requires 2, 2 and 4 units of the respective resources and 1 unit of the second product requires 4, 2 and 0 units of the respective resources. It is known that the first product gives a profit of 2 monetary units per unit and the second 3. Formulate the linear programming problem. How many units of each product should be manufactured for maximizing the profit?
(A) 16 (B) 28
(C) 24 (D) 36

40. The semi-vertical angle of a right circular cone of given slant height and maximum volume is-
(A) $\tan^{-1}2$ (B) $\tan^{-1}\left(\sqrt{2}\right)$
(C) $\tan^{-1}\left(\frac{1}{2}\right)$ (D) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

SECTION – C

*This section contains **10 Multiple Choice Questions** number **41 to 50, out of which 5 questions are based on case studies.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

41. If $f(x) = -1 + |x-1|, -1 \le x \le 3$ and $g(x) = 2 - |x+1|, -2 \le x \le 2$, then for $x \in (0,1)$, $(fog)(x)$ equals

(A) $x-1$ (B) $1-x$

(C) $x+1$ (D) $-(x+1)$

42. The minimum value of the function $\frac{40}{3x^4+8x^3-18x^2+60}$ is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

(C) $\frac{40}{53}$ (D) None of these

43. If $f(x) = 1 + 2\sin x + 3\cos^2 x \left(0 \le x \le \frac{2\pi}{3}\right)$ is

(A) minimum at $x = \frac{\pi}{2}$ (B) maximum at $x = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(C) minimum at $x = \frac{\pi}{3}$ (D) minimum at $x = \sin^{-1}\left(\frac{1}{3}\right)$

44. If $x = (\sec\theta - \cos\theta)$ and $y = \sec^n\theta - \cos^n\theta$, then $\left(\frac{dy}{dx}\right)^2$ equals

(A) $\frac{y^2+4}{n^2(x^2+4)}$ (B) $\frac{y^2+4}{n(x^2+4)}$

(C) $\frac{n^2(y^2+4)}{x^2+4}$ (D) None of these

45. The radius of a right circular cylinder of greatest curved surface which can be inscribed in a given right circular cone is

(A) one third that of the cone (B) $\frac{1}{\sqrt{2}}$ times that of the cone

(C) $\frac{2}{3}$ that of the cone (D) $\frac{1}{2}$ that of the cone

CASE STUDY
(46 – 50)

The fuel cost per hour for running a train is proportional to the square of the speed it generates in km per hour. If the fuel costs Rs 48 per hour at speed 16 km per hour and the fixed charges to run the train amount to Rs 1200 per hour. Assume the speed of the train as υ km/h.

Based on the given information, answer the following questions

46. Given that the fuel cost per hour is k times the square of the speed of the train generates in km/h, the value of k is:

(A) $\frac{16}{3}$ (B) $\frac{1}{3}$

(C) 3 (D) $\frac{3}{16}$

47. If the train has traveled a distance of 500 km, then the total coast of running the train is given by function:

(A) $\frac{15}{16}\upsilon + \frac{600000}{\upsilon}$ (B) $\frac{375}{4}\upsilon + \frac{600000}{\upsilon}$

(C) $\frac{5}{16}\upsilon^2 + \frac{150000}{\upsilon}$ (D) $\frac{3}{16}\upsilon + \frac{600000}{\upsilon}$

48. The most economical speed to run the train is

(A) 18 km/h (B) 5 km/h

(C) 80 km/h (D) 40 km/h

49. The fuel cost for the train to travel 500 km at the most economical speed is:

(A) Rs. 3750 (B) Rs. 750

(C) Rs. 7500 (D) Rs. 75000

50. The total cost of the train to travel 500 km at the most economical speed is:
(A) Rs. 3750 (B) Rs. 75000
(C) Rs. 7500 (D) Rs. 15000